Microscopic insight into the vibrational nature of ^{114,116}Cd isotopes

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(Received 10 December 1998; published 13 September 1999)

The yrast spectra, quadrupole deformation parameter (β_2), occupation probabilities, intrinsic static quadrupole moments, and $B(E2)$ transition probabilities are calculated for ^{114,116}Cd isotopes by carrying out cranked-Hartree-Bogoliubov calculations. These calculations have been performed by employing a quadrupole–quadrupole-plus-pairing model of residual interaction operating in a reasonably large valence space outside the ^{76}Sr core. Our calculations show that the systematics of low-lying levels in ^{114,116}Cd depend upon the simultaneous increase in the occupation probabilities of the $(d_{5/2})_\nu$ and $(h_{11/2})_\nu$ orbits and also on the subshell closure of the $(p_{1/2})_\pi$, $(p_{1/2})_\nu$, and $(g_{9/2})_\nu$ orbits. It turns out that the vibrational character of the Cd isotopes, in general, is due to the nonpolarizability of the $Z=40$ and $N=50$ core. [S0556-2813(99)05209-7]

PACS number(s): 21.10.Ky, 21.60.Jz, 27.60.+j

From fission fragment studies [1], it has been established that there occur large deformations for the neutron-rich Zr and Mo isotopes with $N \geqslant 60$. If one looks at the systematics of the $^{96-104}$Zr isotopes, it is observed that the Zr isotopes with $N<60$ are nearly spherical and those with $N \geqslant 60$ are deformed. The shape transition is very sudden at $N=60$. The mechanism for the sudden onset of large deformations in the Zr and Mo isotopic mass chains at $N=60$ has been studied by a number of theoretical groups. Federman and Pittel [2,3] have explained the emergence of large deformation in Zr and Mo at $N=60$ in terms of the strong deformation-producing tendency of *n-p* interaction when acting between spin-orbit partner orbits—in this case, the $(g_{9/2})_\pi$ and $(g_{7/2})_\nu$ orbits. The subscript π stands for proton and ν for neutron. They have suggested that at $N \geqslant 60$ protons from the $(p_{1/2})_\pi$ orbits are promoted into the $(g_{9/2})_\pi$ orbit and simultaneously the neutrons start filling the $(g_{7/2})_\nu$ orbit. As a result, the *n-p* interaction between protons in the $(g_{9/2})_\pi$ orbit and neutrons in the $(g_{7/2})_\nu$ orbit comes into operation and causes the deformation to occur. The emergence of deformation at $N \geqslant 60$ in the Zr and Mo isotopes was also explained by Khosa, Tripathi, and Sharma [4] in terms of the sudden occupation of low-*k* components of the $(h_{11/2})_\nu$ orbit.

If we now consider the neutron-rich mass chain of cadmium isotopes with $N>60$, it is observed that all these nuclei are nearly vibrational in nature. As typical examples of the nuclei in this mass chain, let us consider ^{114,116}Cd. It is observed that the E_2-E_0 energy gap ≈ 0.55 MeV, and also their excitation energy spectra show a triplet of 0_2^+, 2_2^+, and 4_1^+ states at nearly twice the energy of $(E_{2_1}-E_{0_1})$, which is typical of vibrational type of nuclei. However, if we analyze the situation in the Cd isotopes with $N>60$, in the light of the theoretical explanation put forth by Federman and Pittel [2] and by Khosa, Tripathi, and Sharma [4] in the case of Zr and Mo isotopic mass chains, one should also expect the emergence of large deformations in this mass chain at least for $N>60$, which is not otherwise observed experimentally. This nonobservation creates a paradoxical situation in the Cd isotopes. The question is why does a large deformation not emerge in the Cd isotopes for $N>60$? To unravel and understand the reason for the nonemergence of a large deformation and for the occurrence of vibrational character of the Cd isotopes becomes the motivating factor for the present work.

Nuclei ^{114,116}Cd as typical examples, have been chosen for such a study. For these nuclei, a microscopic study has been carried out involving calculation of the quadrupole deformation parameters β_2, pairing gap parameters Δ_π and Δ_ν for protons and neutrons, yrast spectra, $B(E2; J_i^+ \rightarrow J_f^+)$ values, and occupation probabilities of valence orbits.

