Gauging of equations method. II. Electromagnetic currents of three identical particles

A. N. Kvinikhidze* and B. Blankleider
Department of Physics, The Flinders University of South Australia, Bedford Park, SA 5042, Australia
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The gauging of equations method, introduced in the preceding paper, is applied to the four-dimensional integral equations describing the strong interactions of three identical relativistic particles. In this way we obtain gauge-invariant expressions for all possible electromagnetic transition currents of the identical three-particle system. In the three-nucleon system with no isospin violation, for example, our expressions describe the electromagnetic form factors of ^{3}H, $pd\to pd\gamma$, γ^3He$\to pd$, γ^3He$\to ppn$, etc. A feature of our approach is that gauge invariance is achieved through the coupling of the photon to all possible places in the (nonperturbative) strong interaction model. Moreover, once the proper identical particle symmetry is incorporated into the integral equations describing the strong interactions, the gauging procedure automatically provides electromagnetic transition currents with the proper symmetry. In this way the gauging of equations method results in a unified description of strong and electromagnetic interaction of strongly interacting systems.
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I. INTRODUCTION

In the preceding paper [1] (referred to as I in the following), we have introduced the gauging of equations method as a means of incorporating an external electromagnetic field into descriptions of quarks or hadrons whose strong interactions are described nonperturbatively by integral equations. The feature of this method is that it couples an external photon to all possible places in the strong interaction model despite its nonperturbative nature. Gauge invariance in our approach is therefore implemented in the way prescribed by quantum field theory (QFT). In I we demonstrated the gauging procedure in the example of three distinguishable particles. In this paper we would like to demonstrate the same method as applied to the case of indistinguishable particles where the strong interaction equations have the added complexity of identical particle symmetry.

As in I, the discussion here is restricted to the case where the three strongly interacting particles have no coupling to two-body channels. Thus we have in mind identical particle systems like three quarks qqq or three nucleons NNN. This is not an essential restriction, and indeed we have recently applied the gauging of equations method to the πNN system where coupling to the NN channel is included [2]. However, the purpose of this paper (together with I) is to present the basic details of the gauging of equations method, and as such, coupling to two-body channels presents an unnecessary complication. We note that the main results of this work have previously been summarized in conference proceedings [3].

In dealing with identical particles, one is faced with the problem of incorporating the proper particle-exchange symmetry into the equations describing both their strong and electromagnetic interactions. In quantum mechanics the standard procedure is to explicitly symmetrize (or antisymmetrize) the corresponding distinguishable particle equations. However, such a procedure is not justified within a field theoretic approach. As the basis of our approach here is QFT, we present a derivation of the strong interaction equations for three identical particles that is consistent with QFT, and that is therefore very different from the derivations found in standard texts on the quantum mechanical three-body problem [4].

Once the identical particle equations for the strong interactions are derived, it is a feature of the gauging of equations method that it may be applied directly to these equations, thereby automatically generating electromagnetic transition currents with the proper symmetry. Thus the main effort in generating practical expressions for the transition currents reduces down to a careful choice for the identical particle strong interaction equations that are to be gauged. Here we also show that for three identical particles an optimal choice is provided by an equation of Alt-Grassberger-Sandhas (AGS) form [5], but which is, however, different from the AGS equation used previously in the literature for three-nucleon calculations.

II. GAUGING THE THREE-PARTICLE GREEN FUNCTION

The gauging of equations method, introduced in I and used there to gauge the equations of three distinguishable particles, does not change when the particles are identical. Indeed, the main steps taken in I to derive the electromagnetic transition currents can be repeated for identical particles, although it is necessary to guarantee the proper identical particle symmetry of all perturbative diagrams at each step of the derivation. In this respect, it should be noted that we do not follow the common procedure of symmetrizing or antisymmetrizing the distinguishable particle results. Such a procedure is strictly valid only within the context of second quantization in quantum mechanics, while here the theoretical framework is that of relativistic quantum field theory. Instead we follow the standard rules of QFT for constructing

*On leave from Mathematical Institute of Georgian Academy of Sciences, Tbilisi, Georgia.

Green functions for identical particles. The details of these rules as applied to few-body integral equations have been given by us in Ref. [6].

The strong interactions of three identical particles are described in quantum field theory by the Green function G defined by

$$(2\pi)^4\delta^4(p_1'+p_2'+p_3'-p_1-p_2-p_3)G(p_1'p_2'p_3';p_1p_2p_3)$$
$$=\int d^4y_1d^4y_2d^4y_3d^4x_1d^4x_2d^4x_3 \times e^{i(p_1'y_1+p_2'y_2+p_3'y_3-p_1x_1-p_2x_2-p_3x_3)} \times\langle 0|T\Psi(y_1)\Psi(y_2)\Psi(y_3)\overline{\Psi}(x_1)\overline{\Psi}(x_2)\overline{\Psi}(x_3)|0\rangle. \quad (1)$$

Here Ψ and $\overline{\Psi}$ are Heisenberg fields, T is the time ordering operator, and $|0\rangle$ is the physical vacuum state. The interaction of this three-particle system with an external electromagnetic field is then described by the corresponding seven-point function G^μ defined by

$$G^\mu(k_1k_2k_3;p_1p_2p_3) =\int d^4y_1d^4y_2d^4y_3d^4x_1d^4x_2d^4x_3 \times e^{i(k_1y_1+k_2y_2+k_3y_3-p_1x_1-p_2x_2-p_3x_3)} \times\langle 0|T\Psi(y_1)\Psi(y_2)\Psi(y_3)\overline{\Psi}(x_1)\overline{\Psi}(x_2) \times\overline{\Psi}(x_3)J^\mu(0)|0\rangle, \quad (2)$$

where J^μ is the quantized electromagnetic current operator and e_i is the charge of the ith particle. If the particles are isotopic doublets, then e_i includes an isospin factor, e.g. for nucleons $e_i=\frac{1}{2}[1+\tau_3^{(i)}]e_p$ where τ_3 is the Pauli matrix for the third component of isospin, and e_p is the charge of the proton. The Ward-Takahashi (WT) identity [7], which provides an important constraint on G^μ, takes the same form as in the distinguishable particles case [1]:

$$\begin{aligned} q_\mu G^\mu(k_1k_2k_3;p_1p_2p_3) =&\, i[e_1G(k_1-q,k_2k_3;p_1p_2p_3)\\ &+e_2G(k_1,k_2-q,k_3;p_1p_2p_3)\\ &+e_3G(k_1k_2,k_3-q;p_1p_2p_3)\\ &-G(k_1k_2k_3;p_1+q,p_2p_3)e_1\\ &-G(k_1k_2k_3;p_1,p_2+q,p_3)e_2\\ &-G(k_1k_2k_3;p_1p_2,p_3+q)e_3], \end{aligned} \quad (3)$$

or in the shorthand notation introduced in Ref. [1],

$$q_\mu G^\mu(k_1k_2k_3;p_1p_2p_3) =i\sum_{i=1}^{3}[e_iG(k_i-q;p_1p_2p_3)-G(k_1k_2k_3;p_i+q)e_i]. \quad (4)$$

To be definite, we shall assume that our three identical particles are fermions (for three identical bosons one can simply replace antisymmetric operations by symmetric ones in the following discussion). Then the field theoretic expressions of Eqs. (1) and (2) automatically guarantee the proper antisymmetry of the three-particle Green function G and the seven-point function G^μ. On the other hand, the free Green function G_0 defined by

$$G_0=d_1d_2d_3, \quad (5)$$

where d_i is the dressed propagator of particle i, is not antisymmetric in its particle labels; thus, for identical fermions, G_0 is not equal to the fully disconnected part of G (which we shall denote by G_d). Indeed, it can be easily shown [6] that to obtain G_d, one needs only to antisymmetrize G_0 according to the equation

$$\sum_P G_0(1'2'3',123)=G_d(1'2'3',123), \quad (6)$$

where the sum is over all permutations P of either the initial- or final-state particle labels, and is understood to include a factor $(-1)^P=+1$ or -1 depending on whether the permutation is even or odd, respectively. In Eq. (6) we use a symbolic notation where integers represent the momenta and all quantum numbers of the corresponding particles, with primes distinguishing the final states. To specify permutation sums over just the initial-state momentum labels, we use the letter R (right); similarly, L (left) represents sums over permutations of just the final-state momentum labels. The symbol P will be used only when it makes no difference which sum R or L, is taken. Quantities antisymmetrized in one of these ways will be indicated by the appropriate superscript. Thus, for example, if A is a quantity depending on three initial and three final particle labels, then

$$A^R(1'2'3',123)=A(1'2'3',123)-A(1'2'3',213)+A(1'2'3',231)-\cdots, \quad (7)$$

$$A^L(1'2'3',123)=A(1'2'3',123)-A(2'1'3',123)+A(2'3'1',123)-\cdots, \quad (8)$$

with similar expressions holding for quantities having any number of identical legs. In general we write

$$A^R\equiv\sum_R A, \quad (9)$$

$$A^L\equiv\sum_L A, \quad (10)$$

$$A^P \equiv \sum_P A = A^R = A^L. \tag{11}$$

Defining the kernel K to be the set of all possible three-particle irreducible Feynman diagrams for the $3 \rightarrow 3$ process, we may write the Green function G as [6]

$$G = G_0^P + \frac{1}{3!} G_0 K G, \tag{12}$$

where the 1/3! factor reflects the fact that both G and K are fully antisymmetric in their particle labels. We write the disconnected part of K, indicated by subscript d, in terms of the identical particle two-body potential v:

$$K_d(1'2'3',123) = \sum_{L_c R_c} v(2'3',23) d^{-1}(1) \delta(1',1), \tag{13}$$

where $\delta(1',1)$ represents the momentum conserving Dirac δ function $(2\pi)^4 \delta^4(p_1' - p_1)$, while L_c and R_c indicate that the sum is taken over cyclic permutations of the left labels $(1'2'3')$ and right labels (123), respectively (note that the sum is restricted to cyclic permutations because the potential v is already antisymmetric in its labels [6]).

