

Strange chiral nucleon form factors

Thomas R. Hemmert, Bastian Kubis, and Ulf-G. Meißner

Forschungszentrum Jülich, Institut für Kernphysik (Th), D-52425 Jülich, Germany

(Received 3 May 1999; published 30 August 1999)

We investigate the strange electric and magnetic form factors of the nucleon in the framework of heavy baryon chiral perturbation theory to third order in the chiral expansion. All counterterms can be fixed from data. In particular, the two unknown singlet couplings can be deduced from the parity-violating electron-scattering experiments performed by the SAMPLE and the HAPPEX Collaborations. Within the given uncertainties, our analysis leads to a small and positive electric strangeness radius, $\langle r_{E,s}^2 \rangle = (0.05 \pm 0.09) \text{ fm}^2$. We also deduce the consequences for the upcoming MAMI A4 experiment. [S0556-2813(99)06109-9]

PACS number(s): 13.40.Ks, 12.39.Fe, 13.40.Gp, 14.20.Dh

I. INTRODUCTION

Recently, the first results from parity-violating electron scattering experiments, which allow one to pin down the so-called strange form factors of the nucleon, have become available. The SAMPLE Collaboration has reported the measurement of the strange magnetic moment of the proton [1]. To be precise, they give the strange magnetic form factor in units of nuclear magnetons at a small momentum transfer $Q_S^2 = 0.1 \text{ GeV}^2$

$$G_{\text{SAMPLE}}^{(s)}(Q_S^2) = G_M^{(s)}(Q_S^2) = +0.23 \pm 0.37 \pm 0.15 \pm 0.19. \quad (1)$$

The rather sizable error bars document the difficulty of such types of experiment. The HAPPEX Collaboration has chosen a different kinematics which is more sensitive to the strange electric form factor [2]. Their measurement implies

$$\begin{aligned} G_{\text{HAPPEX}}^{(s)}(Q_H^2) &= G_E^{(s)}(Q_H^2) + 0.39 G_M^{(s)}(Q_H^2) \\ &= 0.023 \pm 0.034 \pm 0.022 \pm 0.026 \end{aligned} \quad (2)$$

with $Q_H^2 = 0.48 \text{ GeV}^2$. There have been many theoretical speculations about the size of the strange form factors, some of them clearly in conflict with the data (for a review, see Ref. [3]). Here, we wish to analyze these data in the framework of chiral perturbation theory. It was shown in [4] that to leading order in the momentum dependence one can make a parameter-free prediction for the momentum dependence of the nucleons' strange magnetic (Sachs) form factor based on the chiral symmetry of QCD solely. The value of the strange magnetic moment, which contains an unknown low-energy constant, can be deduced from the SAMPLE experiment using the momentum dependence derived in [4]. Furthermore, the SU(3) analysis of the baryon octet electromagnetic form factors performed in [5] allows one to pin down the octet component of the strange vector current. We demonstrate here that to one-loop order (more precisely: to third order in the chiral expansion) there is only one additional singlet counterterm, the strength of which can be determined from the value found by HAPPEX. This allows us to give a band for the strange electric form factor and make a prediction for the MAMI A4 experiment [6], which intends to measure

$$G_{\text{MAMI}}^{(s)}(Q_M^2) = G_E^{(s)}(Q_M^2) + 0.22 G_M^{(s)}(Q_M^2), \quad (3)$$

with a four-momentum transfer (squared) $Q_M^2 = 0.23 \text{ GeV}^2$ of approximately half the HAPPEX value.

The strangeness vector current of the nucleon is defined as

$$\begin{aligned} \langle N | \bar{s} \gamma_\mu s | N \rangle &= \langle N | \bar{q} \gamma_\mu (\lambda^0/3 - \lambda^8/\sqrt{3}) q | N \rangle \\ &= (1/3) J_\mu^0 - (1/\sqrt{3}) J_\mu^8 \end{aligned} \quad (4)$$

with $q = (u, d, s)$ denoting the triplet of the light quark fields and $\lambda^0 = I(\lambda^a)$ the unit (the $a = 8$ Gell-Mann) SU(3) matrix. Chiral perturbation theory (ChPT) is a precise tool to investigate such types of low-energy properties of the nucleon [7]. In the past few years, however, it was believed that due to the appearance of higher-order local contact terms with undetermined coefficients, ChPT cannot be used to make any prediction for the strange magnetic moment or the strange electric form factor [8]. However, with the advent of the first SAMPLE and HAPPEX data and renewed theoretical effort the situation has now changed. It could be shown that to third order in small momenta and/or meson mass insertions (we collectively denote these expansion parameters by p), there appear only four low-energy constants (LEC's) in the octet (note that only two combinations of these are relevant here) and two in the singlet current. While the former (two combinations) can be fixed from the isoscalar anomalous magnetic moment and charge radius of the nucleon, the latter two can now be deduced from the pioneering SAMPLE and HAPPEX results.¹

¹As a cautionary remark we mention already here that the momentum transfer in the HAPPEX experiment might be too large to trust the third-order ChPT treatment. However, at the moment we consider the experimental uncertainties associated with the SAMPLE and HAPPEX input into our calculation to be larger than the theoretical uncertainty of truncating the calculation at $O(p^3)$. Ultimately, the situation can be improved by performing the calculation to next order, smaller experimental error bars and utilizing new data at lower q^2 , which should become available within the next few years.

