Bound-state problem of the $NN\Delta$, NNN, and NN systems

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We used the local $N\Delta$, NN, and $NN \leftrightarrow N\Delta$ potentials derived from the chiral quark cluster model to analyze the bound-state problems of the $NN\Delta$, NNN, and NN systems. The last two systems are included for completeness and in order to check the reliability of our model. We found that the $NN\Delta$ system has no bound states although there is evidence of a resonance near the $NN\Delta$ threshold in the $(J,I)=(3/2,1/2)$ channel. We calculated the triton binding energy and the deuteron wave function produced by our model. We also calculated the NN $^1D_2$ amplitude in order to study the effect upon the NN system of the $(j,i)=(2,1)$ $N\Delta$ bound state predicted by the chiral quark cluster model. [S0556-2813(99)05909-9]

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I. INTRODUCTION

This is the third paper of a series whose aim is to investigate the possible existence of two- and three-body bound states of the various systems consisting of nucleons and deltas [1,2]. These are the systems $\Delta\Delta$, $N\Delta$, NN, $\Delta\Delta\Delta$, $N\Delta\Delta$, $NN\Delta$, and NNN. The bound states involving one or more unstable particles will show up in nature as dibaryon and tribaryon resonances. In the case of the two-body systems (dibaryons) they will decay into two nucleons and either one or two pions, while for the three-body case (tribaryons) they will decay into three nucleons and either one, two, or three pions. The systems without Δ's (NN and NNN) are included for completeness as well as to check the reliability of our method since in these cases there is the possibility to compare with other theoretical results and with the known features of the deuteron and triton. In Ref. [1] we discussed the bound-state problems of the $\Delta\Delta$ and $\Delta\Delta\Delta$ systems and in Ref. [2] the $N\Delta$ and $N\Delta\Delta$ systems. Thus, we will now close our study by discussing the bound-state problems of the $NN\Delta$, NNN, and NN systems.

The three-body systems $NN\Delta$ and NNN will be investigated with regard to bound-state solutions. The interest of possible $NN\Delta$ bound states lies in the fact that these states will correspond to the tribaryon resonances with the lowest possible mass since they are resonances that decay into three nucleons and one pion and, therefore, they would be the easiest to detect experimentally. As mentioned before, the NNN bound-state calculation is performed in order to have, within the same model, a complete set of results for the bound-state problem of the two- and three-body systems composed of nucleons and deltas.

We will also study several important consequences of the bound states of the NN and $N\Delta$ systems that are predicted by our model. In the case of the NN state with quantum numbers $(j,i)=(1,0)$ (the deuteron) we will calculate its wave function to see how it compares with those of other models. The $N\Delta$ system possesses a bound state with quantum numbers of angular momentum and isospin $(j,i)=(2,1)$ at zero energy [2]. These quantum numbers and energy correspond precisely to the ones of the nucleon-nucleon $^1D_2$ resonance which led us in Ref. [2] to the identification of this NN resonance as being the consequence of the $N\Delta$ bound state. We will now make this identification more explicit by studying the influence of the $N\Delta$ bound state upon the NN $^1D_2$ channel by including the coupling of the NN system to the $N\Delta$ system.

We use as basic framework for the baryon-baryon interactions the local potentials obtained from the chiral quark cluster model [3,4] as it has been described in Refs. [1,2]. In this model, the basic interaction is at the level of quarks involving only a quark-quark-field (pion or gluon) vertex. Therefore, its parameters (coupling constant, cutoff mass, etc.) are independent of the baryon to which the quarks are coupled, the difference among them being generated by SU(2) scaling, as explained in Ref. [5]. Moreover, quark models provide a definite framework to treat the short-range part of the interaction. The Pauli principle between quarks determines the short-range behavior of the different channels without additional phenomenological assumptions. In this way, even in absence of experimental data, one has a complete scheme which starting from the NN sector allows us to make predictions in the $N\Delta$ and $\Delta\Delta$ sectors. This fact is even more important if one takes into account that the short-range dynamics of the $N\Delta$ and $\Delta\Delta$ systems is to a large extent driven by quark Pauli blocking effects, that do not appear in the NN sector. Pauli blocking acts in a selective way in those channels where the spin-isospin-color degrees of freedom are not enough to accommodate all the quarks of the system [6,7].

In order to perform the NNN and $NN\Delta$ calculations we follow the same procedure that we used with the $\Delta\Delta\Delta$ and $N\Delta\Delta$ cases [1,2] which was taken from the experience gained in the three-nucleon bound-state problem [8,9]. The three-body calculations are performed using a truncated T-matrix approximation where the inputs of the three-body equations are the two-body T matrices truncated such that the orbital angular momentum in the initial and final states is equal to zero. These two-body T matrices, however, have been constructed taking into account the coupling to the $l=2$ states due to the tensor force. This approximation in the case of the three-nucleon system with the NN interaction taken as the Reid soft-core potential leads to a triton binding energy which differs less than 1 MeV from the exact value [8].

