Incremental alignments in the $A \sim 150$ superdeformed region

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It is found that the incremental alignments of superdeformed (SD) bands in nuclei in the $A \sim 150$ region exhibit a simple rule of additivity. This additivity is found to hold with good accuracy even for nuclei that differ by a large number of particles. The effect of neutron and/or proton band crossings and pairing correlations on this additivity are discussed. The additivity property of incremental alignments can be used to predict the energies of transitions in previously unknown SD bands and recent experimental results on SD bands in ^{147}Eu are found to be in excellent agreement with the predicted values. [S0556-2813(99)00109-0]

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Superdeformed (SD) bands in the $A \sim 150$ region have been intensively studied in recent years in terms of both theory and experiment. These bands are characterized by both cascades of regularly spaced, stretched, quadrupole transitions, and correspond to very elongated ellipsoidal shapes with an axis ratio of $\sim 2{:}1$; these structures are stabilized by shell gaps in the single-particle energy spectrum. The microscopic structure of the SD bands has been found to depend predominantly on the so-called intruder orbitals [1]. These high-j, high-$\mathcal{N}$ ($\mathcal{N}$ being the principal oscillator number) particle-like intruder orbitals, brought close to the SD Fermi level by a combination of large deformation and high rotational frequencies, are the proton $\mathcal{N}=6$ and the neutron $\mathcal{N}=7$ states. The occupation numbers of these orbitals are found to have a strong impact on the dynamic moments of inertia, $\mathcal{J}^{(2)}$, and the quadrupole moments [1,2].

In the $A \sim 190$ region, Stephens *et al.* used the near-integer aligned spin of one SD band relative to another SD band in a nearby nucleus as evidence for quantized pseudospin alignment [3], and introduced a quantity, called the incremental alignment, Δi, to describe the SD bands. Δi can be extracted directly from the energies of SD transitions, without the need of, or reference to, the spins of individual SD levels (which are not experimentally known in the $A \sim 150$ region). Thus, Δi depends only on experimental γ-ray energies, E_γ, and is defined as $\Delta i = 2 \times [E_\gamma^B(I') - E_\gamma^A(I)]/[E_\gamma^A(I+2) - E_\gamma^A(I)]$, where B represents the nucleus of interest and A is the reference nucleus [3].

In previous work [4], we have presented a phenomenological description of superdeformed bands in the Eu, Gd, Tb, and Dy isotopes, based on the experimental incremental alignments, Δi, and the associated intruder configurations. The energies of many known SD bands were reproduced rather accurately and we had also made predictions of γ-ray energies of various hithertofore unobserved SD bands. The details of our procedure for calculating the γ-ray energies of these SD bands are presented therein. We found that all the varied effects observed in the evolution of the dynamic moments of inertia $\mathcal{J}^{(2)}$, with the rotational frequency $\hbar\omega$ of the SD bands in the $A \sim 150$ region, are fully attributable to the occupation of specific orbitals, apparently independent of other factors such as mass changes, deformation, pairing, etc.

An additivity property of SD bands in the $A \sim 150$ region was noted some time ago by Ragnarsson [5]. It was shown that for the 4 SD bands then known in the Gd nuclei, the transition energies and the effective alignments (difference of spin at constant frequency), $i_{\rm eff}$, follow a simple rule of additivity. Later, $i_{\rm eff}$ was used as a measure of the contribution of different Nilsson orbitals to assign intruder configurations and relative spins to SD bands in the $A \sim 150$ region [6]. It was concluded that $i_{\rm eff}$ is dominated by the occupation of specific orbitals and that other factors such as the deformation and pairing, etc., have, at best, a weak effect on $i_{\rm eff}$.

Experimental alignments have also been used by Fischer *et al.* [7] in the $A \sim 190$ region of superdeformation: the experimental alignments of orbitals in the odd-A nuclei ^{193}Tl and ^{191}Hg (with respect to the reference nucleus ^{192}Hg) were used to compute the alignments in the odd-odd ^{192}Tl nucleus and good agreement was found between the measured alignments and the values computed from the odd-A neighbors, confirming the validity of the additivity principle for alignments.