II. THEORETICAL FRAMEWORK

In order to obtain the strange electric and magnetic (Sachs) form factors we are calculating the singlet and the octet current matrix element of the nucleon to $O(p^3)$ in SU(3) HBChPT, in the Breit frame (following Refs. [9,10])

$$J_\mu^{(0,8)} = \frac{1}{\mathcal{N}_i \mathcal{N}_f} \bar{u}(p') P_\nu^+ \left[G_E^{(0,8)}(Q^2) v_\mu + \frac{1}{m} G_M^{(0,8)}(Q^2) \right. \\ \left. \times [S_\mu, S_\nu] q^\nu \right] P_\nu^+ u(p), \quad (5)$$

with

$$q_\mu = (p' - p)_\mu, \quad Q^2 = -q^2, \quad \mathcal{N} = \sqrt{\frac{E+m}{2m}}, \quad (6)$$

P_ν^+ being a positive-velocity projection operator and m the nucleon mass. For a more detailed discussion of this expression and the relation to the standard Dirac and Pauli form factors, see, e.g., [10]. From Eq. (5) one can then reconstruct the strangeness form factors as follows:

$$G_{E/M}^{(s)}(Q^2) = \frac{1}{3} G_{E/M}^{(0)}(Q^2) - \frac{1}{\sqrt{3}} G_{E/M}^{(8)}(Q^2). \quad (7)$$

These form factors admit a Taylor expansion around $Q^2 = 0$,

$$G_{E/M}^{(s)}(Q^2) = G_{E/M}^{(s)}(0) - \frac{1}{6} \langle r_{E/M,s}^2 \rangle Q^2 + O(Q^4), \quad (8)$$

in terms of the strange electric/magnetic radii

$$\langle r_{E/M,s}^2 \rangle = -6 \frac{dG_{E/M}^{(s)}(Q^2)}{dQ^2} \Big|_{Q^2=0}. \quad (9)$$

Note that one does not divide by the normalization of the respective form factor (even if it is nonvanishing) as it is usually done in case of the standard electromagnetic Sachs form factors. For that reason, one sometimes also introduces the slope parameters $(\rho_{E/M}^{(s)})^2 = \langle r_{E/M,s}^2 \rangle / 6$ [8].

We give now the relevant HBChPT Lagrangians needed for the calculation. Throughout, we work in the isospin limit $m_u = m_d$. We utilize the covariant derivative acting on the baryon field B in the fundamental representation

$$D_\mu B = \partial_\mu B + [\Gamma_\mu, B] - i \langle v_\mu^{(0)} \rangle B$$

$$\Gamma_\mu = \frac{1}{2} [u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger v_\mu^{(8)} u - \frac{i}{2} u v_\mu^{(8)} u^\dagger + \dots, \quad (10)$$

respectively, the chiral vierbein

$$u_\mu = i u^\dagger (\partial_\mu U - i v_\mu^{(8)} U + i U v_\mu^{(8)}) u^\dagger + \dots, \quad (11)$$

where the quantity $v_\mu^{(8)} [v_\mu^{(0)}]$ corresponds to an external octet [singlet] vector source and $\langle \dots \rangle$ denotes the trace in flavor space. The relevant SU(3) HBChPT Lagrangians then read (we do not show the terms which are only needed for wave-function renormalization)

$$\mathcal{L}_{MB}^{(1)} = \langle \bar{B} i v \cdot DB \rangle + D \langle \bar{B} S^\mu \{u_\mu, B\} \rangle + F \langle \bar{B} S^\mu [u_\mu, B] \rangle, \quad (12)$$

$$\mathcal{L}_{MB}^{(2)} = -\frac{i(1+b^F)}{4m} \langle \bar{B} [S^\mu, S^\nu] [f_{+\mu\nu}^{(8)}, B] \rangle - \frac{ib^D}{4m} \langle \bar{B} [S^\mu, S^\nu] \{f_{+\mu\nu}^{(8)}, B\} \rangle - \frac{i(1+b_0)}{4m} \langle \bar{B} [S^\mu, S^\nu] B \rangle 2 \langle v_{\mu\nu}^{(0)} \rangle \\ - \frac{1}{2m} \langle \bar{B} [D_\mu, [D^\mu, B]] \rangle + \frac{1}{2m} \langle \bar{B} [v \cdot D, [v \cdot D, B]] \rangle, \quad (13)$$