We describe in Sec. II our formalism. In Sec. III we give our results and we present the conclusions in Sec. IV.

II. FORMALISM

A. The two-body interactions

The basic two-body interactions between baryons that are going to be needed in this work are the nucleon-nucleon interaction $V_{NN\to NN}$, the nucleon-delta interaction $V_{N\Delta\to N\Delta}$, and the transition interactions $V_{NN\to N\Delta}$ and $V_{N\Delta\to NN}$. These baryon-baryon interactions were obtained from the chiral quark cluster model developed elsewhere [4]. In this model baryons are described as clusters of three interacting massive (constituent) quarks, the mass coming from the breaking of chiral symmetry. The ingredients of the quark-quark interaction are confinement, one-gluon (OGE), one-pion (OPE), and one-sigma (OSE) exchange terms, and whose parameters are fixed from the NN data. Explicitly, the quark-quark (qq) interaction is

$$V_{qq}(\vec{r}_{ij})=V_{\rm con}(\vec{r}_{ij})+V_{\rm OGE}(\vec{r}_{ij})+V_{\rm OPE}(\vec{r}_{ij})+V_{\rm OSE}(\vec{r}_{ij}), \tag{1}$$

where $\vec{r}_{ij}$ is the ij interquark distance and

$$V_{\rm con}(\vec{r}_{ij})=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_j r_{ij}^2, \tag{2}$$

$$V_{\rm OGE}(\vec{r}_{ij})=\frac{1}{4}\alpha_s\vec{\lambda}_i\cdot\vec{\lambda}_j\left\{\frac{1}{r_{ij}}-\frac{\pi}{m_q^2}\left[1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right]\delta(\vec{r}_{ij}) - \frac{3}{4m_q^2 r_{ij}^3}S_{ij}\right\}, \tag{3}$$

$$V_{\rm OPE}(\vec{r}_{ij})=\frac{1}{3}\alpha_{ch}\frac{\Lambda^2}{\Lambda^2-m_\pi^2}m_\pi\left\{\left[Y(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3}Y(\Lambda r_{ij})\right]\vec{\sigma}_i\cdot\vec{\sigma}_j+\left[H(m_\pi r_{ij}) - \frac{\Lambda^3}{m_\pi^3}H(\Lambda r_{ij})\right]S_{ij}\right\}\vec{\tau}_i\cdot\vec{\tau}_j, \tag{4}$$

$$V_{\rm OSE}(\vec{r}_{ij})=-\alpha_{ch}\frac{4m_q^2}{m_\pi^2}\frac{\Lambda^2}{\Lambda^2-m_\sigma^2}m_\sigma\left[Y(m_\sigma r_{ij}) - \frac{\Lambda}{m_\sigma}Y(\Lambda r_{ij})\right], \tag{5}$$

where

$$Y(x)=\frac{e^{-x}}{x};\quad H(x)=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x). \tag{6}$$

a_c is the confinement strength, the $\vec{\lambda}$'s are the SU(3) color matrices, the $\vec{\sigma}$'s ($\vec{\tau}$'s) are the spin (isospin) Pauli matrices, S_{ij} is the usual tensor operator, m_q (m_π, m_σ) is the quark (pion, sigma) mass, α_s is the qq-gluon coupling constant, α_{ch} is the qq-meson coupling constant and Λ is a cutoff parameter.

TABLE I. Quark model parameters.

m_q(MeV)	313
b(fm)	0.518
α_s	0.485
a_c(MeV fm^{-2})	46.938
$\alpha_{\rm ch}$	0.027
m_σ(fm^{-1})	3.421
m_π(fm^{-1})	0.70
Λ(fm^{-1})	4.2

In order to derive the local $NB\to NB$ interactions ($B=N,\Delta$) from the basic qq interaction defined above we use a Born-Oppenheimer approximation [6]. Explicitly, the potential is calculated as follows:

$$V_{NB(LST)\to NB(L'S'T)}(R)=\xi_{LST}^{L'S'T}(R)-\xi_{LST}^{L'S'T}(\infty), \tag{7}$$

where

$$\xi_{LST}^{L'S'T}(R) = \frac{\left\langle \Psi_{NB}^{L'S'T}(\vec{R})\left|\sum_{i<j=1}^{6}V_{qq}(\vec{r}_{ij})\right|\Psi_{NB}^{LST}(\vec{R})\right\rangle}{\sqrt{\langle\Psi_{NB}^{L'S'T}(\vec{R})|\Psi_{NB}^{L'S'T}(\vec{R})\rangle}\sqrt{\langle\Psi_{NB}^{LST}(\vec{R})|\Psi_{NB}^{LST}(\vec{R})\rangle}}. \tag{8}$$

In the last expression the quark coordinates are integrated out keeping R fixed, the resulting interaction being a function of the N-B distance. The wave function $\Psi_{NB}^{LST}(\vec{R})$ for the two-baryon system is discussed in detail in Refs. [6,7]. The parameters of the model are summarized in Table I.

