Nuclear currents based on the integral form of the continuity equation

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We present an approach to obtain new forms of the nuclear electromagnetic current, which is based on an integral form of the continuity equation. The procedure can be used to restore current conservation in model calculations in which the continuity equation is not verified. In addition, it provides, as a particular result, the so-called Siegert's form of the nuclear current, first obtained by Friar and Fallieros by extending Siegert's theorem to arbitrary values of the momentum transfer. The new currents are explicitly conserved and permit a straightforward analysis of their behavior at both low and high momentum transfers. The results are illustrated with a simple nuclear model which includes a harmonic oscillator mean potential. [S0556-2813(99)02810-1]

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I. INTRODUCTION

One of the oldest and still unsolved problems in nuclear physics is that of the determination of the electromagnetic current operator. The basic difficulty lies in the absence of useful constraints one can impose to this operator. For a long time, the Siegert theorem [1] has played an important role in this respect, mainly because its application allows us to avoid the consideration of two-body currents in the calculations. After the application of the theorem, the effects of such currents can be evaluated in terms of the charge contributions, the corrections of which are *a priori* small. However, the Siegert theorem only applies in the long-wavelength limit and then it is irrelevant for the values of the momentum transfers usually observed in electron scattering experiments.

In principle, one can think that current conservation, and more precisely the continuity equation (CE), could provide the needed procedure. Nevertheless, the task cannot be accomplished due to the impossibility to fix in a unique manner the operator, because those terms given as the rotational of any function are not constrained by the CE.

Using current conservation, the longitudinal current is eliminated in terms of the charge operator, and the electron scattering cross section reads

$$\frac{d\sigma}{d\Omega}=\frac{4\pi\sigma_M}{f_{\rm rec}(2J_i+1)}\left[v_L\sum_\lambda |t_{C\lambda}|^2+v_T\sum_\lambda (|t_{E\lambda}|^2+|t_{M\lambda}|^2)\right], \tag{1}$$

where $t_{C\lambda}$, $t_{E\lambda}$, and $t_{M\lambda}$ are the Coulomb, electric, and magnetic multipoles of the transition. It is common to rewrite the matrix elements of the electric transverse operators in terms of the charge matrix elements, by using the equations

$$\frac{\omega}{q}t_{C\lambda}=-\sqrt{\frac{\lambda}{2\lambda+1}}t_{\lambda-}-\sqrt{\frac{\lambda+1}{2\lambda+1}}t_{\lambda+}, \tag{2}$$

$$t_{E\lambda}=\sqrt{\frac{\lambda+1}{2\lambda+1}}t_{\lambda-}-\sqrt{\frac{\lambda}{2\lambda+1}}t_{\lambda+}. \tag{3}$$

Here we have defined the transition matrix elements

$$t_{\lambda\pm}=\langle J_f\|iT_{\lambda,\lambda\pm1}\|J_i\rangle \tag{4}$$

of the multipole operators

$$T_{JLM}(q)=\int d^3r\, j_L(qr)\mathbf{Y}^M_{JL}(\hat{\mathbf{r}})\cdot\mathbf{J}(\mathbf{r}), \tag{5}$$

where $\mathbf{J}(\mathbf{r})$ is the electromagnetic current operator of the nucleus and $\mathbf{Y}^M_{JL}(\hat{\mathbf{r}})$ labels a vector spherical harmonic.

Equation (2) is the multipole expression of the CE. It relates the charge multipoles to the longitudinal multipoles of the current. Equation (3) is the definition of the electric multipoles. Using Eq. (2) it is possible to eliminate one of the multipoles $t_{\lambda\pm}$ in terms of the Coulomb $t_{C\lambda}$ multipole and substitute it in the expression of $t_{E\lambda}$. In this way one can use the CE constraint even in the calculation of the transverse form factor. In addition, this procedure allows us to minimize the errors coming from an insufficient knowledge of the transverse current (e.g., meson-exchange currents) in electron scattering calculations.

This method of rewriting the matrix elements of the electric transverse operator in terms of the charge matrix elements, has been applied traditionally in many electron scattering calculations. Despite the fact that this ensures that the CE is satisfied, the procedure is completely meaningless in models which do not satisfy the CE [2]. As a consequence, the problem of current conservation in model calculations of electron scattering by nuclei is still under theoretical study [3–5].

A similar situation appears in the relativistic treatment of the off-shell γNN vertex in $(e,e'p)$ reactions. Several prescriptions for the current operators obtained by extrapolating the on-shell currents have been proposed [6]. The corresponding off-shell vertex operators violate current conservation, which is enforced by eliminating the three or zero components using the continuity equation. The problem which arises is that different results are obtained depending on the option chosen [7].

An alternative approach is provided by the so-called Siegert current (SC) developed by Friar and Fallieros [8,9] by extending the Siegert theorem to arbitrary wavelengths. This procedure is based on isolating the components of the current which are constrained by the CE and replacing them by an adequate combination of the charge multipoles. However, the results are not satisfactory [10] because they show a

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pathological behavior for high momentum transfer when the nuclear model considered does not verify current conservation. This has been also pointed out recently by Caparelly and de Passos [3] in RPA and TDA calculations.

Two main conclusions of the work of Friar and Fallieros [9] must be pointed out. First, the SC appears to be preferable to the traditional forms of the nuclear currents even for models which satisfy the CE, because the size of the exchange current operators is considerably reduced. Second, they suggest that well-behaved current operators, satisfying the constraints of current conservation for high momentum transfer, could be determined, but these new forms of the current have not been found up to now.