Defining

$$V_i(1'2'3,123) = v(j'k',jk) d^{-1}(i) \delta(i',i), \tag{14}$$

where (ijk) is a cyclic permutation of (123), we have that

$$K_d = \sum_{P_c} (V_1 + V_2 + V_3), \tag{15}$$

where it makes no difference over which labels, left or right, the cyclic permutations are taken. Unlike the V_i of the distinguishable particle case [see Eq. (61) of I], the one here consists of a two-body potential v that is antisymmetric under the interchange of its initial- or final-state labels. Denoting the connected part of the kernel by K_c, we define the $3 \rightarrow 3$ potential V by

$$V = \frac{1}{2}(V_1 + V_2 + V_3) + \frac{1}{6} K_c. \tag{16}$$

Although V is not fully antisymmetric, it does have the useful symmetry property

$$P_{ij} V P_{ij} = V \tag{17}$$

where P_{ij} is the operator that exchanges the ith and jth momentum, spin, and isospin labels. Since

$$K = \sum_P V, \tag{18}$$

Eq. (12) can be written as

$$G = G_0^P + G_0 V G. \tag{19}$$

Formally, Eq. (19) differs from the equivalent relation for distinguishable particles [Eq. (59) of I] only in the explicit antisymmetrization of the inhomogeneous term. We therefore proceed as for the distinguishable particle case and gauge Eq. (19) directly:

$$G^\mu = G_0^{P\mu} + (G_0 V)^\mu G + G_0 V G^\mu. \tag{20}$$

Before solving this equation for G^μ it is useful to note that

$$G_0^{P\mu} = G_0^{\mu P} = G_0^P G_0^{-1} G_0^\mu. \tag{21}$$

Indeed the combination $G_0^P G_0^{-1}$ plays the role of the antisymmetrization operator since

$$[G_0^P G_0^{-1}](1'2'3,123) = \sum_P \delta(1',1) \delta(2',2). \tag{22}$$

Note that there is no factor $\delta(3',3)$ on the right side of Eq. (22) because an overall momentum conservation delta function has been removed from our expressions. We can therefore write Eq. (20) as

$$G^\mu = (1 - G_0 V)^{-1} \left[G_0^P G_0^{-1} G_0^\mu + G_0^P G_0^{-1} \frac{1}{6} (G_0 V)^\mu G \right], \tag{23}$$

where the inclusion of $G_0^P G_0^{-1}$ in the last term is compensated exactly by 1/6 since V satisfies the symmetry property of Eq. (17) and G is already fully antisymmetric. From Eq. (19) it follows that

$$(1 - G_0 V)^{-1} G_0^P = G, \tag{24}$$

and using this we obtain

$$G^\mu = G G_0^{-1} \left[G_0^\mu + \frac{1}{6} (G_0^\mu V G + G_0 V^\mu G) \right] \tag{25}$$

$$= \frac{1}{6} G [G_0^{-1} G_0^\mu G_0^{-1} (6 G_0 + G_0 V G) + V^\mu G]. \tag{26}$$

In the last equation, we may replace the $6G_0$ by G_0^P because of the antisymmetry of the G outside the square bracket, and using Eq. (19) we finally get that

$$G^\mu = G \Gamma^\mu G, \tag{27}$$

where

$$\Gamma^\mu = \frac{1}{6} (G_0^{-1} G_0^\mu G_0^{-1} + V^\mu) \tag{28}$$

is the electromagnetic vertex function for three identical particles. The extra factor of 1/6 compared with the result for distinguishable particles reflects the fact that here Γ^μ is to be sandwiched between fully antisymmetric functions. Neglecting three-body forces, Eq. (16) implies that

$$V^\mu = \frac{1}{2} (V_1^\mu + V_2^\mu + V_3^\mu), \tag{29}$$

with an extra factor of 1/2 compared with the distinguishable particle case.

Writing Eq. (14) in the shorthand notation

$$V_i = v_i d_i^{-1}, \tag{30}$$

where v_i denotes $v(j'k',jk)$, we may gauge this equation to obtain

$$V_i^\mu = v^\mu d_i^{-1} - v_i \Gamma_i^\mu, \tag{31}$$

where Γ_i^μ is the one-particle electromagnetic vertex function defined by the equation

$$d_i^\mu = d_i \Gamma_i^\mu d_i, \tag{32}$$

and where we have used the fact that $(d_i^{-1})^\mu = -\Gamma_i^\mu$ [1]. Similarly gauging Eq. (5) we find that

$$G_0^{-1} G_0^\mu G_0^{-1} = \sum_{i=1}^{3} \Gamma_i^\mu D_{0i}^{-1}, \tag{33}$$

where

$$D_{0i} = d_j d_k. \tag{34}$$

Using these results in Eq. (28) we can express the electromagnetic vertex function as

$$\Gamma^\mu = \frac{1}{6} \sum_{i=1}^{3} \left(\Gamma_i^\mu D_{0i}^{-1} + \frac{1}{2} v_i^\mu d_i^{-1} - \frac{1}{2} v_i \Gamma_i^\mu \right). \tag{35}$$

All electromagnetic transition currents of three identical particles can be obtained from Eq. (27) by taking appropriate residues at two- and three-body bound-state poles of G. If G admits a three-body bound state, it can be shown that

$$G(p_1'p_2'p_3';p_1p_2p_3) \sim i \frac{\Psi_P(p_1'p_2'p_3')\bar{\Psi}_P(p_1p_2p_3)}{P^2 - M^2} \quad \text{as} \quad P^2 \to M^2, \tag{36}$$

where P is the total momentum, M is the bound-state mass, and Ψ_P is the three-particle bound-state wave function defined by

$$(2\pi)^4 \delta^4(P - p_1 - p_2 - p_3) \Psi_P(p_1p_2p_3) = \int d^4x_1 d^4x_2 d^4x_3 e^{i(p_1x_1 + p_2x_2 + p_3x_3)} \times \langle 0 | T\Psi(x_1)\Psi(x_2)\Psi(x_3) | P \rangle. \tag{37}$$

Here $|P\rangle$ is the eigenstate of the full Hamiltonian corresponding to the three-particle bound state with momentum P^μ.

The three-body bound-state current j^μ is found by taking left and right residues of G^μ at the three-body bound-state poles. By exposing such poles in the field theoretic expression of Eq. (2) one finds that

$$j^\mu = \langle J^\mu \rangle \equiv \langle K | J^\mu(0) | P \rangle, \tag{38}$$

where $K^2 = P^2 = M^2$. To find j^μ for a particular model, one can alternatively use Eq. (27) to take residues at the three-body bound-state poles. In this way one finds that

$$j^\mu = \bar{\Psi}_K \Gamma^\mu \Psi_P. \tag{39}$$

Although this expression is formally identical to the bound-state current for distinguishable particles [Eq. (80) of I], here the vertex function is given by Eq. (35), and the wave function is that for identical particles.

The normalization condition for the wave function in the case of identical particles follows from the fact that

$$G(G_0^{-1} - V)G = G G_0^{-1} G_0^P = 6G. \tag{40}$$

After taking residues at the three-body bound-state poles of G one finds that

$$i \bar{\Psi}_P \frac{\partial}{\partial P^2} (G_0^{-1} - V) \Psi_P = 6. \tag{41}$$

Note that this result differs from the one for distinguishable particles where unity appears on the right-hand side (RHS) [Eq. (82) of I]. This is a consequence of our convention where the same expression, Eq. (37), is used to define the bound-state wave function for both identical and distinguishable particles (in the latter case, however, the fields obtain particle labels).

We can repeat the above procedure and take residues of Eq. (27) at the two-body bound-state poles of G, thereby obtaining electromagnetic transition currents involving two-body bound states. This procedure was carried out in detail for the distinguishable particle case in I. However, as discussed in I, the expressions obtained in this way explicitly involve potentials and gauged potentials, and consequently may not be very convenient for practical calculations. Here we shall therefore forego any further discussion of this procedure, and instead go on to an alternative approach based on the Alt-Grassberger-Sandhas equations [5] which lead, after gauging, to electromagnetic transition currents expressed in terms of t matrices and gauged t matrices.

III. AGS AMPLITUDES FOR IDENTICAL PARTICLES

The AGS equations have long provided a practical way to describe the scattering of three particles in quantum mechanics. Not only do they lead (after one iteration) to equations with a connected kernel, but they also have the feature of having the two-body inputs in terms of t matrices rather than potentials. As we would also like to have these advantages in the case of three relativistic particles, we shall utilize four-dimensional versions of the AGS equations, which for distinguishable particles were given by Eqs. (118) of I.

