$\Delta\Delta$ dibaryon structure in chiral SU(3) quark model

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The structure of the $\Delta\Delta$ dibaryon is studied in terms of the chiral SU(3) quark model. The ground state energy of $\Delta\Delta$ dibaryon is evaluated by considering both the $\Delta\Delta$ and CC (hidden color) channels. The parameter-dependence of the $\Delta\Delta$ dibaryon mass is also investigated. It is shown that the resultant mass of the $\Delta\Delta$ dibaryon is lower than the threshold of the $\Delta\Delta$ channel but still higher than that of the $\Delta N\pi$ channel. [S0556-2813(99)00410-0]

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I. INTRODUCTION

Since the H dibaryon was theoretically predicted by Jaffe in 1977 [1], searching dibaryons both theoretically and experimentally has attracted worldwide attention. Because the dibaryon is a six-quark system confined in a small volume where the one-gluon exchange and the quark exchange play significant roles, it is a good place to see the short-distance behavior of quantum chromodynamics (QCD).

Up to now, various predictions of the H dibaryon have been announced by employing a variety of models [2]. A recent calculation of the H dibaryon by using a chiral SU(3) quark model [3] showed that the overall contributions of chiral SU(3) clouds only offer weak attraction across two baryons, and the mass of the H dibaryon is near the threshold of the $\Lambda\Lambda$ channel. That issue is consistent with most reports given by recent experiments [4].

On the other hand, the chiral SU(3) quark model is quite successful not only in explaining the structures of baryon ground states but also in reproducing nucleon-nucleon (N-N) scattering phase shifts in various partial wave states and hyperon-nucleon (Y-N) cross sections [5] in allowed channels. Thus, in our consideration, analyzing other possible six-quark states by using such models should be quite meaningful. A brief analysis shows that the $\Delta\Delta$ dibaryon (deltaron; $S=3$, $J^{\pi}=3^{+}, T=0$, with S, J, and T being the spin, total angular momentum and isospin, respectively) is an interesting candidate of a dibaryon, because the color-magnetic part of the one-gluon-exchange interaction (CMI) between two Δ clusters is not repulsive. In 1987, Yazaki [6] analyzed systems with two nonstrange baryons in the framework of the cluster model where the one-gluon-exchange interaction and the confining potential between two quarks were considered. The result showed that among the NN, $N\Delta$, and $\Delta\Delta$ systems, the $\Delta\Delta$ system (deltaron with $S=3, T=0$) is the only system in which CMI between two clusters is attractive. Since the Δ is a resonance with a quite wide width and easily decays into $N\pi$, the deltaron might have a large width so that it cannot easily be detected in the experiment, even though it is a bound state of $\Delta\Delta$, except that its mass is below the threshold of the $NN\pi\pi$ channel. Wang *et al.* [7] studied the structure of the deltaron in terms of the quark delocalization model. They found that the deltaron is a deeply bound state with a binding energy of 320–390 MeV, namely, its energy level is below the threshold of the $NN\pi\pi$ channel. In our opinion, the major reason for causing such deeply bound behavior is due to the inconsistency of the employed interaction in their calculation, namely, the form of the confining potential utilized for a pair of quarks located in the same "baryon orbit" is different than the one for the quarks located in two different "baryon orbits." According to the basic principles of quantum mechanics, when and only when the interacting particles in the quantum-mechanical system belong to the mutually orthogonal subspaces, different interactions between particles are allowed. However, in the generating coordinate method (GCM), the left-centered and right-centered orbital wave functions are usually nonorthogonal, and strictly speaking coercively taking two different forms of interaction for different quark pairs is not allowed. Taking two different forms of interaction for different quark pairs belonging to nonorthogonal subspaces to study the bound-state problem, namely, that two interacting clusters are fairly close to each other, would cause substantial deviations. Even a serious delocalization scheme would enlarge these deviations and make the results unreliable.

In this paper, the ground state energy of deltaron is examined in the framework of the resonating group method (RGM) by solving a coupled-channel equation, where the $\Delta\Delta$ and CC (hidden color) channels are included, and the chiral field contributions are considered.

In Sec. II, a brief formalism is demonstrated. The result is presented and discussed in Sec. III. In Sec. IV, the conclusion is drawn.