B. The $N\Delta$ and NN systems

If we consider two baryons N and B ($B=N,\Delta$) in a relative S-state interacting through a potential that contains a tensor force, then there is a coupling to the NB D wave so that the Lippmann-Schwinger equation of the system is of the form

$$t_{ji}^{lsl''s''}(p,p'';E)=V_{ji}^{lsl''s''}(p,p'') + \sum_{l's'}\int_0^\infty p'^2dp'V_{ji}^{lsl's'}(p,p') \times\frac{1}{E-p'^2/2\eta+i\epsilon}t_{ji}^{l's'l''s''}(p',p'';E), \tag{9}$$

where j and i are the angular momentum and isospin of the system, while ls, $l's'$, and $l''s''$ are the initial, intermediate,

TABLE II. NN channels (l_N,s_N) and $N\Delta$ channels (l_Δ,s_Δ) that are coupled together in the $^3S_1$-$^3D_1$, $^1S_0$, and $^1D_2$ NN states.

NN state	j	i	(l_N,s_N)	(l_Δ,s_Δ)
$^3S_1$-$^3D_1$	1	0	(0,1),(2,1)	
$^1S_0$	0	1	(0,0)	(2,2)
$^1D_2$	2	1	(2,0)	(0,1),(2,1),(2,2)

and final orbital angular momentum and spin of the system, respectively. p and η are, respectively, the relative momentum and reduced mass of the two-body system. The coupled channels of orbital angular momentum and spin that contribute in the case of the $N\Delta$ system have been given in Table II of Ref. [2].

Equation (9) will be used also for the case of the NN system with isospin 0. In the case of the NN system with isospin 1 we will take into account the coupling between the NN and $N\Delta$ systems. This will also allow us to study the NN scattering amplitude in the $(j,i)=(2,1)$ channel which is the channel of the $^1D_2$ resonance.

If we call the NN system as channel N and the $N\Delta$ system as channel Δ, then instead of Eq. (9) we will have in this case the set of coupled equations

$$t_{kn;ji}^{lsl''s''}(p_k,p_n;W)=V_{kn;ji}^{lsl''s''}(p_k,p_n)+\sum_{m=N,\Delta}\sum_{l's'}\int_0^\infty p_m^2dp_m \times V_{km;ji}^{lsl's'}(p_k,p_m)G_m(W;p_m) \times t_{mn;ji}^{l's'l''s''}(p_m,p_n;W);\quad k,n=N,\Delta, \tag{10}$$

where W is the invariant mass of the system, $t_{NN;ji}$ is the $NN\rightarrow NN$ scattering amplitude, $t_{\Delta\Delta;ji}$ is the $N\Delta\rightarrow N\Delta$ scattering amplitude, and $t_{N\Delta;ji}$ is the $NN\rightarrow N\Delta$ scattering amplitude. The propagators $G_N(W;p_N)$ and $G_\Delta(W;p_\Delta)$ in Eq. (10) are given by

$$G_N(W;p_N)=\frac{2\eta_{NN}}{k_N^2-p_N^2+i\epsilon}, \tag{11}$$

$$G_\Delta(W;p_\Delta)=\frac{2\eta_{N\Delta}}{k_\Delta^2-p_\Delta^2+i\epsilon}, \tag{12}$$

where the on-shell momenta k_N and k_Δ are defined by

$$W=2\sqrt{m_N^2+k_N^2}=\sqrt{m_N^2+k_\Delta^2}+\sqrt{m_\Delta^2+k_\Delta^2}. \tag{13}$$

If $W>2m_N+m_\pi$, i.e., if one is above the pion-production threshold then one has to include the width of the Δ in the propagator $G_\Delta(W;p_\Delta)$ so that in that case one must have

$$G_\Delta(W;p_\Delta)=\frac{2\eta_{N\Delta}}{k_\Delta^2-p_\Delta^2+i\eta_{N\Delta}\Gamma(W;p_\Delta)}, \tag{14}$$

where the width $\Gamma(W;p_\Delta)$ is due to the decay process $\Delta\rightarrow\pi N$. We will use for this quantity the parametrization [10,11]

$$\Gamma(W;p_\Delta)=\frac{2}{3}0.35q_{\pi N}^3\frac{\sqrt{m_N^2+p_\Delta^2}}{m_\pi^2\sqrt{s_{\pi N}}}, \tag{15}$$

where $s_{\pi N}$ and $q_{\pi N}$ are, respectively, the invariant mass squared and relative momentum of the πN subsystem which are given by

$$s_{\pi N}=W^2+m_N^2-2W\sqrt{m_N^2+p_\Delta^2}, \tag{16}$$

$$q_{\pi N}=\sqrt{\frac{[s_{\pi N}-(m_\pi+m_N)^2][s_{\pi N}-(m_\pi-m_N)^2]}{4s_{\pi N}}}. \tag{17}$$

We give in Table II the channels (l_N,s_N) and (l_Δ,s_Δ) corresponding to the NN and $N\Delta$ systems that are coupled together in the case of three (j,i) states corresponding to the NN channels $^3S_1$-$^3D_1$, $^1S_0$, and $^1D_2$.