$$\mathcal{L}_{MB}^{(3)} = -\frac{d^{101}}{(4\pi F_\phi)^2} \langle \bar{B} [[v^\mu D^\nu, f_{+\mu\nu}^{(8)}], B] \rangle - \frac{d^{102}}{(4\pi F_\phi)^2} \langle \bar{B} \{ [v^\mu D^\nu, f_{+\mu\nu}^{(8)}], B \} \rangle - \frac{d_0^{102}}{(4\pi F_\phi)^2} \langle \bar{B} B \rangle \langle [v^\mu \partial^\nu, 2v_{\mu\nu}^{(0)}] \rangle \\ + \frac{1}{2m} \langle \bar{B} (\gamma_0 \mathcal{B}^{(2)\dagger} \gamma_0 \mathcal{B}^{(1)} + \gamma_0 \mathcal{B}^{(1)\dagger} \gamma_0 \mathcal{B}^{(2)}) B \rangle - \frac{1}{4m^2} \langle \bar{B} \gamma_0 \mathcal{B}^{(1)\dagger} \gamma_0 (i v \cdot D) \mathcal{B}^{(1)} B \rangle + \dots, \quad (14)$$

with

$$f_{+\mu\nu}^{(8)} = u^\dagger (\partial_\mu v_\nu^{(8)} - \partial_\nu v_\mu^{(8)}) u + u (\partial_\mu v_\nu^{(8)} - \partial_\nu v_\mu^{(8)}) u^\dagger \\ v_{\mu\nu}^{(0)} = \partial_\mu v_\nu^{(0)} - \partial_\nu v_\mu^{(0)}, \quad (15)$$

and the matrices $\mathcal{B}^{(1,2)}$ encode the information concerning

the $1/m$ corrections due to transitions between the light and heavy components [9]. Their explicit form for the SU(3) case can be found in Ref. [11]. Furthermore, $F_\phi = (F_\pi + F_k)/2 \approx 100$ MeV is the average pseudoscalar decay constant. We use this value because the difference between the pion and the kaon decay constants only shows up at higher order. For the conventional axial meson-baryon couplings we will use $F=0.5$, $D=0.75$.² The LEC's d^{101} , d^{102} have already been determined in [5] from the electric radii of the proton and the neutron. In contrast to Refs. [4,8] we separate the anomalous and nonanomalous contributions to the magnetic moments, utilizing the path integral formalism of [9]. To make contact with the notation used in Ref. [4], we notice that in the corresponding dimension two LEC's $b_{6b}^{D/F}$ and b_{6c} are related to the ones given above in the following way:

$$b_{6b}^F := 1 + b^F, \quad b_{6b}^D := b^D, \quad b_{6c} := 3(1 + b_0). \quad (16)$$

The first two of these are nothing but the two SU(3) parameters originally introduced by Coleman and Glashow [12] to derive relations between the magnetic moments of the octet baryons. The dimension-two LEC's are finite numbers since loop corrections only start at third order.

With these Lagrangians, we are now in the position to evaluate the strange form factors. Consider first the singlet contributions. To third order in the chiral expansion, these are given entirely in terms of tree graphs and, therefore, take the very simple forms

$$G_E^{(0)}(Q^2) = 3 \left(1 + \frac{1}{(4\pi F_\phi)^2} 2d_0^{102} Q^2 - \frac{1}{4m^2} b_0 Q^2 \right),$$

$$G_M^{(0)}(Q^2) = 3(1 + b_0) = G_M^{(0)}(0) := 3 + \kappa_N^{(0)}, \quad (17)$$

with $\kappa_N^{(0)}$ being the singlet nucleon anomalous magnetic moment. Since there are no loop contributions to this order, the LEC d_0^{102} is finite and scale independent. Furthermore, the last term in the electric form factor is the singlet Foldy term, i.e., we can rewrite the expression for $G_E^{(0)}$ as

$$G_E^{(0)}(Q^2) = 3 \left(1 + \left[\frac{2d_0^{102}}{(4\pi F_\phi)^2} - \frac{\kappa_N^{(0)}}{12m^2} \right] Q^2 \right), \quad (18)$$

where the term in the square brackets is (up to a factor of -18) the singlet electric radius squared, see Eq. (8). Such a structure is of course familiar from the expression for the neutron charge radius where the dominant contribution to the radius comes indeed from the Foldy term. The precise splitting for the strange electric radius will be discussed below. The normalization of $G_{E,M}^{(0)}$ is related to our normalization of the singlet current. It is defined as in [4] with respect to the (valence) quark number and not the baryon number as often done, see, e.g., [8]. There are no loop corrections to the singlet electric charge because the baryon number current is conserved. There are also no loop contributions to the strange electric radius since $v_\mu^{(0)}$ does not couple to the meson cloud and all graphs with couplings to the nucleon are momentum independent to third order. This will change at $\mathcal{O}(p^4)$. The singlet magnetic form factor in Eq. (17) behaves similarly to the isoscalar magnetic form factor in SU(2), i.e., to third order it is entirely given in terms of a dimension-two contact term with no momentum dependence.