The above calculations have been performed in the cranked-Hartree-Bogoliubov (CHB) framework using the standard quadrupole–quadrupole-plus-pairing model (QQPP) of residual interaction of Kumar and Baranger [5] operating in a valence space spanned by the $3s_{1/2}$, $2p_{1/2}$, $2d_{3/2}$, $2d_{5/2}$, $1g_{7/2}$, $1g_{9/2}$, and $1h_{11/2}$ orbits for protons as well as neutrons. The nucleus ^{76}Sr is considered as the inert core. In the CHB formalism, the potential parameters are determined self-consistently. In spite of its several shortcomings, this formalism has been used extensively to investigate the complexity of nuclear spectra (arising from the interplay of single-particle and collective aspects of nuclear motion) as a function of rotational frequency. The spherical single particle energies (SPE) that we have employed are (in MeV) $(2p_{1/2})=-1.3$, $(1g_{9/2})=0.0$, $(2d_{5/2})=5.4$, $(3s_{1/2})=6.4$, $(2d_{3/2})=7.9$, $(1g_{7/2})=10.5$, and $(1h_{11/2})=11.0$. Except for the energies of the $1g_{7/2}$ and $1h_{11/2}$ orbits, this set of input SPE values is nearly the same as that employed in a number of successful shell-model calculations in the $A \approx 100$ region by Vergados and Kuo [6] as well as by Federman, Pittel, and Campos [3].

The two-body effective interactions that have been employed are also of the QQPP type [5]. The strengths for the like-particle (*nn*) as well as for the neutron-proton (*np*) interactions were taken as

$$\chi_{nn}(=\chi_{pp}) = -0.009\,45 \text{ MeV b}^{-4}$$

and

$$\chi_{np} = -0.022 \text{ MeV b}^{-4}.$$

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TABLE I. Comparison of the experimental and calculated yrast levels and quadrupole deformation parameters (β_2) in the ^{114,116}Cd isotopes. Here $\langle Q_0^2\rangle_\pi$ ($\langle Q_0^2\rangle_\nu$) gives the contribution of the protons (neutrons) to the components of the quadrupole moment operator. The quadrupole moments have been calculated in units of b^2, where $b=\sqrt{\hbar/m\omega}$ is the oscillator parameter. In the last two columns of this table, Δ_π and Δ_ν are the pairing gap parameters for protons and neutrons, respectively.

J^π	E_J(MeV) Expt.	E_J(MeV) Th.	$\langle Q_0^2\rangle_\pi$	$\langle Q_0^2\rangle_\nu$	$(\beta_2)_{\text{Expt}}$[a]	$(\beta_2)_{Th}$	Δ_π	Δ_ν
				^{114}Cd				
0^+	0.0	0.0	33.64	37.76	0.1912(35)	0.193	1.57	1.07
2^+	0.56	0.53	33.61	37.64		0.192	1.62	0.81
4^+	1.28	1.36	33.43	36.25		0.189	1.55	0.47
6^+	1.99	1.98	33.25	35.22		0.186	1.48	0.29
8^+	2.67	2.50	33.10	34.51		0.185	1.42	0.18
10^+	3.14	3.04	32.99	34.17		0.184	1.37	0.14
12^+	3.71	3.60	32.84	33.77		0.182	1.33	0.10
				^{116}Cd				
0^+	0.0	0.0	33.76	38.70	0.1907(34)	0.183	1.65	1.41
2^+	0.51	0.50	33.73	38.65		0.182	1.67	1.16
4^+	1.21	1.07	33.67	38.50		0.180	1.66	0.98
6^+	2.03	1.90	33.56	38.27		0.179	1.62	0.92
8^+	2.56	2.54	33.43	37.97		0.177	1.60	0.78
10^+	2.78	3.45	33.25	37.56		0.175	1.56	0.63

[a]Experimental data are from Ref. [19].

Here b is the oscillator parameter. These values for the strength of the $q\cdot q$ interactions compare favorably with the ones employed in the study of deformation systematics in the $A\sim 100$ region by Bharti and Khosa [7].