Our goal here is to extend the discussion of Sec. III C of I to the case of three identical particles. That is, we would like to express the electromagnetic transition currents of all possible processes involving three identical particles in terms of AGS amplitudes and gauged AGS amplitudes. Although the handling of identical particles in the AGS formulation is well documented [4], as far as we know all previous discussions do this by antisymmetrizing the distinguishable particle case. As stated previously, such a procedure is inappropriate for the field theoretic approach undertaken here. In this section we shall therefore define AGS amplitudes for identical particles and relate them to the $3\to 3$ Green function G in a way that is consistent with field theory. Moreover, we do this with the view of gauging our final expressions, a task left to the following sections.

Our starting point here shall be the Green function for three identical particles as given by Eq. (19). The natural way to introduce the AGS operators U_{ij} for three identical particles is via the distinguishable particle case. Recalling that Eq. (19) differs from the one for distinguishable particles in that the inhomogeneous term G_0^P is explicitly antisymmetrized, we are led to introduce a new three-particle Green function G^D defined by the equation

$$G^D = G_0 + G_0 V G^D, \tag{42}$$

where the inhomogeneous term G_0 is not antisymmetrized. By its structure, Eq. (42) looks like the equation for the Green function of three distinguishable particles [Eq. (59) of I] and therefore allows us to define the AGS operators in the standard way. Nevertheless, G^D should not be identified with the distinguishable particle Green function as the V in Eq. (42) is defined in terms of antisymmetric potentials v [see Eqs. (14) and (16)] while the V for distinguishable particles is defined in terms of two-body potentials which are not antisymmetric. The fully antisymmetric Green function G can be obtained from G^D simply by antisymmetrizing:

$$G = G^{DL} = G^{DR} = G^{DP}. \tag{43}$$

Neglecting the three-body force V_c, we now proceed by analogy with the distinguishable particles case and define the AGS operators U_{ij} through the equation

$$G^D = G_i \delta_{ij} + G_i U_{ij} G_j, \tag{44}$$

where G_i satisfies the equation

$$G_i = G_0 + \frac{1}{2} G_0 V_i G_i \tag{45}$$

(note that the inhomogeneous term in the last equation is not antisymmetrized, in contrast to the G_i used for identical particles in Ref. [6]). The factor of 1/2 in Eq. (45) originates from Eq. (16) where it is clear that $1/2V_i$ is the disconnected potential to be identified with the distinguishable particle case. Taking this into account, the AGS equations for the operators U_{ij} become

$$U_{ij} = G_0^{-1}\bar{\delta}_{ij} + \frac{1}{2}\sum_{k=1}^{3} \bar{\delta}_{ik} T_k G_0 U_{kj},$$

$$U_{ij} = G_0^{-1}\bar{\delta}_{ij} + \frac{1}{2}\sum_{k=1}^{3} U_{ik} G_0 T_k \bar{\delta}_{kj}, \tag{46}$$

where the T_i satisfy the equation

$$T_i = V_i + \frac{1}{2} V_i G_0 T_i \tag{47}$$

and are given in terms of the two-body t matrices t_i by

$$T_i = t_i d_i^{-1}. \tag{48}$$

Note that t_i is shorthand for $t(j'k',jk)$ and is fully antisymmetric under the interchange of its initial or final particle labels.

As discussed in I, for gauging purposes it is preferable to work in terms of AGS Green functions

$$\tilde{U}_{ij} = G_0 U_{ij} G_0, \tag{49}$$

which now satisfy the equations

$$\tilde{U}_{ij} = G_0 \bar{\delta}_{ij} + \frac{1}{2}\sum_{k=1}^{3} \bar{\delta}_{ik} G_0 T_k \tilde{U}_{kj},$$

$$\tilde{U}_{ij} = G_0 \bar{\delta}_{ij} + \frac{1}{2}\sum_{k=1}^{3} \tilde{U}_{ik} T_k G_0 \bar{\delta}_{kj}. \tag{50}$$

By using the above equations one can show that

$$G^D = G_0 + \frac{1}{2}\sum_i G_0 T_i G_0 + \frac{1}{4}\sum_{i,k} G_0 T_i \tilde{U}_{ik} T_k G_0. \tag{51}$$

In this way we obtain that

$$G = G_0^P + \frac{1}{2}\sum_i G_0 T_i G_0^P + \frac{1}{4}\sum_{i,k} G_0 T_i \tilde{U}_{ik} T_k G_0^P \tag{52}$$

$$= G_0^P + \frac{1}{2}\sum_i G_0^P T_i G_0 + \frac{1}{4}\sum_{i,k} G_0^P T_i \tilde{U}_{ik} T_k G_0. \tag{53}$$

By taking appropriate residues, either of these relations allows us to obtain the scattering amplitudes for all possible processes in the system of three identical particles. To be specific, let us choose Eq. (52) to discuss the taking of residues. As only the last term in Eq. (52) is connected, only this term need be considered for the purposes of extracting physical three-particle amplitudes. We therefore define

$$G_c = \frac{1}{4}\sum_{i,k} G_0 T_i \tilde{U}_{ik} T_k G_0^P. \tag{54}$$

By writing out this sum explicitly and making use of the symmetry properties of $\tilde{U}_{ik}$ discussed in Appendix A, it is possible to rewrite Eq. (54) in terms of just one AGS-like Green function $\tilde{U}$:

$$G_c = \sum_{L_c R_c} G_0 T_1 \tilde{U} T_1 G_0. \tag{55}$$

A detailed derivation of Eq. (55) is presented in Appendix C. The full Green function can then be written as

$$G=G_0^P+\sum_{L_cR_c}(G_0T_1G_0+G_0T_1\tilde{U}T_1G_0). \tag{56}$$

As discussed in Appendix C, there is a variety of ways to choose $\tilde{U}$ without affecting the value of G. One form that has previously been used in three-nucleon calculations [8] is

$$\tilde{U}=\frac{1}{2}\tilde{X}, \tag{57}$$

with $\tilde{X}$ obeying the equation [see Eq. (C4)]

$$\tilde{X}=G_0\mathcal{P}+\frac{1}{2}\mathcal{P}G_0T_1\tilde{X}, \tag{58}$$

where $\mathcal{P}=P_{12}P_{31}+P_{31}P_{12}$ is the sum of two successive cyclic permutations. The choice for $\tilde{U}$ specified by Eq. (57) is unsatisfactory for our purposes since the presence of a sum of two permutations in both the inhomogeneous term and kernel of Eq. (58) makes the gauging of this expression particularly cumbersome. Fortunately there is another way to choose $\tilde{U}$ that avoids these difficulties. As shown in Appendix C, we can take

$$\tilde{U}=-\tilde{Z}P_{12} \tag{59}$$

where the AGS-like Green function $\tilde{Z}$ obeys the equation

$$\tilde{Z}=G_0-G_0P_{12}T_1\tilde{Z}, \tag{60}$$

with no permutation sums involved. This is the form for $\tilde{U}$ that we shall use in the next section for the purposes of gauging. By displaying all momentum variables in Eq. (55) it is easy to see that the connected part of the Green function can be written directly in terms of $\tilde{Z}$ as

$$G_c=\sum_{L_cR_c}G_0T_1\tilde{Z}T_2G_0. \tag{61}$$

A. $Nd\to Nd$ amplitude

For notational purposes, we shall refer to our three identical strongly interacting fermions as a ''nucleons'' (NNN) although the true identity of these particles is arbitrary. Similarly we refer to a two-body bound state as a ''deuteron'' (d) and a three-body bound state as ''^{3}H.'' This enables us to write the various reactions that can take place between any three identical particles in a familiar way.

Using this notation, we can obtain the amplitude for $Nd\to Nd$, by follow the usual procedure of taking residues at the two-body bound-state poles of G. Indeed for identical particles it can be shown that if quantum field theory admits the existence of two-body bound states, then the Green function $G(k_1k_2k_3;p_1p_2p_3)$ possesses poles with respect to any of the variables $(k_i+k_j)^2$ or $(p_i+p_j)^2$. To be definite, consider the variables $(k_2+k_3)^2$ and $(p_2+p_3)^2$ for the three-particle system. In the vicinity of the corresponding two-body poles, only the connected part of G contributes and we have that

$$G_c(k_1k_2k_3;p_1p_2p_3)\sim i\frac{\psi_{K_1}(k_2k_3)d(k_1)}{(k_2+k_3)^2-m^2}T_{dd}(k_1K_1;p_1P_1)i\frac{d(p_1)\bar{\psi}_{P_1}(p_2p_3)}{(p_2+p_3)^2-m^2}, \tag{62}$$

where $K_1=k_2+k_3$, $P_1=p_2+p_3$, ψ_{K_1} is the deuteron wave function [defined analogously to Eq. (37)], m is the mass of the deuteron, and $T_{dd}(k_1K_1;p_1P_1)$ is the physical scattering amplitude for $N(p_1)+d(P_1)\to N(k_1)+d(K_1)$. The same scattering amplitude could be picked out in the other channels, for example,

$$G_c(k_1k_2k_3;p_1p_2p_3)\sim i\frac{\psi_{K_1}(k_2k_3)d(k_1)}{(k_2+k_3)^2-m^2}T_{dd}(k_1K_1;p_3P_3)i\frac{d(p_3)\bar{\psi}_{P_3}(p_1p_2)}{(p_1+p_2)^2-m^2}, \tag{63}$$

where $T_{dd}(k_1K_1;p_3P_3)$ depends on the variables p_3, P_3 in just the same way as $T_{dd}(k_1K_1;p_1P_1)$ depends on p_1, P_1. On the other hand, G_c is given by Eq. (55) which when written out explicitly reads

