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Transition strength sums and quantum chaos

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For the embedded Gaussian orthogonal ensemble of random matrices, the strength sums generated by a transition operator acting on an eigenstate vary with the excitation energy as the ratio of two Gaussians. This general result is compared to exact shell-model calculations of Gamow-Teller strength sums in nuclei. Good agreement is obtained in the chaotic domain of the spectrum, and strong deviations are observed as nuclear motion approaches a regular regime. Thus transition strength sums seem to be a new statistic sensitive to the chaoticity of the system. [S0556-2813(99)51411-8]

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In the last 15 years there has been an explosive growth in the use of random matrix theories for quantum systems, particularly in the context of quantum chaos [1]. It is now well known that the fluctuation properties of energy levels in quantum systems whose classical analogs are chaotic agree with the predictions of the Gaussian orthogonal ensemble (GOE) of random matrix theory, while quantum analogs of classically integrable systems display the features of Poisson statistics [2,3]. However, in the chaotic and intermediate domains, much less is known about the structure and correlations of stationary wave-function amplitudes and transition strengths generated by the action of a transition operator on an eigenstate [4].

In this work we study whether the statistical properties of the total Gamow-Teller (GT) strength, as function of the excitation energy E of nuclear states, can be related to regular or chaotic features of nuclear dynamics. This strength is a relevant quantity in astrophysics for presupernova evolution and stellar collapse. In fact, the smoothed behavior of the total GT strength vs E will be adequate for many astrophysical purposes, and it will give information about order-chaos transitions, just as energies, wave-function amplitudes, and transition strengths. Our purpose is to formulate, for the smoothed GT strength sums, a statistical theory in terms of the embeded Gaussian orthogonal ensemble of random matrices (EGOE), test it in terms of exact shell-model calculations, and study its behavior in chaotic and in regular domains. It should be mentioned that the EGOE smoothed forms gave birth to the so-called statistical nuclear spectroscopy [2,5–8], and there are recent studies of this in atoms [9], molecules and solids [10], and mesoscopic systems [11,12].

The EGOE(k) is defined in m-particle spaces [i.e., in the $\binom{\mathcal{N}}{m}$ dimensional space generated by distributing m fermions over $\mathcal{N}$ single particle states] with a GOE representation in k-particle space for k-body operators (usually $k \ll m$). Two important results given by EGOE are that, in strongly interacting shell-model spaces (essentially in $0\hbar\omega$ spaces), (i) the state densities $I(E)=\langle\langle\delta(H-E)\rangle\rangle$ take the Gaussian form [5] and (ii) with the strength $R(E,E') = |\langle E'|\mathcal{O}|E\rangle|^2$ generated by a transition operator $\mathcal{O}$ in the H-diagonal basis, the bivariate strength densities $I_{biv;\mathcal{O}}(E,E')=\langle\langle\mathcal{O}^\dagger\delta(H-E')\mathcal{O}\delta(H-E)\rangle\rangle=I'(E')|\langle E'|\mathcal{O}|E\rangle|^2 I(E)$ take bivariate Gaussian form [6]; here and elsewhere $\langle\langle\cdots\rangle\rangle$ denotes trace (similarly $\langle\cdots\rangle$ denotes average). Though the EGOE forms in (i) and (ii) are derived by evaluating the averages over fixed-m spaces, in a large number of numerical shell-model examples, it is verified that they apply equally well in fixed-m, mT, and mJT spaces [2,5–8]. In practice, Edgeworth corrections are added to the Gaussian forms. One important byproduct of (ii) is that the transition strength sum density $\langle\langle\mathcal{O}^\dagger\mathcal{O}\delta(H-E)\rangle\rangle$, which is a marginal density of the bivariate strength density, takes a Gaussian form, since the marginal of a bivariate Gaussian is a Gaussian. Therefore, using (i) and (ii), it is immediately seen that transition strength sums vary with excitation energy as the ratio of two Gaussians. Given $K=\mathcal{O}^\dagger\mathcal{O}$, the transition strength sum is given by the expectation value $\langle K\rangle^E$, and can be written in terms of the expectation value density $\rho_K(E)$ [6,8],