Another additivity property of SD bands has been described recently by Satuła *et al.* [2]. They have shown that the quadrupole and hexadecapole moments can be described as arising from the sum of independent contributions from single-particle (hole) states around the doubly magic SD core of ^{152}Dy. This additivity has since been extended experimentally in that the quadrupole moment of the yrast SD band in ^{142}Sm, Q_0 [^{142}Sm], was well reproduced [8] using Q_0 [^{152}Dy] as the reference case.

In this paper, we show that the incremental alignments similarly exhibit a simple rule of additivity of contributions from individual configurations and investigate this additivity in detail in an attempt to answer the following questions: What is the typical accuracy of additivity of incremental alignments? Is it orbital dependent? Does the additivity of incremental alignments show fluctuations as a function of rotational frequency (or spin)? Will this additivity still hold with reasonable accuracy when the configurations of the SD bands under comparison differ by several (especially, high-$\mathcal{N}$ intruder) orbitals? Does this additivity break down if a proton and/or neutron band crossing is encountered? What is the effect of pairing correlations on this additivity?

As pointed out earlier, Δi depends only on the γ-ray en-

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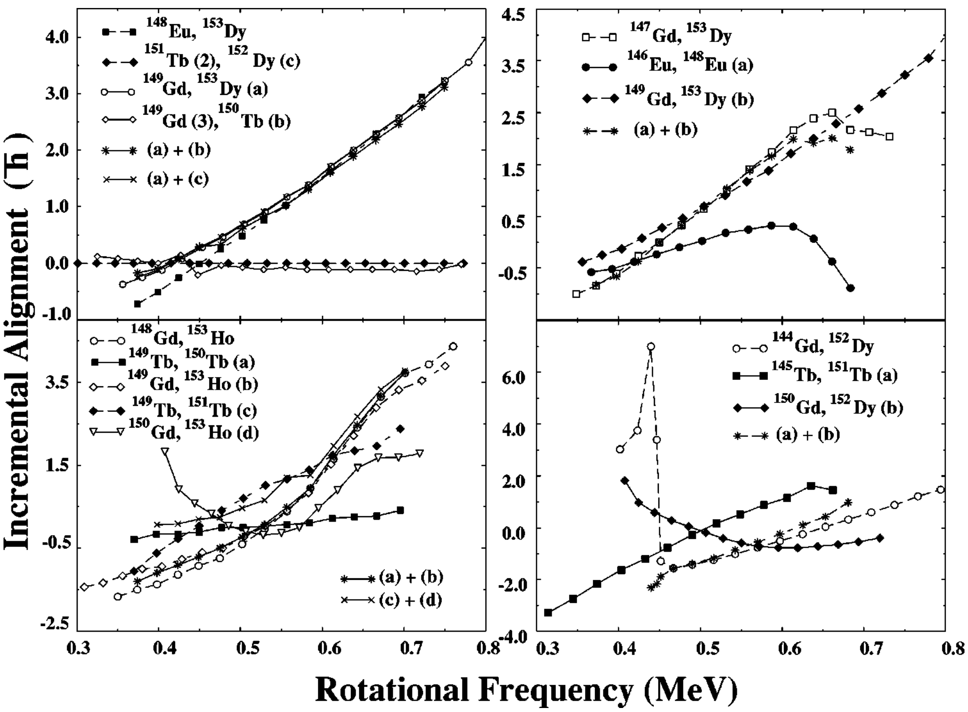

FIG. 1. Typical examples of the additivity of the incremental alignments for nuclei with $\Delta n \geqslant 4$. Δn is the number of extra particles (or holes) that separate the nuclei A and B, where B is the studied nucleus and A is the nucleus of reference.

ergies, E_γ, of the transitions in corresponding SD bands (actually, the differences between these transition energies). Our procedure consists of four steps: (i) calculate the incremental alignment Δi_d of the nucleus B vs A [where $B = A + \alpha p + \beta n$; α (β) is the number of extra protons (neutrons)]; (ii) find two pairs of nuclei (C, D) and (E, F) where $D = C + \gamma p$ and $F = E + \lambda n$, such that $\alpha + \beta = \gamma + \lambda$, and the proton (neutron) orbitals that characterize the difference between A and B are the same as those in C and D (E and F); (iii) calculate the alignments Δi_p of D vs C, and Δi_n of F vs E; (iv) compare the incremental alignment Δi_d to the sum $\Delta i_p + \Delta i_n$.