$$G_c(k_1k_2k_3;p_1p_2p_3)=\sum_{L_cR_c}\int dk_2'\,dp_2'\,D_0(k_2k_3)t(k_2k_3;k_2'k_3')\times\tilde{U}(k_1k_2'k_3';p_1p_2'p_3')t(p_2'p_3';p_2p_3)D_0(p_2p_3), \tag{64}$$

where $k_2'+k_3'=K_1$ and $p_2'+p_3'=P_1$. In Eq. (64), D_0 is the free two-body propagator, and the poles at $(k_2+k_3)^2=m^2$ and $(p_2+p_3)^2=m^2$ are contained in the two-body t matrices $t(k_2k_3;k_2'k_3')$ and $t(p_2'p_3';p_2p_3)$, respectively. In particular,

$$t(k_2k_3;k_2'k_3')\sim i\frac{\phi_{K_1}(k_2k_3)\bar{\phi}_{K_1}(k_2'k_3')}{K_1^2-m^2}, \tag{65}$$

where ϕ and $\bar{\phi}$ are two-body bound-state vertex functions defined by the equations

$$\psi=G_0\phi,\quad \bar{\psi}=\bar{\phi}G_0. \tag{66}$$

Note that the two-body t matrix satisfies the integral equation

$$t=v+\frac{1}{2}vD_0t. \tag{67}$$

With particle i as spectator, this equation when multiplied by d_i^{-1} gives Eq. (47). The full two-body Green function D is given by analogy with Eq. (12) as

$$D=D_0^P+\frac{1}{2}D_0vD. \tag{68}$$

The normalization condition for our two-body wave function is therefore given by

$$i\bar{\psi}_P \frac{\partial}{\partial P^2}\left(D_0^{-1}-\frac{1}{2}v\right)\psi_P=2, \tag{69}$$

a fact that follows from the same argument that led to Eq. (41) but adapted to the case of two identical particles.

Comparing Eq. (62) with Eq. (64) in the vicinity of the two poles gives the Nd elastic scattering amplitude as

$$\begin{aligned} T_{dd}(k_1K_1;p_1P_1) &= \int dk_2'\,dp_2'\,d^{-1}(k_1)\bar{\phi}_{K_1}(k_2'k_3') \\ &\times \tilde{U}(k_1k_2'k_3';p_1p_2'p_3')\phi_{P_1}(p_2'p_3')d^{-1}(p_1). \end{aligned} \tag{70}$$

This equation can be written symbolically as

$$T_{dd}=d_1^{-1}\bar{\phi}_1\tilde{U}\,\phi_1 d_{p_1}^{-1} \tag{71}$$

or in terms of deuteron wave functions as

$$T_{dd}=\bar{\psi}_1 U\,\psi_1. \tag{72}$$

B. $Nd\to NNN$ amplitude

The amplitude for the breakup reaction $Nd\to NNN$ is found by taking the residue of the Green function G at the initial-state deuteron pole. Choosing momentum variables as above, we may express the connected part of the Green function G_c in the vicinity of the initial bound-state pole as

$$\begin{aligned} G_c(k_1k_2k_3;p_1p_2p_3) &\sim d(k_1)d(k_2)d(k_3)\,T_{0d}(k_1k_2k_3;p_1P_1) \\ &\times i\frac{d(p_1)\bar{\psi}_{P_1}(p_2p_3)}{(p_2+p_3)^2-m^2}, \end{aligned} \tag{73}$$

where $(p_2+p_3)^2\to m^2$. This relation defines the breakup amplitude $T_{0d}(k_1k_2k_3;p_1P_1)$. Comparing this with the behavior of Eq. (64) in the vicinity of the same pole we deduce that

$$\begin{aligned} T_{0d}(k_1k_2k_3;p_1P_1) &= \sum_{L_c}\int dk_2'\,dp_2' d^{-1}(k_1)t(k_2k_3;k_2'k_3') \\ &\times \tilde{U}(k_1k_2'k_3';p_1p_2'p_3')\phi_{P_1}(p_2'p_3')d^{-1}(p_1). \end{aligned} \tag{74}$$

Written symbolically this gives

$$T_{0d}=\sum_{L_c} T_1\tilde{U}\phi_1 d_1^{-1}=\sum_{L_c} T_1 G_0 U\psi_1. \tag{75}$$

Using Eq. (59) and Eq. (60) we may eliminate T_1 from the last equation to obtain

$$T_{0d}=\sum_{L_c}(P_{12}ZP_{12}-G_0^{-1})\psi_1. \tag{76}$$

IV. GAUGING THE IDENTICAL PARTICLE AGS EQUATIONS

Having developed the necessary expressions describing three identical particles in the purely strong interaction sector, we are now ready to carry out the gauging procedure that will generate the coupling to an external electromagnetic field. We follow the same procedure as used for distinguishable particles in Sec. III C of I.

A. $Nd\to Nd$ transition current

The $Nd\to Nd$ electromagnetic transition current j^μ_{dd} describes, for example, the process $Nd\to\gamma Nd$. To obtain the expression for j^μ_{dd} we write Eq. (71) as

$$\tilde{T}_{dd}=\bar{\phi}_1\tilde{U}\,\phi_1, \tag{77}$$

where

$$\tilde{T}_{dd}=d_1T_{dd}d_1. \tag{78}$$

Gauging Eq. (77) gives

$$\tilde{T}^\mu_{dd}=\bar{\phi}^\mu_1\tilde{U}\phi_1+\bar{\phi}_1\tilde{U}\phi^\mu_1+\bar{\phi}_1\tilde{U}^\mu\phi_1. \tag{79}$$

The $Nd\to Nd$ electromagnetic transition current is then given by

$$j^\mu_{dd}=d_1^{-1}(\bar{\phi}^\mu_1\tilde{U}\phi_1+\bar{\phi}_1\tilde{U}\phi^\mu_1+\bar{\phi}_1\tilde{U}^\mu\phi_1)d_1^{-1}. \tag{80}$$

In Eq. (79), $\bar{\phi}^\mu_1$ and ϕ^μ_1 are the gauged two-body bound-state vertex functions which follow from the solution of the two-body problem for particles 2 and 3. Dropping the spectator particle label, the bound-state equation for ϕ is

$$\phi=\frac{1}{2}vD_0\phi. \tag{81}$$

Gauging this equation and using Eq. (68) gives

$$\phi^\mu=\frac{1}{4}D_0^{-1}D[vD_0]^\mu\phi. \tag{82}$$

By using the fact that

$$D=G_0^P+D_0tD_0, \tag{83}$$

the previous equation can also be written as

$$\phi^\mu=\left(1+\frac{1}{2}tD_0\right)\left(D_0^{-1}D_0^\mu D_0^{-1}+\frac{1}{2}v^\mu\right)\psi-D_0^{-1}D_0^\mu D_0^{-1}\psi, \tag{84}$$

which is the identical particle version of Eq. (38) of I.

To determine j^μ_{dd} all that is left is to specify a practical expression for $\tilde{U}^\mu$. If we make the choice $\tilde{U}=-\tilde{Z}P_{12}$ as in

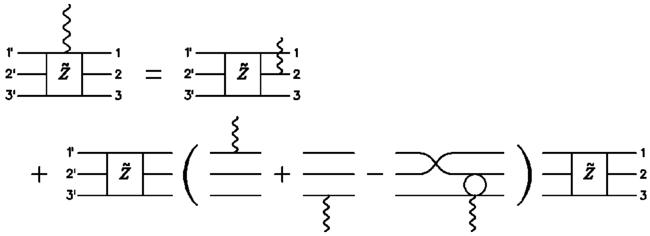

FIG. 1. Graphical representation of Eq. (87) for the gauged AGS-like Green function $\tilde{Z}^\mu$.

Eq. (59), then $\tilde{U}^\mu = -\tilde{Z}^\mu P_{12}$ and the problem reduces to that of gauging Eq. (60) in order to obtain $\tilde{Z}^\mu$. Equation (60) is a relatively simple equation that has only one type of disconnectedness in the kernel, and unlike the corresponding equation for the distinguishable particle case [see Eqs. (126) of I], Eq. (60) is not a matrix equation. We write Eq. (60) as

$$\tilde{Z} = G_0 - D_0 t P_{12} \tilde{Z}, \tag{85}$$

where it is to be understood that $t = t_2$ and $D_0 = d_3 d_1$. Gauging this equation gives

$$\tilde{Z}^\mu = G_0^\mu - D_0^\mu t P_{12} \tilde{Z} - D_0 t^\mu P_{12} \tilde{Z} - D_0 t P_{12} \tilde{Z}^\mu, \tag{86}$$

so that

$$(1 + D_0 t P_{12}) \tilde{Z}^\mu = G_0^\mu - D_0^\mu t P_{12} \tilde{Z} - D_0 t^\mu P_{12} \tilde{Z}.$$

With the help of Eq. (85) we then obtain

$$\tilde{Z}^\mu = \tilde{Z} G_0^{-1} G_0^\mu - \tilde{Z} G_0^{-1} D_0^\mu t P_{12} \tilde{Z} - \tilde{Z} G_0^{-1} D_0 t^\mu P_{12} \tilde{Z}.$$