As mentioned before, for the solution of the three-body system we will use only the component of the T matrix obtained from the solution of Eqs. (9) and (10) with $l=l''=0$. For that purpose we define the S-wave truncated amplitude which in the case of the $N\Delta$ system and the NN system with isospin 0 is defined from the solutions of Eq. (9) by

$$t_{si}(p,p'';E)\equiv t_{si}^{0s0s}(p,p'';E), \tag{18}$$

and in the case of the NN system with isospin 1 is defined from the solutions of Eq. (10) by

$$t_{si}(p,p'';E)\equiv t_{NN;si}^{0s0s}(p,p'';W), \tag{19}$$

where

$$E=\frac{k_N^2}{2\eta_{NN}}, \tag{20}$$

and W and k_N related through Eq. (13).

C. The $NN\Delta$ and NNN systems

The numerical solution of the bound-state problem in the case of the $NN\Delta$ system will be obtained using the same formalism that was used in Ref. [2] for the case of the $N\Delta\Delta$ system since in both cases one is dealing with a system with two identical particles and a third one which is different. The two-body channels that contribute to the three-body equations in the case of the $NN\Delta$ system are given in Table III for all the possible states in which the three baryons are in S waves.

The numerical solution of the bound-state problem in the case of the NNN system will be obtained using the same formalism that was used in Ref. [1] for the case of the $\Delta\Delta\Delta$ system since in both cases one is dealing with a system of three identical particles. The two-body channels that contribute to the three-body equations in the case of the NNN sys-

TABLE III. Two-body $N\Delta$ channels (s_2,i_2) and two-body NN channels (s_1,i_1) that contribute to a given $NN\Delta$ state with total spin S and isospin I.

S	I	(s_2,i_2)	(s_1,i_1)
1/2	1/2	(1,1)	
1/2	3/2	(1,1),(1,2)	(1,0)
1/2	5/2	(1,2)	
3/2	1/2	(1,1),(2,1)	(0,1)
3/2	3/2	(1,1),(1,2),(2,1),(2,2)	(1,0),(0,1)
3/2	5/2	(1,2),(2,2)	(0,1)
5/2	1/2	(2,1)	
5/2	3/2	(2,1),(2,2)	(1,0)
5/2	5/2	(2,2)	

tem are given in Table IV for all the possible states in which the three nucleons are in S waves.

III. RESULTS

We will start by discussing the predictions of our model for the stable systems NN and NNN and afterwards we discuss the unstable system $NN\Delta$.

A. The NN system

As we already reported in Ref. [2], the NN system in this model has only one bound state which corresponds to the deuteron, and with a binding energy of 3.13 MeV. That energy disagrees with the experimental value 2.225 MeV which is also the value obtained using the quark-quark interaction Eqs. (1)–(6) together with the resonating group method (RGM). This discrepancy in the binding energy is due to the use of the Born-Oppenheimer approximation (7),(8), which is needed in order to solve the bound-state problem of the three-body systems since in the RGM that would be extremely complicated technically. The difference between the RGM value and the Born-Oppenheimer approximation value is less than 1 MeV, which gives us confidence on the reliability of our method for the two-baryon system. Since we want to apply the NN interaction to study some standard features of few-nucleon systems such as the deuteron wave function and the triton binding energy we have readjusted our model of the ${}^3S_1$-${}^3D_1$ interaction such that it gives the correct binding energy for the deuteron. This was achieved by slightly increasing the cutoff parameter Λ (see Table I) from 4.20 to 4.28 fm^{-1}.

We compare in Fig. 1 the S- and D-wave components of the deuteron wave function (in momentum space) predicted