At first glance this might appear to be a trivial result of the type $[(x\text{-}z) = (x\text{-}y) + (y\text{-}z)]$ and, indeed, that would be true if one were to always consider the same intermediate nucleus y, or if all the dynamic moments of inertia $\mathcal{J}^{(2)}$ involved were identical. Neither of these conditions is existent in the cases investigated in this work. The $\mathcal{J}^{(2)}$'s of the SD bands in this region are very different (indeed, the variations in the $\mathcal{J}^{(2)}$'s are used to assign specific intruder orbitals to the SD bands) and, in our prescription, the pairs (A, B), (C, D), and (E, F) may be very different from each other, as long as they satisfy the condition (ii) above. For instance, if we consider (^{148}Eu, ^{153}Dy) = (A, B), $\alpha + \beta = 5$. This alignment corresponds to the contribution of the proton [301]1/2 natural parity orbital, the $\pi 6_3$ and $\pi 6_4$ intruder orbitals and the neutron $\nu 7_2$ and $\nu 7_3$ intruder orbitals.[1] In the second step, then, we may choose the pairs [^{151}Tb (band 2), ^{152}Dy] = (C, D) ($\gamma = 1$), and (^{149}Gd, ^{153}Dy) = (E, F) ($\lambda = 4$). The pair (C, D) gives the contribution of the π [301]1/2 orbital to the alignment, and the pair (E, F) the alignment of the four remaining intruder orbitals. These six nuclei are quite different from each other (in the sense of not being immediate neighbors). However, $\gamma + \lambda = \alpha + \beta = 5$, and the neutron and proton orbitals mentioned above for (A, B) are the same ones that are involved in (C, D) and (E, F). As can be seen in Fig. 1, the incremental alignment of ^{148}Eu vs ^{153}Dy is reproduced very well by the sum [(^{151}Tb (band 2) vs ^{152}Dy) + (^{149}Gd vs ^{153}Dy)], except for a small deviation at very low frequencies. In fact, a similar result is obtained if the pair [^{151}Tb (band 2), ^{152}Dy] is replaced by, for example, [^{149}Gd (band 3), ^{150}Tb].

Many such combinations of incremental alignments have been investigated for SD bands in the ^{144}Eu [10], ^{147,148}Eu [11], $^{144-150}$Gd [12], $^{149-151}$Tb [12,13], $^{151-155}$Dy [12], ^{153}Ho [14], and ^{154}Er [12] nuclei. The rule of additivity involving nuclei with mass difference up to four units has been shown previously to work very well and typical examples were discussed in Ref. [4]. We have now extended the procedure to SD bands involving configurations that differ by a larger number of particles (holes). The particles under consideration may occupy either intruder or nonintruder orbitals. Some selected cases are shown in Fig. 1, and typical examples are listed in Table I. Indeed, for most of the studied nuclei, the final incremental alignments are reproduced with good accuracy by their sums of the alignments of the respective intermediate steps, $\Sigma \Delta i_m$, implying that the incremental alignments can be expressed as sums of individual contributions carried by individual particles or holes. It would appear, then, that the incremental alignment depends predominantly on the occupation of specific single-particle orbitals.

[1]The single-particle orbitals have been labeled by means of the corresponding asymptotic Nilsson quantum numbers. For more details, see Ref. [9], for example.

TABLE I. Typical examples of additivity of the incremental alignments for nuclei with $\Delta n \geqslant 4$ (Δn is the number of extra particles or holes between the nuclei *A* and *B*, where *B* is the studied nucleus and *A* is the nucleus of reference), their respective sums and the root-mean-squared deviations (averaged over the whole SD band) between the sum of intermediate alignments and the direct alignment.