$$\langle K\rangle^E=[d\rho(E)]^{-1}\left[\sum_{\alpha\varepsilon E}\langle E\alpha|K|E\alpha\rangle\right]=I_K(E)/I(E)$$
$$=\rho_K(E)/\rho(E)\xrightarrow{\text{EGOE}}\overline{\rho_K(E)}/\overline{\rho(E)}=\rho_{K:\mathcal{G}}(E)/\rho_{\mathcal{G}}(E), \tag{1}$$

$$\rho(E)=\langle\delta(H-E)\rangle=d^{-1}I(E)=d^{-1}\langle\langle\delta(H-E)\rangle\rangle,$$

$$\rho_K(E)=\langle K\delta(H-E)\rangle=d^{-1}I_K(E)$$
$$=d^{-1}\langle\langle K\delta(H-E)\rangle\rangle;\quad K=\mathcal{O}^\dagger\mathcal{O}.$$

In Eqs. (1) d is the dimensionality, ''$\mathcal{G}$'' stands for Gaussian, and the bars over $\rho(E)$ and $\rho_K(E)$ indicate the ensemble average (smoothed) with respect to EGOE. In deriving Eqs. (1) it is assumed that the smothed form of $\rho_K(E)/\rho(E)$ reduces to the ratio of smoothed forms of $\rho_K(E)$ and $\rho(E)$. This result ignores the fluctuations in both $\rho_K(E)$ and $\rho(E)$, and the rms error due to neglect of the fluctuations is given in terms of the number of principal components or the inverse participation ratio for the transition operator $\mathcal{O}$

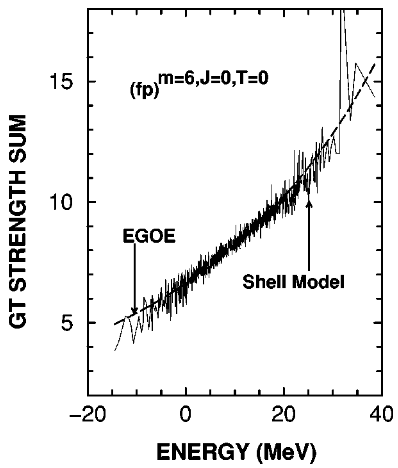

FIG. 1. Gamow-Teller (GT) strength sum versus excitation energy (E) for the 814 dimensional six particle (fp)-shell space with $J=0$, $T=0$. The exact shell-model results for the realistic KB3 interaction are compared with the EGOE predictions given by Eq. (1).

[2,7,13]. Note that the smoothed EGOE form for $\langle K\rangle^E$ takes into account (K,H) and (K,H^2) correlations, which define the centroid ϵ_K and width σ_K of $\rho_K(E)$; $\epsilon_K=\langle KH\rangle/\langle K\rangle$ and $\sigma_K^2=\langle KH^2\rangle/\langle K\rangle-\epsilon_K^2$.

Equation (1) expresses a very general result. In order to study its domain of validity we carried out detailed shell-model tests using the operator that generates GT strength sums. The GT transition operator $\mathcal{O}_{GT}$ is given by

$$\mathcal{O}^{(\pm)}_{GT;\mu}=\sum_{i=nucleons}\sigma_\mu(i)t_\pm(i), \tag{2}$$

where $t_\pm$ converts a neutron into a proton and vice versa. The total GT strength originating from an initial state at energy E to all final states is given by the expectation value $\langle K(GT)\rangle^E$ of the operator $K(GT)$, where

$$K^{(\pm)}(GT)=\sum_\mu \mathcal{O}^{(\pm)\dagger}_{GT;\mu}\mathcal{O}^{(\pm)}_{GT;\mu}. \tag{3}$$

In this paper we restrict ourselves to $T=0$ states of $N=Z$ nuclei and, therefore, $K(GT)=K^{(+)}(GT)=K^{(-)}(GT)$.