We now discuss J_μ^8 . The corresponding octet components are of course implicitly contained in Ref. [5] since the electromagnetic current is an appropriate combination of triplet and octet components. Indeed, to this order the octet form factor can be calculated from the sum of the physical proton and neutron form factors and at this order happens to be equal to the isoscalar electromagnetic form factor of the nucleon,

$$G_{E/M}^{(8)}(Q^2) = \sqrt{3} [G_{E/M}^p(Q^2) + G_{E/M}^n(Q^2)]$$

$$= \sqrt{3} G_{E/M}^{I=0}(Q^2) + \mathcal{O}(p^4). \quad (19)$$

After standard renormalization to take care of the divergences as detailed in [11], the corresponding octet electric form factor can thus be written as

$$G_E^{(8)}(Q^2) = \sqrt{3} + \frac{\sqrt{3}}{(4\pi F_\phi)^2} \left\{ \left[\frac{1}{12} + \frac{85}{108} D^2 - \frac{17}{18} DF + \frac{17}{12} F^2 + \left(\frac{1}{2} + \frac{5}{6} \left(\frac{5}{3} D^2 - 2DF + 3F^2 \right) \right) \ln \left(\frac{M_K}{\mu} \right) \right] Q^2 + \left[\left(\frac{5}{3} D^2 - 2DF + 3F^2 \right) \left(2M_K^2 + \frac{5}{4} Q^2 \right) + 3 \left(M_K^2 + \frac{1}{4} Q^2 \right) \right] I_E^K(Q^2) + 2 \left(d^{101}(\mu) - \frac{1}{3} d^{102}(\mu) \right) Q^2 \right\} + \frac{\sqrt{3}}{4m^2} \left(\frac{1}{3} b^D - b^F \right) Q^2, \quad (20)$$

where

$$I_E^K(Q^2) = \frac{1}{3} \int_0^1 dx \ln \left(1 + x(1-x) \frac{Q^2}{M_K^2} \right), \quad (21)$$

²Note that the symbol D is used for the covariant derivative and for one of the axial coupling constants. From the context it is, however, always obvious which one is meant.

with $M_K = 494$ MeV the kaon mass and μ is the scale of dimensional regularization. Throughout, we set $\mu = 1$ GeV

and the scale-dependent LEC's are also given at that scale. They can be evaluated for any other scale making use of the β functions given in Ref. [11]. The corresponding octet radius can be written as

$$\begin{aligned} \langle r_{8,E}^2 \rangle = & -\frac{\sqrt{3}(b^D - 3b^F)}{2m^2} - \frac{1}{32\sqrt{3}\pi^2 F_\phi^2} \left(7(5D^2 - 6DF \right. \\ & + 9F^2) + 9 + 72d^{102}(\mu) - 24d^{101}(\mu) \\ & \left. + 2[5(D^2 - 6DF + 9F^2) + 9] \ln \frac{M_K}{\mu} \right), \end{aligned} \quad (22)$$

using

$$I_E^K(Q^2) = \frac{1}{18} \frac{Q^2}{M_K^2} + O\left(\frac{Q^4}{M_K^4}\right). \quad (23)$$

Similarly, the magnetic octet form factor takes the form

$$\begin{aligned} G_M^{(8)}(Q^2) = & \sqrt{3} \left(1 - \frac{1}{3} b^D + b^F \right) - \frac{\sqrt{3}m}{16\pi F_\phi^2} \left\{ \left(\frac{5}{3} D^2 - 2DF \right. \right. \\ & \left. \left. + 3F^2 \right) \left[M_K + \left(M_K^2 + \frac{1}{4} Q^2 \right) I_M^K(Q^2) \right] \right\}, \end{aligned} \quad (24)$$

where

$$I_M^K(Q^2) = \int_0^1 \frac{dx}{\sqrt{M_K^2 + x(1-x)Q^2}}. \quad (25)$$

To further disentangle the momentum dependence of this form factor, we bring it into the following compact form:

$$\begin{aligned} G_M^{(8)}(Q^2) = & \sqrt{3} + \kappa^{(8)} - \frac{2\sqrt{3}}{3} \frac{\pi m M_K}{(4\pi F_\phi)^2} (5D^2 - 6DF \\ & + 9F^2) f(Q^2), \end{aligned} \quad (26)$$

with the octet anomalous magnetic moment

$$\kappa^{(8)} = \sqrt{3} \left(b_F - \frac{1}{3} b_D - \frac{m M_K}{24\pi F_\phi^2} (5D^2 - 6DF + 9F^2) \right), \quad (27)$$

and the function $f(Q^2)$ given in Ref. [4]. To this order, we have $\kappa^{(8)} = \sqrt{3}(\kappa_p + \kappa_n)$ due to Eq. (19). This relation is trivially fulfilled if one fits the LEC's b_D and b_F to the neutron and proton magnetic moments using the third-order formula. In fact, the form of $G_M^{(8)}$ as given in Eqs. (24) and (26) differs by the loop contribution to the magnetic moments. This difference is, however, of higher order. In what follows, we will work with the form of the octet form factor given in Eq. (24). We remark that to this order in the chiral expansion, the momentum dependence of the magnetic octet form factor completely determines the one of the strange magnetic form factor.

