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Search for evidence of two-photon exchange in new experimental high momentum transfer data on electron deuteron elastic scattering

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The possible contribution of two-photon exchange to electron deuteron elastic scattering at relatively high momentum transfer is discussed. This study was motivated by the high precision data recently obtained at the Jefferson Laboratory. Using general arguments, based on crossing symmetry for the processes $e^-+h\to e^-+h$ and $e^++e^-\to h+\overline{h}$, we find a parametrization for the angular dependence of the interference between the one- and two-photon exchanges in the differential cross section for elastic ed scattering in terms of a new kinematical variable and compare our findings to the recent data. [S0556-2813(99)50210-0]

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The study of the structure of hadrons and nuclei with electromagnetic probes is based on the validity of the one-photon mechanism for elastic and inelastic electron-hadron scattering. On the basis of a well established formalism, the measured cross sections and polarization observables can be directly related to the electromagnetic form factors and structure functions [1]. The validity of this approach is based on the assumption that the possible two-photon contribution is small. The relative contribution of two-photon exchange, from simple counting in α, would be of the order of the fine structure constant, $\alpha=e^2/4\pi\simeq 1/137$: for eh scattering, where h is a hadron, any contribution of two-photon exchange through its interference with the main (i.e., one-photon) mechanism would not exceed 1%. On the other hand, more than 25 years ago it was observed [2–5] that the simple rule of α counting for the estimation of the relative role of two-photon contribution to the amplitude of elastic ed scattering does not hold at large momentum transfer. Using a Glauber approach for the calculation of multiple scattering contributions [6], it appeared that the relative role of two-photon exchange can increase significantly in the region of high momentum transfer. This effect can be observed in particular in ed-elastic scattering, due to the steep decrease of the deuteron form factors. It has to be even larger for heavy nuclei (i.e., ^{3}He or ^{4}He) and it would manifest already at momentum transfer of the order of 1 GeV^2, in particular in the region of diffractive minima. The argument is that not only the degree of the fine structure constant α is important in the estimation of the relative role of the two-photon exchange, but also the value of the momentum transfer. The standard calculations of radiative corrections for eh scattering contain the contribution of two-photon exchange where most of the transferred momentum is carried by one photon, while the other photon has very small momentum. Our considerations concern the case when the momentum transfer is shared between two photons.

In Ref. [2] the two-photon amplitude is purely imaginary, at least at very small scattering angles, so it cannot interfere with the one-photon exchange amplitude in the differential cross section of unpolarized particles scattering. However, in this case, the polarization observables in elastic ed scattering have to be large, in particular the T-odd polarization observables, as they take their maximum value if the relative phase of the interfering amplitudes is near $\pi/2$. But the predicted increasing of the two-photon mechanism is so large that it may be observed in the differential cross section of elastic ed scattering, at relatively large momentum transfer square, $q=8-12$ fm^{-1}. In this region new results from Jefferson Laboratory, obtained with a very good accuracy [7,8], can be compared with the previous SLAC results [9].

The dedicated experiments completed up until now [10] have not shown any deviation from the one-photon expectation, and the Rosenbluth separation, when done, shows a linear dependence of the cross section for different angles, at a fixed value of the momentum transfer, q. However the two-photon contribution has been recently experimentally observed in the domain of very small energies in atomic physics [11,12]. The accuracy of elastic scattering experiments is smaller and the corresponding theoretical calculations of two-photon contributions (called *dispersion corrections*) [13] are very complicated.

In the last experiments completed at the Jefferson Laboratory, on ed-elastic scattering no test of the validity of the one-photon mechanism, like the Rosenbluth separation, was done and no dedicated polarization measurement exists in this range of momentum transfer. The following general formula holds for the differential cross section of elastic scattering of an unpolarized electron by an unpolarized target:

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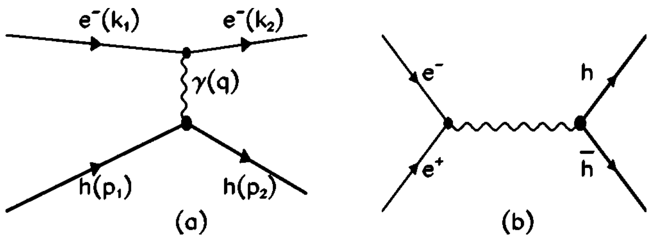