TABLE I. Coefficients between the physical basis states and the symmetry basis states.

	$[6][33]_{30}$	$[42][33]_{30}$
$(\Delta\Delta)_{30}$	$\sqrt{\frac{1}{5}}$	$\sqrt{\frac{4}{5}}$
$(CC)_{30}$	$\sqrt{\frac{4}{5}}$	$-\sqrt{\frac{1}{5}}$

TABLE II. Binding energy B and rms $\overline{R}$ of the deltaron $B=-(E_{\text{deltaron}}-2M_{\Delta})$, $\overline{R}=\sqrt{\langle r^2\rangle}$.

		$\Delta\Delta(L=0)$	$\Delta\Delta\begin{pmatrix}L=0\\+2\end{pmatrix}$	$\Delta\Delta$ CC $(L=0)$	$\Delta\Delta$ CC $\begin{pmatrix}L=0\\+2\end{pmatrix}$
OGE	B (MeV)	29.8	29.9	41.0	42.0
	$\overline{R}$ (fm)	0.92	0.92	0.87	0.87
OGE+π,σ	B (MeV)	50.2	62.6	68.6	79.7
	$\overline{R}$ (fm)	0.87	0.86	0.84	0.83
OGE+SU(3)	B (MeV)	18.4	22.5	31.7	37.3
	$\overline{R}$ (fm)	1.01	1.00	0.92	0.92

II. BRIEF FORMALISM

As is well known, in the traditional quark potential model, V_{qq} consists of two parts: the one-gluon-exchange potential (OGE) governing the short-range interaction and the confining force dominating mainly the long range interaction. In spite of the unclear source of the constituent quark mass, this semiphenomenological model achieved great success in explaining the properties of heavy quarkonia. But this model met problems in light quark systems, especially in understanding the N-N interaction. The major perplexity is the lack of media, which can alternatively be treated by introducing the constraint of chiral symmetry which is very important in the strong interaction [5]. In the chiral SU(3) quark model, starting from the linear expression of the chiral-quark coupling Lagrangian, the chiral-quark-interaction Hamiltonian can be written as

$$H_I^{\text{ch}}=g_{\text{ch}}F(q^2)\overline{\Psi}\left(\sum_{a=0}^{8}\sigma_a\lambda_a+i\sum_{a=0}^{8}\pi_a\lambda_a\gamma_5\right)\Psi, \tag{1}$$

where σ_a denotes four scalar meson fields σ, σ', κ, and ϵ, respectively, π_a represents four pseudoscalar meson fields η_1, π, K, and η_8, respectively, and $F(q^2)$ is a form factor [5]. Then, the chiral-quark-coupling-induced interaction between quarks can be derived as

$$V_{ij}^{ch}=V_{ij}^{PS}+V_{ij}^{S}, \tag{2}$$

where

$$V_{ij}^{PS}=C(g_{ch},m_{\pi_a},\Lambda)\frac{m_{\pi_a}^2}{12m_im_j}[f_1(m_{\pi_a},\Lambda,r_{ij})(\vec{\sigma}_i\cdot\vec{\sigma}_j)+f_2(m_{\pi_a},\Lambda,r_{ij})S_{ij}](\lambda_i^a\lambda_j^a)_f \tag{3}$$

and

$$V_{ij}^{S}=-C(g_{ch},m_{\sigma_a},\Lambda)f_3(m_{\sigma_a},\Lambda,r_{ij})(\lambda_i^a\lambda_j^a)_f, \tag{4}$$

with V_{ij}^{PS} and V_{ij}^{S} being the pseudoscalar- and scalar-field-induced interactions, respectively. The expressions of f_i and C can be found in Ref. [5]. In expressions (3) and (4), the coupling constant g_{ch} is a unique constant and can be fixed by the following relation:

$$\frac{g_{ch}^2}{4\pi}=\frac{g_{NN\pi}^2}{4\pi}\frac{9}{25}\frac{m_q^2}{M_N^2}. \tag{5}$$

The Hamiltonian of the system in the chiral SU(3) quark model reads

$$H=\sum_i T_i-T_G+\sum_{i<j}V_{ij}, \tag{6}$$

where V_{ij} includes one-gluon-exchange interaction, confinement potential and chiral-quark-field-coupling-induced interactions

$$V_{ij}=V_{ij}^{\text{OGE}}+V_{ij}^{\text{conf}}+V_{ij}^{\text{ch}}. \tag{7}$$

In Eq. (7), V_{ij}^{OGE} is taken in the usual form and V_{ij}^{conf} is chosen in the quadratic form [5]

$$V_{ij}^{\text{conf}}=-(\lambda_i^a\lambda_j^a)_c(a_{ij}r_{ij}^2+a_{ij0}). \tag{8}$$

The coupling constant of OGE and the strength of confinement potential are determined by the stability condition of nucleon and the mass difference between Δ and N [3,5].

