Lessons to be learned from the coherent photoproduction of pseudoscalar mesons

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We study the coherent photoproduction of pseudoscalar mesons—particularly of neutral pions—placing special emphasis on the various sources that put into question earlier nonrelativistic-impulse-approximation calculations. These include final-state interactions, relativistic effects, off-shell ambiguities, and violations to the impulse approximation. We establish that while distortions play an essential role in the modification of the coherent cross section, the uncertainty in our results due to the various choices of optical-potential models is relatively small (at most 30%). By far the largest uncertainty emerges from the ambiguity in extending the many on-shell-equivalent representations of the elementary amplitude off the mass shell. Indeed, relativistic impulse-approximation calculations that include the same pionic distortions, the same nuclear-structure model, and two sets of elementary amplitudes that are identical on-shell, lead to variations in the magnitude of the coherent cross section by up to factors of 5. Finally, we address qualitatively the assumption of locality implicit in most impulse-approximation treatments, and suggest that the coherent reaction probes—in addition to the nuclear density—the polarization structure of the nucleus. [S0556-2813(99)03410-X]

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I. INTRODUCTION

The coherent photoproduction of pseudoscalar mesons has been advertised as one of the cleanest probes for studying how nucleon-resonance formation, propagation, and decay get modified in the many-body environment; for current experimental efforts see Ref. [1]. The reason behind such optimism is the perceived insensitivity of the reaction to nuclear-structure effects. Indeed, many of the earlier nonrelativistic calculations suggest that the full nuclear contribution to the coherent process appears in the form of its matter density [2–5]—itself believed to be well constrained from electron-scattering experiments and isospin considerations.

Recently, however, this simple picture has been put into question. Among the many issues currently addressed—and to a large extent ignored in all earlier analyses—are background (nonresonant) processes, relativity, off-shell ambiguities, nonlocalities, and violations to the impulse approximation. We discuss each one of them in the manuscript. For example, background contributions to the resonance-dominated process can contaminate the analysis due to interference effects. We have shown this recently for the η-photoproduction process, where the background contribution (generated by ω-meson exchange) is in fact larger than the corresponding contribution from the $D_{13}(1520)$ resonance [6]. In that same study, as in a subsequent one [7], we suggested that—by using a relativistic and model-independent parametrization of the elementary $\gamma N \to \eta N$ amplitude—the nuclear-structure information becomes sensitive to off-shell ambiguities. Further, the local assumption implicit in most impulse-approximation calculations, and used to establish that all nuclear-structure effects appear exclusively via the matter density, has been lifted by Peters, Lenske, and Mosel [8]. An interesting result that emerges from their work on coherent η photoproduction is that the $S_{11}(1535)$ resonance—known to be dominant in the elementary process but predicted to be absent from the coherent reaction [4]—appears to make a non-negligible contribution to the coherent process. Finally, to our knowledge, a comprehensive study of possible violations to the impulse approximation, such as the modification to the production, propagation, and decay of nucleon resonances in the nuclear medium, has yet to be done.

In this paper we concentrate—in part because of the expected abundance of new, high-quality experimental data—on the coherent photoproduction of neutral pions. The central issue to be addressed here is the off-shell ambiguity that emerges in relativistic descriptions and its impact on extracting reliable resonance parameters; no attempt has been made here to study possible violations to the impulse approximation or to the local assumption. Indeed, we carry out our calculations within the framework of a relativistic impulse approximation model. However, rather than resorting to a nonrelativistic reduction of the elementary $\gamma N \to \pi^0 N$ amplitude, we keep intact its full relativistic structure [9]. As a result, the lower components of the in-medium Dirac spinors are evaluated dynamically in the Walecka model [10].

Another important ingredient of the calculation is the final-state interactions of the outgoing pion with the nucleus. We address the pionic distortions via an optical-potential model of the pion-nucleus interaction. We use earlier models of the pion-nucleus interaction plus isospin symmetry—since these models are constrained mostly from charged-pion data—to construct the neutral-pion optical potential. However, since we are unaware of a realistic optical-potential model that covers the Δ-resonance region, we have extended the low-energy work of Carr, Stricker-Bauer, and McManus [11] to higher energies. In this way we have attempted to keep at a minimum the uncertainties arising from the optical potential, allowing concentration on the impact of the off-shell ambiguities to the coherent process. A paper discussing

this extended optical-potential model will be presented shortly [12]. Finally, we use an elementary $\gamma N \rightarrow \pi^0 N$ amplitude extracted from the most recent phase-shift analysis of Arndt, Strakovsky, and Workman [13].

Our paper has been organized as follows. In Sec. II and in the Appendix we discuss in some detail the pion-nucleus interaction and its extension to the Δ-resonance region. Section III is devoted to the central topic of the paper: the large impact of the off-shell ambiguity on the coherent cross section. Section IV includes a qualitative discussion on several important mechanisms that go beyond the impulse-approximation framework, but that should, nevertheless, be included in any proper treatment of the coherent process. Finally, we summarize in Sec. V.

II. PIONIC DISTORTIONS

Pionic distortions play a critical role in all studies involving pion-nucleus interactions. These distortions are strong and, thus, modify significantly any process relative to its naive plane-wave limit. Indeed, it has been shown in earlier studies of the coherent pion photoproduction process—and verified experimentally [14]—that there is a large modification of the plane-wave cross section once distortions are included. Because of the importance of the pionic distortions, any realistic study of the coherent reaction must invoke them from the outset. However, since a detailed microscopic model for the distortions has yet to be developed, we have resorted to an optical-potential model. This semiphenomenological choice implies some uncertainties. Thus, pionic distortions represent the first challenge in dealing with the coherent photoproduction processes.

We have used earlier optical-potential models of the pion-nucleus interaction, supplemented by isospin symmetry, to construct the π^0-nucleus optical potential. Moreover, we have extended the low-energy work of Carr, Stricker-Bauer, and McManus [11] to the Δ-resonance region. Most of the formal aspects of the optical potential have been reserved to the Appendix and to a forthcoming publication [12]. Here we proceed directly to discuss the impact of the various choices of optical potentials on the coherent cross section.