Nucleus of interest and the reference	Typical intermediate sums	Average rms difference between Δi_d and $\Sigma \Delta i_m$ (in $\hbar$)
^{148}Gd vs ^{152}Dy	^{149}Tb vs ^{151}Tb+^{149}Gd vs ^{151}Dy	0.08
	^{149}Tb vs ^{151}Tb+^{150}Gd vs ^{152}Dy	0.12
^{148}Gd vs ^{153}Ho	^{149}Tb vs ^{150}Tb+^{149}Gd vs ^{153}Ho	0.28
	^{147}Eu vs ^{148}Eu+^{149}Gd vs ^{153}Ho	0.25
	^{149}Tb vs ^{151}Tb+^{150}Gd vs ^{153}Ho	0.77
^{147}Eu vs ^{153}Ho	^{148}Eu vs ^{150}Tb+^{149}Tb vs ^{153}Ho	0.15
	^{149}Tb vs ^{150}Tb+^{148}Eu vs ^{153}Ho	0.21
^{147}Eu vs ^{152}Dy	^{148}Gd vs ^{149}Gd+^{148}Eu vs ^{152}Dy	0.20
	^{148}Eu vs ^{150}Tb+^{149}Tb vs ^{152}Dy	0.22
^{148}Eu vs ^{153}Dy	^{149}Gd (band 3) vs ^{150}Tb+^{149}Gd vs ^{153}Dy	0.22
	^{151}Tb (band 2) vs ^{152}Dy+^{149}Gd vs ^{153}Dy	0.30
^{148}Eu vs ^{154}Dy	^{147}Eu vs ^{149}Tb+^{150}Tb vs ^{154}Dy	0.25
	^{147}Eu vs ^{150}Tb+^{151}Tb vs ^{154}Dy	0.23
^{147}Gd vs ^{153}Dy	^{146}Eu vs ^{148}Eu+^{149}Gd vs ^{153}Dy	0.17
^{144}Gd vs ^{152}Dy	^{145}Tb vs ^{151}Tb+^{150}Gd vs ^{152}Dy	3.15 [a]
^{144}Eu vs ^{150}Tb	^{145}Gd vs ^{149}Gd+^{147}Eu vs ^{149}Tb	0.64 [b]

[a] If the three lowest frequencies are omitted then the rms is only $0.31\hbar$.
[b] If the first frequency is omitted then the rms is only $0.20\hbar$.

(A few discrepancies have been observed in this rule of additivity and those are discussed below.)

An interesting result is that the additivity of alignment is good up to $\hbar\omega_c \sim 0.6$ MeV for the ^{146,147}Gd (corresponding to $N=82$, 83) yrast SD bands, then starts shifting (see Fig. 1). These nuclei were found to exhibit an interaction between the [651]1/2 and [642]5/2 neutron orbitals (both located below the Fermi surface) precisely at this rotational frequency [15]. Therefore, we conclude that the interaction between these $N=6$ neutron orbitals breaks the rule of the additivity of the incremental alignment. Also, the results show that the crossing of the $\nu 7_1$ intruder and the $[514]9/2^-$ $(\alpha=-)$ orbitals observed in ^{144}Gd and ^{144}Eu [10] (corresponding to $N=80$ and 81) leads to a large discrepancy on the additivity below $\hbar\omega_c \sim 0.45$ MeV. Nonetheless, the ''direct'' and ''summed'' alignments are in close agreement above the backbending.

In our previous work, we have shown that predictions about energies of SD bands could be made in the $A\sim150$ region using incremental alignments [4]. However, it appeared that this scheme did not work very well for $N \leqslant 83$ since more than one set of energies were obtained for the calculated SD bands. We had suggested that the observed discrepancy could be associated with the interaction between the [651]1/2 and [642]5/2 neutron orbitals. The present work provides further evidence supporting that suggestion.

It is noteworthy that the calculated incremental alignment Δi_d of [^{148}Eu vs ^{154}Dy] SD bands is strikingly similar to the sums of the alignments for [(^{147}Eu vs ^{149}Tb)+(^{150}Tb vs ^{154}Dy)] $(\Delta i_{s1}+\Delta i_{s2})$. The nucleus ^{154}Dy has three more high-$\mathcal{N}$ intruder orbitals occupied than ^{148}Eu. Because of their large intrinsic angular momenta and quadrupole moments, these orbitals are believed to strongly polarize the ^{148}Eu SD core. Effects of this polarization were observed in the differences in the $\mathcal{J}^{(2)}$ and the quadrupole moments. Therefore, one would expect that because of the ''filling-up'' of these orbitals, the incremental alignment of ^{148}Eu vs ^{154}Dy would be different, at least in magnitude, from $\Delta i_{s1} + \Delta i_{s2}$. However, no discernible differences have been observed.