According to the analysis in Ref. [8], it would be effective if one chooses the two-cluster configurations as the dibaryon's model space. Referring to the result in Ref. [9], the basis functions of the two-cluster configuration (namely, the physical basis function) and the six-quark-cluster configuration (namely, the symmetry basis function) in the case of the $S=3$ and $T=0$ state have a certain relation. We show it in Table I. In the table, the CC channel has the form

TABLE III. Deltaron binding energy B(MeV) with different parameters. $B=-(E_{\text{deltaron}}-2M_{\Delta})$.

	$(\Delta\Delta+CC)$	$(L=0+L=2)$		
$B(\text{OGE}+\pi,\sigma)$	79.7	97.1	97.9	113.4
$B[\text{OGE+SU(3)}]$	37.3	64.2	52.4	79.2
b_N(fm)	0.505	0.60	0.505	0.60
m_σ(MeV)	625	625	550	550

TABLE IV. Binding energy B of deltaron with error-function-like confinement. ($b_N=0.505$ fm, $m_\sigma=625$ MeV.) $B=-(E_{\text{deltaron}}-2M_\Delta)$.

		$\Delta\Delta(L=0)$	$\Delta\Delta\begin{pmatrix}L=0\\+2\end{pmatrix}$	$\begin{matrix}\Delta\Delta\\CC\end{matrix}$ $(L=0)$	$\begin{matrix}\Delta\Delta\\CC\end{matrix}\begin{pmatrix}L=0\\+2\end{pmatrix}$
OGE	B (MeV)	20.9	21.0	38.1	39.1
OGE+π,σ	B (MeV)	44.4	56.8	71.4	81.8
OGE+SU(3)	B (MeV)	13.3	17.5	32.5	38.1

$$|CC\rangle_{S=3,T=0}=-\frac{1}{2}|\Delta\Delta\rangle_{S=3,T=0}+\frac{\sqrt{5}}{2}A_{\text{STC}}|\Delta\Delta\rangle_{S=3,T=0}, \tag{9}$$

where A_{STC} stands for the antisymmetrizer in the spin-isospin-color space.

In this investigation, the mixture of the $L=0$ and $L=2$ states which shows the effects of the tensor forces in OGE and chiral field induced potentials are also considered, namely, the two-channels-four-state, $\Delta\Delta(L=0)$, $\Delta\Delta(L=2)$, $CC(L=0)$, and $CC(L=2)$, calculation is performed. The corresponding matrix elements of spin-isospin-color operators are given in the Appendix.

III. RESULT AND DISCUSSION

In the coupled-channel bound-state calculation, one must carefully eliminate forbidden states, which may spoil the numerical calculation. In the deltaron case, there exists a state, which has the zero eigenvalue of the normalization operator $\langle N\rangle=0$ due to the Pauli blocking effect. It reads

$$|\Psi\rangle_{\text{forbidden}}=|\Delta\Delta\rangle-\frac{1}{2}|CC\rangle. \tag{10}$$

Performing an off-shell transformation, this nonphysical degree of freedom can be eliminated and the reliable result can be achieved.

The model parameters in Ref. [5], with which the empirical N-N scattering phase shifts and Y-N cross sections can be well reproduced, is employed. The resultant deltaron binding energy and the corresponding root-mean-square radius (rms) are presented in Table II. For comparison, the following cases are also considered: OGE only and OGE plus π and σ fields [the chiral SU(2) quark model]. The calculations are carried out in four combinations: $\Delta\Delta(L=0)$, $\Delta\Delta(L=0$ and $2)$, $\Delta\Delta+CC(L=0)$, and $\Delta\Delta+CC(L=0$ and $2)$. The results are all tabulated in Table II. It is shown that the binding energy of the deltaron is indeed lower than the threshold of the $\Delta\Delta$ channel. It is several tens MeV in all the cases. Since the deltaron mass is still higher than the mass of $N\Delta\pi$, the deltaron would not be a narrow width dibaryon.