Results. The large effect of distortions can be easily seen in Fig. 1. The left panel of the graph (plotted on a linear scale) shows the differential cross section for the coherent photoproduction of neutral pions from ^{40}Ca at a laboratory energy of $E_\gamma = 168$ MeV. The solid line displays our results using a relativistic distorted-wave impulse approximation (RDWIA) formalism, while the dashed line displays the corresponding plane-wave result (RPWIA). The calculations have been done using a vector representation for the elementary $\gamma N \rightarrow \pi^0 N$ amplitude. Note that this is only one of the many possible representations of the elementary amplitude that are equivalent on-shell. A detailed discussion of these off-shell ambiguities is deferred to Sec. III. At this specific photon energy—one not very far from threshold—the distortions have more than doubled the value of the differential cross section at its maximum. Yet, the shape of the angular distribution seems to be preserved. However, upon closer examination (the right panel of the graph shows the same

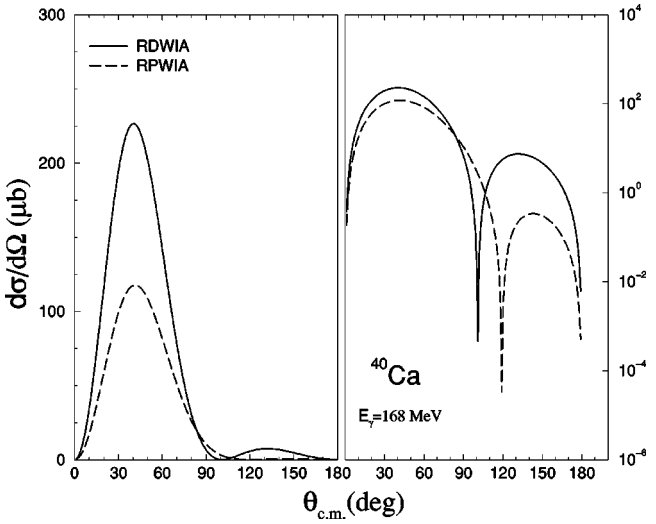

FIG. 1. Differential cross section for the coherent pion photoproduction reaction from ^{40}Ca at $E_\gamma = 168$ MeV using the vector representation for the elementary amplitude with (solid line) and without (dashed line) the inclusion of distortions. Results on the left(right) panel are plotted using a linear(logarithmic) scale.

calculations on a logarithmic scale) we observe that the distortions have caused a substantial back-angle enhancement due to a different sampling of the nuclear density, relative to the plane-wave calculation. This has resulted in a small—but not negligible—shift of about 10° in the position of the minima. The back-angle enhancement, with its corresponding shift in the position of the minimum, has been seen in our calculations also at different incident photon energies.

The effect of distortions on the total photoproduction cross section from ^{40}Ca as a function of the photon energy is displayed in Fig. 2. The behavior of the the distorted cross

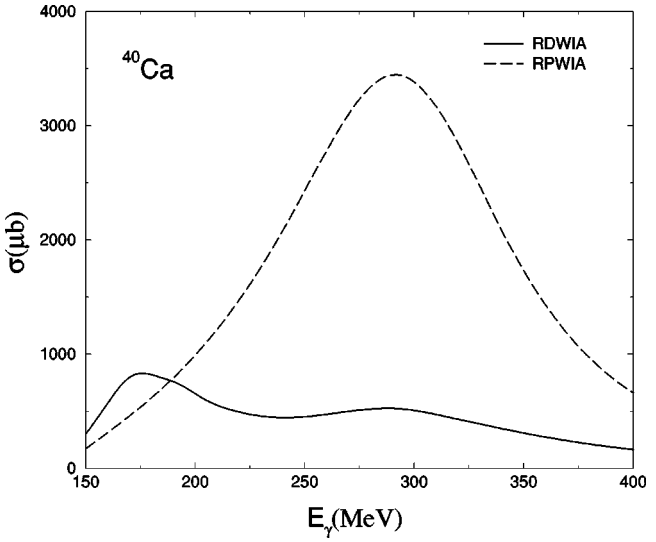

FIG. 2. Total cross section for the coherent pion photoproduction reaction from ^{40}Ca as a function of the photon energy in the laboratory frame with (solid line) and without (dashed line) including pionic distortions. A vector representation for the elementary part of the amplitude is used.

section is explained in terms of a competition between the attractive real (dispersive) part and the absorptive imaginary part of the optical potential. Although the optical potential encompasses very complicated processes, the essence of the physics can be understood in terms of Δ-resonance dominance. Ironically, the behavior of the dispersive and the absorptive parts are caused primarily by the same mechanism: Δ-resonance formation in the nucleus. The mechanism behind the attractive real part is the scattering of the pion from a single nucleon—which is dramatically increased in the Δ-resonance region. In contrast, the absorptive imaginary part is the result of several mechanisms, such as nucleon knock-out, excitation of nuclear states, and two-nucleon processes. At very low energies some of the absorptive channels are not open yet, resulting in a small imaginary part of the potential. This in turn provides a chance for the attractive real part to enhance the coherent cross section. As the energy increases, specifically in the Δ-resonance region, a larger number of absorptive channels become available leading to a large dampening of the cross section. Although the attractive part also increases around the Δ-resonance region, this increase is more than compensated by the absorptive part, which greatly reduces the probability for the pion to interact elastically with the nucleus.

Since understanding pionic distortions constitutes our first step towards a comprehensive study of the coherent process, it is instructive to examine the sensitivity of our results to various theoretical models. To this end, we have calculated the coherent cross section using different optical potentials, all of which fit π-nucleus scattering data as well as the properties of pionic atoms. We have started by calculating the coherent cross section using the optical potential developed by Carr and collaborators [11]. It should be noted that although our optical potential originates from the work of Carr and collaborators, there are still significant differences between the two sets of optical potentials. Some of these differences arise in the manner in which some parameters are determined. Indeed, in our case parameters that have their origin in pion–single-nucleon physics have been determined from a recent π-N phase shift analysis [15], while Carr and collaborators have determined them from fits to pionic-atom data. Moreover, we have included effects that were not explicitly included in their model, such as Coulomb corrections when fitting to charge-pion data.

In addition to the above potentials, we have calculated the coherent cross section using a simple four-parameter Kisslinger potential of the form

$$2\omega U = -4\pi\left[b_{\rm eff}\rho(r) - c_{\rm eff}\vec{\nabla}\rho(r)\cdot\vec{\nabla} + c_{\rm eff}\frac{\omega}{2M_N}\nabla^2\rho(r)\right]. \tag{1}$$

Note that we have used two different sets of parameters for this Kisslinger potential, denoted by K1 and K2 [11]. Both sets of parameters were constrained by π-nucleus scattering data and by the properties of pionic atoms. However, while the K1 fit was constrained to obtain $b_{\rm eff}$ and $c_{\rm eff}$ parameters that did not deviate much from their pionic-atom values, the K2 fit allowed them to vary freely, so as to obtain the best possible fit.

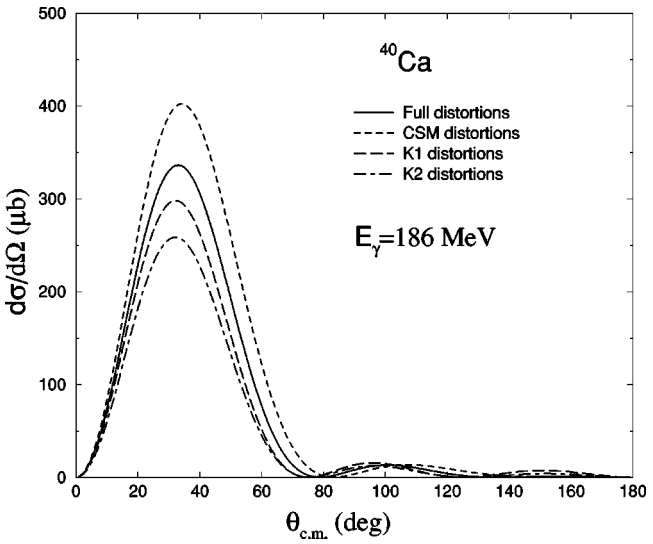

FIG. 3. Differential cross section for the coherent pion-photoproduction reaction from ^{40}Ca at $E_\gamma = 186$ MeV (resulting in the emission of a 50 MeV pion) using different optical-potential models. All of these models are equivalent insofar as they fit properties of pionic atoms and π-nucleus scattering data. A vector representation for the elementary part of the amplitude is used.