Putting pieces together, the strange electric form factor of the nucleon takes the form

$$\begin{aligned} G_E^{(s)}(Q^2) = & \frac{1}{(4\pi F_\phi)^2} \left\{ -\left[\frac{1}{12} + \frac{85}{108} D^2 - \frac{17}{18} DF + \frac{17}{12} F^2 + \left(\frac{1}{2} + \frac{5}{6} \left(\frac{5}{3} D^2 - 2DF + 3F^2 \right) \right) \ln \left(\frac{M_K}{\mu} \right) \right] Q^2 \right. \\ & - \left[\left(\frac{5}{3} D^2 - 2DF + 3F^2 \right) \left(2M_K^2 + \frac{5}{4} Q^2 \right) + 3 \left(M_K^2 + \frac{1}{4} Q^2 \right) \right] I_E^K(Q^2) - 2 \left(d^{101}(\mu) - \frac{1}{3} d^{102}(\mu) - d_0^{102} \right) Q^2 \left. \right\} \\ & - \frac{1}{4m^2} \left(b_0 + \frac{1}{3} b^D - b^F \right) Q^2. \end{aligned} \quad (28)$$

The strange electric radius can readily be deduced from Eq. (28), singlet and octet radii given before, see Eqs. (18),(22), via

$$\langle r_{E,s}^2 \rangle = \frac{1}{3} \langle r_{E,0}^2 \rangle - \frac{1}{\sqrt{3}} \langle r_{E,8}^2 \rangle. \quad (29)$$

In this formula, one could express the terms $\sim b^{D,F}$ by the octet magnetic moment. This again differs from the expression one derives from Eq. (28) by terms of higher order. Given the rather sizable uncertainty of the present data, we refrain from discussing these differences here. Clearly, the last term in Eq. (28) is nothing but the (strange) Foldy term.

For completeness we also give the strange magnetic form factor found in [4]

$$\begin{aligned} G_M^{(s)}(Q^2) = & \mu_N^{(s)} + \frac{\pi m M_K}{(4\pi F_\phi)^2} \frac{2}{3} (5D^2 - 6DF + 9F^2) \\ & \times \left[\frac{4M_K^2 + Q^2}{4M_K \sqrt{Q^2}} \arctan \left(\frac{\sqrt{Q^2}}{2M_K} \right) - \frac{1}{2} \right], \end{aligned} \quad (30)$$

where we have introduced the strange magnetic moment of the nucleon

$$\mu_N^{(s)} = b_0 + \frac{1}{3}b^D - b^F + \frac{mM_K}{24\pi F_\phi^2}(5D^2 - 6DF + 9F^2). \quad (31)$$

We remark that to the order we are working, the strange form factors are identical for the proton and the neutron. This is expected since symmetry breaking only sets in at second order and thus should only show up in a complete fourth-order calculation.

To summarize this section, we have given explicit expressions for the strange (Sachs) form factors of the nucleon comprising the various contributions from tree and one-loop graphs. To third order in small momenta, there appear four octet and two singlet LEC's. This has been observed before [8]. The octet LEC's can be fixed from standard electromagnetic nucleon and hyperon properties as detailed in Ref. [5]. The two singlet LEC's play very different roles. One of them enters directly the strange electric radius (d_0^{102}), the other one (b_0) can be fixed from the strange magnetic moment of the nucleon. This is the reason why to this order the Q^2 dependence of the strange magnetic form factor could be predicted without unknown parameters in [4]. It is obvious that the two results from SAMPLE and HAPPEX are sufficient to pin down the singlet LEC's (within some ranges due to the presently large experimental uncertainties).

III. RESULTS AND DISCUSSION

We are now in the position to determine the various LEC's and consequently the strange form factors of the nucleon. To deal with the systematic, statistical, and theoretical errors given by the SAMPLE and HAPPEX Collaborations, we add these in quadrature and thus use

$$G_{\text{SAMPLE}}^{(s)}(Q_S^2) = 0.23 \pm 0.44, \quad (32)$$

$$G_{\text{HAPPEX}}^{(s)}(Q_H^2) = 0.023 \pm 0.048. \quad (33)$$

Together with the LEC's $b^{D,F}$ and $d^{101,102}$ fixed from the proton and neutron magnetic moments and charge radii, respectively [5],

$$\begin{aligned} b^D &= 3.92, & b^F &= 2.92, \\ d^{101}(1 \text{ GeV}) &= -1.06, \\ d^{102}(1 \text{ GeV}) &= 1.70, \end{aligned} \quad (34)$$

we can easily deduce the LEC's b_0 and d_0^{102} (assuming that we can use the third-order chiral expansion at the momentum of the HAPPEX experiment, see the first footnote),