Using

$$G_0^{-1} D_0 = d^{-1} \quad \text{and} \quad G_0^\mu = d^\mu D_0 + d D_0^\mu,$$

where $d = d_2$ and $d^\mu = d_2^\mu$, then gives

$$\tilde{Z}^\mu = \tilde{Z} d^{-1} d^\mu + \tilde{Z} D_0^{-1} D_0^\mu G_0^{-1} (G_0 - D_0 t P_{12} \tilde{Z}) - \tilde{Z} d^{-1} t^\mu P_{12} \tilde{Z}$$

and therefore

$$\tilde{Z}^\mu = \tilde{Z} d^{-1} d^\mu + \tilde{Z} (D_0^{-1} D_0^\mu D_0^{-1} d^{-1} - d^{-1} t^\mu P_{12}) \tilde{Z}. \tag{87}$$

This equation, illustrated in Fig. 1, describes the attachment of photons at all possible places in the multiple-scattering series of three identical particles. As such, it forms the central result in the gauged identical three-quark or three-hadron problem. The structure of this equation may seem surprising in that the second line is gauged only when it is right external [the first term on the RHS of Eq. (87)] whereas the first and third lines are gauged everywhere (because $\tilde{Z}$ contains all possible diagrams, including G_0). That there is no inconsistency can be seen graphically from Fig. 2. There we show a contribution to the term $\tilde{Z}(D_0^{-1} D_0^\mu D_0^{-1} d^{-1})\tilde{Z}$ where the single-scattering contribution to each $\tilde{Z}$ is used and the intermediate line 1 is gauged. Because this is a Feynman diagram, the gauging of the intermediate line 1 is the same as the gauging of the left external line 2. To be noted is the crucial role the permutation operator P_{12} plays in the equation for $\tilde{Z}$, Eq. (60)—this operator is responsible for the crossing of the lines in the single-scattering contributions shown in Fig. 2.

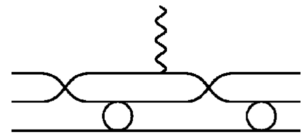

FIG. 2. A contribution to the term $\tilde{Z}(D_0^{-1} D_0^\mu D_0^{-1} d^{-1})\tilde{Z}$ showing how it leads to the gauging of the left external particle 2.

B. $Nd \to NNN$ transition current

The t matrix T_{0d} for the breakup process $Nd \to NNN$ was given in Eq. (76). We follow the same procedure as given in Sec. III C 4 for distinguishable particles. Thus we do not gauge T_{0d} directly but instead introduce the Green function quantity

$$\tilde{T}_{0d} = G_0 T_{0d} d_1 = \sum_{L_c} (P_{12} \tilde{Z} P_{12} - G_0) \phi_1. \tag{88}$$

The electromagnetic current for $Nd \to \gamma NNN$ is then given by

$$j_{0d}^\mu = G_0^{-1} \tilde{T}_{0d}^\mu d_1^{-1}. \tag{89}$$

Gauging $\tilde{T}_{0d}$, one obtains

$$j_{0d}^\mu = \sum_{L_c} (P_{12} G_0^{-1} \tilde{Z}^\mu G_0^{-1} P_{12} - G_0^{-1} G_0^\mu G_0^{-1}) \psi_1 + \sum_{L_c} (P_{12} Z P_{12} - G_0^{-1}) D_{01} \phi_1^\mu. \tag{90}$$

C. Three-nucleon bound-state current

For three identical nucleons the electromagnetic bound-state current j^μ was expressed in terms of the two-nucleon potential v and gauged potential v^μ by Eq. (35) and Eq. (39). This was achieved by expressing the seven-point function as $G^\mu = G \Gamma^\mu G$ and then taking the left and right residues of G^μ at the three-body bound-state poles. The connection with j^μ follows from the general structure of G^μ in the vicinity of these poles:

$$G^\mu = \frac{i \Psi_K}{K^2 - M^2} j^\mu \frac{i \bar{\Psi}_P}{P^2 - M^2}. \tag{91}$$

In this subsection we shall determine an alternative expression for j^μ that is given in terms of the two-nucleon t matrix t and gauged t matrix t^μ. To do this, we again take left and right residues of G^μ at the three-body bound-state poles;

however, this time we use an expression for G^μ which is obtained by gauging Eq. (56) where G is written in terms of the AGS Green function $\tilde{U}$. For the purpose of taking residues it is sufficient to gauge just the connected part of G. Thus, making the choice $\tilde{U}=-\tilde{Z}P_{12}$, we are led to the gauging of Eq. (61):

$$G_c^\mu=\sum_{L_cR_c}[(G_0T_1)^\mu\tilde{Z}T_2G_0+G_0T_1\tilde{Z}^\mu T_2G_0+G_0T_1\tilde{Z}(T_2G_0)^\mu]. \tag{92}$$

As discussed in Appendix C 2, $\tilde{Z}$ contains the three-nucleon bound-state pole with the pole structure being given by Eq. (C22). The bound-state poles of G_c^μ can therefore be revealed by simply using Eq. (87) to express the middle term of Eq. (92) (containing $\tilde{Z}^\mu$) in terms of $\tilde{Z}$ factors. It is seen that only this middle term contains both the left and right bound-state poles; thus, in the vicinity of these poles one can write the seven-point function as

$$G^\mu=\sum_{L_cR_c}G_0T_1\frac{i\Psi_2^K\bar{\Psi}_1^K}{K^2-M^2}(D_0^{-1}D_0^\mu D_0^{-1}d^{-1}-d^{-1}t^\mu P_{12})\frac{i\Psi_2^P\bar{\Psi}_1^P}{P^2-M^2}T_2G_0. \tag{93}$$

Now using the results of Appendix B, we have that $\Psi_K=\Sigma_{L_c}G_0T_1\Psi_2^K$ and $\bar{\Psi}_P=\Sigma_{R_c}\bar{\Psi}_1^PT_2G_0$. Comparing with Eq. (91) and using $\Psi_2=-P_{12}\bar{\Psi}_1$ then gives the desired expression

$$j^\mu=\bar{\Psi}_1^K[d_2^{-1}t_2^\mu-(\Gamma_1^\mu d_3^{-1}+\Gamma_3^\mu d_1^{-1})d_2^{-1}P_{12}]\Psi_1^P. \tag{94}$$

Using the bound-state equations for Ψ_1 and $\bar{\Psi}_1$ [Eq. (B11) and Eq. (B12)], it is easy to see that

$$\bar{\Psi}_1^K\Gamma_1^\mu d_3^{-1}d_2^{-1}P_{12}\Psi_1^P=\bar{\Psi}_1^K\Gamma_2^\mu d_3^{-1}d_1^{-1}P_{12}\Psi_1^P. \tag{95}$$

We may thus write Eq. (94) as

$$j^\mu=\bar{\Psi}_1^KP_{12}d_1^{-1}t_1^\mu P_{12}\Psi_1^P-\bar{\Psi}_1^K(\Gamma_2^\mu d_3^{-1}d_1^{-1}+\Gamma_3^\mu d_1^{-1}d_2^{-1})P_{12}\Psi_1^P, \tag{96}$$

whose form corresponds to the bound-state current given by us in Ref. [9] in the context of the spectator approach to the three-nucleon system.

Equation (96) may appear to be in a form where the first term on the RHS corresponds to the two-body interaction current and the second term corresponds to the one-body current. Yet this is not the case since t^μ in fact contains both types of contribution. To see this explicitly we gauge Eq. (67) for t in this way obtaining

$$t^\mu=\frac{1}{2}tD_0^\mu t+\left(1+\frac{1}{2}tD_0\right)v^\mu\left(1+\frac{1}{2}D_0t\right). \tag{97}$$

The first term on the RHS corresponds to a one-body current while the second term gives the two-body interaction current. Thus the total one-body current contribution to the three-nucleon bound-state current is

$$j^\mu_{\text{one-body}}=\bar{\Psi}_1^K\left[P_{12}\frac{1}{2}d_1^{-1}t_1D_0^\mu t_1P_{12}-(\Gamma_2^\mu d_3^{-1}d_1^{-1}+\Gamma_3^\mu d_1^{-1}d_2^{-1})P_{12}\right]\Psi_1^P. \tag{98}$$

Again using the bound-state equations for Ψ_1 and $\bar{\Psi}_1$, we may write this result as

$$j^\mu_{\text{one-body}}=\bar{\Psi}_1^K(\Gamma_2^\mu d_3^{-1}d_1^{-1}+\Gamma_3^\mu d_1^{-1}d_2^{-1})\left(\frac{1}{2}-P_{12}\right)\Psi_1^P. \tag{99}$$