Results for the coherent photoproduction cross section from ^{40}Ca at a photon energy of $E_\gamma = 186$ MeV (resulting in the emission of a 50 MeV pion) for the various optical-potential models are shown in Fig. 3. In the plot, our results are labeled full-distortions (solid line) while those of Carr, Stricker-Bauer, and McManus as CSM (short dashed line); those obtained with the four-parameter Kisslinger potential are labeled K1 (long-dashed line) and K2 (dot-dashed line), respectively. It can be seen from the figure that our calculation differs by at most 30% relative to the ones using earlier forms of the optical potential. Note that we have only presented results computed using the vector parameterization of the elementary amplitude. Similar calculations done with the tensor amplitude (not shown) display optical-model uncertainties far smaller (of the order of 5%) than the ones reported in Fig. 3. In conclusion, although there seems to be a non-negligible uncertainty arising from the optical potential, these uncertainties pale in comparison to the large off-shell ambiguity, to be discussed next.

III. OFF-SHELL AMBIGUITY

The study of the coherent reaction represents a challenging theoretical task due to the lack of a detailed microscopic model of the process. Indeed, most of the models used to date rely on the impulse approximation: the assumption that the elementary $\gamma N \rightarrow \pi N$ amplitude remains unchanged as the process is embedded in the nuclear medium. Yet, even a detailed knowledge of the elementary amplitude does not guarantee a good understanding of the coherent process. The main difficulty stems from the fact that there are, literally, an

infinite number of equivalent on-shell representations of the elementary amplitude. These different representations of the elementary amplitude—although equivalent on-shell—can give very different results when evaluated off-shell. Of course, this uncertainty is present in many other kind of nuclear reactions, not just in the coherent photoproduction process. Yet, this off-shell ambiguity comprises one of the biggest, if not the biggest, hurdle in understanding the coherent photoproduction of pseudoscalar mesons.

A. Formalism

Before discussing the off-shell ambiguity, let us set the background by introducing some model-independent results for the differential cross section. Using the relativistic formalism developed in our earlier work [6], the differential cross section in the center-of-momentum frame (c.m.) for the coherent photoproduction of pseudoscalar mesons is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm c.m.}=\left(\frac{M_T}{4\pi W}\right)^2\left(\frac{q_{\rm c.m.}}{k_{\rm c.m.}}\right)\times\left(\frac{1}{2}k_{\rm c.m.}^2 q_{\rm c.m.}^2\sin^2\theta_{\rm c.m.}\right)|F_0(s,t)|^2, \quad (2)$$

where M_T is the mass of the target nucleus. Note that W, $\theta_{\rm c.m.}$, $k_{\rm c.m.}$, and $q_{\rm c.m.}$ are the total energy, scattering angle, photon, and π-meson momenta in the c.m. frame, respectively. Thus, all dynamical information about the coherent process is contained in the single Lorentz-invariant form factor $F_0(s,t)$; this form-factor depends on the Mandelstam variables s and t.

We now proceed to compute the Lorentz invariant form factor in a relativistic impulse approximation. In order to do so, we need an expression for the amplitude of the elementary process: $\gamma N\to\pi^0 N$. We start by using the ''standard'' model-independent parametrization given in terms of four Lorentz- and gauge-invariant amplitudes [4,9]. That is,

$$T(\gamma N\to\pi^0 N)=\sum_{i=1}^{4}A_i(s,t)M_i, \quad (3)$$

where $A_i(s,t)$ are scalar functions of s and t, and for the Lorentz structure of the amplitude we use the standard set

$$M_1=-\gamma^5\not\epsilon\not k, \quad (4a)$$

$$M_2=2\gamma^5[(\epsilon\cdot p)(k\cdot p')-(\epsilon\cdot p')(k\cdot p)], \quad (4b)$$

$$M_3=\gamma^5[\not\epsilon(k\cdot p)-\not k(\epsilon\cdot p)], \quad (4c)$$

$$M_4=\gamma^5[\not\epsilon(k\cdot p')-\not k(\epsilon\cdot p')]. \quad (4d)$$

This form, although standard, is only one particular choice for the elementary amplitude. Many other choices—all of them equivalent on shell—are possible. Indeed, we could have used the relation—valid only on the mass shell,

$$M_1=-\gamma^5\not\epsilon\not k=\frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\mu k_\nu\sigma_{\alpha\beta}$$
$$=\frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\mu k_\nu\frac{Q_\alpha}{M_N}\gamma_\beta-\frac{1}{2M_N}\gamma^5[\not\epsilon(k\cdot p)-\not k(\epsilon\cdot p)]$$
$$-\frac{1}{2M_N}\gamma^5[\not\epsilon(k\cdot p')-\not k(\epsilon\cdot p')], \quad (5)$$

to obtain the following representation of the elementary amplitude:

$$T(\gamma N\to\pi^0 N)=\sum_{i=1}^{4}B_i(s,t)N_i, \quad (6)$$

where the new invariant amplitudes and Lorentz structures are now defined as

$$B_1=A_1,\ \ N_1=\frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\epsilon_\mu k_\nu\frac{Q_\alpha}{M_N}\gamma_\beta, \quad (7a)$$

$$B_2=A_2,\ \ N_2=M_2=2\gamma^5[(\epsilon\cdot p)(k\cdot p')-(\epsilon\cdot p')(k\cdot p)], \quad (7b)$$

$$B_3=A_3-A_1/2M_N,\ \ N_3=M_3=\gamma^5[\not\epsilon(k\cdot p)-\not k(\epsilon\cdot p)], \quad (7c)$$

$$B_4=A_4-A_1/2M_N,\ \ N_4=M_4=\gamma^5[\not\epsilon(k\cdot p')-\not k(\epsilon\cdot p')]. \quad (7d)$$

Note that we have introduced the four-momentum transfer $Q^\mu\equiv(k-q)^\mu=(p'-p)^\mu$. Although clearly different, Eqs. (3) and (6) are totally equivalent on-shell: no observable measured in the elementary $\gamma N\to\pi^0 N$ process could distinguish between these two forms. We could go on. Indeed, it is well known that a pseudoscalar and a pseudovector representation are equivalent on shell. That is, we could substitute the pseudoscalar vertex in N_2 and M_2 by a pseudovector one:

$$\gamma^5=\frac{\not Q}{2M_N}\gamma^5. \quad (8)$$

The possibilities seem endless.