FIG. 1. One-photon approximation for the crossed channels $e^- + h \to e^- + h$ and $e^- + e^+ \to h + \bar{h}$.

$$\frac{d\sigma}{d\Omega_e} = \sigma_0 \left[A(q) \cot^2 \frac{\theta_e}{2} + B(q) \right], \tag{1}$$

where $A(q)$ and $B(q)$ are real functions of the single variable q, called structure functions (SF), and θ_e is the electron scattering angle in LAB-system. The SF's $A(q)$ and $B(q)$ are quadratic combinations of the charge, quadrupole and magnetic form factors, respectively G_C, G_Q, and G_M:

$$A(q) = G_C^2 + \frac{8}{9}\tau^2 G_Q^2 + \frac{2}{3}\tau G_M^2, \quad B(q) = \frac{4}{3}\tau(1+\tau) G_M^2,$$

$$\tau = -\frac{q^2}{4M^2},$$

where M is the deuteron mass.

This linear $\cot^2(\theta_e/2)-$dependence (''Rosenbluth fit'') holds only in the framework of the one-photon exchange mechanism. As a result, the Rosenbluth fit allows on one hand, the separation of the structure functions $A(q)$ and $B(q)$, and on the other hand, the test of the validity of one-photon mechanism. For this, it is necessary to measure the differential cross section at least at three different values of the scattering angle at a fixed value of q. This procedure was previously used during the measurements of ep- and ed-elastic scattering.

Studies of distortion effects induced by the static Coulomb field of the target in quasi-elastic reactions [14] on heavy nuclei show that the Rosenbluth separation might still be valid, but care must be taken in the interpretation of the structure functions. The quantitative results are model dependent and effects are expected to be small in case of elastic sacttering on light nuclei. Our approach here differs essentially, as we consider two ''hard'' photons, which share each half of the transferred momentum. We look for model independent estimations for possible deviations from the Rosenbluth formula (1), and derive a general formula based only on the properties of crossing symmetry. No model for the electromagnetic form factors of hadrons or for the dynamics of the 2γ-exchange is needed. The crossing symmetry provides a relation between different channels of a process. In our case we compare the elastic $e^- + h \to e^- + h$ scattering with e^+e^--annihilation: $e^+ + e^- \to \bar{h} + h$, in one-photon approximation (Fig. 1), where the crossing symmetry can be expressed by the following relation between the matrix elements $\mathcal{M}$ of the crossed processes:

$$\overline{|\mathcal{M}(eh \to eh)|^2} = f(s,t) = \overline{|\mathcal{M}(e^+e^- \to \bar{h}h)|^2}. \tag{2}$$

The line over $\mathcal{M}$ denotes the sum over the polarizations of all particles (in initial and final states). The Mandelstam variables s and t are defined as follows (for the scattering channel):

$$s = (k_1 + p_1)^2 = M^2 + 2E_1 M \geq M^2, \quad m_e = 0,$$

$$t = (k_1 - k_2)^2 = q^2, \quad q^2 < 0, \tag{3}$$

where m_e is the electron mass and the particle 4-momenta are shown in Fig. 1. The annihilation channel $e^+ + e^- \to \bar{h} + h$, and the scattering channel $e^\pm + h \to e^\pm + h$ correspond to different kinematical regions for the variables s and t.

In the center of mass of the $e^+ + e^- \to \bar{h} + h$ (this channel is preferable for the kinematical analysis), we can write $t = 4\tilde{\epsilon}^2$, $s = M^2 - 2\tilde{\epsilon}^2 + 2\tilde{\epsilon}\sqrt{\tilde{\epsilon}^2 - M^2}\cos\tilde{\theta}$, where $\tilde{\epsilon}$ is the energy of the initial lepton (or final hadron), and $\tilde{\theta}$ is the hadron production angle. The presence of a single virtual photon in the reaction $e^+e^- \to \gamma^* \to \bar{h}h$ constrains the total angular momentum $\mathcal{J}$ and the P parity for the $\bar{h}h$ system to take only one possible value, $\mathcal{J}^P = 1^-$, the quantum number of the photon. Therefore, in the framework of the one-photon approximation, the $\cos\tilde{\theta}$ dependence of $\overline{|\mathcal{M}(eh \to eh)|^2}$ can be predicted in a general form:

$$\overline{|\mathcal{M}(eh \to eh)|^2} = a(t) + b(t)\cos^2\tilde{\theta}, \tag{4}$$

where $a(t)$ and $b(t)$ are definite quadratic combinations of the electromagnetic form factors for the hadron h. This specific and simple $\cos\tilde{\theta}$ dependence for the universal function $f(s,t)$ is due to the C invariance of the electromagnetic hadron interaction, which allows only even degrees of $\cos\tilde{\theta}$. Moreover the degree of the $\cos\tilde{\theta}$ polynomial is limited to the second order, due to the spin of the virtual photon.