In this paper we present a new approach to obtain nuclear conserved currents. Specifically we show that the SC appears as a particular case of a more general family of current operators which can be obtained from the CE. Though all of them produce identical results in models conserving the current, they permit us to formulate different prescriptions for restoring current conservation in electron scattering model calculations. In addition, our formalism allows us to obtain explicitly the asymptotic behavior of these new currents.

Our method deals with the CE written in integral form (Sec. II) from which the new currents can be obtained directly both in coordinate and momentum spaces. Our approach generalizes the Friar and Fallieros procedure and, as a consequence, permits us to find, *a priori*, infinite ways of restoring the CE in models which do not conserve the current. In Sec. III we show how to do it and we study in detail the low and high momentum transfer behavior of the conserved currents found as well as that of their multipoles. To finish, in Sec. IV we show some examples in a simple nuclear model. The conclusions are drawn in Sec. V.

II. INTEGRAL CONTINUITY EQUATION

A. Coordinate space

Usually, the CE is written in differential form as

$$\nabla\cdot\mathbf{J}(\mathbf{r}) = -i[H,\rho(\mathbf{r})], \tag{6}$$

where H is the Hamiltonian and ρ and $\mathbf{J}$ are the nuclear charge and current densities, respectively. This equation restricts the value of $\nabla\cdot\mathbf{J}$, once the density ρ and the Hamiltonian H of the system are known, but, as mentioned above, it does not permit to fix completely the current because the rotational terms do not contribute to the divergence.

The key point in our development consists in writing an integral equation equivalent to the CE. To do that, we start with the relation

$$\frac{1}{r}\frac{d}{dr}[r^2\mathbf{J}(\mathbf{r})] = 2\mathbf{J}(\mathbf{r}) + (\mathbf{r}\cdot\nabla)\mathbf{J}(\mathbf{r}). \tag{7}$$

Taking into account the vector identity

$$\nabla\times[\mathbf{r}\times\mathbf{J}(\mathbf{r})] = [\nabla\cdot\mathbf{J}(\mathbf{r})]\mathbf{r} - 2\mathbf{J}(\mathbf{r}) - \mathbf{r}\cdot\nabla\mathbf{J}(\mathbf{r}),$$

one can rewrite Eq. (7) as

$$\frac{d}{dr}[r^2\mathbf{J}(\mathbf{r})] = r\{[\nabla\cdot\mathbf{J}(\mathbf{r})]\mathbf{r} - \nabla\times[\mathbf{r}\times\mathbf{J}(\mathbf{r})]\}. \tag{8}$$

Integrating in the radial direction and using the CE (6) we obtain

$$\begin{aligned} r^2\mathbf{J}(\mathbf{r}) - r_0^2\mathbf{J}(\mathbf{r}_0) &= \int_{r_0}^{r} dr\, r\{[\nabla\cdot\mathbf{J}(\mathbf{r})]\mathbf{r} - \nabla\times[\mathbf{r}\times\mathbf{J}(\mathbf{r})]\} \\ &= -\int_{r_0}^{r} dr\, r\{i[H,\rho(\mathbf{r})]\mathbf{r} + \nabla\times[\mathbf{r}\times\mathbf{J}(\mathbf{r})]\}. \end{aligned} \tag{9}$$

This equation permits to obtain the integral continuity equation (ICE)

$$\mathbf{J}(\mathbf{r}) = \frac{r_0^2}{r^2}\mathbf{J}(\mathbf{r}_0) - \frac{1}{r^2}\int_{r_0}^{r} dr\, r\{i[H,\rho(\mathbf{r})]\mathbf{r} + \nabla\times[\mathbf{r}\times\mathbf{J}(\mathbf{r})]\}. \tag{10}$$

The ICE is equivalent to the CE and if current conservation is fulfilled, the electromagnetic current is recovered by computing the right-hand side of the equation. The ICE is simplified by choosing a value of $\mathbf{r}_0$ for which $r_0^2\mathbf{J}(\mathbf{r}_0)=0$, such as, for instance, $r_0=0$ or ∞. In addition, the fact that the above expression of the current is manifestly conserved, permits us to use it as a prescription to restore the CE. This point will be discussed in the next section.

It is useful to write the ICE in an equivalent form. One starts from the equation

$$\frac{d}{d\alpha}[\alpha^2\mathbf{J}(\alpha\mathbf{r})] = 2\alpha\mathbf{J}(\alpha\mathbf{r}) + \alpha^2\mathbf{r}\cdot\nabla\mathbf{J}(\alpha\mathbf{r}) = \alpha\mathbf{C}(\alpha\mathbf{r}), \tag{11}$$

where we have introduced the adimensional parameter α and defined the auxiliary current $\mathbf{C}$. It is easy to check that

$$\mathbf{C}(\mathbf{r}) \equiv -i[H,\rho(\mathbf{r})]\mathbf{r} - \nabla\times[\mathbf{r}\times\mathbf{J}(\mathbf{r})]. \tag{12}$$