Given the fact that there are many—indeed infinite—equivalent parametrizations of the elementary amplitude on-shell, it becomes ambiguous on how to take the amplitude off the mass shell. In this work we have examined this off-shell ambiguity by studying the coherent process using the ''tensor'' parametrization, as in Eq. (3), and the ''vector'' parametrization, as in Eq. (6). Denoting these parameterizations as tensor and vector originates from the fact that for the coherent process from spherical nuclei (such as the ones considered here) the respective cross sections become sensitive to only the tensor and vector densities, respectively. Indeed, the tensor parametrization yields a coherent amplitude that depends exclusively on the ground-state tensor density [6]:

$$[\rho_T(r)\hat r]^i=\sum_{\alpha}^{\rm occ}\bar{\mathcal{U}}_\alpha(\mathbf{x})\sigma^{0i}\mathcal{U}_\alpha(\mathbf{x});$$

$$\rho_T(r)=\sum_a^{\rm occ}\left(\frac{2j_a+1}{4\pi r^2}\right)2g_a(r)f_a(r), \tag{9}$$

where $\mathcal{U}_\alpha(\mathbf{x})$ is an in-medium single-particle Dirac spinor, $g_a(r)$ and $f_a(r)$ are the radial parts of the upper and lower components of the Dirac spinor, respectively, and the above sums run over all the occupied single-particle states.

The vector parametrization, on the other hand, leads to a coherent amplitude that depends on timelike-vector—or matter—density of the nucleus, which is defined as

$$\rho_V(r)=\sum_\alpha^{\rm occ}\bar{\mathcal{U}}_\alpha(\mathbf{x})\gamma^0\mathcal{U}_\alpha(\mathbf{x});$$

$$\rho_V(r)=\sum_a^{\rm occ}\left(\frac{2j_a+1}{4\pi r^2}\right)[g_a^2(r)+f_a^2(r)]. \tag{10}$$

In determining these relativistic ground-state densities, we have used a mean-field approximation to the Walecka model [10]. In doing so, we have maintained the full relativistic structure of the process. In the Walecka model, one obtains three nonvanishing ground state densities for spherical, spin-saturated nuclei. These are the timelike-vector and tensor densities defined earlier, and the scalar density given by

$$\rho_S(r)=\sum_\alpha^{\rm occ}\bar{\mathcal{U}}_\alpha(\mathbf{x})\mathcal{U}_\alpha(\mathbf{x});$$

$$\rho_S(r)=\sum_a^{\rm occ}\left(\frac{2j_a+1}{4\pi r^2}\right)[g_a^2(r)-f_a^2(r)]. \tag{11}$$

All other ground-state densities—such as the pseudoscalar and axial-vector densities—vanish due to parity conservation. This is one of the appealing features of the coherent reaction; because of the conservation of parity, the coherent process becomes sensitive to only one (A_1) of the possible four, elementary amplitudes. Moreover, this suggests that, depending on the adopted parametrization of the elementary amplitude, only matrix elements of either M_1 or N_1 [Eq. (4a) or Eq. (7a)] between in-medium Dirac spinors must be evaluated. Because of the structure of these operators, gauge invariance is still maintained in the coherent process. It is important to note that the three nonvanishing relativistic ground-state densities are truly independent and constitute fundamental nuclear-structure quantities. The fact that in the nonrelativistic framework [2–5,14] only one density survives (the scalar and vector densities become equal and the tensor density becomes dependent on the vector one) is due to the limitation of the approach. Indeed, in the nonrelativistic framework one employs free Dirac spinors to carry out the nonrelativistic reduction of the elementary amplitude. Hence, any evidence of possible medium modifications to the ratio of lower-to-upper components of the Dirac spinors is lost.

Before presenting our results we should mention a ''conventional'' off-shell ambiguity. In the vector parametrization of Eq. (6) the amplitude includes the four-momentum transfer $Q=(k-q)$. While the photon momentum $\mathbf{k}$ is well defined, the asymptotic pion three-momentum $\mathbf{q}$ is different—because of distortions—from the pion momentum immediately after the photoproduction process. Since the ''local'' pion momentum in the interaction region is the physically relevant quantity, we have replaced the asymptotic pion momentum $\mathbf{q}$ by the pion-momentum operator $(-i\nabla)$. Thus, in evaluating the scattering matrix element $T_\pm=\langle\pi(q);A(p')|J^\mu|A(p);\gamma(k,\epsilon_\pm)\rangle$, we arrive at an integral of the form:

$$\varepsilon^{ijm0}\epsilon_i k_j\int d^3x[\nabla\phi_q^{(-)*}(\mathbf{x})]_m e^{i\mathbf{k}\cdot\mathbf{x}}\frac{\rho_V(r)}{2M_N}$$

$$=\pm(2\pi)^{3/2}\frac{|\mathbf{k}|}{M_N}\sum_{l=1}^{\infty}\sqrt{\frac{l(l+1)}{2l+1}}Y_{l,\pm1}(\hat{q})$$

$$\times\int r^2dr\rho_V(r)R_l(r), \tag{12}$$

where

$$R_l(r)=j_{l+1}(kr)\left[\frac{d}{dr}-\frac{l}{r}\right]\phi_{l,q}^{(+)}(r)$$

$$+j_{l-1}(kr)\left[\frac{d}{dr}+\frac{l+1}{r}\right]\phi_{l,q}^{(+)}(r). \tag{13}$$

Note that we have introduced the distorted pion wave function $\phi_q^{(\pm)}(\mathbf{x})$, the spherical Bessel functions of order $l\pm1$, and the $\pm$ sign for positive/negative circular polarization of the incident photon. Moreover, adopting the $\mathbf{q}\rightarrow-i\nabla$ prescription, has resulted, as in the tensor case [6], in no s-wave $(l=0)$ contribution to the scattering amplitude. This is also in agreement with the earlier nonrelativistic calculation of Ref. [2]. Finally, we have obtained the four Lorentz- and gauge-invariant amplitudes $A_i(s,t)$ for the elementary process from the phase-shift analysis of the VPI group [13].

B. Results

Based on the above formalism, we present in Fig. 4 the differential cross section for the coherent photoproduction of neutral pions from ^{40}Ca at a photon energy of $E_\gamma=230$ MeV using a relativistic impulse approximation approach. Both tensor and vector parametrizations of the elementary amplitude were used. The off-shell ambiguity is immense; factors of two (or more) are observed when comparing the vector and tensor representations. It is important to stress that these calculations were done by using the same nuclear-structure model, the same pionic distortions, and two elementary amplitudes that are identical on-shell. The very large discrepancy between the two theoretical models emerges from the dynamical modification of the Dirac spinors in the nuclear medium, and not from changes to the elementary production amplitude (assessing the impact of medium modifications to the elementary amplitude remains an important open question). Indeed, in the nuclear medium the tensor density—which is linear in the lower-component of the Dirac spinors [see Eq. (9)]—is strongly enhanced due to the presence of a large scalar potential (the so-called ''$M^\star$ effect''). In contrast, the conserved vector density is insensitive to the $M^\star$