The channel coupling effect is much larger than the L state mixing effect which is caused by the tensor interaction. The largest binding energy of deltaron appears in the OGE +SU(2) case. It means that the π and σ chiral fields offer substantial attractions across two Δ clusters, so that the deltaron becomes bounder. The overall effects of SU(3) chiral fields would reduce the deltaron binding energy from the one in the SU(2) case.

The parameter-dependence of the deltaron binding energy is also investigated. The effect of the σ mass is examined first. In general, the mass of σ can be estimated by the following relation [10]:

TABLE V. Coefficients of spin-flavor-color operators.

$\hat{O}_{ij}$		$\begin{matrix}\Delta\Delta\\\Delta\Delta\end{matrix}$	$\begin{matrix}\Delta\Delta\\CC\end{matrix}$	$\begin{matrix}CC\\CC\end{matrix}$	$\hat{O}_{ij}$		$\begin{matrix}\Delta\Delta\\\Delta\Delta\end{matrix}$	$\begin{matrix}\Delta\Delta\\CC\end{matrix}$	$\begin{matrix}CC\\CC\end{matrix}$
	1	27	0	27		$\hat{O}_{ij}$	9	0	9
	P_{36}	-3	-12	-21	$\vec{\sigma}_i\cdot\vec{\sigma}_j$	$\hat{O}_{ij}P_{36}$	-1	-4	-7
	$\hat{O}_{12}$	-72	0	-18		$\hat{O}_{12}$	9	0	-9
	$\hat{O}_{36}$	0	0	-36		$\hat{O}_{36}$	-15	0	-3
$\lambda_i^c\cdot\lambda_j^c$	$\hat{O}_{12}P_{36}$	8	32	2	$\Sigma_{k=1}^3\lambda_i^F(k)\lambda_j^F(k)$	$\hat{O}_{12}P_{36}$	-1	-4	11
and	$\hat{O}_{13}P_{36}$	8	32	20	and	$\hat{O}_{13}P_{36}$	-1	-4	5
$(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\lambda_i^c\cdot\lambda_j^c)$	$\hat{O}_{16}P_{36}$	8	-4	20	$(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\Sigma_{k=1}^3\lambda_i^F(k)\lambda_j^F(k))$	$\hat{O}_{16}P_{36}$	-1	8	5
	$\hat{O}_{14}P_{36}$	-4	2	35		$\hat{O}_{14}P_{36}$	3	6	0
	$\hat{O}_{36}P_{36}$	-16	8	32		$\hat{O}_{36}P_{36}$	7	4	1
factor		$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	factor		$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

TABLE VI. Coefficients of spin-flavor-color operators (tensor part).

	$\hat{O}_{ij}$	$\Delta\Delta$ $\Delta\Delta$	$\Delta\Delta$ CC	CC CC		$\hat{O}_{ij}$	$\Delta\Delta$ $\Delta\Delta$	$\Delta\Delta$ CC	CC CC
$(\vec{\sigma}_i\vec{\sigma}_j)_2$	$\hat{O}_{ij}$	54	0	54					
	$\hat{O}_{ij}P_{36}$	-6	-24	-42					
	$\hat{O}_{12}$	-144	0	-36		$\hat{O}_{12}$	18	0	-18
	$\hat{O}_{36}$	0	0	-72		$\hat{O}_{36}$	-30	0	-6
	$\hat{O}_{12}P_{36}$	16	64	4		$\hat{O}_{12}P_{36}$	-2	-8	22
$(\vec{\sigma}_i\vec{\sigma}_j)_2(\lambda_i^c\cdot\lambda_j^c)$	$\hat{O}_{13}P_{36}$	16	64	40	$(\vec{\sigma}_i\vec{\sigma}_j)_2(\Sigma_{k=1}^3\lambda_i^F(k)\lambda_j^F(k))$	$\hat{O}_{13}P_{36}$	-2	-8	10
	$\hat{O}_{16}P_{36}$	16	-8	40		$\hat{O}_{16}P_{36}$	-2	16	10
	$\hat{O}_{14}P_{36}$	-8	4	70		$\hat{O}_{14}P_{36}$	6	12	0
	$\hat{O}_{36}P_{36}$	-32	16	64		$\hat{O}_{36}P_{36}$	14	8	2
factor		$\frac{1}{27}\sqrt{\frac{14}{5}}$	$\frac{1}{27}\sqrt{\frac{14}{5}}$	$\frac{1}{27}\sqrt{\frac{14}{5}}$	factor		$\frac{1}{9}\sqrt{\frac{14}{5}}$	$\frac{1}{9}\sqrt{\frac{14}{5}}$	$\frac{1}{9}\sqrt{\frac{14}{5}}$