The equivalent expression of Eq. (4) for the scattering channel, using crossing symmetry properties, can be found by rewriting $\cos\tilde{\theta}$ in terms of the invariant variables s and t:

$$\cos^2\tilde{\theta} = 1 + \frac{st + (s - M^2)^2}{t\left(\frac{t}{4} - M^2\right)}. \tag{5}$$

Substituting Eq. (5) in Eq. (4), we find the expression for $f(s,t)$ [in terms of the SF's $a(t)$ and $b(t)$ for the annihilation channel]. Let us consider the function $f(s,t)$ in the physical region of the scattering channel. The following relation follows from Eqs. (3) and (5):

$$\cos^2\tilde{\theta} = 1 + \frac{\cot^2\frac{\theta_e}{2}}{1+\tau}. \tag{6}$$

This one-to-one correspondence between $\cos^2\tilde{\theta}$ (in the annihilation channel) and $\cot^2(\theta_e/2)$ (in the scattering channel) explains the origin of the linear $\cot^2(\theta_e/2)$-dependence of the differential cross section for any eh process. This dependence results from the one-photon mechanism for elastic eh scattering.

This formalism allows us to easily describe and parametrize possible effects of the two-photon mechanism. Again it is more convenient to consider the annihilation channel.

The presence of 2γ in the intermediate state $e^+ + e^- \to 2\gamma \to \bar{h} + h$ can induce any value of the total angular momentum and space parity in the annihilation channel, because the relative 3-momentum for the 2γ state is nonzero, contrary to the case of the one-photon mechanism. However, one can draw a general consideration about the $\cos\tilde{\theta}$ dependence of the interference contribution to the differential cross section for the annihilation channel, $\overline{\mathrm{Re}\,\mathcal{M}_1\mathcal{M}_2^*}$, where $\mathcal{M}_1$ and $\mathcal{M}_2$ are the matrix elements corresponding to the one- and two-photon exchanges. The $\bar{h}h$ system, produced through 1γ and 2γ exchanges has different values of C-parity, because $C(\gamma)=-1$ and $C(2\gamma)=+1$. Therefore the above-mentioned interference must be an *odd* function of $\cos\tilde{\theta}$: $\overline{\mathrm{Re}\,\mathcal{M}_1\mathcal{M}_2^*}=\cos\tilde{\theta}(a_0+a_2\cos^2\tilde{\theta}+\dots)$. After translating this dependence to the scattering channel, we can find from the previous crossing symmetry considerations:

$$\frac{d\sigma}{d\Omega_e}(e^-h\to e^-h)=\sigma_0[a_0+x^2a+\alpha x(i_o+i_2x^2+..)], \tag{7}$$

where we introduce a new kinematical variable:

$$x=\sqrt{1+\frac{\cot^2\frac{\theta_e}{2}}{1+\tau}},$$

which seems the most adequate one for the study of possible deviations from the standard Rosenbluth formula. In the particular case of electron scattering at small angles, where $\cot^2(\theta_e/2)\gg 1$, one can write

$$\frac{d\sigma}{d\Omega_e}(e^-h\to e^-h)=\sigma_0\left(A\cot^2\frac{\theta_e}{2}+B+C\cot\frac{\theta_e}{2}+D\cot^3\frac{\theta_e}{2}+\dots\right), \tag{8}$$

where the t-dependent coefficients C and D, which characterize the contribution of the interference between one- and two-photon mechanisms in $e^-+h\to e^-+h$ processes, have order α, relative to the main contributions A and B. In the presence of a two-photon mechanism, if the electron is detected at small angles, the cubic term D, would give the largest contribution. And the values of $A(q)$ extracted for measurements at different electron scattering angles, using the standard one-photon procedure, would be different. This might explain, in principle, the difference among existing sets of data, which are taken at different angles and different initial energies, in particular between the new results [7,8].