Exact shell-model calculations for the total GT strength have been carried out for all the $J=0$ states of ^{46}V in the 814 dimensional $(1f2p)^{m=6,J=0,T=0}$ space. The calculations were performed with the NATHAN code of the Strasbourg-Madrid group, using the effective interaction KB3, which successfully reproduces experimental binding energies, excitation spectra, and transition strengths for nuclei in this region [14]. On the other hand, the expectation value density $\rho_{K(GT):\mathcal{G}}$ for the $K(GT)$ operator is constructed in terms of its centroid and width and, similarly, the state density Gaussian. Then, using Eqs. (1), the smoothed form of the GT strength sum as a function of excitation energy is constructed and compared with exact shell-model results.

In Fig. 1 it is seen that the smoothed EGOE curve describes very well the shell-model results, except at the edges of the spectra. Thus it seems that the agreement is good in the chaotic region and that the deviations are just in the ground-state region, where the states are not sufficiently complex (chaotic). Similar deviations are observed at the upper end due to the finite shell-model space.

In order to verify this interpretation of the agreement as a consequence of chaoticity, next we study this question via $\langle K(GT)\rangle^E$ when the Hamiltonian changes through a parameter from a symmetry preserving, i.e., regular Hamiltonian to a chaotic one. In the situation that a given one- plus two-body H is a Wigner spin-isospin $SU(4)$-ST invariant, the eigenvalues and eigenvectors are given easily by $SU(4)$-ST algebra. The eigenstates are labeled, for a given number m of valence nucleons, by L, S, J, T, and the $SU(4)$ irreducible representations. In addition, as the GT operator is a generator of the $SU(4)$ group, the $SU(4)$ algebra directly gives $\langle K(GT)\rangle^E$ in terms of $SU(4)$-ST quantum numbers. Decomposing a given Hamiltonian H into the $SU(4)$-ST invariant $H_{SU(4)\text{-}ST}$ [15], and the remaining part that mixes $SU(4)$-ST states, one can construct the H_λ Hamiltonian,

$$H_\lambda=H_{SU(4)\text{-}ST}+\lambda(H-H_{SU(4)\text{-}ST}). \tag{4}$$

As λ varies from 0 to 1, there is order [good $SU(4)$-ST] to chaos (given by full realistic H) transition, since it is well known that realistic Hamiltonians generate, except at the ends of the spectrum, chaotic (complex) states [1,2,4].

Calculations for ^{24}Mg in the 325 dimensional $(2s1d)^{m=8,J=0,T=0}$ space were performed using the Rochester–Oak Ridge shell-model code and Kuo's two-body interaction [16] with ^{17}O single-particle energies in H. Figure 2 shows the total GT strength vs excitation energy for $\lambda=0$, 0.3, 0.5, and 1 cases. For the realistic nuclear interaction $(\lambda=1)$ the results are in good agreement with the EGOE curve except at the edges, i.e., the statistical theory gives a good smoothed description in the chaotic domain, just as we saw for ^{46}V. For the $SU(4)$-ST Hamiltonian ($\lambda=0$), there are many degenerate states and the shape of the GT strength sums is very different from the shape for the realistic interaction. It is clearly seen that the EGOE curve is not a good smooth aproximation to the exact results in the case of regular motion.

As a quantitative measure of the similarity between EGOE and shell-model results we can use the mean square deviation χ^2 between their total GT strengths at the shell-model energies and their corresponding values in the smoothed EGOE curve. Figure 3 shows this χ^2 value as a function of λ. It is reduced by an order of magnitude when λ makes the transition from order to chaos, confirming our interpretation of the EGOE and shell-model agreement as a measure of chaoticity.