By integrating Eq. (11) from $\alpha=\alpha_0$ to $\alpha=1$ we obtain

$$\mathbf{J}(\mathbf{r}) = \alpha_0^2\mathbf{J}(\alpha_0\mathbf{r}) + \int_{\alpha_0}^{1} d\alpha\, \alpha\mathbf{C}(\alpha\mathbf{r}) \tag{13}$$

and introducing the form of $\mathbf{C}$ given by Eq. (12) we have

$$\begin{aligned} \mathbf{J}(\mathbf{r}) = \alpha_0^2\mathbf{J}(\alpha_0\mathbf{r}) &- i\left[H, \int_{\alpha_0}^{1} d\alpha\,\alpha^2\rho(\alpha\mathbf{r})\right]\mathbf{r} \\ &- \nabla\times\left[r\times\int_{\alpha_0}^{1} d\alpha\,\alpha\mathbf{J}(\alpha\mathbf{r})\right]. \end{aligned} \tag{14}$$

As for Eq. (10), this equation simplifies by selecting a value of α_0 such as $\alpha_0^2\mathbf{J}(\alpha_0\mathbf{r})=0$. It is of particular interest to choose $\alpha_0=\infty$. In this case we can write the current as

$$\mathbf{J}(\mathbf{r}) = \mathbf{J}_c(\mathbf{r}) + \mathbf{J}_m(\mathbf{r}), \tag{15}$$

where

$$\mathbf{J}_c(\mathbf{r}) \equiv i\left[H, \int_1^\infty d\alpha\, \alpha^2 \rho(\alpha\mathbf{r})\right]\mathbf{r} = i\left[H, \int_0^1 \frac{d\lambda}{\lambda^4}\rho(\mathbf{r}/\lambda)\right]\mathbf{r}, \tag{16}$$

$$\mathbf{J}_m(\mathbf{r}) \equiv \nabla\times\left(\mathbf{r}\times\int_1^\infty \alpha\mathbf{J}(\alpha\mathbf{r})\right) = \nabla\times\left(\mathbf{r}\times\int_0^1 \frac{d\lambda}{\lambda^3}\mathbf{J}(\mathbf{r}/\lambda)\right). \tag{17}$$

These currents coincide with those obtained by Friar and Fallieros [see Eqs. (8a)–(8d) and (4c) in Ref. [9]]. Then, Eq. (14) can be considered as an extension of the Siegert's current formulated by these authors. Other values of the parameter α_0 fulfilling the relation $\alpha_0^2\mathbf{J}(\alpha_0\mathbf{r})=0$ (such as, e.g., $\alpha_0=0$) provide new currents potentially useful to restore the CE in model calculations. On the other hand, the procedure we have followed is considerably simpler.

To finish this discussion, we remember that if CE is satisfied, the different currents one can obtain from Eq. (14) coincide. Then, for the two particular cases $\alpha_0=0$ and ∞, we have

$$\mathbf{J}(\mathbf{r}) = \int_0^1 d\alpha\, \alpha\mathbf{C}(\alpha\mathbf{r}) = \int_\infty^1 d\alpha\, \alpha\mathbf{C}(\alpha\mathbf{r}), \tag{18}$$

and, as a result,

$$\int_0^\infty d\alpha\, \alpha\mathbf{C}(\alpha\mathbf{r}) = 0. \tag{19}$$

This is a global consequence of the CE and is verified by any conserved current.

B. Momentum space

In order to study the behavior of the currents as a function of the momentum transfer $\mathbf{q}$, it is useful to see how the equations obtained in the previous subsection read in momentum space. The corresponding version of the ICE can be obtained by calculating the Fourier transform in Eq. (10). However, we derive it again to illustrate the differences between coordinate and momentum spaces in what refers to the procedure followed.

We start with the equation analogous to Eq. (7):

$$\frac{d}{dq}[q\mathbf{J}(\mathbf{q})] = \mathbf{J}(\mathbf{q}) + \mathbf{q}\cdot\nabla_{\mathbf{q}}\mathbf{J}(\mathbf{q}). \tag{20}$$

If we consider the vector relation

$$\nabla_{\mathbf{q}}[\mathbf{q}\cdot\mathbf{J}(\mathbf{q})] = \mathbf{J}(\mathbf{q}) + \mathbf{q}\cdot\nabla_{\mathbf{q}}\mathbf{J}(\mathbf{q}) + \mathbf{q}\times\nabla_{\mathbf{q}}\times\mathbf{J}(\mathbf{q}),$$

we obtain from Eq. (20)

$$\frac{d}{dq}[q\mathbf{J}(\mathbf{q})] = \nabla_{\mathbf{q}}[\mathbf{q}\cdot\mathbf{J}(\mathbf{q})] - \mathbf{q}\times\nabla_{\mathbf{q}}\times\mathbf{J}(\mathbf{q}),$$

and, by inserting here the CE in momentum space

$$\mathbf{q}\cdot\mathbf{J}(\mathbf{q}) = [H, \rho(\mathbf{q})], \tag{21}$$

we have

$$\frac{d}{dq}[q\mathbf{J}(\mathbf{q})] = [H, \nabla_{\mathbf{q}}\rho(\mathbf{q})] - \mathbf{q}\times\nabla_{\mathbf{q}}\times\mathbf{J}(\mathbf{q}) \equiv -\mathbf{C}(\mathbf{q}). \tag{22}$$

Here we have introduced the auxiliary current $\mathbf{C(q)}$ which, as it is easy to check, is the Fourier transform of the current $\mathbf{C(r)}$ defined in Eq. (12). The ICE in momentum space is then obtained by integrating Eq. (22)

$$\mathbf{J}(\mathbf{q}) = \frac{q_0}{q}\mathbf{J}(\mathbf{q}_0) - \frac{1}{q}\int_{q_0}^q dq\,\mathbf{C}(\mathbf{q}) \tag{23}$$

$$= \frac{q_0}{q}\mathbf{J}(\mathbf{q}_0) + \frac{1}{q}\int_{q_0}^q dq\{[H, \nabla_{\mathbf{q}}\rho(\mathbf{q})] - \mathbf{q}\times\nabla_{\mathbf{q}}\times\mathbf{J}(\mathbf{q})\}. \tag{24}$$

As in the case of coordinate space, this ICE reduces by choosing a value q_0 for which $q_0\mathbf{J}(\mathbf{q}_0)=0$.