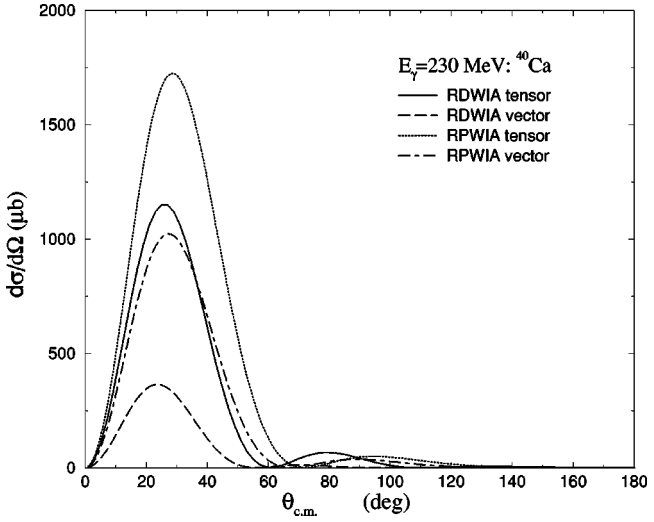

FIG. 4. Differential cross section for the coherent pion photoproduction reaction from ^{40}Ca at $E_\gamma = 230$ MeV with (RDWIA) and without (RPWIA) pionic distortions. Tensor and vector parametrizations of the elementary amplitude are used.

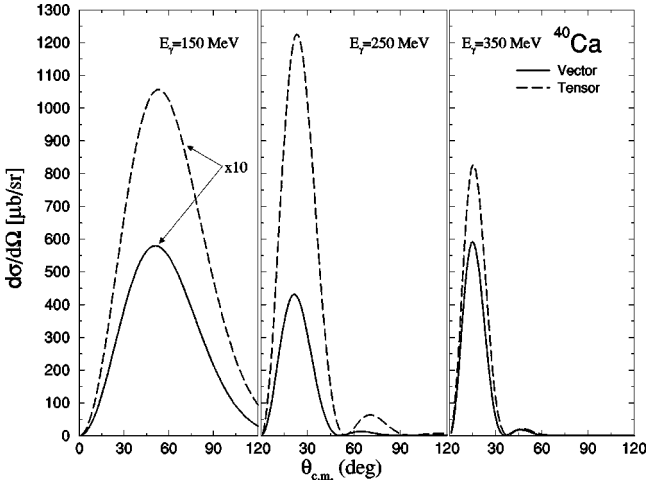

FIG. 5. Differential cross section for the coherent pion photoproduction reaction for ^{40}Ca at a variety of photon energies using a RDWIA formalism. Tensor (dashed line) and vector (solid line) parametrizations of the elementary amplitude are used.

effect. Yet the presence of the large scalar—and vector—potentials in the nuclear medium is essential in accounting for the bulk properties of nuclear matter and finite nuclei [10]. While the nuclear structure model used here is known to yield the smallest $M^\star$ and, thus, the largest discrepancy between the vector and tensor results, the qualitative picture should remain unchanged, even as more realistic nuclear-structure models are incorporated. Moreover, the large discrepancy between the calculations cannot be attributed to an improper treatment of gauge invariance, as gauge invariance is strictly maintained in all of our calculations [see Eq. (5)].

We have compared our theoretical results to preliminary and unpublished data (not shown) provided to us courtesy of Krusche [16]. The data follows the same shape as our calculations but the experimental curve seems to straddle between the two calculations, although the vector calculations appears closer to the experimental data. This behavior—a closer agreement of the vector calculation to data—has been observed in all of the comparisons that we have done so far.

In Fig. 5 we present results for the differential cross section from ^{40}Ca at a variety of photon energies, while in Fig. 6 we display results for the total cross section. By examining these graphs one can infer that the tensor parametrization always predicts a large enhancement of the cross section—irrespective of the photon incident energy and the scattering angle—relative to the vector predictions. As stated earlier, this large enhancement is inextricably linked to the corresponding in-medium enhancement of the lower components of the nucleon spinors. Moreover, the convolution of the tensor and vector densities with the pionic distortions give rise to similar qualitative, but quite different quantitative, behavior on the energy dependence of the corresponding coherent cross sections.

In order to explore the A dependence of the coherent process, we have also calculated the cross section from ^{12}C at various photon energies. This is particularly relevant for our present discussion, as ^{12}C displays an even larger off-shell ambiguity than ^{40}Ca. In Fig. 7 we show the differential cross section for the coherent process from ^{12}C at a photon energy of $E_\gamma = 173$ MeV. The off-shell ambiguity for this case is striking; at this energy the tensor result is five times larger than the vector prediction. The additional enhancement observed here relative to ^{40}Ca is easy to understand on the basis of some of our earlier work [7]. Indeed, we have shown in our study of the coherent photoproduction of η mesons, that if one artificially adopts an in-medium ratio of upper-to-lower components identical to the one in free space, then the tensor and vector densities are no longer independent; rather, they become related by

$$\rho_T(Q) = -\frac{Q}{2M_N}\rho_V(Q). \tag{14}$$

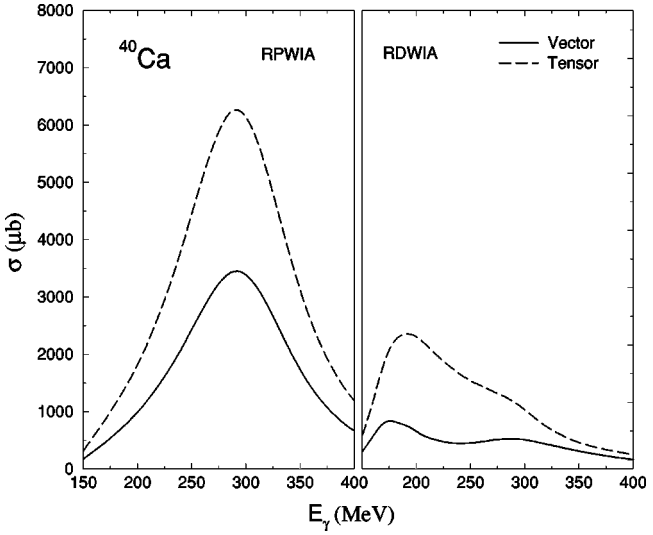

FIG. 6. Total cross section for the coherent pion photoproduction reaction from ^{40}Ca as a function of the photon energy with (right panel) and without (left panel) pionic distortions. Tensor (dashed line) and vector (solid line) parametrizations of the elementary amplitude are used.

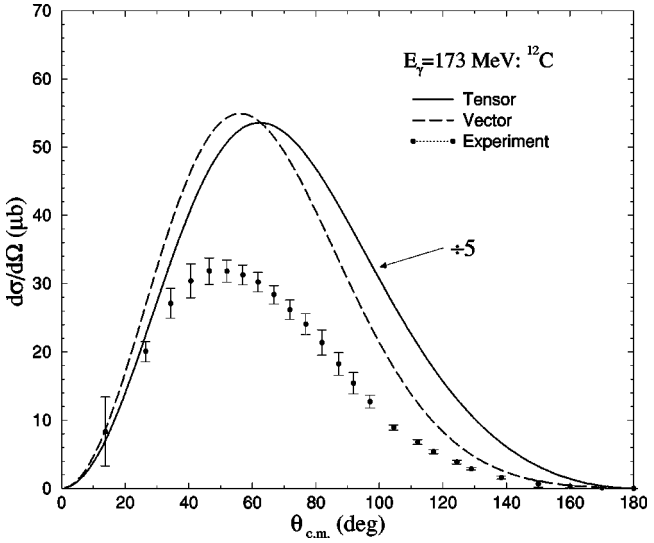

FIG. 7. Differential cross section for the coherent pion photoproduction reaction from ^{12}C at $E_\gamma = 173$ MeV. Tensor (dashed line) and vector (solid line) parametrizations of the elementary amplitude are used. The experimental data is from Ref. [17].