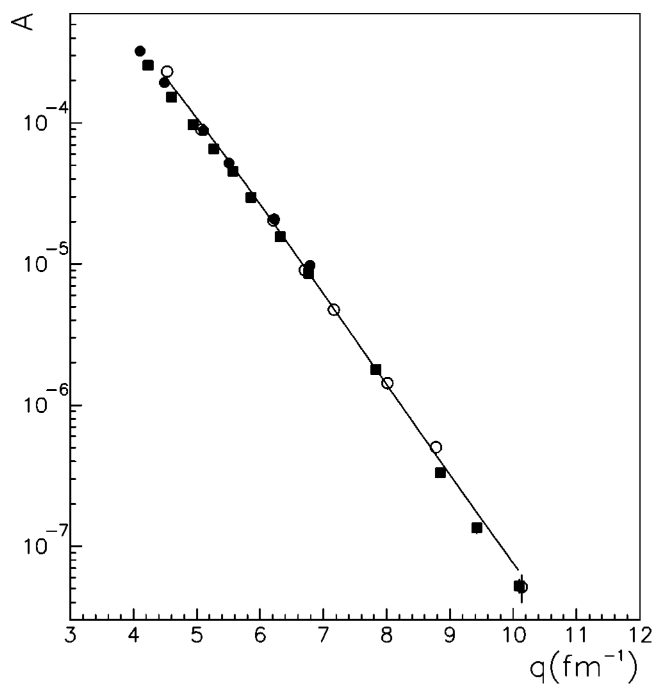

FIG. 2. Data set for the structure function A used in the analysis: solid circles from [7]; solid squares from [8]; open circles from [9]. The line is the result of the reference fit on the base of the data from [9] (see text).

We will consider here the ed-elastic scattering data from three experiments [7–9]. The $B(q)$ contribution can be neglected in the kinematical conditions of all these experiments. In order to compare different sets of data, in the spirit of the Rosenbluth fit, we should have cross sections at different angles and at the same momentum transfer. Unfortunately this information is not experimentally available, but the data in [9] have been taken at the same θ_e. In this case we can interpolate the cross section, for $\theta_e=8°$, as a function of the momentum transfer squared, according to the following expression:

$$\frac{d\sigma}{d\Omega_e}=\left(\sigma_0\cot^2\frac{\theta_e}{2}\right)\Bigg|_{\theta_e=8°}\frac{p_1}{(1+q^2/p_2)^{p_3}}, \tag{9}$$

where q is expressed in fm^{-1}. The best fit is obtained for $p_1=0.006\pm0.002$, $p_2=57\pm15\ \mathrm{fm}^{-2}$, and $p_3=11\pm1$. We attribute to the fitted cross section a global error of $\pm5\%$ (Fig. 2).

This allows us to express the cross section at one electron scattering angle ($\theta_e=8°$), for any value of q corresponding to the other points, and it will serve as reference. In the kinematic conditions of the new experiments [7,8], where θ_e is larger, we shall use the following formula, which is equivalent to Eq. (7):