The strength for the pairing interaction was fixed through the approximate relation $G=(18-21)/\text{nucleon}$. The value of $|G|$ was taken to be 0.18 MeV for ^{114,116}Cd. The calculation of the energy of the yrast levels has been carried out in the CHB framework. This formalism has been discussed in detail by Goodman [8].

Our calculations reveal that the $(p_{1/2})_{\pi,\nu}$ and $(g_{9/2})_\nu$ orbits are full in the ^{114,116}Cd isotopes. It appears that the emergence of large deformation in these nuclei is prohibited because of the $(p_{1/2})_{\pi,\nu}$ and $(g_{9/2})_\nu$ subshell closure and this property could be thought to be responsible for the vibrational nature of Cd isotopes. Moreover, it turns out from our calculations that the low-lying experimental yrast spectra upto $12\hbar$ in ^{114}Cd and $8\hbar$ in ^{116}Cd are satisfactorily reproduced. The results on the quadrupole deformation parameter (β_2) show a decrease of deformation with angular momentum along the yrast states. The increase in the occupation probabilities of the $(d_{5/2})_\nu$ and $(h_{11/2})_\nu$ orbits is responsible for the low-lying systematics of the 2^+, 4^+, and 6^+ states in the ^{114,116}Cd isotopes.

In Table I, some of the results of the CHB calculations for ^{114,116}Cd are presented. The calculated quadrupole deformation parameters (β_2) for the ground states are in satisfactory agreement with the experimental values. The values of $(\beta_2)_{\text{Th}}$ are calculated from the values of intrinsic quadrupole moments using standard formulas [9]. The theoretical value for this quantity (hence, the collectivity) is seen to decrease as we move along the yrast states. Concerning the pairing gap parameters Δ_π and Δ_ν for protons and neutrons, the calculated results show that Δ_π suffers a very slow decrease as we move along the yrast states whereas the decrease in the Δ_ν values is quite fast. For example, the value of Δ_ν for the 0^+ state in ^{114}Cd is 1.07 and its value for the 12^+ state is 0.10 MeV. The values of pairing gap parameters have been calculated using the standard formulas given by Lane [10]. In Table I, also a comparison of the energy values of the observed yrast states and those calculated in the cranking framework has been presented. In the case of ^{114}Cd, what has been reproduced is the energy spectra up to the 12^+ state within an accuracy of about 0.17 MeV; however, in the case of ^{116}Cd, the states up to the 8^+ state are very well reproduced. The experimental data for these isotopes are taken from Refs. [11–14].

It is important to comment on why the proton pairing gap remains relatively constant from $J^\pi=0^+$ to $J^\pi=12^+$, while the neutron pairing gap almost disappears for ^{114}Cd and is significantly reduced for ^{116}Cd. It was suggested by Banerjee, Mang, and Ring [15] and Fleckner *et al.* [16] that there is a large breakdown of pairing field when a pair of nucleons gets decoupled in a high-j orbit due to Coriolis antipairing effect. The signature of the breakdown of pairing field is indicated by the decrease in the pairing gap parameter. Besides this effect, the decrease in the pairing gap parameter depends on the percentage of realignment of the nucleon pair. The decoupling of the nucleon pair is most favored when a high-j orbit with small m components appears at the Fermi surface. On examination of the proton part of the wave function, what gets revealed is that the high-j orbit—the $(h_{11/2})_\pi$ orbit in the present context—does not appear in the neighborhood of the Fermi level for protons and conse-

TABLE II. The values of occupation probabilities V_j^2 [normalized to $(2j+1)$] of neutrons in the ^{114,116}Cd isotopes.

J^π	$s_{1/2}$	$p_{1/2}$	$d_{3/2}$	$d_{5/2}$	$g_{7/2}$	$g_{9/2}$	$h_{11/2}$
			^{114}Cd				
0^+	0.62	1.00	0.56	0.84	0.36	0.99	0.39
2^+	0.61	1.00	0.56	0.85	0.36	0.99	0.39
4^+	0.61	1.00	0.56	0.89	0.37	1.00	0.35
6^+	0.61	1.00	0.57	0.91	0.38	1.00	0.34
8^+	0.61	1.00	0.57	0.92	0.38	1.00	0.33
10^+	0.59	1.00	0.58	0.92	0.38	0.99	0.33
12^+	0.58	1.00	0.58	0.91	0.38	0.99	0.33
			^{116}Cd				
0^+	0.64	1.00	0.56	0.92	0.38	1.00	0.50
2^+	0.61	1.00	0.56	0.91	0.38	1.00	0.50
4^+	0.60	1.00	0.56	0.91	0.40	1.00	0.50
6^+	0.58	1.00	0.57	0.91	0.39	1.00	0.50
8^+	0.57	1.00	0.58	0.90	0.40	1.00	0.50
10^+	0.56	1.00	0.58	0.90	0.40	1.00	0.50