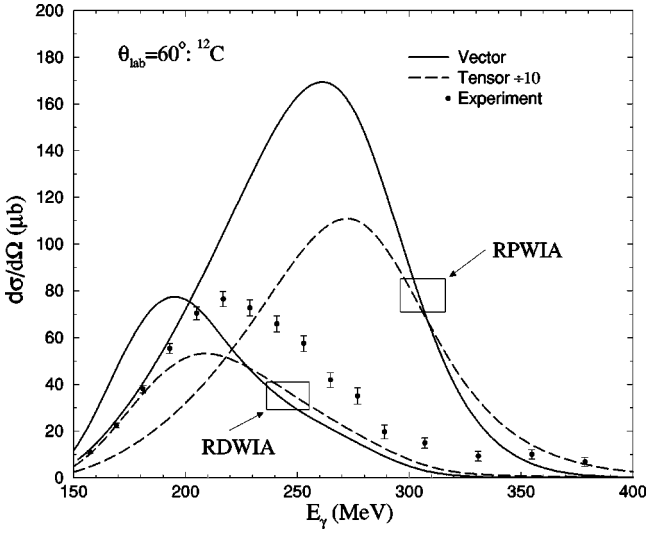

FIG. 8. Differential cross section for the coherent pion photoproduction reaction from ^{12}C as a function of photon energy at a fixed laboratory angle of $\theta_{\rm lab} = 60°$, with and without pionic distortions. Tensor (dashed lines) and vector (solid lines) parametrizations of the elementary amplitude are used. The experimental data is from Ref. [20].

However, this relation was proven to be valid only for closed-shell nuclei. As ^{12}C is an open-shell nucleus (closed $p^{3/2}$ but open $p^{1/2}$ orbitals) an additional enhancement of the tensor density—above and beyond the $M^\star$ effect—was observed. Figure 7 also shows a comparison of our results with the experimental data of Ref. [17]. It is clear from the figure that the vector representation is closer to the data; note that the tensor calculation has been divided by a factor of 5. Even so, the vector calculation also overestimates the data by a considerable amount.

For further comparison with experimental data we have calculated the coherent cross section from ^{12}C at photon energies of $E_\gamma = 235$, 250, and 291 MeV. In Table I we have collated our calculations with experimental data published by Arends and collaborators [18] for $E_\gamma = 235$ and 291 MeV, and with data presented by Booth [19] and Nagl, Devanathan, and Überall [14] for $E_{\gamma{\rm lab}} = 250$ MeV. The experimental data exhibits similar patterns as our calculations (not shown) but the values of the maxima of the cross section are different. The tensor calculations continue to predict large enhancement factors (of 5 and more) relative to the vector calculations. More importantly, these enhancement factors are in contradiction with experiment. The experimental data appears to indicate that the maximum in the differential cross section from ^{12}C is largest at about 250 MeV, while our calculations predict a maximum around 295 MeV. It is likely that this energy ''shift'' might be the result of the formation and propagation of the Δ resonance in the nuclear medium. Clearly, in an impulse-approximation framework, medium modifications to the elementary amplitude—arising from changes in resonance properties—cannot be accounted for. Yet, a binding-energy correction of about 40 MeV due to the Δ-nucleus interaction has been suggested before. Indeed, such a shift would also explain the discrepancy in the position of our theoretical cross sections in ^{40}Ca, relative to the (unpublished) data by Krusche and collaborators [16]. Moreover, such a shift—albeit of only 15 MeV—was invoked by Peters, Lenske, and Mosel [8] in their recent calculation of the coherent pion-photoproduction cross section. Yet, a detailed study of modifications to hadronic properties in the nuclear medium must go beyond the impulse approximation; a topic outside the scope of the present work. However, a brief qualitative discussion of possible violations to the impulse approximation is given in the next section.

TABLE I. Maxima of the differential cross section (in μb) for the coherent pion photoproduction reaction from ^{12}C at various energies.

E_γ (MeV)	Tensor	Vector	Experiment
235	694	116	105
250	731	133	190
291	786	186	175

We conclude this section by presenting in Figs. 8 and 9, a comparison between our plane- and distorted-wave calculations with experimental data for the coherent cross section from ^{12}C as a function of photon energy for a fixed angle of $\theta_{\rm lab} = 60°$. The experimental data from MAMI is contained in the doctoral dissertation of Schmitz [20].

Perhaps the most interesting feature in these figures is the very good agreement between our RDWIA calculation using the vector representation and the data—if we were to shift our results by $+25$ MeV. Indeed, this effect is most clearly appreciated in Fig. 9, where the shifted calculation is now represented by the dashed line. In our treatment of the coherent process, the detailed shape of the cross section as a function of energy results from a delicate interplay between several effects arising from (a) the elementary amplitude—

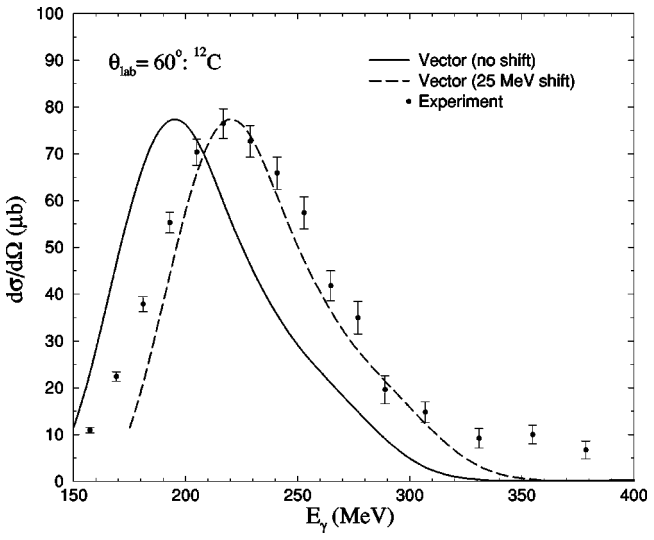

FIG. 9. Differential cross section for the coherent pion photoproduction reaction from ^{12}C as a function of photon energy at a fixed laboratory angle of $\theta_{\rm lab}=60°$, with pionic distortions and using only a vector parametrization of the elementary amplitude. The same calculation—including a shift of 25 MeV is also included (dashed line). The experimental data is from Ref. [20].

which peaks at the position of the delta resonance ($E_\gamma \simeq 340$ MeV from a free nucleon and slightly lower here because of the optimal prescription [6]), (b) the nuclear form factor—which peaks at low-momentum transfer, and (c) the pionic distortions—which strongly quench the cross sections at high energy, as more open channels become available. We believe that the pionic distortions (see Sec. II) as well as the nuclear form factor have been modeled accurately in the present work. The elementary amplitude, although obtained from a recent phase-shift analysis by the VPI group [13], remains one of the biggest uncertainties, as no microscopic model has been used to estimate possible medium modifications to the on-shell amplitude. Evidently, an important modification might arises from the production, propagation, and decay of the Δ resonance in the nuclear medium. Indeed, a very general result from hadronic physics, obtained from analyses of quasielastic (p,n) and $(^3{\rm He},t)$ experiments [21], is that the position of the Δ peak in nuclear targets is lower relative to the one observed from a free proton target.