D. Current conservation

To prove current conservation for observables expressed in terms of $\tilde{Z}^\mu$, it is useful to first deduce the WT identity for $\tilde{Z}^\mu$. To do this we write Eq. (87) as

$$\tilde{Z}^\mu=\tilde{Z}d_2^{-1}d_2^\mu+\tilde{Z}\Lambda_2^\mu\tilde{Z}, \tag{100}$$

where

$$\Lambda_2^\mu=(\Gamma_1^\mu d_3^{-1}+\Gamma_3^\mu d_1^{-1})d_2^{-1}-d_2^{-1}t_2^\mu P_{12}, \tag{101}$$

and then follow the procedure of I in Sec. III B 2. Thus we introduce the quantities

$$\hat{e}_i(k_1k_2k_3,p_1p_2p_3)=ie_i(2\pi)^{12}\delta^4(k_i-p_i-q)\times\delta^4(k_j-p_j)\,\delta^4(k_k-p_k), \tag{102}$$

where ijk represent a cyclic ordering of 123. This allows us to write the WT identities for the gauged two-body potential and gauged one-particle propagator in three-particle space in terms of commutators as

$$q_\mu v_i^\mu I_i=[\hat{e}_j+\hat{e}_k,v_i],\quad q_\mu d_i^\mu I_jI_k=[\hat{e}_i,d_i], \tag{103}$$

where I_i, I_j, and I_k are unit operators in the space of particles i, j, and k respectively. Using Eqs. (103), it is then easy to see that

$$q_\mu t_2^\mu I_2=[\hat{e}_3+\hat{e}_1,t_2],\quad q_\mu\Gamma_iI_jI_k=-[\hat{e}_i,d_i^{-1}]. \tag{104}$$

The first term on the RHS of Eq. (100) contracted with q_μ thus gives

$$q_\mu\tilde{Z}d_2^{-1}d_2^\mu=q_\mu\tilde{Z}\Gamma_2^\mu d_2=-\tilde{Z}[\hat{e}_2,d_2^{-1}]d_2=-\tilde{Z}[\hat{e}_2,G_0^{-1}]G_0, \tag{105}$$

while for the last term of Eq. (100) we first deduce that

$$q_\mu\Lambda_2^\mu=-[\hat{e}_3+\hat{e}_1,G_0^{-1}]-[\hat{e}_3+\hat{e}_1,t_2]d_2^{-1}P_{12}. \tag{106}$$

By using

$$\tilde{Z}^{-1}=G_0^{-1}+d_2^{-1}t_2P_{12}, \tag{107}$$

which follows from Eq. (85), the previous equation reduces to

$$q_\mu\Lambda_2^\mu=-[\hat{e}_3+\hat{e}_1,\tilde{Z}^{-1}]+(G_0^{-1}-\tilde{Z}^{-1})(\hat{e}_1-\hat{e}_2). \tag{108}$$

The WT identity for the last term of Eq. (100) thus becomes

$$q_\mu\tilde{Z}\Lambda_2^\mu\tilde{Z}=[\hat{e}_3+\hat{e}_1,\tilde{Z}]-(\hat{e}_1+\hat{e}_2)\tilde{Z}+\tilde{Z}G_0^{-1}(\hat{e}_1-\hat{e}_2)\tilde{Z}. \tag{109}$$

To simplify this expression further we use Eq. (107) and the fact that $[\hat{e}_2,\hat{t}_2]=0$, to obtain

$$\begin{aligned}\tilde{Z}G_0^{-1}\hat{e}_1\tilde{Z}&=\hat{e}_1\tilde{Z}-\tilde{Z}d_2^{-1}t_2P_{12}\hat{e}_1\tilde{Z}=\hat{e}_1\tilde{Z}-\tilde{Z}d_2^{-1}\hat{e}_2t_2P_{12}\tilde{Z}\\&=\hat{e}_1\tilde{Z}-\tilde{Z}G_0^{-1}\hat{e}_2G_0d_2^{-1}t_2P_{12}\tilde{Z}=\hat{e}_1\tilde{Z}+\tilde{Z}G_0^{-1}\hat{e}_2\tilde{Z}\\&\quad-\tilde{Z}G_0^{-1}\hat{e}_2G_0.\end{aligned} \tag{110}$$

Substituting this result into Eq. (109), we obtain

$$q_\mu\tilde{Z}\Lambda_2^\mu\tilde{Z}=[\hat{e},\tilde{Z}]+\tilde{Z}[\hat{e}_2,G_0^{-1}]G_0, \tag{111}$$

where $\hat{e}=\hat{e}_1+\hat{e}_2+\hat{e}_3$. Combining Eq. (111) with Eq. (105) we finally obtain the WT for $\tilde{Z}^\mu$:

$$q_\mu\tilde{Z}^\mu=[\hat{e},\tilde{Z}]. \tag{112}$$

The expression for the three-nucleon bound-state current j^μ, Eq. (94), can be written as

$$j^\mu=\bar{\Psi}_1^K\Lambda_2^\mu\Psi_2^P \tag{113}$$

and on comparison with Eq. (100) is recognized to be the result of taking simultaneous residues of $\tilde{Z}^\mu$ at the initial and final bound-state poles. Taking such simultaneous residues of Eq. (112), the left-hand side of this equation gives $q_\mu j^\mu$, while the right-hand side gives zero since $\tilde{Z}$ has only a single pole; in this way we obtain the current conservation equation for the bound-state current: $q_\mu j^\mu=0$.

To prove that the $Nd\to Nd$ electromagnetic transition current of Eq. (80) is conserved, we write the WT identity for ϕ_1^μ and $\bar{\phi}_1^\mu$ [see Eqs. (153) of I] in three-particle space as

$$q_\mu\phi_1^\mu I_1=(\hat{e}_2+\hat{e}_3)\phi_1,\quad q_\mu\bar{\phi}_1^\mu I_1=-\bar{\phi}_1(\hat{e}_2+\hat{e}_3), \tag{114}$$

and note that Eq. (112) implies

$$q_\mu\tilde{U}^\mu=[\hat{e},\tilde{U}], \tag{115}$$

where $\tilde{U}=-\tilde{Z}P_{12}$. Using the last three equations, we obtain from Eq. (80) that

$$\begin{aligned}q_\mu j_{dd}^\mu&=d_1^{-1}\bar{\phi}_1[\hat{e}_1,\tilde{U}]\phi_1d_1^{-1}=d_1^{-1}\hat{e}_1d_1\bar{\phi}_1d_2d_3Ud_2d_3\phi_1\\&\quad-\bar{\phi}_1d_2d_3Ud_2d_3\phi_1d_1\hat{e}_1d_1^{-1}.\end{aligned} \tag{116}$$

By shifting momentum arguments, the $\hat{e}_1$ factors stop the cancellation of external d_1^{-1} terms with the neighboring d_1 propagators contained in $\tilde{U}$. Thus for on-mass-shell nucleons $d_1^{-1}=0$, and both terms on the RHS of Eq. (116) become zero.

E. Normalization condition and charge conservation

The normalization condition for the three-body bound-state wave function that was given in Eq. (41) follows almost immediately from the basic integral equation for G, Eq. (19). The expression obtained gives the normalization condition in terms of the three-body potential V and therefore in terms of input two-body potentials. The disadvantage of these quantities has already been discussed. To obtain the normalization condition without explicit reference to V we follow a similar procedure to that for G, but instead use the AGS Green function $\tilde{Z}$. From Eq. (107) it follows that

$$\tilde{Z}=\tilde{Z}(G_0^{-1}+d_2^{-1}t_2P_{12})\tilde{Z}. \tag{117}$$

Using the pole structure of $\tilde{Z}$ given in Eq. (C22) and then taking residues, one obtains

$$i\bar{\Psi}_1^P\frac{\partial}{\partial P^2}(G_0^{-1}+d_2^{-1}t_2P_{12})\Psi_2^P=1, \tag{118}$$

which is the normalization condition expressed in terms of the input two-body t matrix and Faddeev components of the bound-state wave function.

Analogous to the normalization condition for the three-body bound-state wave function is a condition known as ''charge conservation'' for the three-body bound-state electromagnetic current. The field theoretic definition of the three-body bound-state current is given by Eq. (38). From this expression, the translational invariance of the electromagnetic current operator, and the fact that $|P\rangle$ is an eigenstate of the charge operator, it is easy to show that

$$j^\mu(P,P)=2eP^\mu, \tag{119}$$

where e is the charge of the three-body bound state. Equation (119) constitutes the charge conservation condition and corresponds to the fact that the full charge of the three particles is being probed by the external electromagnetic probe. As such, it is an essential condition that needs to be satisfied by any model calculation. Within our model the expression for j^μ is given by Eq. (96). Assuming the Ward identities for the one- and two-body input, it is a matter of simple algebra to prove that this expression does indeed satisfy the charge conservation condition of Eq. (119). Such a proof has already been given by us in Ref. [9] for the case of the three-nucleon bound-state current within the spectator approach. As there is a direct one-to-one correspondence between Eq. (96) and the bound-state current in the spectator approach [Eq. (25) of Ref. [9]], this proof need not be repeated here.

V. SUMMARY

We have applied the gauging of equations method, introduced in the preceding paper for distinguishable particles [1], to the integral equations describing the strong interactions of three identical relativistic particles. For simplicity of presentation, we restricted the discussion to identical particle systems like that of three quarks (qqq) or three nucleons (NNN) where there is no strong interaction coupling to two-particle channels. Once the strong interaction equations are specified, the gauging of equations method couples an external photon to all possible places in the strong interaction model, while at the same time preserving the proper identical particle symmetry of the original equations. In this way we have obtained gauge-invariant expressions for the various electromagnetic transition currents of such identical three-particle systems.