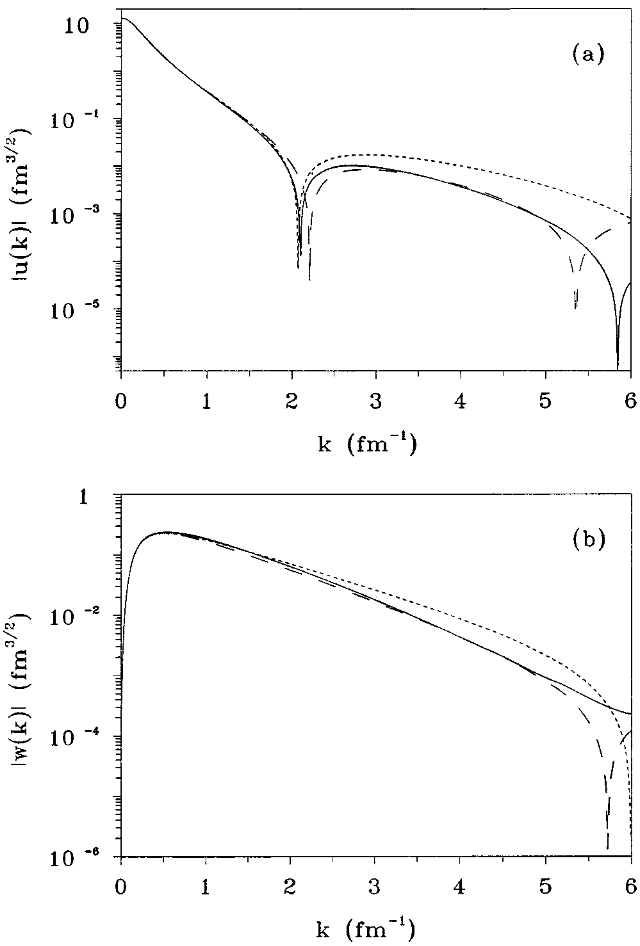


FIG. 1. The deuteron wave function of our model (solid line) compared with the ones of Paris (dashed line) and Bonn (dotted line) potentials. (a) S wave, (b) D wave.

by the readjusted model with the ones of the Paris [12] and Bonn [13] potentials. Our wave function has a D-state probability of 5.2%. In Figs. 2 and 3 we show the NN ${}^3S_1$ and ${}^3D_1$ phase shifts with the original cutoff parameter $\Lambda = 4.20$ fm^{-1} (dashed line) and the ones produced by the readjusted model with $\Lambda = 4.28$ fm^{-1} (solid line).[1]

We now move to the NN isospin 1 channels. In this case one has the possibility of coupling to the $N\Delta$ system. As it has been shown in Refs. [14,15] this approach leads to a satisfactory description of the NN ${}^1S_0$ channel when the chiral quark cluster model defined by Eqs. (1)–(6) is used within the framework of the resonating group method. In our model, which uses the Born-Oppenheimer approximation (7),(8), we expect to get somewhat more attraction as was the case with the ${}^3S_1$-${}^3D_1$ channel previously discussed. Indeed, we find that the virtual bound state becomes a real bound state with a very small binding energy of 0.046 MeV. We show in Fig. 4 as a dashed line the NN ${}^1S_0$ phase shift

TABLE IV. Two-body NN channels (s,i) that contribute to a given NNN state with total spin S and isospin I.

S	I	(s,i)
1/2	1/2	(1,0),(0,1)
1/2	3/2	(0,1)
3/2	1/2	(1,0)

[1] The ${}^3S_1$ and ${}^3D_1$ phase shifts shown in Ref. [2] corresponding to the model with $\Lambda = 4.20$ fm^{-1} had a minor numerical error. The correct results are the ones given here by the dashed line in Figs. 2 and 3.

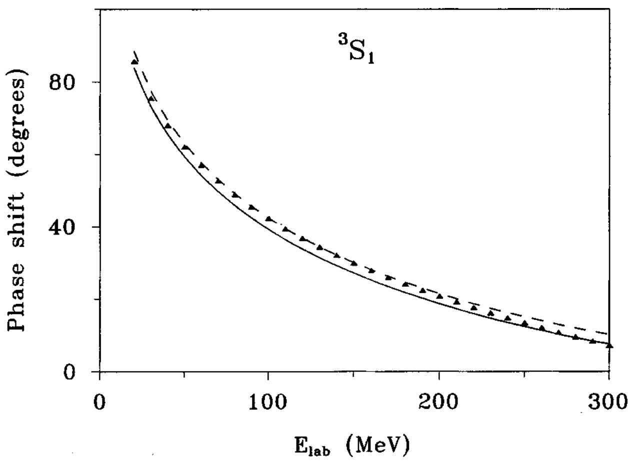


FIG. 2. The NN $^3S_1$ phase shift as a function of laboratory kinetic energy. The dashed line is the result of the standard model with $\Lambda=4.20$ fm^{-1} and the solid line is the result of the readjusted model with $\Lambda=4.28$ fm^{-1}. The data are from Ref. [18].

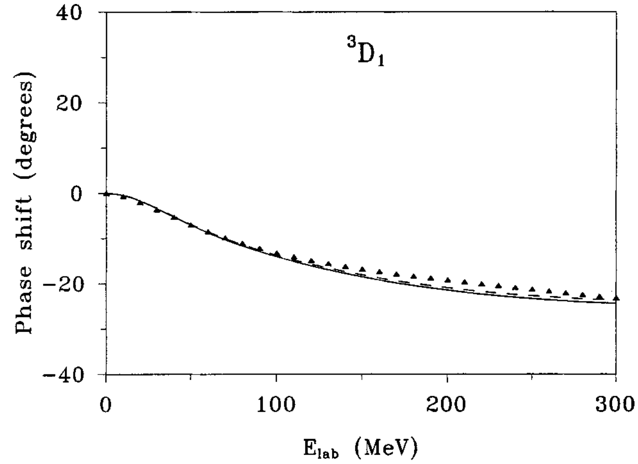


FIG. 3. The NN $^3D_1$ phase shift as a function of laboratory kinetic energy. The dashed line is the result of the standard model with $\Lambda=4.20$ fm^{-1} and the solid line is the result of the readjusted model with $\Lambda=4.28$ fm^{-1}. The data are from Ref. [18].