quently the decoupling of a pair of protons is unfavored. Because of this fact, the values of Δ_π for all the cadmium isotopes remains constant. In the case of neutrons, the high-j orbit ($j=h_{11/2}$, $m=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$) is partially occupied and appears in the neighborhood of the Fermi level. An examination of the neutron wave function shows that there is a realignment of neutrons taking place involving the $h_{11/2}$ orbit. The decoupling of a pair of neutrons becomes larger and larger as we go to higher angular momentum states. Because the occupation probability of the $(h_{11/2})_\nu$ orbit for ^{114}Cd is smaller than for ^{116}Cd, the decoupling of a pair of neutrons and breakdown of collective pairing in neutrons take place faster in ^{114}Cd as compared to ^{116}Cd. In the case of ^{114}Cd the $m=\frac{1}{2},\frac{3}{2}$ components of $(h_{11/2})_\nu$ orbit are fully occupied whereas in ^{116}Cd the $m=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ components are completely occupied.

From the systematics of the observed low-lying states in ^{114,116}Cd, one observes that the low-lying 2^+, 4^+, and 6^+ states are at nearly the same energy. To understand such systematics, we present in Table II the results of occupation probabilities V_j^2 [normalized to $(2j+1)$] of various neutron subshells for ^{114,116}Cd. An examination of the occupation probabilities of the $(d_{5/2})_\nu$ and $(h_{11/2})_\nu$ orbits shows an increase from their values of 0.84 and 0.39, respectively, to 0.92 and 0.50 as one goes from ^{114}Cd to ^{116}Cd. It may be noted that $(d_{5/2})_\nu$ orbit is more than half full in the ^{114,116}Cd isotopes and an increase in its occupation probability should be accompanied by a decrease in collectivity. Similarly, the increase in the occupation probability of $(h_{11/2})_\nu$ orbit should result in an increase of deformation or should produce a decrease in the energy of the 2_1^+ state as we go from ^{114}Cd to ^{116}Cd. Thus, the simultaneous increase in the occupation probabilities of $(d_{5/2})_\nu$ and $(h_{11/2})_\nu$ orbits causes these two effects to partly neutralize each other. This cancellation is the reason that the 2_1^+, 4_1^+, and 6_1^+ states in these two nuclei are nearly at the same energy. Besides this we have verified that the $(p_{1/2})_\pi$, $(p_{1/2})_\nu$, and $(g_{9/2})_\nu$ subshells are fully occupied in both isotopes. This feature of nonpolarizability of the $(p_{1/2})_\pi$, $(p_{1/2})_\nu$, and $(g_{9/2})_\nu$ subshells is also found to be exhibited by other $^{108-112}$Cd isotopes. These results imply that the $Z=40$ and $N=50$ core remains unpolarized for the cadmium isotopes. It was suggested by Sugita and Arima [17] that for the emergence of large deformations in the Zr-Mo region, it is important to have holes in the $(p_{1/2})_\pi$ orbit. In the Federman and Pittel [2] mechanism (for the emergence of large deformation in the Zr isotopes at $N \geqslant 60$) it was said that there is a promotion of protons from $(p_{1/2})_\pi$ orbit to $(g_{9/2})_\pi$ at $N \geqslant 60$. This promotion naturally causes the $Z=40$ core to get polarized. This type of trend is not observed in the cadmium isotopes as a result of which these nuclei tend to be vibrational in nature. Thus, the emergence of large deformations in these isotopes is prohibited because of the nonpolarizability of $(p_{1/2})_\pi$, $(p_{1/2})_\nu$, and $(g_{9/2})_\nu$ shells.