In order to study the effect of the proton crossings on the rule of the additivity of the incremental alignments, we have considered the nucleus ^{153}Ho [14]. The yrast SD band of this nucleus is, so far, the only yrast SD band known in the $A \sim 150$ region that undergoes a proton band crossing [14]. Indeed, a large increase in the $\mathcal{J}^{(2)}$ was observed and was explained by the crossing of the [770]1/2 and [530]1/2 proton orbitals located above the proton shell gap $Z=66$ [14]. But as shown in Fig. 1, the ''direct'' alignment of ^{147}Eu vs ^{153}Ho is very similar to the sum [(^{148}Eu vs ^{150}Tb)+(^{149}Tb vs ^{153}Ho)]. Also ^{148}Gd vs ^{153}Ho$=\Delta i_d$ is in good agreement with the sum [(^{149}Tb vs ^{150}Tb)+(^{149}Gd vs ^{153}Ho)]$=\Delta i_{s1}$. Therefore, we can conclude that, in contrast with the $N=80-83$ Gd isotopes, this proton crossing does not lead to any (or, at best, very weak) deviation from the additivity rule of incremental alignments. A notable exception is that the use of the ^{150}Gd yrast SD band leads to a large discrepancy at low frequency. In fact, bigger values of $\Delta i_{s2}=$[(^{149}Tb vs ^{151}Tb)+(^{150}Gd vs ^{153}Ho)] compared to Δi_d are obtained except for at higher frequencies. One possible explanation for this discrepancy could be the strong pairing correlations observed in ^{150}Gd at low frequencies [16].

TABLE II. Comparison between the predicted and experimental γ-ray energies of the yrast SD band in ^{147}Eu and its signature partner, and predictions for the γ-ray energies of the two signature-partner SD bands in ^{151}Gd (see text for more details).

^{147}Eu ($B1$) Prediction	^{147}Eu ($B1$) Experiment	^{147}Eu ($B2$) Prediction	^{147}Eu ($B5$) [a] Experiment	^{151}Gd ($B1$) Prediction	^{151}Gd ($B2$) Prediction
	737.3				
784.6	790.6				
837.7	842.3	833.8	835.9	871	893
890.7	892.3	885.7	889.0	910	934
944.7	946.8	938.9	940.5	960	979
999.0	1001.3	993.6	994.7	1005	1025
1055.4	1056.3	1048.8	1048.6	1049	1071
1111.1	1112.5	1106.4	1103.7	1094	1118
1169.0	1169.4	1161.2	1155.4	1144	1166
1226.4	1226.6	1219.2	1222.8	1191	1216
1284.9	1284.2	1277.5	1276.0	1241	1267
1343.1	1342.7	1334.2	1331.8	1292	1318
1402.5	1401.6	1393.1	1388.3	1345	1374
1460.7	1460.5	1450.5	1447.6	1397	1427
1519.3	1519.3	1507.4	1506.9	1443	1481
	1578.5		1561.1	1504	
	1637.5		1620.5		

[a]The signature partner of the ^{147}Eu yrast SD band has been termed $B5$ in Ref. [11].

To assess quantitatively the extent of similarity between the direct alignments Δi_d, and the respective sums, Δi_s $(=\Sigma\Delta i_m)$, of the alignments of the intermediate steps, we have calculated the root-mean-square (rms) differences between Δi_d and Δi_s over the entire SD bands for each pair of alignments compared in this work. These (rms) differences are also shown in Table I. In general, these (rms) differences are very small, clearly establishing that the additivity of the alignments is maintained with good accuracy. Notice that due to the discrepancies discussed above, larger values for the rms differences are obtained in case of ^{144}Eu and the $N=80-83$ Gd nuclei.