We can also obtain the ICE in momentum space equivalent to Eq. (14). To do this we introduce an adimensional parameter λ such that

$$\frac{\partial}{\partial\lambda}[\lambda\mathbf{J}(\lambda\mathbf{q})] = -\mathbf{C}(\lambda\mathbf{q}). \tag{25}$$

Integrating this equation in the interval $[\lambda_0,1]$, and taking into account the definition of the current $\mathbf{C}$ given in Eq. (22), we obtain for the ICE in momentum space

$$\mathbf{J}(\mathbf{q}) = \lambda_0\mathbf{J}(\lambda_0\mathbf{q}) - \int_{\lambda_0}^1 d\lambda\,\mathbf{C}(\lambda\mathbf{q}) \tag{26}$$

$$= \lambda_0\mathbf{J}(\lambda_0\mathbf{q}) + \left[H, \int_{\lambda_0}^1 d\lambda(\nabla_{\mathbf{q}}\rho)(\lambda\mathbf{q})\right] - \mathbf{q}\times\int_{\lambda_0}^1 d\lambda\,\lambda(\nabla_{\mathbf{q}}\times\mathbf{J})(\lambda\mathbf{q}). \tag{27}$$

It is worthwhile to note that the second and third terms in this equation coincide with the Fourier transforms of the $\mathbf{J}_c$ and $\mathbf{J}_m$ currents defined in Eqs. (16) and (17). In addition, it can be shown that, for $\lambda_0=0$, the corresponding currents introduced by Friar and Fallieros [9] are recovered.

Finally, the condition analogous to Eq. (19) is found by considering the values $\lambda_0=0$ and ∞ in the ICE (27):

$$\int_0^\infty d\lambda\,\mathbf{C}(\lambda\mathbf{q}) = 0. \tag{28}$$

The integral in this equation allows us to analyze the asymptotic behavior of the currents in momentum space in case the CE is not fulfilled. We discuss this point in the next section.

C. Multipoles of the current

In this subsection we obtain the corresponding expressions for the multipoles of the current. As it is known [11], the electric and magnetic multipoles are linear combinations of the multipole operators defined in Eq. (5). Taking into account Eq. (13) we have

$$T_{JLM}(q)=\int d^3r j_L(qr)\mathbf{Y}^M_{JL}(\hat{\mathbf{r}})\cdot\left[\alpha_0^2\mathbf{J}(\alpha_0\mathbf{r})+\int_{\alpha_0}^1 d\alpha\,\alpha\mathbf{C}(\alpha\mathbf{r})\right]. \tag{29}$$

An easy calculation permits us to write

$$T_{JLM}(q)=\frac{1}{\alpha_0}T_{JLM}\left(\frac{q}{\alpha_0}\right)+\int_{\alpha_0}^1 d\alpha\frac{1}{\alpha^2}C_{JLM}\left(\frac{q}{\alpha}\right), \tag{30}$$

where we have defined the multipoles of the auxiliary current $\mathbf{C}$ as

$$C_{JLM}(q)\equiv\int d^3r j_L(qr)\mathbf{Y}^M_{JL}(\hat{\mathbf{r}})\cdot\mathbf{C}(\mathbf{r}). \tag{31}$$

Finally, writing the integral (30) in terms of the variable $\lambda=1/\alpha$ we obtain

$$T_{JLM}(q)=\lambda_0 T_{JLM}(\lambda_0 q)+\int_1^{\lambda_0}d\lambda C_{JLM}(\lambda q), \tag{32}$$

where $\lambda_0=1/\alpha_0$. Hence, the multipoles of the current $\mathbf{J}$ are given by integrating the multipoles $C_{JLM}(q)$ of the auxiliary current $\mathbf{C}(\mathbf{r})$. This equation is the one corresponding to Eq. (26) verified by the current $\mathbf{J}(\mathbf{q})$, and which is, therefore, satisfied by each one of its multipoles separately.

Introducing in Eq. (32) the variable $q'=\lambda q$ we finally have

$$T_{JLM}(q)=\frac{q_0}{q}T_{JLM}(q_0)+\frac{1}{q}\int_q^{q_0}dq' C_{JLM}(q'), \tag{33}$$

where we have defined $q_0=\lambda_0 q$.

By choosing the values $q_0=0$ and ∞, for which the first term in Eq. (33) vanishes, we obtain a global condition for the integral of the multipoles C_{JLM}

$$\int_0^\infty dq C_{JLM}(q)=0. \tag{34}$$

As for the current, this condition is fulfilled when the CE is satisfied.

III. PRESCRIPTIONS FOR RESTORING THE CONTINUITY EQUATION

The equations derived in the previous section assume that current conservation is verified. Often, the nuclear current is not conserved in electron scattering calculations, where a known density operator $\rho(\mathbf{r})$, obtained as the sum of one-body single-nucleon densities, and a current operator $\mathbf{J}^{(\mathrm{NC})}$, such as

$$\nabla\cdot\mathbf{J}^{(\mathrm{NC})}(\mathbf{r})\neq -i[H,\rho(\mathbf{r})], \tag{35}$$

are considered.