$$\frac{d\sigma}{d\Omega_e}=\sigma_0[A(q)(x^2-1)(1+\tau)+C(q)x+D(q)x^3].$$

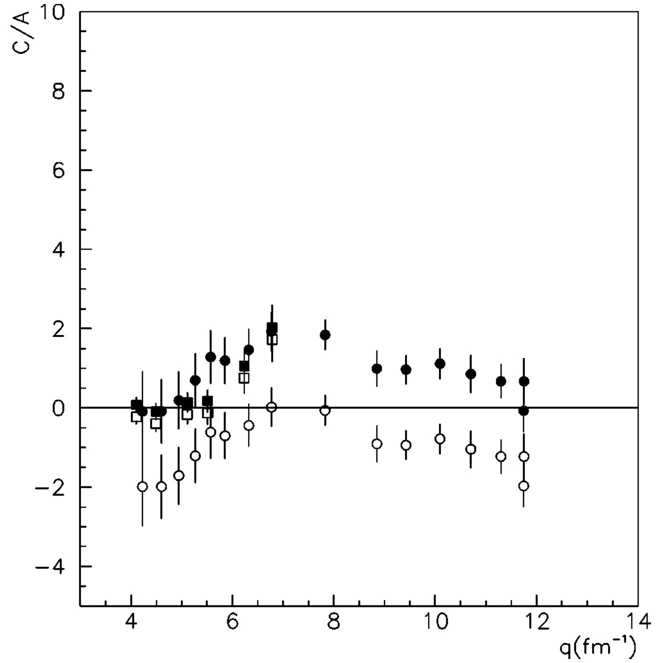

FIG. 3. Ratio C/A for the two sets of data as a function of $q(\mathrm{fm}^{-1})$: squares from [7]; circles from [8], corresponding solid symbols after renormalization (see text).

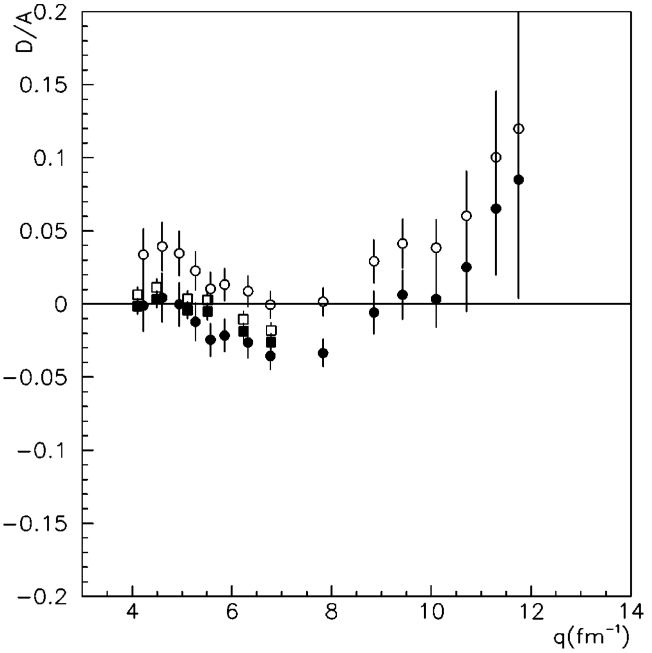

FIG. 4. Ratio D/A for the two sets of data as a function of $q(\mathrm{fm}^{-1})$: squares from [7]; circles from [8], corresponding solid symbols after renormalization (see text).

The x-odd contribution can now be estimated from the cross sections at two angles for the same q, one value being given from the fit, Eq. (9), and the other by the recent data [7,8]. We can then calculate separately the linear contribution or the cubic contribution. It is expected that deviation from the linear $\cot^2(\theta_e/2)$ formula would not appear for $q \leqslant 5\ \mathrm{fm}^{-1}$. The resulting ratios C/A and D/A are reported in Figs. 3 and 4 as functions of q (open squares [7] and open circles [8]). The first set of data [7] extends up to $q = 6.8\ \mathrm{fm}^{-1}$ and the last two points show a deviation from zero. The data from Ref. [8] extend up to higher values of momentum transfer, $q = 12.4\ \mathrm{fm}^{-1}$. If we rescale these ratios in order to have zero deviation at low q (corresponding solid symbols), attributing this shift to systematic errors in the measurement, the data agree nicely: at large momentum transfer these ratios deviate from zero and show a dependence on the transferred momentum, which could result from two photon exchange.

While this cannot be considered as definite evidence for the presence of 2γ exchange in ed-elastic scattering, it is the first attempt to obtain a quantitative upper limit of a possible 2γ contribution using a parametrization of the 2γ term and the existing experimental data. The quality of the data is clearly reflected in these ratios. On the other hand, the discrepancy between the two new sets of data does not appear to be explained by this contribution. This analysis does not exclude a detectable 2γ contribution in a dedicated experiment. Therefore our estimation can be used as a guideline for future experiments. For example, the difference in the cross section of electron and positron scattering on the deuteron, in sign and absolute value, can be derived from the odd contribution mentioned above:

$$\frac{\left[\dfrac{d\sigma}{d\Omega_e}(e^- d) - \dfrac{d\sigma}{d\Omega_e}(e^+ d)\right]}{\left[\dfrac{d\sigma}{d\Omega_e}(e^- d) + \dfrac{d\sigma}{d\Omega_e}(e^+ d)\right]} = \frac{C \cot\dfrac{\theta_e}{2} + D \cot^3\dfrac{\theta_e}{2}}{A \cot^2\dfrac{\theta_e}{2} + B}.$$