$$b_0 = 0.06 \pm 0.44, \quad d_0^{102} = -2.20 \pm 0.20, \quad (35)$$

leading to the singlet magnetic moment and electric radius of

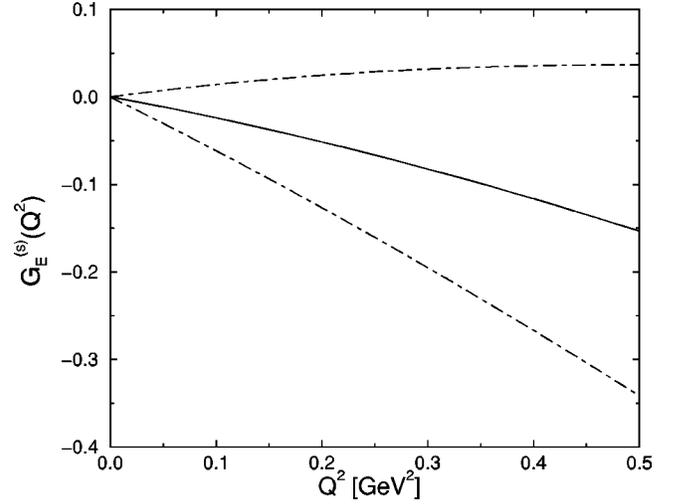


FIG. 1. Electric strangeness form factor of the nucleon. The solid band gives the prediction based on the central values of the LEC's and the dot-dashed lines reflect the possible (conservative) range due to the uncertainties.

$\kappa_N^{(0)} = 0.18$ and $\langle r_{0,E}^2 \rangle = 1.96 \text{ fm}^2$.³ We remark that the value for d_0^{102} is of natural size, i.e., of order one, and that the uncertainty reflects only the experimental errors. For b_0 , the central value appears somewhat small but it can be considered natural within its sizeable uncertainty. With these numbers, we can now evaluate the strange form factors. In what follows, we will always give a central value (cv) based on the central values of b_0 and d_0^{102} and a range, which is given by the lower and upper bounds we can get from combining the uncertainties $\pm \delta b_0$ and $\pm \delta d_0^{102}$ in all possible ways (for the electric form factor). We consider this a conservative estimate of the theoretical uncertainty within the accuracy of the calculation presented here. It does in no way reflect an estimate about the possible accuracy when one goes to higher order in the chiral expansion. Such an error is difficult to estimate since at present only very few systematic studies in three-flavor baryon ChPT exist (in the sense that all possible terms at a given order have been retained and that the counter terms can be fixed without any modeling; for a recent review, see [13]).

Consider first the strange electric form factor. It is shown in Fig. 1 for the central values of the LEC's (solid line) and the band displayed by the dot-dashed lines gives the theoretical uncertainty as explained above. We remark again that this band is presumably too wide, i.e., if one were to perform an analysis based on correlated uncertainties, this band would shrink. We remark that these uncertainties are dominated by the uncertainty in d_0^{102} , whereas the error in b_0 leads only to moderate changes. This means that the contribution from the Foldy term to the strange electric form factor

³Note that this value appears unnaturally large due to our normalization of the singlet current. For the more conventional normalization to the baryon number, it would have to be divided by a factor of 3.

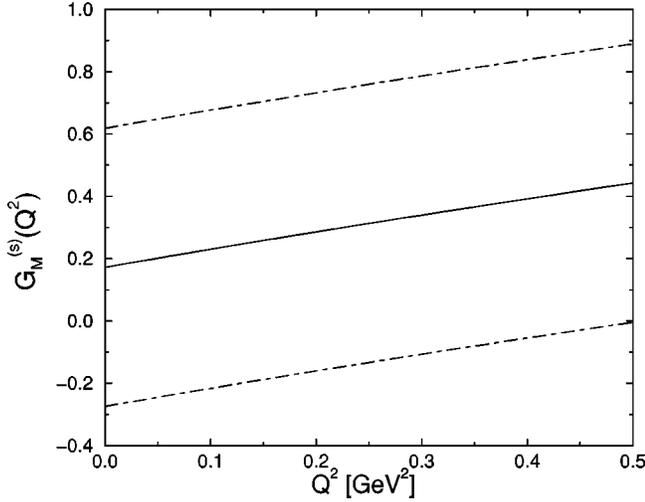


FIG. 2. The strange magnetic form factor. For notations, see Fig. 1.

is of much less importance as, e.g., in the case of the neutron charge form factor. From the form factor we readily deduce the strange electric radius as defined in Eq. (9). We find

$$\langle r_{E,s}^2 \rangle = (0.05 \pm 0.09) \text{ fm}^2, \quad (36)$$

which is a fairly small and *positive* number, and even given the sizable uncertainty, is on the lower side of predictions based on dispersive approaches including maximal OZI violation [14,15]. It is more compatible with models that include $\pi\rho$ [16] or $\bar{K}K$ [17] continuum contributions in the isoscalar spectral functions besides the vector meson poles (ω, ϕ, \dots). Furthermore, we remark that the central value for the strange electric radius agrees in size but not in sign with the quark model calculation of Ref. [18]. Note also that from the octet current the strange electric radius inherits the chiral singularity $\sim \ln(M_K)$, cf. Eq. (28). The corresponding octet radius is $\langle r_{8,E}^2 \rangle = 1.04 \text{ fm}^2$. It is also worth to point out that the momentum dependence of the strange electric form factor is rather different from the one of the neutron charge form factor, which also vanishes at zero momentum transfer.