Here we use the ICE to restore the CE. Starting with the nonconserved current $\mathbf{J}^{(\mathrm{NC})}$ and the charge density, we calculate the auxiliary current $\mathbf{C}^{(\mathrm{NC})}$ as given by Eq. (22) and define new currents according to Eq. (23). In particular we consider the values $q_0=0$ and ∞ for which $q_0\mathbf{J}(\mathbf{q}_0)=0$ and we obtain

$$\mathbf{J}^{(0)}(\mathbf{q})=-\frac{1}{q}\int_0^q dq'\,\mathbf{C}^{(\mathrm{NC})}(\mathbf{q}'), \tag{36}$$

$$\mathbf{J}^{(\infty)}(\mathbf{q})=\frac{1}{q}\int_q^\infty dq'\,\mathbf{C}^{(\mathrm{NC})}(\mathbf{q}'), \tag{37}$$

where $\mathbf{q}'=(q'/q)\mathbf{q}$. As discussed in Sec. II B, the current $\mathbf{J}^{(0)}(\mathbf{q})$ is the one derived by Friar and Fallieros [9] and afterwards used by Friar and Haxton [10] to calculate the electron scattering form factors.

The second current $\mathbf{J}^{(\infty)}(\mathbf{q})$ constitutes a new possible prescription in order to restore the CE. Both currents coincide only if the model current is conserved; if CE is not verified, both prescriptions are in general different, and one has

$$\mathbf{J}^{(\infty)}(\mathbf{q})-\mathbf{J}^{(0)}(\mathbf{q})=\frac{1}{q}\int_0^\infty dq'\,\mathbf{C}^{(\mathrm{NC})}(\mathbf{q}')\neq 0. \tag{38}$$

The asymptotic behavior of the new currents can be determined from their definitions in Eqs. (36),(37). It can be summarized in the following properties:

$$q\to 0:\quad \mathbf{J}^{(0)}(\mathbf{q})\sim -\mathbf{C}^{(\mathrm{NC})}(0),\quad \mathbf{J}^{(\infty)}(\mathbf{q})=O(1/q), \tag{39}$$

$$q\to\infty:\quad \mathbf{J}^{(0)}(\mathbf{q})=O(1/q),\quad \mathbf{J}^{(\infty)}(\mathbf{q})\sim\mathbf{C}^{(\mathrm{NC})}(\mathbf{q}), \tag{40}$$

The first condition is equivalent to the Siegert theorem. In fact, from the definition of $\mathbf{C}(\mathbf{q})$ in Eq. (22) we have

$$-\mathbf{C}^{(\mathrm{NC})}(0)=[H,(\nabla_{\mathbf{q}}\rho)(0)]=i[H,\mathbf{d}], \tag{41}$$

where $\mathbf{d}$ is the electric dipole momentum of the system

$$\mathbf{d}=\int d^3r\,\mathbf{r}\rho(\mathbf{r}). \tag{42}$$

Therefore the current $\mathbf{J}^{(0)}$ verifies the Siegert theorem because it equals the time derivative of the electric dipole momentum of the system.

The second current, $\mathbf{J}^{(\infty)}(\mathbf{q})$, however, does not satisfy this theorem. In fact, from its definition, we have

$$\lim_{q\to 0} q\mathbf{J}^{(\infty)}(\mathbf{q})=\int_0^\infty dq\,\mathbf{C}^{(\mathrm{NC})}(\mathbf{q})\neq 0. \tag{43}$$

Then, as established in Eq. (39), this current diverges in the origin as $O(1/q)$ if the original current is not conserved. Therefore it should be discarded as a physical current for low momentum transfer.

On the other hand, a similar nonphysical behavior is shown by the Friar and Fallieros current $\mathbf{J}^{(0)}$ for large momentum transfer, because

$$\lim_{q\to\infty} q\mathbf{J}^{(0)}(\mathbf{q}) = -\int_0^\infty dq \mathbf{C}^{(\mathrm{NC})}(\mathbf{q}) \neq 0. \tag{44}$$

This current goes as $O(1/q)$ for large q.

Instead, the current $\mathbf{J}^{(\infty)}$ works well in the large q limit. From Eqs. (36),(37) we can write

$$\mathbf{J}^{(\infty)}(q) = \mu \int_0^\mu d\mu' \frac{1}{\mu'^2} \mathbf{C}^{(\mathrm{NC})}(\mathbf{q}') \overset{q\to\infty}{\to} \mathbf{C}^{(\mathrm{NC})}(\mathbf{q}), \tag{45}$$

where the change of variables $\mu = 1/q$, $\mu' = 1/q'$ has been used. Then the behavior of $\mathbf{J}^{(\infty)}$ for $q\to\infty$ is the same as that of the auxiliary current $\mathbf{C}^{(\mathrm{NC})}$, which shows the adequate one because it is built from the physical charge and current density operators and its definition does not include integrations, but just derivatives and products. Its good asymptotic properties for large momentum transfer made this current to be *a priori* useful in this regime in electron scattering calculations, without the handicap of the pathological behavior shown by $\mathbf{J}^{(0)}$. The existence of such a current was suggested in Ref. [10], but, to the best of our knowledge, this hypothesis has not been proved up to now.