The order to chaos transition as λ increases is clearly illustrated by the the $P(s)$ distribution of nearest-neighbor level spacings and the spectral rigidity Δ_3, which are commonly used statistics to distinguish order from chaos in quantum systems [1,2]. Figure 4 shows $P(s)$ and Δ_3 for $\lambda=0.3$, 0.5, and 1. The $P(s)$ distribution is related to short-range correlations, and Δ_3, to long-range correlations between energy levels. At $\lambda=0.3$, $P(s)$ is already very close to the GOE limit, apparently indicating a quick onset of chaos

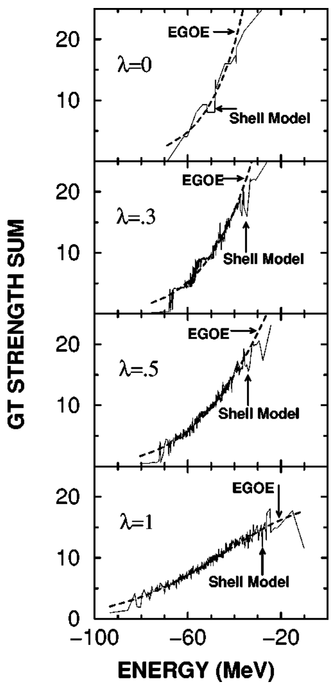

FIG. 2. Gamow-Teller (GT) strength sum versus excitation energy (E) for 325 dimensional eight particle (ds)-shell space with $J=0$, $T=0$. The exact shell-model results for an $SU(4)$-ST invariant Hamiltonian ($\lambda=0$), for a realistic Kuo interaction ($\lambda=1$) and for the interpolating cases with $\lambda=0.3$ and 0.5 are compared with the EGOE predictions given by Eq. (1).

even for a smaller fraction of the realistic nuclear force, but Δ_3 is still relatively far from the GOE limit. Since the total GT strength is the sum of strengths from one parent state to all the states of the daughter nucleus, it is a quantity associated with long-range correlations between nuclear states and it can be expected to behave more like Δ_3. In Fig. 3 it is indeed seen that χ^2 is three times larger at $\lambda=0.3$ than at $\lambda=1$. Finally, the system becomes fully chaotic at $\lambda=0.5$, where $P(s)$, Δ_3, and χ^2 are all close to their chaotic limits.

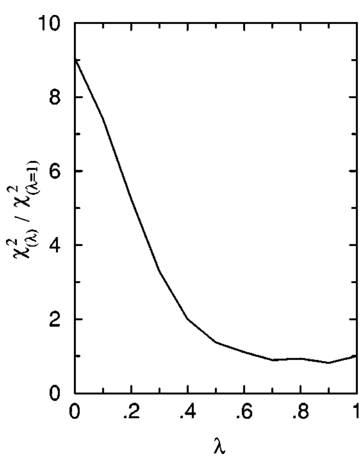

FIG. 3. Mean square deviation $\chi^2_{(\lambda)}/\chi^2_{(\lambda=1)}$ between the exact shell-model and EGOE predictions for GT strength sums for all the $(ds)^{m=8,J=0,T=0}$ states, versus the interpolating parameter λ in H_λ defined by Eq. (4).