which shows the presence of the bound state in the fact that $\delta_{E=0}=180°$. Thus, in order to improve the description of the phase shift and to be able to study other systems we readjusted our model of the $^1S_0$ channel by increasing the cutoff parameter Λ from 4.20 to 4.38 fm^{-1}. We show in Fig. 4 as solid line the NN $^1S_0$ phase shift produced by the readjusted model which leads to a good description of the low-energy data. The slightly different tuning of the cutoff for the $^1S_0$ and $^3S_1$ partial waves resembles the different value of the sigma-meson parameters used by the Bonn potential for the same channels, in order to achieve a correct description of the low-energy data for both partial waves [13].

Since the purpose of our entire study [1,2] is the investigation of bound states between nucleons and deltas when two baryons can be in a S state we must consider also the NN $^1D_2$ channel for which as seen in Table II there is a S state in the $N\Delta$ system. As was shown in Ref. [2], the chiral quark cluster model defined by Eqs. (1)–(8) predicts a $N\Delta$ bound state right at the $N\Delta$ threshold which corresponds to an invariant energy of 2.17 GeV. That energy coincides with the mass of the $^1D_2$ resonance [16,17]. We show in Fig. 5 the real and imaginary parts of the NN $^1D_2$ amplitude obtained from our model (solid line) as compared with the experimental ones given by the most recent amplitude analysis [18] (dashed line). As one sees from this figure there is only qualitative agreement between the theoretical and experimental amplitudes. An important question is the position of the resonance (corresponding to the energy where Im F is maximum) which is given in the experimental amplitude by 2.17 GeV and in our model by 2.16 GeV. Since the bound state of the $N\Delta$ system lies at 2.17 GeV it is interesting that at least within our model the position of the resonance in the NN system corresponds closely with that of the bound state in the $N\Delta$ system.

B. The NNN system

As a test of the reliability of our model in the case of the three-baryon system we solved the NNN bound-state problem. We found that of the states of Table IV only the state with $(S,I)=(\frac{1}{2},\frac{1}{2})$, that is the triton, has a bound state. Since the triton binding energy is very sensitive to the description of the two-body channels, we considered both our standard value $\Lambda=4.20$ fm^{-1} as well as the readjusted $\Lambda=4.28$ fm^{-1} for the $^3S_1$-$^3D_1$ channel and $\Lambda=4.38$ fm^{-1} for the $^1S_0$ channel. Since our standard model has too much attraction in the case of the NN channels we expect that it will also have too much attraction in the NNN sector. We are, therefore, not surprised when we find that it produces a triton binding energy of 10.89 MeV ($B_{\rm exp}=8.49$ MeV). Our readjusted model, on the other hand, gives rise to a triton binding energy of 6.90 MeV. This difference of about 4 MeV is a measure of the uncertainty in the case of the three-baryon system due to the use of the Born-Oppenheimer approximation. For comparison, we notice that the triton binding energy for the Reid-soft-core potential in the truncated T-matrix approximation is 6.58 MeV [2].

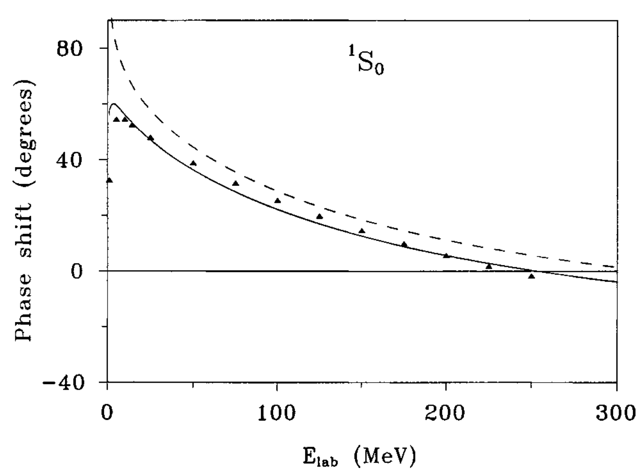
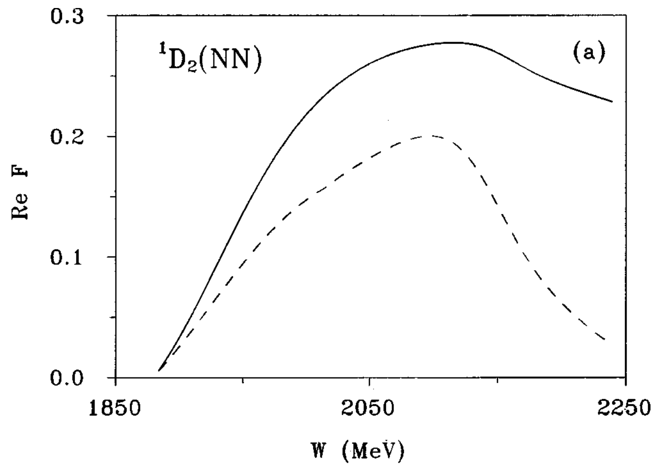


FIG. 4. The NN $^1S_0$ phase shift as a function of laboratory kinetic energy. The dashed line is the result of the standard model with $\Lambda=4.20$ fm^{-1} and the solid line is the result of the readjusted model with $\Lambda=4.38$ fm^{-1}. The data are from Ref. [18].