In order to have a better understanding of the properties of these two currents, we can also obtain the asymptotic behavior in coordinate space. A straightforward calculation, similar to that developed for momentum space, permits us to write

$$r\to 0: \quad \mathbf{J}^{(0)}(\mathbf{r}) = O(1/r^2), \quad \mathbf{J}^{(\infty)}(\mathbf{r}) \sim \mathbf{J}^{(\mathrm{NC})}(\mathbf{r}), \tag{46}$$

$$r\to\infty: \quad \mathbf{J}^{(0)}(\mathbf{r}) \sim -\mathbf{C}^{(\mathrm{NC})}(\mathbf{r}), \quad \mathbf{J}^{(\infty)}(\mathbf{r}) = O(1/r^2). \tag{47}$$

As we can see from these equations, both currents have also the opposite roles. The current $\mathbf{J}^{(0)}$ is well behaved at large distances, but it diverges near the origin. On the other hand, for $r\to 0$, the current $\mathbf{J}^{(\infty)}$ goes as $\mathbf{J}^{(\mathrm{NC})}$, which is supposed to be well behaved at short distances, but it does not reach zero fast enough for $r\to\infty$.

Finally, we summarize the asymptotic properties of the multipoles $T^M_{JL}(q)$. By defining the multipoles of the two currents $\mathbf{J}^{(0)}$ and $\mathbf{J}^{(\infty)}$ as

$$T^{(0)}_{JLM}(q) = -\frac{1}{q}\int_0^q dq' C^{(\mathrm{NC})}_{JLM}(q'), \tag{48}$$

$$T^{(\infty)}_{JLM}(q) = \frac{1}{q}\int_q^\infty dq' C^{(\mathrm{NC})}_{JLM}(q'), \tag{49}$$

respectively, we find the following asymptotic behavior for these two sets of multipoles

$$q\to 0: \quad T^{(0)}_{JLM}(q) \sim -C^{(\mathrm{NC})}_{JLM}(q), \quad T^{(\infty)}_{JLM}(q) = O(1/q), \tag{50}$$

$$q\to\infty: \quad T^{(0)}_{JLM}(q) = O(1/q), \quad T^{(\infty)}_{JLM}(q) \sim C^{(\mathrm{NC})}_{JLM}(q). \tag{51}$$

These properties are similar to the ones verified by the currents in momentum space given by Eqs. (39),(40).

The multipoles $T^{(0)}$ are well behaved for low momentum transfer, where the Siegert theorem applies, but fail for high momentum transfer. As a consequence, the transition matrix elements corresponding to the current $\mathbf{J}^{(0)}$ do not go to zero fast enough as q goes to ∞, because the bound nuclear wave functions have an exponential-like behavior at large q values. This is the reason of the pathology observed in this regime in the electric multipoles computed with the current developed by Friar and Fallieros [10,3].

On the other hand, the multipoles $T^{(\infty)}$ do not work well near the origin (they diverge as $1/q$), but are well behaved for high momentum transfer. Hence they can safely be used in this regime in model calculations.

IV. A SIMPLE MODEL CALCULATION

In order to illustrate the results quoted in the last section, we have used a very simple model to analyze different transitions in ^{16}O and ^{39}K and which was previously considered in Ref. [2]. In this model, the nuclear structure is described by means of a single-particle Hamiltonian of the form

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V_0 + \frac{\hbar^2}{2m}\frac{r^2}{b^4} + V_{\mathrm{LS}}\,\mathbf{l}\cdot\mathbf{s}.$$

The eigenfunctions R_{nl} are harmonic oscillator functions and the corresponding eigenvalues are

$$E_{nlj} = \frac{\hbar^2}{mb^2}\left(2n + l - \frac{1}{2}\right) + V_0 + \frac{1}{2}V_{\mathrm{LS}}\left[j(j+1) - l(l+1) - \frac{3}{4}\right].$$

The nuclear charge density operator is taken to be the usual one,

$$\rho(\mathbf{r}) = \sum_{k=1}^{A} \frac{1+\tau_3^k}{2}\delta(\mathbf{r}-\mathbf{r}_k),$$

while in the nuclear current density operator we have included the well-known convection and spin-magnetization one-body terms

$$\mathbf{J}^{\mathrm{C}}(\mathbf{r}) = \sum_{k=1}^{A} \frac{1}{2M_k}\frac{1}{i}\frac{1+\tau_3^k}{2}[\delta(\mathbf{r}-\mathbf{r}_k)\nabla_{\mathbf{r}_k} + \nabla_{\mathbf{r}_k}\delta(\mathbf{r}-\mathbf{r}_k)], \tag{52}$$

$$\mathbf{J}^{\mathrm{M}}(\mathbf{r}) = \sum_{k=1}^{A}\left(\mu_{\mathrm{P}}\frac{1+\tau_3^k}{2} + \mu_{\mathrm{N}}\frac{1-\tau_3^k}{2}\right)\nabla\times[\delta(\mathbf{r}-\mathbf{r}_k)\boldsymbol{\sigma}^k], \tag{53}$$

as well as the so-called spin-orbit current

TABLE I. Parameters of the potential used in the toy model considered to discuss the violation of the CE (see text).