Looking at the variation of the root-mean-squared difference as a function of the ''mass difference'' between the nucleus of interest and the reference nucleus, Δn, one finds that (i) the values of the rms averaged over the entire SD band remain smaller than 0.10 $\hbar$ for $\Delta n \leqslant 4$;[2] (ii) the values of rms are between 0.15 $\hbar$ and 0.3 $\hbar$ for $\Delta n \geqslant 5$ except, as stated before, when the specific neutron crossings mentioned earlier are involved (for example, the rms values are very high for ^{144}Gd vs ^{152}Dy). Further, the use of ^{150}Gd yrast SD band as a reference leads to higher values of rms (for example, $\sim$0.8 for ^{148}Gd vs ^{153}Ho).

These observations suggest that the proton $\pi 6_3, 6_4$, and $\pi 7_1$, and neutron $\nu 7_1, \nu 7_2$, and $\nu 7_3$ intruder orbitals have no (or, at best, very weak) effect on the additivity of the incremental alignments. Similar results are obtained for the natural-parity (nonintruder) neutron [521]3/2, [402]5/2, and proton [301]1/2 and [404]9/2 orbitals. At the same time, significant discrepancies in the additivity of incremental alignment are observed due to the interaction involving the [651]1/2 and [642]5/2 neutron orbitals on one hand *and* $\nu 7_1$ and $\nu[514]9/2^-$ $(\alpha=-)$ on the other.

Properties other than incremental alignments, such as changes in mass, shapes, and pairing, etc., would be expected to affect the observed behavior of the SD bands. However, the fact that the alignment, obtained directly from the experimental transition energies, appears to depend predominantly on the occupation of specific single-particle orbitals, and that the additivity property holds with very good accuracy, suggest that these other contributions are acting in a mutually compensatory manner and that the main contribution to the alignment comes from the pure single-particle effects. Our results lead, therefore, to a simple picture of single-particle motion at extreme conditions, similar to that proposed in Ref. [2] for the quadrupole moments. The quality of this description is found to be very good, as indicated by the low root-mean-squared differences. It is worth noting that both the experimental (see, for example, Refs. [17,18]) and calculated [2] quadrupole moments are averaged over many transition energies (i.e., assumed constant for each band), so that there are no fluctuations as a function of the rotational frequency. The incremental alignment, on the other hand, shows a varied behavior as a function of the rotational frequency due to the orbital dependence and still the additivity is maintained over the entire range, as long as the neutron band crossings or the pairing correlations are not involved. It is hoped that more theoretical effort will be de-

[2]The typical errors in incremental alignments due to experimental uncertainties in the associated γ-ray energies are $\sim 0.02\hbar$, so that the errors in the differences between Δi_d and $\Sigma\Delta i_m$ are $\gtrsim 0.03\hbar$, depending on the number of intermediate steps involved.

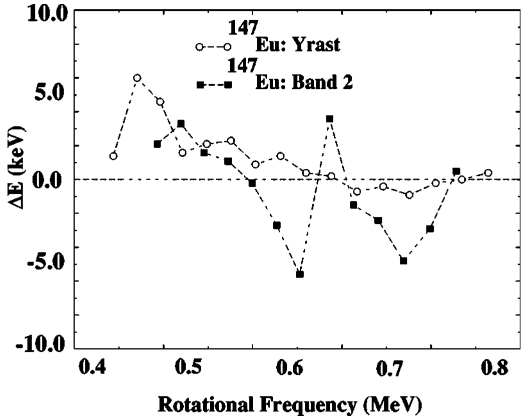

FIG. 2. Comparison between the experimental and the predicted γ-ray energies of the yrast and signature-partner SD bands in ^{147}Eu.

voted to the understanding of the physics underlying these empirical observations.