However, it is also well known that such a shift is not observed when the Δ resonance is excited electromagnetically [21]. This apparent discrepancy has been attributed to the different dynamic responses that are being probed by the two processes. In the case of the hadronic process, it is the (pionlike) spin-longitudinal response that is being probed, which is known to get ''softened'' (shifted to lower excitation energies) in the nuclear medium. Instead, quasielastic electron scattering probes the spin-transverse response—which shows no significance energy shift. Unfortunately, in our present local-impulse-approximation treatment it becomes impossible to assess the effects associated with medium modifications to the Δ resonance. A detailed study of possible violations to the impulse approximation and to the local assumption remains an important open problem for the future (for a qualitative discussion see Sec. IV).

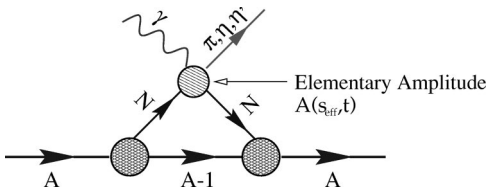

FIG. 10. Pictorial representation of the impulse approximation for the coherent photoproduction of pseudoscalar mesons. Note that the elementary amplitude is evaluated using the optimal prescription (see, for example, Ref [6]).

IV. VIOLATIONS TO THE IMPULSE APPROXIMATION

In this section we address an additional ambiguity in the formalism, namely, the use of the impulse approximation. The basic assumption behind the impulse approximation is that the interaction in the medium is unchanged relative to its free-space value. The immense simplification that is achieved with this assumption is that the elementary interaction now becomes model independent, as it can be obtained directly from a phase-shift analysis of the experimental data (see, for example, Ref. [13]). The sole remaining question to be answered is the value of s at which the elementary amplitude should be evaluated, as now the target nucleon is not free but rather bound to the nucleus (see Fig. 10). This question is resolved by using the ''optimal'' prescription of Gurvitz, Dedonder, and Amado [22], which suggests that the elementary amplitude should be evaluated in the Breit frame. Then, this optimal form of the impulse approximation leads to a factorizable and local scattering amplitude—with the nuclear-structure information contained in a well-determined vector form factor. Moreover, as the final-state interaction between the outgoing meson and the nucleus is well constrained from other data, a parameter-free calculation of the coherent photoproduction process ensues.

This form of the impulse approximation has been used with great success in hadronic processes, such as in (p,p') and (p,n) reactions, and in electromagnetic processes, such as in electron scattering. Perhaps the main reason behind this success is that the elementary nucleon-nucleon or electron-nucleon interaction is mediated exclusively by t-channel exchanges—such as arising from γ, π, or σ exchange. This implies that the local approximation (i.e., the assumption that the nuclear-structure information appears exclusively in the form of a local nuclear form factor) is well justified. For the coherent process this would also be the case if the elementary amplitude would be dominated by the exchange of mesons, as in the last Feynman diagram in Fig. 11. However, it is well known—at least for the kinematical region of current interest—that the elementary photoproduction process is dominated by resonance ($N^\star$ or Δ) formation, as in the s-channel Feynman diagram of Fig. 11. This suggests that the coherent reaction probes, in addition to the nuclear density, the polarization structure of the nucleus (depicted by the ''bubbles'' in Fig. 11). As the polarization structure of the nucleus is sensitive to the ground- as well as to the excited-

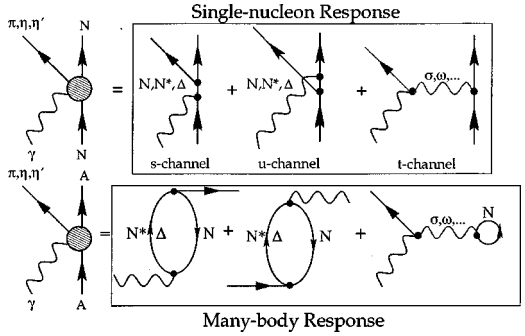

FIG. 11. Characteristic *s*-, *u*-, and *t*-channel Feynman diagrams for the photoproduction of pseudoscalar mesons from a single nucleon (upper panel) and—coherently—from the nucleus (lower panel).

state properties of the nucleus, its proper inclusion could lead to important corrections to the local impulse-approximation treatment. Indeed, Peters, Lenske, and Mosel have lifted the local assumption and have reported—in contrast to all earlier local studies—that the $S_{11}(1535)$ resonance does contribute to the coherent photoproduction of η mesons. Clearly, understanding these additional contributions to the coherent process is an important area for future work.

V. CONCLUSIONS

We have studied the coherent photoproduction of pseudoscalar mesons in a relativistic-impulse-approximation approach. We have placed special emphasis on the ambiguities underlying most of the current theoretical approaches. Although our conclusions are of a general nature, we have focused our discussions on the photoproduction of neutral pions due to the ''abundance'' of data relative to the other pseudoscalar channels.

We have employed a relativistic formalism for the elementary amplitude as well as for the nuclear structure. We believe that, as current relativistic models of nuclear structure rival some of the most sophisticated nonrelativistic ones, there is no longer a need to resort to a nonrelativistic reduction of the elementary amplitude. Rather, the full relativistic structure of the coherent amplitude should be maintained [6,7].

We have also extended our treatment of the pion-nucleus interaction to the Δ-resonance region. Although most of the details about the optical potential will be reported shortly [12], we summarize briefly some of our most important findings. As expected, pionic distortions are of paramount importance. Indeed, we have found factors-of-2 enhancements (at low energies) and up to factors-of-5 reductions (at high energies) in the coherent cross section relative to the plane-wave values. Yet, ambiguities arising from the various choices of optical-model parameters are relatively small; of at most 30%.

By far the largest uncertainty in our results emerges from the ambiguity in extending the many—actually infinite—equivalent representations of the elementary amplitude off the mass shell. While all these choices are guaranteed to give identical results for on-shell observables, they yield vastly different predictions off-shell. Yet, it is worth mentioning that the off-shell ambiguity emerges mainly from our insistence in using the impulse approximation. With an effective microscopic model—calibrated to reproduce two- and many-body scattering amplitudes—the off-shell ambiguity can, to a large extent, be removed. This task, however, remains a formidable one—forcing us, as well as most of the other theoretical approaches, to rely on the impulse approximation.