Nucleus	V_0 [MeV]	b [fm]	V_{LS} [MeV]
^{16}O	-53.6	1.67	-4.20
^{40}Ca	-55.7	1.80	-1.90

$$\mathbf{J}^{\mathrm{LS}}(\mathbf{r}) = \frac{1}{2} V_{\mathrm{LS}} \sum_{k=1}^{A} \frac{1+\tau_3^k}{2} \delta(\mathbf{r}-\mathbf{r}_k) \boldsymbol{\sigma}^k \times \mathbf{r}_k . \tag{54}$$

In the previous equations M_k labels the mass of the k-nucleon, $\mathbf{S}^k = \boldsymbol{\sigma}^k/2$ is its spin and $\tau_3^k = 1$ or -1 according to whether this nucleon is a proton or neutron, respectively. Finally, μ_{P} (μ_{N}) is the proton (neutron) magnetic moment.

The model built in this way satisfies the CE. Within it we calculate the electric multipoles corresponding to the current

$$\mathbf{J}^{(\mathrm{PD})}(\mathbf{r}) = \mathbf{J}^{\mathrm{C}}(\mathbf{r}) + \mathbf{J}^{\mathrm{M}}(\mathbf{r}) + \mathbf{J}^{\mathrm{LS}}(\mathbf{r}) \tag{55}$$

for some transitions in ^{16}O and ^{39}K and the results are considered as ''pseudodata.'' The parameters V_0, V_{LS}, and b used in the calculations are shown in Table I. They were fixed in order to reproduce the energies of the single-particle states around the Fermi level in the two double closed-shell nuclei (^{16}O and ^{40}Ca) of interest.

In what follows we discuss the point relative to the restoration of the CE by using the currents $\mathbf{J}^{(0)}$ and $\mathbf{J}^{(\infty)}$ defined in Eqs. (36) and (37), respectively. We have simulated the usual situation of a model not verifying the CE by eliminating the spin-orbit current from the nuclear current operator:

$$\mathbf{J}^{(\mathrm{NC})}(\mathbf{r}) = \mathbf{J}^{\mathrm{C}}(\mathbf{r}) + \mathbf{J}^{\mathrm{M}}(\mathbf{r}). \tag{56}$$

We are interested in analyzing the goodness of the approach sketched in the previous section in retrieving the ''true'' results.

We focus our attention on the two following electric transitions: $(0^+ \to 2s_{1/2} 1p_{1/2}^{-1})_{1^-}$ in ^{16}O and $(1d_{3/2}^{-1} \to 2s_{1/2}^{-1})_{2^+}$ in ^{39}K. In particular, we study the electric multipoles

$$t_{EJ}^{(\kappa)}(q) = \sqrt{\frac{J+1}{2J+1}} \langle J_{\mathrm{f}} \| i T_{JJ-1}^{(\kappa)}(q) \| J_{\mathrm{i}} \rangle - \sqrt{\frac{J}{2J+1}} \langle J_{\mathrm{f}} \| i T_{JJ+1}^{(\kappa)}(q) \| J_{\mathrm{i}} \rangle, \tag{57}$$

where the multipole operators $T_{JJ\pm1}$ are given by Eq. (5) and where (κ) stands for (PD), (0), (∞), and (NC).

In Fig. 1 we compare the multipoles $|t_{EJ}^{(\kappa)}(q)|$ for $(\kappa) = (0)$ (full curves) and (∞) (dashed curves) with the results of the calculation done with the full model, $(\kappa) = (\mathrm{PD})$ (dashed-dotted curves). The asymptotic behavior discussed in the previous section is now apparent. The calculations performed with $\mathbf{J}^{(0)}$, that is the Siegert's current of Friar and Fallieros [9], are right at low q, but differ notably from the ''exact values'' in the high q region, in agreement with the findings of Refs. [10,3]. On the contrary, the multipoles corresponding to $\mathbf{J}^{(\infty)}$ are wrong for small momentum transfer, providing the correct behavior for large q.