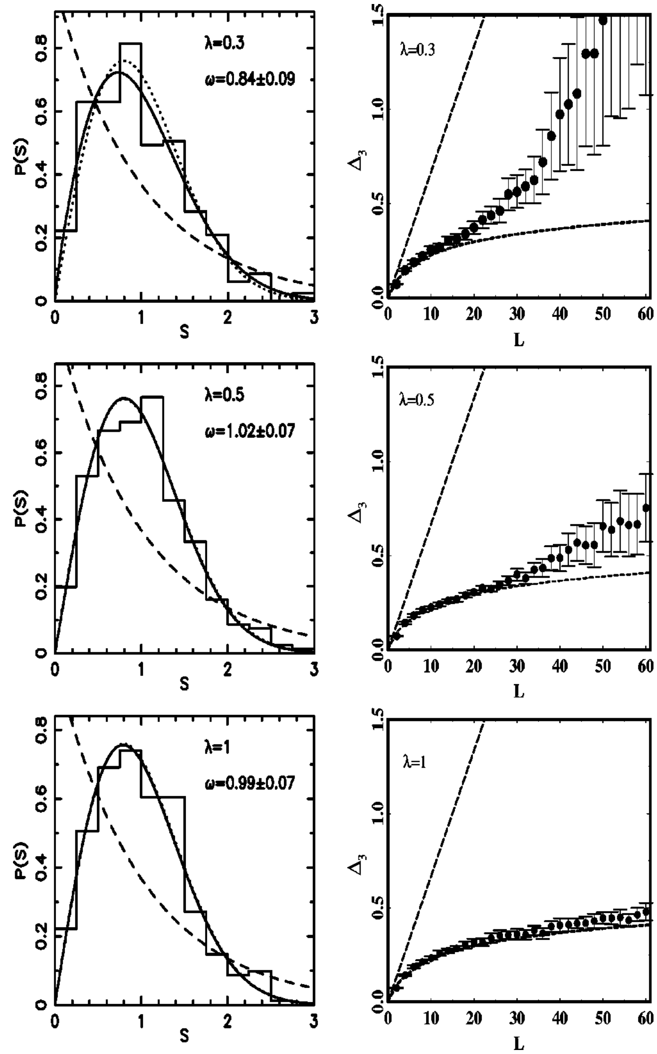

FIG. 4. Distribution of nearest-neighbor spacings $P(s)$ and averaged spectral rigidity $\Delta_3(L)$ for the $(ds)^{m=8,J=0,T=0}$ levels in ^{24}Mg, for various values of the interpolating parameter λ in H_λ defined by Eq. (4). The dashed lines represent Poisson; dotted lines, GOE predictions; and solid lines, best fit Brody distributions with parameter ω [2]. The error bars give the standard deviation of the Δ_3 average over overlaping intervals of length L.

In conclusion, we find that the GT strength sum as a function of excitation energy is an observable which is sensitive to the regular or chaotic regimes of nuclear motion. The transition from order to chaos has been studied as the nuclear Hamiltonian H_λ goes from the $SU(4)$-ST limit for $\lambda=0$ to a realistic interaction for $\lambda=1$. It has been shown that as nuclear dynamics approach the chaotic regime, the GT strength sums approach EGOE predictions. The EGOE (equivalently statistical spectroscopy) smoothed strength sum for a transition operator varies with excitation energy as the ratio of two Gaussians, and agrees very well with exact shell-model predictions in the chaotic domain of the spectrum for a realistic nuclear Hamiltonian, while important deviations are observed at the edges, where nuclear states are not chaotic.

The chaoticity of the system has been studied using the

$P(s)$ distribution and the Δ_3 statistic as well. As the order to chaos transition takes place, $P(s)$ approaches very quickly the Wigner surmise limit. In contrast, the GT strength sums behave rather like the Δ_3 statistic, approaching more slowly the EGOE and GOE limits. This similarity is probably due to the fact that both statistics are related to long-range correlations between energy levels or wave functions. This and other related questions require investigations using deformed embedded random matrix ensembles [H_λ in Eq. (4) is a member of a deformed EGOE and [4] gives other examples].

Although we have presented results for the GT strength sums only, similar conclusions are seen to be valid for other transition operators as well. A more comprehensive account of these results will be presented elsewhere.

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[13] The mean square deviation $\hat{\Sigma}^2(E)$ at energy E in the smoothed expectaion value $\overline{\mathcal{M}(E)}=\overline{\langle \mathcal{O}^\dagger \mathcal{O}\rangle^E}$ follows by using the Porter-Thomas assumption for strength fluctuations. In terms of the bivariate strength density $\rho_{biv;\mathcal{O}}(E,E')$ and the state density $\rho'(E')$, it is easy to show that (with d' the dimensionality of E' space),

$$\hat{\Sigma}^2(E)/(\overline{\mathcal{M}(E)})^2=\frac{2}{d'}\left\{\left(\int dE'\,\overline{\rho_{biv;\mathcal{O}}(E,E')}\right)^2\right\}^{-1}\times\int dE'\,\frac{\overline{\overline{(\rho_{biv;\mathcal{O}}(E,E'))^2}}}{\rho'(E')}.$$

See Ref. [2] and note that the inverse of $\hat{\Sigma}^2(E)/[\overline{\mathcal{M}(E)}]^2$ is, to within a constant, nothing but $(\mathrm{NPC})_E$ [7].

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