Two essentially different integral equations were gauged. The first was the integral equation for the three-particle Green function G, Eq. (12). This equation has a disconnected kernel that is defined in terms of the two-body potential v. As a result, the electromagnetic transition currents that follow are themselves expressed in terms of v and the gauged potential v^μ. Although formally correct, such expressions for the transition currents may not be the most useful for practical calculations. We have therefore considered an alternative formulation of the strong interaction problem in terms of four-dimensional versions of the well-known AGS equations. Such integral equations have a kernel that is connected (after one iteration) and are expressed in terms of two-body t matrices. For identical particles there are many ways to define the AGS amplitude, all giving the same three-body Green function G. We have introduced an AGS amplitude that satisfies a particularly simple AGS equation, Eq. (60), where the inhomogeneous term consists of a single unpermuted term. This equation is ideal for our gauging procedure. By contrast, the AGS equation used previously in three-nucleon calculations [8] is inconvenient for gauging purposes as both its inhomogeneous term and kernel consist of sums over two different permutations. After gauging, our AGS equation gives practical expressions in terms of the two-body t matrix t and gauged t matrix t^μ, for all the electromagnetic transition currents of three identical particles. In this sense, the gauging of equations method has provided a unified description of the strong and electromagnetic interactions of the identical three-particle system.

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APPENDIX A: SYMMETRY PROPERTIES OF $\tilde{U}_{ij}$ FOR IDENTICAL PARTICLES

Here we derive some useful properties of the AGS Green functions $\tilde{U}_{ik}$ in the case of identical particles. Although these properties are well known, we include them here both for completeness and to show how they may be derived within the formalism used in this paper.

Using the definition of $\tilde{U}_{ik}$ given in Eq. (44), it is easy to show that

$$\tilde{U}_{ik}=\overline{\delta}_{ik}G_0+\frac{1}{2}\sum_j \overline{\delta}_{ij}G_0V_jG_0\overline{\delta}_{jk}+\frac{1}{4}\sum_{jn}\overline{\delta}_{ij}G_0V_jG^DV_nG_0\overline{\delta}_{nk}. \tag{A1}$$

For the case $k=1$ we explicitly have that

$$\tilde{U}_{11}=\frac{1}{2}G_0(V_2+V_3)G_0+\frac{1}{4}G_0(V_2+V_3)G^D(V_2+V_3)G_0,$$

$$\tilde{U}_{21}=G_0+\frac{1}{2}G_0V_3G_0+\frac{1}{4}G_0(V_1+V_3)G^D(V_2+V_3)G_0,$$

$$\tilde{U}_{31}=G_0+\frac{1}{2}G_0V_2G_0+\frac{1}{4}G_0(V_1+V_2)G^D(V_2+V_3)G_0. \tag{A2}$$

From Eqs. (17) and (42) it follows that G^D commutes with all elements of the symmetry group S_3: $P_{ij}G^DP_{ij}=G^D$ (of course for G there is the stronger symmetry property $P_{ij}G=GP_{ij}=-G$). Now using the fact that

$$P_{23}V_1P_{23}=V_1,\quad P_{23}V_2P_{23}=V_3,\quad P_{23}V_3P_{23}=V_2, \tag{A3}$$

the following two relations are easily deduced from Eqs. (A2):

$$P_{23}\tilde{U}_{21}P_{23}=\tilde{U}_{31},\quad P_{23}\tilde{U}_{31}P_{23}=\tilde{U}_{21}. \tag{A4}$$

In a similar way one can deduce how any AGS Green function transforms under the simultaneous exchange of two corresponding left- and right-particle labels. The following is a complete list for the case $k=1$:

$$P_{12}\tilde{U}_{11}P_{12}=\tilde{U}_{22},\quad P_{12}\tilde{U}_{21}P_{12}=\tilde{U}_{12},\quad P_{12}\tilde{U}_{31}P_{12}=\tilde{U}_{32},$$

$$P_{23}\tilde{U}_{11}P_{23}=\tilde{U}_{11},\quad P_{23}\tilde{U}_{21}P_{23}=\tilde{U}_{31},\quad P_{23}\tilde{U}_{31}P_{23}=\tilde{U}_{21},$$

$$P_{31}\tilde{U}_{11}P_{31}=\tilde{U}_{33},\quad P_{31}\tilde{U}_{21}P_{31}=\tilde{U}_{23},\quad P_{31}\tilde{U}_{31}P_{31}=\tilde{U}_{13}.$$

The corresponding transformations for $k=2$ and $k=3$ can be written down by inspection.

APPENDIX B: THREE-BODY BOUND-STATE WAVE FUNCTION FOR IDENTICAL PARTICLES

The three-body Green function G is defined by Eq. (1). If the field theory admits a three-body bound state, then G's behavior in the vicinity of the bound-state pole is given by Eq. (36) where the bound-state wave function Ψ_P is defined in Eq. (37). These equations are true whether the three particles are identical or not.

In the identical particle case the bound-state wave equation follows from Eq. (19), $G=G_0^P+G_0VG$, by taking residues at the three-body bound-state pole:

$$\Psi=G_0V\Psi, \tag{B1}$$

where V is given by Eq. (16). In the absence of three-body forces,

$$V=\frac{1}{2}(V_1+V_2+V_3), \tag{B2}$$

and the Faddeev wave function components are defined by

$$\Psi_i=\frac{1}{2}G_0V_i\Psi, \tag{B3}$$

so that

$$\Psi=\Psi_1+\Psi_2+\Psi_3. \tag{B4}$$

Using Eq. (47) it then follows from Eq. (B3) that the Ψ_i satisfy the coupled equations

$$\Psi_i=\frac{1}{2}G_0T_i(\Psi_j+\Psi_k), \tag{B5}$$

where ijk are cyclic.

By writing Eq. (B3) explicitly showing particle labels, for example,

$$\Psi_1(123)=\frac{1}{2}D_0(23)v(23,2'3')\Psi(12'3'), \tag{B6}$$

the following symmetry relations are easily deduced:

$$\begin{aligned}
&P_{23}\Psi_1=-\Psi_1, \quad P_{12}\Psi_1=-\Psi_2, \quad P_{31}\Psi_1=-\Psi_3,\\
&P_{31}\Psi_2=-\Psi_2, \quad P_{23}\Psi_2=-\Psi_3, \quad P_{12}\Psi_2=-\Psi_1,\\
&P_{12}\Psi_3=-\Psi_3, \quad P_{31}\Psi_3=-\Psi_1, \quad P_{23}\Psi_3=-\Psi_2.
\end{aligned} \tag{B7}$$

These symmetry relations enable us to write Eq. (B4) as

$$\begin{aligned}
\Psi(123)&=\Psi_1(123)+\Psi_2(123)+\Psi_3(123)\\
&=\Psi_1(123)+\Psi_1(231)+\Psi_1(312),
\end{aligned} \tag{B8}$$

or in general for any fixed i,

$$\Psi=\sum_{P_c}\Psi_i, \tag{B9}$$

the sum being over cyclic permutations of the particle labels. Similarly the symmetry relations (B7) enable us to write Eqs. (B5) for $i=1$ as

$$\Psi_1(123)=D_0(23)t(23,2'3')\Psi_1(2'3'1) \tag{B10}$$

or in general, for any i,

$$\Psi_i=-G_0T_iP_{ij}\Psi_i=-G_0T_iP_{ik}\Psi_i. \tag{B11}$$

Similarly,

$$\bar{\Psi}_i=-\bar{\Psi}_iP_{ij}T_iG_0=-\bar{\Psi}_iP_{ik}T_iG_0. \tag{B12}$$

APPENDIX C: THE GREEN FUNCTION $\tilde{Z}$

In dealing with identical particles, only one AGS-like Green function $\tilde{U}$ is needed to describe all possible processes of the three-body system. The connected part of the three-body Green function, G_c, is expressed in terms of $\tilde{U}$ according to Eq. (55). In this appendix we present a derivation of Eq. (55) and show how various forms for $\tilde{U}$ can be specified. One of these involves the AGS-like Green function $\tilde{Z}$ which, because of its simplicity, is chosen for the purposes of gauging. The pole structure of $\tilde{Z}$ is also discussed.

1. Derivation

Writing out the sums in Eq. (54) and showing particle labels explicitly, this equation can be written as

$$\begin{aligned}
G_c(123,1'2'3')=&\frac{1}{4}D_0(23)t(23,2''3'')[\tilde{U}_{11}(12''3'',1'2'''3''')t(2'''3''',2'3')D_0(2'3')\\
&+\tilde{U}_{12}(12''3'',1'''2'3''')t(3'''1''',3'1')D_0(3'1')+\tilde{U}_{13}(12''3'',1'''2'''3')t(1'''2''',1'2')D_0(1'2')]P\\
&+\frac{1}{4}D_0(31)t(31,3''1'')[\tilde{U}_{21}(1''23'',1'2'''3''')t(2'''3''',2'3')D_0(2'3')\\
&+\tilde{U}_{22}(1''23'',1'''2'3''')t(3'''1''',3'1')D_0(3'1')+\tilde{U}_{23}(1''23'',1'''2'''3')t(1'''2''',1'2')D_0(1'2')]P\\
&+\frac{1}{4}D_0(12)t(12,1''2'')[\tilde{U}_{31}(1''2''3,1'2'''3''')t(2'''3''',2'3')D_0(2'3')\\
&+\tilde{U}_{32}(1''2''3,1'''2'3''')t(3'''1''',3'1')D_0(3'1')+\tilde{U}_{33}(1''2''3,1'''2'''3')t(1'''2''',1'2')D_0(1'2')]P,
\end{aligned} \tag{C1}$$

where integrals over double- and triple-primed momenta are understood. The symbols P in the above equation indicate sums over all permutations of the initial (right) particle labels. Carrying out these permutation sums and taking into account the symmetries of the $\tilde{U}_{ij}$ discussed in Appendix A, this equation can be simplified to read

$$\begin{aligned} G_c(123,1'2'3') &= \frac{1}{2} P_c D_0(23) t(23,2''3'') \tilde{X}(12''3'',1'2'''3''') \\ &\quad \times t(2'''3''',2'3') D_0(2'3') P_c, \end{aligned} \tag{C2}$$

where the symbols P_c indicate sums over cyclic permutations of both initial- and final-state particle labels, and where the Green function $\tilde{X}(123,1'2'3')$ is defined by

$$\begin{aligned} \tilde{X}(123,1'2'3') &= \tilde{U}_{11}(123,1'2'3') + \tilde{U}_{12}(123,3'1'2') \\ &\quad + \tilde{U}_{12}(132,2'1'3'). \end{aligned} \tag{C3}$$