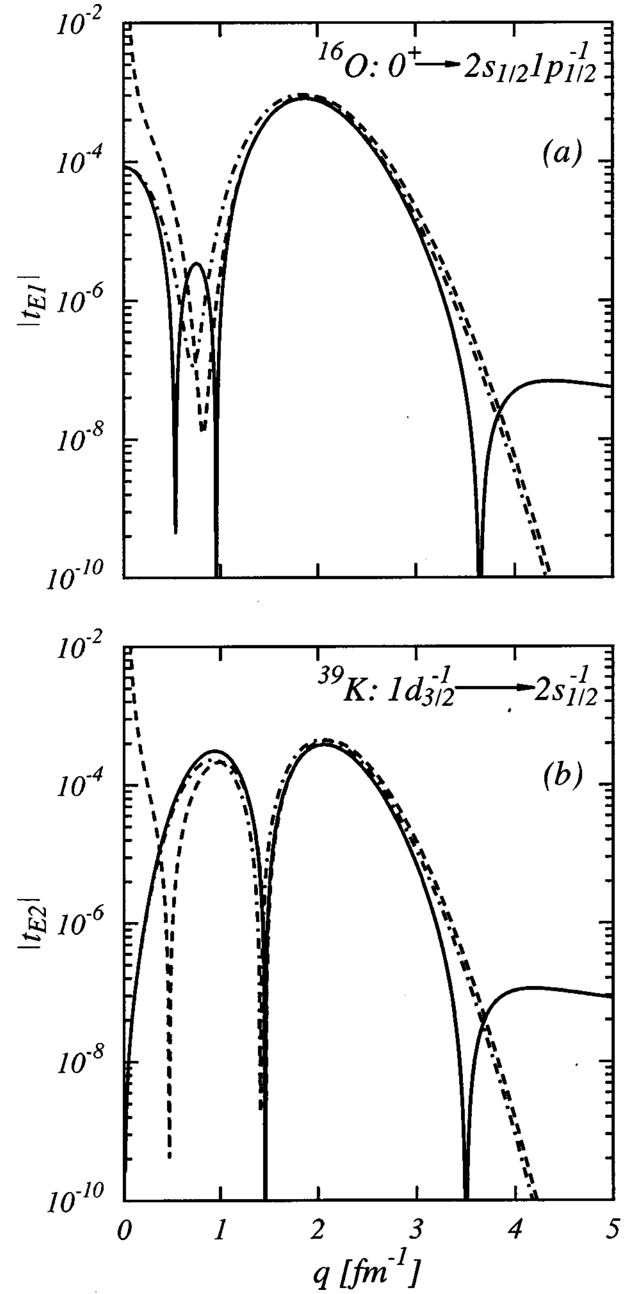

FIG. 1. Absolute value of the multipoles t_{EJ} given by Eq. (57) as a function of the momentum transfer q, for the two transition considered in this work. Dot-dashed curves correspond to the ''exact'' model. Full and dashed curves have been obtained for the currents $\mathbf{J}^{(\lambda_0)}$ for $\lambda_0 = 0$ and ∞, respectively.

In order to have a better idea of the goodness of both calculations, we show in Fig. 2 [upper panels (a),(b)] the quantity

$$\Delta^{(\kappa)}(q) \equiv |t_{EJ}^{(\mathrm{PD})}(q)| - |t_{EJ}^{(\kappa)}(q)| \tag{58}$$

for $(\kappa) = (0)$ (solid curves), (∞) (dashed curves), and (NC) (dotted curves).

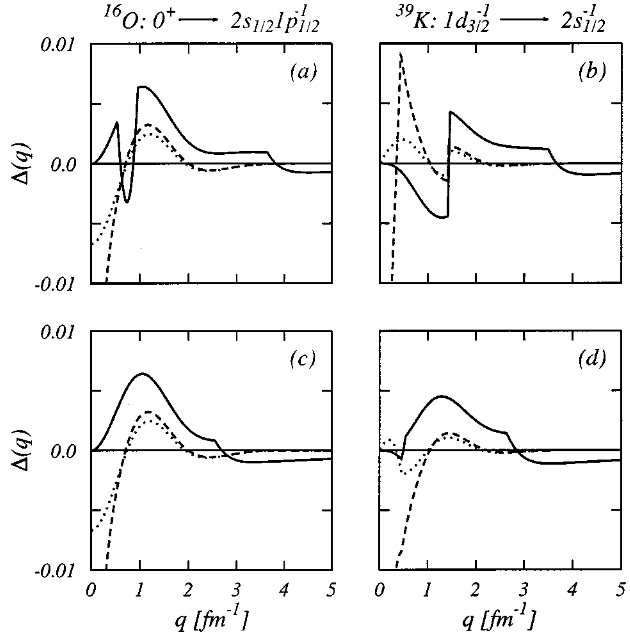

FIG. 2. Differences Δ given by Eq. (58) for the two transitions considered in this work. Full and dashed curves have been obtained for the currents $\mathbf{J}^{(\lambda_0)}$ for $\lambda_0 = 0$ and ∞, respectively. Dotted curves correspond to the model in which the current is not conserved [that is, by ignoring the spin-orbit current (54)]. Upper panels (a),(b) include the magnetization current (53) while this current has been not included in the calculations plotted in the lower panels (c),(d).

As we can see, this last calculation provides the better result except at very low q, where we know it violates the Siegert theorem. However, for $q > 1\ \mathrm{fm}^{-1}$ is closer to the ''pseudodata'' than the other two calculations.

It is worthwhile to note the situation of the Siegert's current (solid curves). Although this current provides the right behavior of the electric multipoles at very low momentum transfer, they show a considerable disagreement with the ''pseudodata'' for q above 1 fm^{-1}, where the new current we have obtained for $(\kappa) = (\infty)$ (dashed curves) gives rise to more accurate results.

In order to go deeper in the analysis, we have done new calculations without considering the magnetization current [see lower panels (c),(d)]. As we know, this current is not affected by the CE and by ignoring it we can test the importance of this piece of the current in the results. However, we can see that the situation does not change too much and similar comments to those made above can be stated for these new calculations.

V. CONCLUSIONS

In this work we have developed a method to obtain new forms of the nuclear electromagnetic current. The approach is based on the integral form of the continuity equation and produces, as a particular case, the Siegert's current developed by Friar and Fallieros. As in this case, the new currents can be used to restore current conservation in those model calculations in which the continuity equation is not fulfilled. In addition, our procedure permits us to understand in an easy way the asymptotic behavior shown by the currents.

We have illustrated the method by means of a simple nuclear model based on a harmonic oscillator potential which includes a spin-orbit term. The results obtained show that, at least in the cases studied, the multipoles calculated by using two of the new currents do not produce better results than those found with a model in which current is not conserved. This puts some doubts concerning the procedures of ''restoring the continuity equation.''

This situation is similar to the one found in relativistic calculations of quasielastic electron scattering by nuclei [6] where no tractable approach to treating the off-shell dependence rigorously exists. This makes inevitable the *ad hoc* modifications of the currents in order to recover conservation, but it is not possible to decide which one of the different prescriptions is the better.

In any case, our approach permits other possibilities to define new currents which deserve a more careful analysis. Work in this direction is in progress.

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