It will be appropriate at this stage to understand the causes for the nonpolarizability of the $(p_{1/2})_\pi$ and $(g_{9/2})_\nu$ subshells in cadmium isotopes. The reason for the nonpolarizability of the $Z=40$ core is the fact that the low m components of the neighboring valence orbit [that is, the $(g_{9/2})_\pi$ orbit] are fully occupied and the two-body residual interaction is not able to scatter protons from the $(p_{1/2})_\pi$ orbit to single-particle orbitals having large values of m. Such a scattering will need a large excitation energy. Similarly, in the case of neutrons, the presence of neutrons in the neighboring $(d_{5/2})_\nu$ orbit prevents the polarization of the $g_{9/2}$ neutron subshell and thus prevents the polarization of the $N=50$ core. In the zirconium region [4], the $Z=40$ core is easily polarizable because the low-m components of $(g_{9/2})_\pi$ orbit are empty and protons in the $(p_{1/2})_\pi$ orbit need small energy in getting scattered to these small-m component orbitals of the $(g_{9/2})_\pi$ subshell. They can easily pick up this energy from the residual two-body interaction.

We have examined the goodness of the CHB wave function by calculating the $B(E2)$ values. It has been shown [18]

that the intrinsic electric quadrupole moments are related to the $B(E2;J_i^+ \rightarrow J_f^+)$ by the relation

$$B(E2;J_i^+ \rightarrow J_f^+) = [5/(16\pi)] \begin{bmatrix} J_i^+ & 2 & J_f^+ \\ 0 & 0 & 0 \end{bmatrix}^2 \times [e_\pi \langle Q_0^2 \rangle_\pi + e_\nu \langle Q_0^2 \rangle_\nu]^2. \quad (1)$$

In Table III, we present a comparison of the observed $B(E2;J_i^+ \rightarrow J_f^+)$ values with the values calculated by substituting in relation (1) the values of $\langle Q_0^2 \rangle_\pi$ and $\langle Q_0^2 \rangle_\nu$ for ^{114,116}Cd given in Table I. It is satisfying to note that the calculated $B(E2)$ estimates are in good agreement with the experiments [14,19] for the $0_1^+ \rightarrow 2_1^+$ transitions provided one chooses $e_{\rm eff}=0.20$ for ^{114,116}Cd. For higher transitions, the agreement is poor.

To summarize, the systematics of low-lying states in ^{114,116}Cd are found to arise due to the simultaneous increase in the occupation probabilities of the $(d_{5/2})_\nu$ and $(h_{11/2})_\nu$ orbits and also due to the subshell closure of the $(p_{1/2})_\pi$, $(p_{1/2})_\nu$, and $(g_{9/2})_\nu$ orbits. It turns out that the vibrational character of the Cd isotopes, in general, is due to the nonpolarizability of the $Z=40$ and $N=50$ core. The proton pairing gap parameters (Δ_π) are found to be nearly constant with change of angular momentum, whereas the neutron pairing gap parameters Δ_ν are seen to show a decreasing trend.

TABLE III. Comparison of relative theoretical and experimental $B(E2;J_i^+ \rightarrow J_f^+)$ values in the ^{114,116}Cd isotopes. For protons, the effective charge is $e_\pi = 1 + e_{\rm eff}$; and, for neutrons, it is $e_\nu = e_{\rm eff}$. Only one parameter $e_{\rm eff}$ is introduced in the calculations. The values of the oscillator parameter have been calculated from the relation $b = 1.01A^{1/6}$ fm. The entries presented in the third column correspond to the calculated values of $B(E2)$ values for $e_{\rm eff}=0.20$.

Nucleus	Transition $(J_i^+ \rightarrow J_f^+)$	$B(E2; J_i^+ \rightarrow J_f^+)_{\rm Th.}$ e^2b^2	$B(E2; J_i^+ \rightarrow J_f^+)_{\rm Expt.}$ e^2b^2
^{114}Cd	$0_1^+ \rightarrow 2_1^+$	0.559	0.550(20)[a]
	$2_1^+ \rightarrow 4_1^+$	0.297	
^{116}Cd	$0_1^+ \rightarrow 2_1^+$	0.571	0.560(20)[a]
	$2_1^+ \rightarrow 4_1^+$	0.293	0.35(7)[b]

[a]From Ref. [19].
[b]From Ref. [14].

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