Because of the importance of this problem for hadron electrodynamics, not only the measurements of the differential cross section are necessary, but the study of polarization phenomena as well. In the case of very different spin structures of one- and two-photon mechanisms, the polarization phenomena have to be, in particular, sensitive to interference effects, but not the differential cross section (with unpolarized particles). The same is also correct if the 2γ amplitude is essentially imaginary. In this case the polarization observables of T-odd nature take the largest values.

We discuss briefly the polarization phenomena induced by $1\gamma \otimes 2\gamma$ interference for elastic ed scattering. Only one experiment was dedicated to the meaurement of the vector deuteron polarization P_y in unpolarized ed-elastic scattering at $q = 3.65\ \mathrm{fm}^{-1}$ [15]. The result was $P_y = 0.075 \pm 0.088$.

The recent progress in developing polarized deuterons targets [16] and in the polarimetry of the produced deuterons [17] makes large accuracy measurements of vector and tensor deuteron polarization possible. Although measuring tensor polarization is more difficult than vector polarization, the tensor polarization can also be used as a test of one-photon mechanism. Let us write the corresponding formulas for the different components t_{2q}, $q = 0,1,2$ of the tensor polarization of the scattered deuteron, which are valid for the one-photon mechanism:

$$Wt_{20}=-\frac{1}{2}\left[\frac{8}{3}\tau G_C G_Q+\frac{8}{9}\tau^2 G_Q^2+\frac{\tau}{3}\left(1+2(1+\tau)\tan^2\frac{\theta_e}{2}\right)G_M^2\right],$$

$$Wt_{21}=-\frac{2}{\sqrt{3}}\tau\left(\tau+\tau^2\sin^2\frac{\theta_e}{2}\right)^{1/2}G_M G_Q\sec\frac{\theta_e}{2},$$

$$Wt_{22}=-\frac{1}{2}\sqrt{3}\tau G_M^2,\ \text{and}\ W=A(q)+\tan^2\frac{\theta_e}{2}B(q).$$

One can see, that in this approach, the component Wt_{22} and the θ_e-dependent contribution to Wt_{20} are determined only by the deuteron magnetic form factor, which can be found on the basis of the SF $B(q)$ (from the backward ed scattering). Therefore, these two terms can be derived from the SF $B(q)$, only. Such relations between form factors and polarization observables are valid only in one-photon approximation. Any inconsistency between measured values and values expected from these relations can be considered as evidence of the presence of two-photon exchange.

A similar procedure can be suggested for polarization phenomena in elastic electron-proton scattering. Here we also have one example of asymmetry in $\vec{e}+\vec{p}$ scattering from the two possible independent ones, which is induced by the proton polarization in the direction of the 3-momentum of the virtual photon, and which is determined by G_{Mp}^2, only. This form factor can be measured at large momentum transfer with good accuracy.

In conclusion, several authors [2–5] have suggested that, with increasing momentum transfer, the role of two-photon exchange could become very important. Up until now two-photon exchange in electron scattering by protons or deuterons has not been experimentally observed.

We have given here an attempt to evaluate the presence of two ''hard'' photon contributions to ed-elastic scattering through a deviation from the linear dependence in $\cot^2(\theta_e/2)$ of the cross section using a Rosenbluth fit, which has been parametrized in a model independent way according to crossing symmetry considerations.

Let us summarize here the possible methods to test the presence of 2γ exchange in elastic hadron scattering.

(i) Comparison of the cross section for scattering of unpolarized electrons and positrons (by protons or deuterons) in the same kinematical conditions.

(ii) Deviation from a straight line on the Rosenbluth plane: cross section versus $\cot^2(\theta_e/2)$.

(iii) Specific polarization phenomena (a) appearance of T-*odd* polarization observables and (b) violation of definite relations between polarization observables (T even) and the SF $B(q)$ for ep and ed scattering.

The Rosenbluth fit (i.e., the differential cross section with unpolarized particles) and polarization phenomena, especially the T-odd polarization observables, seem the most perspective ways to realize this program. These two types of measurements bring complementary and independent pieces of information.

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