In Ref. [4], we have shown that the additivity property of incremental alignments can be employed to reproduce the transition energies in known SD bands, as also to predict those for many so far unknown bands. A dramatic experimental confirmation of some of those predictions has since become available. Using two experiments performed with Gammasphere, Haslip *et al.* [11] have reported two SD bands in ^{148}Eu and five SD bands in ^{147}Eu. In Table II, we list the predictions for the γ-ray energies of the yrast and signature-partner SD bands in ^{147}Eu, as well as the experimentally observed energies of these bands, as reported in Ref. [11]. The γ-ray energy differences between the experimental values and the calculated ones are plotted in Fig. 2. It is clear that the experimental energies and the calculated values are, indeed, very close. The rms difference between the predicted energies and the experimental values for the yrast band is $\sim$2.2 keV. Indeed, if the first three values (corresponding to the lowest rotational frequencies) are omitted, the rms difference is only $\sim$1.2 keV. The rms difference for the signature partner of the yrast band is slightly larger ($\sim$2.9 keV); still, the overall agreement between the predictions and the experimental values is unprecedented. As reported in Ref. [4], similar agreement (rms differences $\sim$3 keV) is seen also between the predicted and experimentally observed values in other known SD bands in this region.[3] This success in the predictability of the γ-ray energies of SD bands (to about $\sim$1 keV in the example of the ^{147}Eu yrast band stated above) is very intriguing and points to the need of further investigations.

The somewhat larger discrepancy between the experimental and calculated energies at low frequency for the ^{147}Eu yrast SD band is indicative of pairing correlations becoming important in this frequency range. In fact, a similar effect has been observed in all of the calculated SD bands. As was pointed out in Ref. [4], our procedure works so well primarily because the effects of pairing are minimal (except at the lowest frequencies) in the $A\sim150$ SD region, in contrast with the SD bands in the $A\sim190$ region where pairing effects contribute substantially [19]. The sudden ''jump'' observed in the energy-difference plot for band 2 at $\hbar\omega\sim0.6$ MeV (see Fig. 2) appears to correspond to an irregularity observed in the moment of inertia, $\mathcal{J}^{(2)}$, of this band at this frequency [11].

So far predictions were made only for nuclei located below the neutron SD gap $N=86$. Emboldened somewhat with the success of this procedure, we have extended our calculations beyond this SD magic number and considered the nucleus ^{151}Gd ($N=87$). Of all the known SD nuclei in the $A\sim150$ region, only ^{152}Tb and ^{153}Dy correspond to $N=87$. The ^{153}Dy yrast SD band correspond to the $^{152}\mathrm{Dy}\otimes\nu7^3$ intruder configuration, while the two SD bands known in ^{152}Tb correspond to $^{151}\mathrm{Tb}\otimes\nu[402]5/2$ (with the $\nu7_3$ orbital being empty). Bands of similar configurations are observed also in other nuclei such as ^{149}Gd, ^{151}Tb, and ^{153}Dy. Based on our procedure and the SD bands involving the $\nu[402]5/2$ orbital, the γ-ray energies of two signature-partner SD bands calculated in ^{151}Gd are listed in Table II. We keenly await a test of these predictions as more experimental data becomes available on SD bands in this region.

In summary, a systematic study of the additivity of the incremental alignment of SD bands in the $A\sim150$ region has been presented. The results show that this additivity holds with rather good accuracy, over the entire range of SD bands under study, with the configurations of the bands involved differing by a large number of particles. The neutron band crossing involving the natural-parity $\mathcal{N}=6$ orbitals (the $\nu7_1$ and $\nu[514]9/2$ ($\alpha=-$) orbitals) leads to breakdown of this additivity after (before) the backbending. The crossing by the proton intruder orbital [770]1/2, on the other hand, has a very weak effect on this additivity. It is also suggested that pairing correlations, where present, lead to similar discrepancies. The newly reported experimental energies of the yrast and signature-partner SD bands of ^{147}Eu are found to be in excellent agreement with our predictions, providing strong support for our procedure for calculating SD band energies on the basis of the additivity of incremental alignments and the associated intruder configurations. Finally, predictions were extended beyond the $N=86$ SD gap, to the $N=87$ SD nucleus ^{151}Gd.

Note added in proof. Experimental results on SD bands in ^{151}Gd have since become available [20]. The observed energies of bands 1 and 2 are in very good agreement with the predicted values listed in Table II (rms) differences <3 keV).

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[3]A 3-keV rms difference is $\leqslant$0.3% of the average γ-ray energy in the SD bands under consideration.

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