In this work we have investigated two on-shell-equivalent representations of the elementary amplitude: a tensor and a vector. The tensor representation employs the ''standard'' form of the elementary amplitude [4,9] and generates a coherent photoproduction amplitude that is proportional to the isoscalar tensor density. However, this form of the elementary amplitude, although standard, is not unique. Indeed, through a simple manipulation of operators between on-shell Dirac spinors, the tensor representation can be transformed into the vector one, so-labeled because the resulting coherent amplitude becomes proportional now to the isoscalar vector density. The tensor and vector densities were computed in a self-consistent, mean-field approximation to the Walecka model [10]. The Walecka model is characterized by the existence of large Lorentz scalar and vector potentials that are responsible for a large enhancement of the lower components of the single-particle wave functions. This so-called ''$M^\star$ enhancement'' generates a large increase in the tensor density, as compared to a scheme in which the lower component is computed from the free-space relation. No such enhancement is observed in the vector representation, as the vector density is insensitive to the $M^\star$ effect. As a result, the tensor calculation predicts coherent photoproduction cross sections that are up to a factor of 5 larger than the vector results. These large enhancement factors are not consistent with existent experimental data. Still, it is important to note that the vastly different predictions of the two models have been obtained using the same pionic distortions, the same nuclear-structure model, and two sets of elementary amplitudes that are identical on-shell.

Finally, we addressed—in a qualitative fashion—violations to the impulse approximation. In the impulse approximation one assumes that the elementary amplitude may be used without modification in the nuclear medium. Moreover, by adopting the optimal prescription of Ref. [22], one arrives at a form for the coherent amplitude that is local and factorizable. Indeed, such an optimal form has been used extensively—and with considerable success—in electron and nucleon elastic scattering from nuclei. We suggested here that the reason behind such a success is the *t*-channel dominance of these processes. In contrast, the coherent-photoproduction process is dominated by resonance formation in the *s* channel. In the nuclear medium a variety of processes may affect the formation, propagation, and decay of these resonances. Thus, resonant-dominated processes may not be amenable to treatment via the impulse approximation. Further, in *s*-channel–dominated processes, it is not the local nuclear density that is probed, but rather, it is the (nonlocal) polarization structure of the nucleus. This can lead to important deviations from the naive local picture. Indeed, by relaxing the local assumption, Peters and collaborators

have reported a non-negligible contribution from the $S_{11}(1535)$ resonance to the coherent photoproduction of η mesons [8], in contrast to all earlier local studies.

In summary, we have studied a variety of sources that challenge earlier studies of the coherent photoproduction of pseudoscalar mesons. Without a clear understanding of these issues, erroneous conclusions are likely to be extracted from the wealth of experimental data that will soon become available. What will be the impact of these calculations on our earlier work on the coherent photoproduction of η mesons [6,7] is hard to predict. Yet, based on our present study it is plausible that the large enhancement predicted by the tensor form of the elementary amplitude might not be consistent with the experimental data. In that case, additional calculations using the vector form will have to be reported. Moreover, this should be done within a framework that copes simultaneously with all other theoretical ambiguities. Indeed, many challenging and interesting lessons have yet to be learned before a deep understanding of the coherent-photoproduction process will emerge.

ACKNOWLEDGMENTS

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APPENDIX: PION-NUCLEUS OPTICAL POTENTIAL

The form of the optical potential is derived using a semiphenomenological formalism that uses a parameterized form of the elementary $\pi N \to \pi N$ amplitude that is assumed to remain unchanged in the nuclear medium (impulse approximation). However, the elementary amplitude does not encompass the many other processes that can occur in the many-body environment, such as multiple scattering, true pion absorption, Pauli blocking, and Coulomb (in the case of charged-pion scattering) interactions. The corrections resulting from these processes are of second and higher order relative to the strength of the first-order expression given by the impulse approximation. To account for these corrections, the impulse approximation form of the optical potential is modified to arrive at a pion-nucleus optical potential—applicable from threshold up to the delta-resonance region—of the form

$$2\omega U = -4\pi\left[p_1 b(r) + p_2 B(r) - \vec{\nabla} Q(r)\cdot\vec{\nabla} - \frac{1}{4}p_1 u_1 \nabla^2 c(r) - \frac{1}{4}p_2 u_2 \nabla^2 C(r) + p_1 y_1 \tilde{K}(r)\right], \tag{A1}$$

where

$$b(r) = \overline{b_0}\rho(r) - \epsilon_\pi b_1 \delta\rho(r), \tag{A2a}$$

$$B(r) = B_0\rho^2(r) - \epsilon_\pi B_1 \rho(r)\,\delta\rho(r), \tag{A2b}$$

$$c(r) = c_0\rho(r) - \epsilon_\pi c_1 \delta\rho(r), \tag{A2c}$$

$$C(r) = C_0\rho^2(r) - \epsilon_\pi C_1 \rho(r)\,\delta\rho(r), \tag{A2d}$$

$$Q(r) = \frac{L(r)}{1+(4\pi/3)\lambda L(r)} + p_1 x_1 \acute{c}\rho(r), \tag{A2e}$$

$$L(r) = p_1 x_1 c(r) + p_2 x_2 C(r), \tag{A2f}$$

$$\tilde{K}(r) = \frac{3}{5}\left(\frac{3\pi^2}{2}\right)^{2/3} c_0 \rho^{5/3}(r), \tag{A2g}$$

and with

$$\overline{b_0} = b_0 - p_1 \frac{A-1}{A}(b_0^2 + 2b_1^2) I, \tag{A3a}$$

$$\acute{c} = p_1 x_1 \frac{1}{3} k_0^2 (c_0^2 + 2c_1^2) I. \tag{A3b}$$

In the above expressions, the set $\{p_1,\ u_1,\ x_1,$ and $y_1\}$ represents various kinematic factors in the effective π-N system (pion-nucleon mechanisms), and the set $\{p_2,\ u_2,$ and $x_2\}$ represents the corresponding kinematic factors in the π-$2N$ system (pion-two-nucleon mechanisms). These kinematic factors have been derived using the relativistic potential model [23] with no recourse to nonrelativistic approximations and it includes nucleus recoil. The set of parameters $\{b_0,\ b_1,\ c_0,$ and $c_1\}$ originates from the $\pi N \to \pi N$ elementary amplitudes while all other parameters—excluding the kinematic factors—have their origin in the second and higher order corrections to the optical potential. These first-order parameters have been determined from a recent π-N phase-shift analysis [15], in contrast to the approach by Carr and collaborators in which they were fit to pionic-atom results. In spite of this difference, the parameters determined by the two methods match nicely. Nuclear effects enter in the optical potentials through the nuclear density $\rho(r)$, and through the neutron-proton density difference $\delta\rho(r)$. Moreover, A is the atomic number, λ is the Ericson-Ericson effect parameter, k_0 is the pion lab momentum, ω is the pion energy in the pion-nucleus center of mass system, and I is the so-called $1/r_{\text{correlation}}$ function. The B and C parameters arise from true pion absorption. A detailed account of this optical potential will be the subject of a paper that will be submitted for publication shortly [12].

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