We now turn to the strange magnetic form factor. Its momentum dependence was already discussed in Ref. [4], but having fixed the LEC b_0 within a certain range here, we now have an absolute prediction for $G_M^{(s)}(Q^2)$. This is shown in Fig. 2. The rather wide band shown in Fig. 2 reflects the sizeable uncertainty of the SAMPLE result. The central value of the so-determined strange magnetic moment is, however, positive [4]

$$\mu_N^{(s,\text{cv})} = 0.18, \quad (37)$$

and is thus at odds with most model calculations (see, e.g., Table I in [19]). If one uses the relation of Ref. [4] that relates the momentum dependence of the strange magnetic form factor to the one of the isoscalar magnetic nucleon form factor, the deduced strange magnetic moment would still be positive but very close to zero. This shows that there is still some room for improving the theoretical description of the

TABLE I. Predictions for the combination of strange form factors to be measured at MAMI by the A4 Collaboration. The central values of the singlet LEC's are denoted by the asterisk.

d_0^{102}	b_0	$G_{\text{MAMI}}^{(s)}(Q_M^2)$
-2.20*	0.06*	0.007
-2.00	0.50	0.134
-2.00	-0.38	-0.002
-2.40	0.50	0.017
-2.40	-0.38	-0.119

magnetic form factor. The magnetic radius is uniquely fixed in terms of well-known low-energy parameters [4],

$$\begin{aligned} \langle r_{M,s}^2 \rangle &= -\frac{\pi m}{(4\pi F_\phi)^2 M_K} \frac{1}{3} (5D^2 - 6DF + 9F^2) \\ &= -0.14 \text{ fm}^2. \end{aligned} \quad (38)$$

The slope is identical for a proton or a neutron target, it is *negative* and to this order independent of the strange magnetic moment $\mu_N^{(s)}$. The radius has the very reasonable behavior that in the limit of very heavy kaons $M_K \rightarrow \infty$ it goes to zero, whereas it explodes in the chiral limit $M_K \rightarrow 0$.

We now turn to the MAMI experiment, which attempts to measure $G_{\text{MAMI}}^{(s)}(Q_M^2) = G_E^{(s)}(Q_M^2) + 0.22G_M^{(s)}(Q_M^2)$ at a four-momentum transfer (squared) of $Q_M^2 = 0.23 \text{ GeV}^2$. This value of Q^2 is much better suited for the chiral expansion. We find, however, that at this value of the momentum transfer, there are sizeable cancellations between the electric and the magnetic contributions. The prediction for the various combinations of the singlet LEC's are given in Table I. The corresponding results for a small Q^2 interval ($0.20 \leq Q^2 \leq 0.24 \text{ GeV}^2$) are shown in Fig. 3. Here, the uncertainty band is given by almost equal shares from the uncertainty in b_0 and the one in d_0^{102} .

IV. SUMMARY AND CONCLUSIONS

We have calculated the form factors of the strange vector current of the nucleon in the framework of chiral perturbation theory, updating the analysis of Ref. [8]. To third order in the chiral expansion, there appear six low-energy constants. Four of these can be trivially deduced from the neutron and proton charge radii and magnetic moments. The remaining two singlet couplings can be determined from the recent SAMPLE and HAPPEX measurements of combinations of the strange form factors. The crucial *assumption* here is that we can apply the chiral expansion at a momentum transfer as large as the one in the HAPPEX experiment, i.e., at $Q^2 = 0.48 \text{ GeV}^2$. With this cautionary remark in mind, the pertinent results of our study can be summarized as follows:

The singlet LEC's given in Eq. (35) are of natural size. The error given reflects the sizeable uncertainty of the experimental values obtained by SAMPLE and HAPPEX. To obtain theoretical uncertainties of the LEC's, we have added the various experimental errors in quadrature.

For the central values of the LEC's, the strange electric

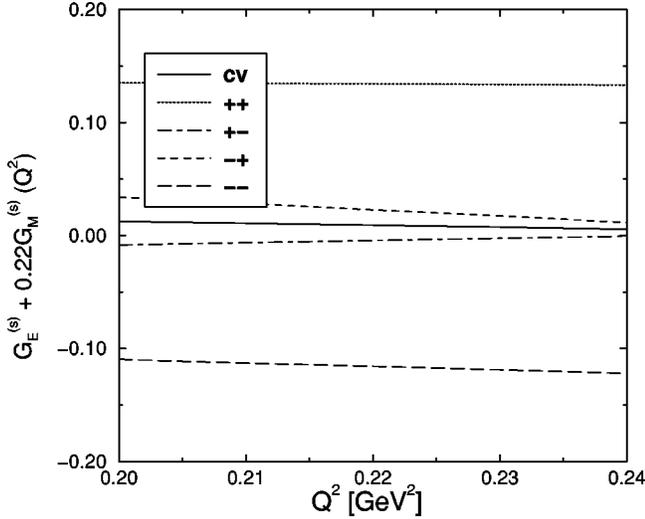


FIG. 3. $G_E^{(s)}(Q^2) + 0.22G_M^{(s)}(Q^2)$ for momentum transfer squared between 0.20 and 0.24 GeV^2 . The solid line refers to the central values of the LEC's b_0 , d_0^{102} and the other lines (dotted, dashed, . . .) to the various combinations of the LEC's within their uncertainties as given in the inset (here, the first \pm refers to $\pm \delta d_0^{102}$ and the second to $\pm \delta b_0$, see, also, Table I).