Note the symmetry relation $\tilde{X} = P_{23}\tilde{X}P_{23}$. The last two terms of Eq. (C3) contribute equally to Eq. (C2); nevertheless, we do not define $\tilde{X}$ as $\tilde{U}_{11}(123,1'2'3') + 2\tilde{U}_{12}(123,3'1'2')$ as this would not make it easy to write an integral equation for $\tilde{X}$. Using the AGS equations for $\tilde{U}_{11}$ and $\tilde{U}_{12}$, it follows that the $\tilde{X}$ of Eq. (C3) satisfies the integral equation

$$\begin{aligned} \tilde{X}(123,1'2'3') &= G_0(123,3'1'2') + G_0(132,2'1'3') \\ &\quad + \frac{1}{2} D_0(12) t(12,1''2'') \tilde{X}(31''2'',1'2'3') \\ &\quad + \frac{1}{2} D_0(31) t(31,3''1'') \tilde{X}(23''1'',1'2'3'). \end{aligned} \tag{C4}$$

Writing Eq. (C2) in the shorthand form as

$$G_c = \frac{1}{2} P_c G_0 T_1 \tilde{X} T_1 G_0 P_c, \tag{C5}$$

we have in this way derived Eq. (55) with $\tilde{U}$ being specifically given by $\tilde{U} = 1/2\tilde{X}$. This form for $\tilde{U}$ has been used by Glöckle *et al.* in practical calculations [8]. However, it is possible to choose other forms for $\tilde{U}$, and as we show below, one of these is particularly simple.

Since the two-body t matrices in Eq. (C2) are fully antisymmetric, we can equally well write this equation as

$$\begin{aligned} G_c(123,1'2'3') &= \frac{1}{2} P_c D_0(23) t(23,2''3'') \tilde{Y}(12''3'',1'2'''3''') \\ &\quad \times t(2'''3''',2'3') D_0(2'3') P_c, \end{aligned} \tag{C6}$$

where

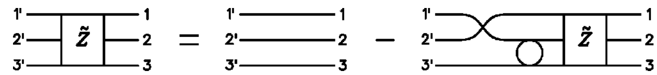

FIG. 3. Illustration of Eq. (C10) showing how the final state momenta of the AGS-like Green function $\tilde{Z}$ are permuted in the integral term.

$$\tilde{Y}(123,1'2'3') = \frac{1}{2}[\tilde{X}(123,1'2'3') - \tilde{X}(132,1'2'3')]. \tag{C7}$$

We note that $\tilde{Y}$ has the symmetry property $P_{23}\tilde{Y} = \tilde{Y}P_{23} = -\tilde{Y}$ and satisfies the integral equation

$$\begin{aligned} \tilde{Y}(123,1'2'3') &= \frac{1}{2}[G_0(123,3'1'2') + G_0(123,2'3'1') \\ &\quad - G_0(132,3'1'2') - G_0(132,2'3'1')] \\ &\quad + \frac{1}{2} D_0(12) t(12,1''2'') \tilde{Y}(31''2'',1'2'3') \\ &\quad + \frac{1}{2} D_0(31) t(31,3''1'') \tilde{Y}(23''1'',1'2'3'). \end{aligned} \tag{C8}$$

Although this appears to be a more complicated equation than Eq. (C4), it is easily shown that $\tilde{Y}$ can alternatively be specified as

$$\begin{aligned} \tilde{Y}(123,1'2'3') &= \frac{1}{2}[\tilde{Z}(123,3'1'2') + \tilde{Z}(123,2'3'1') \\ &\quad - \tilde{Z}(132,3'1'2') - \tilde{Z}(132,2'3'1')], \end{aligned} \tag{C9}$$

where $\tilde{Z}$ satisfies the especially simple equation

$$\begin{aligned} \tilde{Z}(123,1'2'3') &= G_0(123,1'2'3') \\ &\quad + D_0(31) t(31,3''1'') \tilde{Z}(23''1'',1'2'3'). \end{aligned} \tag{C10}$$

This equation constitutes a threefold reduction in the size of the kernel in comparison with the original AGS equations, Eqs. (50). One can write Eq. (C10) in shorthand form as

$$\tilde{Z} = G_0 - G_0 T_2 P_{12} \tilde{Z} = G_0 - G_0 P_{12} T_1 \tilde{Z}, \tag{C11}$$

the last version of which is illustrated in Fig. 3.

It is recognized that the kernel of this equation is identical to the kernel of the bound-state equation for Ψ_2. As Eq. (C11) implies $\tilde{Z}^{-1} = G_0^{-1} + T_2 P_{12}$, it also follows that

$$\tilde{Z} = G_0 - \tilde{Z} T_2 P_{12} G_0 = G_0 - \tilde{Z} P_{12} T_1 G_0, \tag{C12}$$

which is an alternative equation for $\tilde{Z}$ whose kernel is identical to the kernel of the bound-state equation for $\overline{\Psi}_1$.

Substituting Eq. (C9) into Eq. (C6) and taking into account the antisymmetry of the two-body t matrices we obtain

$$G_c(123,1'2'3') = \frac{1}{2} P_c D_0(23) t(23,2''3'') \times [\tilde{Z}(12''3'',3'''1'2''') + \tilde{Z}(12''3'',2'''3'''1')] \times t(2'''3''',2'3') D_0(2'3') P_c . \tag{C13}$$

The two $\tilde{Z}$ terms in Eq. (C13) in fact contribute equally to G_c. This can be seen by using Eq. (C12) to write

$$\tilde{Z}(123,3'1'2') = G_0(123,3'1'2') + \tilde{Z}(123,1''2''3') t(1''2'',1'2') D_0(1'2'), \tag{C14}$$

$$\tilde{Z}(123,2'3'1') = G_0(123,2'3'1') + \tilde{Z}(123,3''1''2') t(3''1'',3'1') D_0(3'1'). \tag{C15}$$

When used in Eq. (C13), the antisymmetry of the two-body t matrices makes the last terms of these equations identical, while the cyclic permutation operators P_c ensure the identity of their inhomogeneous terms. Using the antisymmetry of t's once more, we arrive at our final form for the AGS Green function $\tilde{U}$ that is to be used in Eq. (55):

$$\tilde{U} = -\tilde{Z} P_{12}, \tag{C16}$$

where $\tilde{Z}$ is given by Eq. (C10). In view of the numerical simplicity of the equation for $\tilde{Z}$, it is this form for $\tilde{U}$ that we choose in this paper for the purposes of gauging.

2. Pole structure of $\tilde{Z}$

If the three-body system admits a three-body bound state, it is clear from Eq. (55) that $\tilde{U}$ and, therefore, $\tilde{Z}$ have a pole at the bound-state mass M. Writing the pole structure of $\tilde{Z}$ as

$$\tilde{Z}(123,1'2'3') \sim i \frac{R(123,1'2'3')}{P^2 - M^2}, \tag{C17}$$

it is our goal to deduce the explicit form of the residue $R(123,1'2'3')$.

Taking the residue of Eq. (C10) at the bound-state pole gives the equation

$$R(123,1'2'3') = D_0(31) t(31,3''1'') R(23''1'',1'2'3'). \tag{C18}$$

With the initial momenta fixed, this equation coincides with the equation for $\Psi_2(123)$, the third Faddeev bound-state wave function component—see Eq. (B11). On the other hand, we can use Eq. (C12) to write

$$\tilde{Z}(123,1'2'3') = G_0(123,1'2'3') + \tilde{Z}(123,2''3''1') t(2''3'',2'3') D_0(2'3'). \tag{C19}$$

Taking residues of this equation gives

$$R(123,1'2'3') = R(123,2''3''1') t(2''3'',2'3') D_0(2'3'), \tag{C20}$$

which for fixed final momenta coincides with the equation for $\bar{\Psi}_1(1'2'3')$—see Eq. (B12). We may thus write $R(123,1'2'3') = C\Psi_2(123)\bar{\Psi}_1(1'2'3')$ where C is a constant. To determine C we use $\tilde{U} = -\tilde{Z}P_{12}$ in Eq. (55) and take residues of both sides of the equation, in this way obtaining

$$i\Psi\bar{\Psi} = -P_c G_0 T_1 C \Psi_2 \bar{\Psi}_1 P_{12} T_1 G_0 P_c . \tag{C21}$$

Using the results of Appendix B it is then easily seen that $C = i$. We have thus shown that the pole structure of $\tilde{Z}$ is given by

$$\tilde{Z}(123,1'2'3') \sim i \frac{\Psi_2(123)\bar{\Psi}_1(1'2'3')}{P^2 - M^2}. \tag{C22}$$

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