$$m_\sigma^2=(2m_q)^2+m_\pi^2. \tag{11}$$

Thus, $m_\sigma=600-700$ MeV is reasonable. In Ref. [5], the mass of σ was taken to be 625 MeV. For comparison, the results with $m_\sigma=550$ MeV which is a value in limit are also tabulated in Table III. It is shown that the result has visible change when m_σ varies in a reasonable region. Then, the influence from the baryon size parameter b_N which greatly affects the confinement strength is studied. In the N-N scattering calculation, b_N was chosen to be 0.505 fm and consequently the confinement strength $a_c=54.34$ MeV/fm^2 [5]. Now, another set of parameters, is employed. The resultant binding energies are also tabulated in Table III. It is seen that there is visible but not qualitative b_N dependence of the deltaron binding energy.

It is well known that in the two-color-singlet-cluster system, the form and the strength of the confining potential do not affect the resultant quantities much. Now, to study the deltaron structure, one has to include the hidden-color channel CC to enlarge the model space. However, once the CC channel is considered, the color Van der Waals force appears. To eliminate this unreasonable force, one used an error-function-like confining potential to take the color screening effect, namely, the nonperturbative QCD effect, into account [11]. Therefore, it is necessary to examine the stability of the resultant binding energy with respect to the form of the confining potential in the presence of the hidden-color state. For this purpose, we also adopt an error-function-like confining potential

$$V_{ij}^{\text{erf-conf}}=-(\lambda_i\cdot\lambda_j)_c\left[a_{ij0}+a_{ij}erf\left(\frac{r}{l_{cs}}\right)\right], \tag{12}$$

where l_{cs} denotes the color screening length and is taken to be 2.0 fm in the deltaron structure calculation. The results are shown in Table IV. From the table, one sees that the resultant binding energy and rms of deltaron are quite similar to those in the quadratic confinement case, namely, the bound state property would not change much when the color screening effect is counted.

IV. CONCLUSION

The binding energy and corresponding root mean square radius of the deltaron are examined in the framework of chiral quark model. The result shows that the binding energy of the deltaron is stably ranged around several tens MeV even the mass of σ and the value of the baryon size parameter b_N are varied in reasonable regions, and the form of the confinement is changed to include the color screening effect, except in the case of SU(2) chiral quark model with $b_N=0.6$ fm and $m_\sigma=550$ MeV, where the binding energy of the deltaron is about 113 MeV. This means that the mass of the deltaron is always smaller than that of $\Delta\Delta$, but larger than that of $\Delta N\pi$. As a conclusion, we report that although the model space is enlarged, the color screening effect is considered and the contributions from various chiral fields are included, the binding energy of deltaron is always retained around several tens MeV. Thus, we dispute the existence of a deeply bound deltaron dibaryon state with the narrow width.

APPENDIX

Table V shows the coefficients for operators involving spin, flavor, and color.

Table VI shows the relevant coefficients for the tensor part. Here, the following identities hold:

$$\langle\lambda_i^F(8)\lambda_j^F(8)\rangle=\frac{1}{3}\langle 1\rangle,$$

$$\langle(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\lambda_i^F(8)\lambda_j^F(8))\rangle=\frac{1}{3}\langle\vec{\sigma}_i\cdot\vec{\sigma}_j\rangle,$$

$$\langle(\vec{\sigma}_i\vec{\sigma}_j)_2(\lambda_i^F(8)\lambda_j^F(8))\rangle=\frac{1}{3}\langle(\vec{\sigma}_i\vec{\sigma}_j)_2\rangle.$$

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