form factor of the nucleon is negative, cf. Fig. 1. The band given in the figure reflects the worst case scenario of combining the uncertainties in the singlet LEC's (i.e., an analysis based on correlated errors would give a smaller uncertainty). To this order in the chiral expansion, the proton and neutron strange electric form factor are equal with small and *positive*

radius, $\langle r_{E,s}^2 \rangle = (0.05 \pm 0.09) \text{ fm}^2$, due to cancellations between the octet and the singlet contributions.

The strange magnetic form factor was already discussed in detail in Ref. [4]. In Fig. 2 we show the absolute prediction based on input from the SAMPLE result. The corresponding central value of the strange magnetic moment is $\mu_N^{(s)} = 0.18$ with an uncertainty as given in Eq. (32). The corresponding strange magnetic radius is given entirely in terms of well-known parameters, $\langle r_{M,s}^2 \rangle = -0.14 \text{ fm}^2$. Here, the singlet and octet terms add.

The predictions for the MAMI A4 experiment, which intends to measure $G_E^{(s)} + 0.22G_M^{(s)}$ at $Q^2 = 0.23 \text{ GeV}^2$, are collected in Table I. For the central values of the LEC's, the resulting number is fairly small due to cancellations between the electric and magnetic contributions. Due to these cancellations, varying the LEC's within their uncertainties does not allow for a precise prediction.

We have shown that heavy baryon chiral perturbation theory can indeed be used to analyze the strange form factors of the nucleon. Our study should be considered exploratory due to the fairly large momentum transfer involved in the HAPPEX experiment. However, with the on-going activities at BATES, Jefferson Lab, and MAMI we should soon have an improved data base which will allow us to make better use of the chiral symmetry constraints for the strangeness vector current matrix elements in the nucleon. Higher-order calculations (possibly involving the decuplet) are also needed [20].

ACKNOWLEDGMENT

We thank Nathan Isgur for a useful conversation.

-
- [1] SAMPLE Collaboration, B. Mueller *et al.*, Phys. Rev. Lett. **78**, 3824 (1997).
 - [2] HAPPEX Collaboration, K.A. Aniol *et al.*, Phys. Rev. Lett. **82**, 1096 (1999).
 - [3] M.J. Musolf *et al.*, Phys. Rep. **239**, 1 (1994).
 - [4] T.R. Hemmert, Ulf-G. Meißner, and S. Steininger, Phys. Lett. B **437**, 184 (1998).
 - [5] B. Kubis, T.R. Hemmert, and Ulf-G. Meißner, Phys. Lett. B **456**, 240 (1999).
 - [6] MAMI A4 Collaboration, D. von Harrach *et al.* (unpublished).
 - [7] T.R. Hemmert, Few Body Syst., Suppl. **11**, 78 (1999).
 - [8] M.J. Ramsey-Musolf and H. Ito, Phys. Rev. C **55**, 3066 (1997).
 - [9] V. Bernard, J. Kambor, N. Kaiser, and Ulf-G. Meißner, Nucl. Phys. **B388**, 315 (1992).
 - [10] V. Bernard, H.W. Fearing, T.R. Hemmert, and Ulf-G. Meißner, Nucl. Phys. **A635**, 121 (1998); **A642**, 563(E) (1998).
 - [11] G. Müller and Ulf-G. Meißner, Nucl. Phys. **B492**, 379 (1997).
 - [12] S. Coleman and S.L. Glashow, Phys. Rev. Lett. **6**, 423 (1961).
 - [13] Ulf-G. Meißner, in *Chiral Dynamics: Theory and Experiment*, edited by A.M. Bernstein, D. Drechsel, and Th. Walcher (Springer-Verlag, Heidelberg, 1998).
 - [14] R.L. Jaffe, Phys. Lett. B **229**, 275 (1989).
 - [15] H.-W. Hammer, Ulf-G. Meißner, and D. Drechsel, Phys. Lett. B **367**, 323 (1996).
 - [16] Ulf-G. Meißner, V. Mull, J. Speth, and J.W. Van Orden, Phys. Lett. B **408**, 381 (1997).
 - [17] M.J. Ramsey-Musolf and H.-W. Hammer, Phys. Rev. Lett. **80**, 2539 (1998); H.-W. Hammer and M.J. Ramsey-Musolf, Phys. Lett. B **416**, 5 (1998); hep-ph/9903367.
 - [18] P. Geiger and N. Isgur, Phys. Rev. D **55**, 299 (1997).
 - [19] SAMPLE Collaboration, E.J. Beise *et al.*, nucl-ex/9602001.
 - [20] S. Puglia and M.J. Ramsey-Musolf (unpublished).