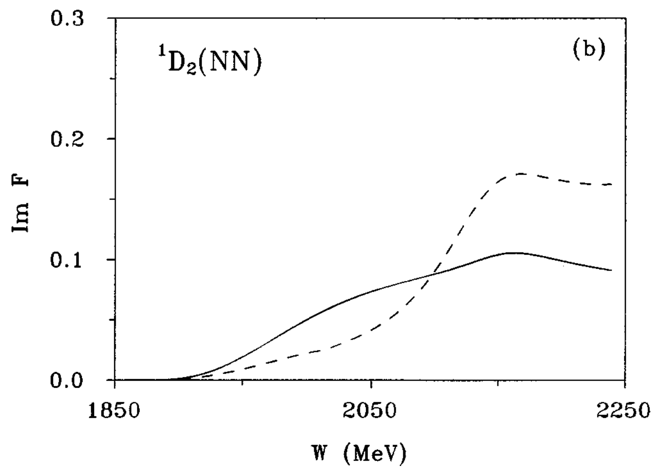


FIG. 5. The NN $^1D_2$ amplitude as a function of the invariant energy. The solid line is the result obtained from our model and the dashed line is the result of the amplitude analysis of Ref. [18]. (a) Real part, (b) imaginary part.

C. The $NN\Delta$ system

One may have hoped to find several bound states in this system due to the facts that the $N\Delta$ two-body subsystem has a bound state in the channel $(s_2,i_2)=(2,1)$ and the NN two-body subsystem has a bound state in the channel $(s_1,i_1)=(1,0)$ and an almost-bound state in the channel $(s_1,i_1)=(0,1)$. This is not the case however, and as a matter of fact none of the nine possible three-body states given in Table III is bound. The reason is the following: As we have already pointed out in Ref. [2] the $N\Delta$ two-body channels $(s_2,i_2)=(1,1)$ and $(s_2,i_2)=(2,2)$ present Pauli blocking [6] and, therefore, they have a strong repulsive barrier at short distances in the S-wave central interaction. Therefore, in all the three-body states of Table III where these $N\Delta$ repulsive channels contribute they wipe out any possibility of binding. The only three-body states that do not have this problem are $(S,I)=(\frac{1}{2},\frac{5}{2})$ and $(S,I)=(\frac{5}{2},\frac{1}{2})$ but in these states there is no contribution from the attractive NN channels and, therefore, one does not have enough attraction to bind the system.

The three-body state which is closer to being bound is the $(S,I)=(\frac{3}{2},\frac{1}{2})$ which has a repulsive contribution from the $N\Delta$ channel $(s_2,i_2)=(1,1)$ and attractive contributions from the $N\Delta$ channel $(s_2,i_2)=(2,1)$ and from the NN channel $(s_1,i_1)=(0,1)$. We show in Fig. 6 the Fredholm determinant of this state as a function of energy near the $NN\Delta$ threshold (a bound state would exist if the Fredholm determinant had passed through zero at an energy below threshold). The solid line is the result of our standard model and the dashed line is the result obtained using the readjusted NN $^1S_0$ interaction described before. As seen from this figure the state is almost bound. That means that the $(S,I)=(\frac{3}{2},\frac{1}{2})$ state is very near the $NN\Delta$ threshold and, therefore, it represents the tribaryon resonance with the lowest possible mass.

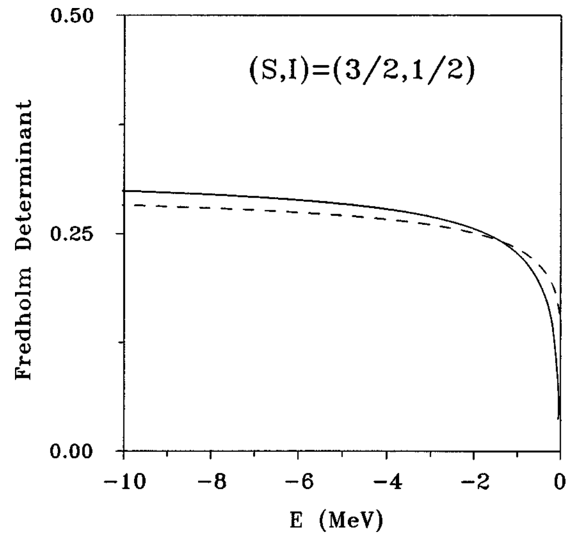


FIG. 6. The Fredholm determinant of the $NN\Delta$ system in the state $(S,I)=(\frac{3}{2},\frac{1}{2})$ as a function of energy. The solid line is the result of our standard model and the dashed line is the result using for the NN $^1S_0$ interaction the readjusted model described in the text.

IV. CONCLUSIONS

We have studied the bound-state solutions of the systems NN and $N\Delta$. In the case of the stable NN system we obtained the bound-state wave function and showed that it is quite similar to that predicted by other models [12,13]. In the case of the unstable $N\Delta$ system where there is a bound state with zero binding energy, we calculated its effect upon the NN $^1D_2$ amplitude and we found that it gives rise to a NN resonance at an invariant energy close to that of the $N\Delta$ bound state.

We also studied the bound-state problems of the three-body systems NNN and $NN\Delta$. In the case of the NNN system we found that (as it should be) there is a bound state in the triton channel $(S,I)=(\frac{1}{2},\frac{1}{2})$. In the case of the $NN\Delta$ system we found that there are no bound-state solutions in any of the allowed states although in the case of the state $(S,I)=(\frac{3}{2},\frac{1}{2})$, we found evidence that a resonance lies near the $NN\Delta$ threshold. This would be the tribaryon resonance with the lowest possible mass and, therefore, the one more easy to detect experimentally.

With this paper we have now concluded our theoretical investigation on the possible existence of bound states for the two- and three-body systems composed of nucleons and deltas. Putting together the results of Refs. [1,2] and the ones obtained here we conclude that the two-body systems NN, $N\Delta$, and $\Delta\Delta$ have one, one, and six bound states, respectively. The bound states of the unstable systems $N\Delta$ and $\Delta\Delta$ correspond to dibaryon resonances that decay into two nucleons and one pion and two nucleons and two pions, respectively. The $N\Delta$ bound state with $(j,i)=(2,1)$ and $M \approx 2.17$ GeV is the dibaryon resonance with the lowest possible mass and the one which seems to be well confirmed by experiment. The six $\Delta\Delta$ bound states correspond to dibaryon resonances with masses between 2.33 and 2.46 GeV. With respect to the three-body systems the NNN has one and the $\Delta\Delta\Delta$ has six bound states, while the $NN\Delta$ has one resonance near threshold and the $N\Delta\Delta$ has two resonances near threshold. The predicted $NN\Delta$ state with $(S,I)=(\frac{3}{2},\frac{1}{2})$ and $M\approx 3.4$ GeV is the tribaryon resonance with the lowest mass and, therefore, the one that would be more easy to detect experimentally.

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[1] H. Garcilazo, F. Fernández, A. Valcarce, and R. D. Mota, Phys. Rev. C **56**, 84 (1997).
[2] R. D. Mota, F. Fernández, A. Valcarce, and H. Garcilazo, Phys. Rev. C **59**, R46 (1999).
[3] A. Faessler, F. Fernández, G. Lübeck, and K. Shimizu, Phys. Lett. **112B**, 201 (1982); M. Oka and K. Yazaki, Prog. Theor. Phys. **66**, 556, 572 (1980).
[4] F. Fernández, A. Valcarce, U. Straub, and A. Faessler, J. Phys. G **19**, 2013 (1993); A. Valcarce, A. Buchmann, F. Fernández, and A. Faessler, Phys. Rev. C **50**, 2246 (1994).
[5] G. E. Brown and W. Weise, Phys. Rep. **22**, 279 (1975).
[6] A. Valcarce, F. Fernández, P. González, and V. Vento, Phys. Rev. C **52**, 38 (1995).
[7] A. Valcarce, F. Fernández, and P. González, Phys. Rev. C **56**, 3026 (1997).
[8] E. P. Harper, Y. E. Kim, and A. Tubis, Phys. Rev. Lett. **28**, 1533 (1972).
[9] G. H. Berthold, H. Zankel, L. Mathelitsch, and H. Garcilazo, Nuovo Cimento A **93**, 89 (1986).
[10] H. Garcilazo and M. T. Peña, Phys. Rev. C **44**, 2311 (1991).
[11] H. Garcilazo, T. Mizutani, M. T. Peña, and P. U. Sauer, Phys. Rev. C **42**, 2315 (1990).
[12] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[13] R. Machleidt, K. Holinde, and C. Elster, Phys. Rep. **149**, 1 (1987).
[14] A. Valcarce, A. Faessler, and F. Fernández, Phys. Lett. B **345**, 367 (1995).
[15] L. Dai and Z. Y. Zhang, J. Phys. G **23**, 1663 (1997).
[16] N. Hoshizaki, Prog. Theor. Phys. **60**, 1796 (1978); **61**, 129 (1979).
[17] A. Yokosawa, Phys. Rep. **64**, 47 (1980).
[18] R. A. Arndt, C. H. Oh, I. I. Strakovski, R. L. Workman, and F. Dohrmann, Phys. Rev. C **56**, 3005 (1997).