# $d\pi\pi$ decay of the $d^*$ dibaryon

Chun Wa Wong
*Department of Physics and Astronomy, University of California at Los Angeles, Los Angeles, California 90095-1547*
(Received 16 February 1999; published 10 August 1999)

The $d^*\to d\pi\pi$ partial decay width has been calculated in a wave-function model for $d^*$ and $d$. It is found to be smaller than a previous estimate by a factor of 7. A previously proposed dependence on dibaryon sizes is confirmed. The large reduction found is caused partly by a change in the $d^*$ size used and partly by the need to match momenta in pion emissions, a feature not included in the previous estimate.
[S0556-2813(99)03309-9]

PACS number(s): 14.20.Pt, 25.40.Ep

Years ago, Goldman *et al.* [1] (referred to below as GMSSW) estimated the $d\pi\pi$ partial decay width of the dibaryon $d^*(J^\pi=3^+,T=0)$ to be about 20 eV at the dibaryon mass $m^*$ 100 MeV above the $d\pi\pi$ threshold of about 2150 MeV/$c^2$. Though small, this width is of some interest as the total pion production cross section of $d^*$ from deuteron can be related to it. The result estimated by GMSSW is 0.1 $\mu$b at an incident pion momentum of 580 MeV/$c$.

To estimate the decay width, GMSSW used an effective $dd^*\pi\pi$ vertex of the form $(g/m_\pi^2)d^*_{\{\mu\nu\lambda\}}d^\lambda\partial^\mu\pi_1\partial^\nu\pi_2$, where $\{\mu\nu\lambda\}$ are symmetrized indices. The effective coupling constant $g$ can be related to the effective coupling constant $g_{\pi N\Delta}$ for $\Delta\to N\pi$ decay by the expression

$$g\approx g^2_{\pi N\Delta}(m_\pi/M_N)^2 f(d^*\to\Delta\Delta)f(d\to NN), \tag{1}$$

where the vertex overlap factors are taken to be $f(d\to NN)\sim 1$ and $f(d^*\to\Delta\Delta)\sim(r^*/r_d)^3\sim 2^{-3}$, where $r_d$ ($r^*$) is the deuteron ($d^*$) radius. In this approximate treatment, one of the vertex factors can always be set to one, since it is only the volume ratio that matters. The reduction shown for the second vertex factor represents the estimated difficulty of finding the same three quarks in each baryon in the smaller size in $d^*$ when the baryon was originally in the larger bound state that was $d$. This GMSSW approach to the $d\pi\pi$ partial width is elegant, but it has an Achilles heel, namely, the inability to improve on the given choice of the vertex factor. It is therefore of some interest to determine if the reported estimate is reliable.

Recently, I have estimated the $NN$ and $\pi NN$ partial widths of $d^*$ using a perturbative method based on a baryon wave function for $d^*$ [2,3]. The purpose of this Brief Report is to illustrate how this more systematic approach can be applied to the calculation of the $d\pi\pi$ partial width. The calculated width shows the size effect proposed by GMSSW, but depends additionally and strongly on the momentum matching required in the pion emissions, a feature that is completely missing in the GMSSW approach.

For an $S$-wave deuteron state described by a realistic Bonn $C$ wave function [4] and a three-term Gaussian approximation to a two-center baryon wave function for $d^*$, the width calculated here is about 3 eV at $m^*=2250$ MeV/$c^2$. This estimate is smaller than that reported by GMSSW by a factor of 7. About half of this reduction is caused by my use of a smaller volume ratio, while the remaining half of the discrepancy comes from the effect of momentum matching among the wave functions required in the pion emissions.

The $d^*\to d\pi\pi$ decay in the lowest-order perturbation theory can be visualized as the decay of the two off-shell $\Delta$'s shown in Fig. 1 in time-ordered diagrams. The resulting decay width is, according to the Fermi golden rule [5],

$$\Gamma=\frac{1}{(2\pi)^5}\int\langle|(V_1G_{12}V_2)_{\rm fi}|^2\rangle_{\rm spin}E_1E_2E_3dE_1dE_2d^2\Omega_1d\phi_2, \tag{2}$$

where $V_i$ is a pion-emission vertex from a baryon, and $E_1$ and $E_2$ are the pion energies in the $d^*$ rest frame. $G_{12}$ is the sum of Green functions for the two diagrams labeled $a$ and $b$. To avoid possible zeros in the energy denominators $D_i$, $i=a,b$, where, for example,

$$D_a=m^*-E_1(\mathbf{k}_1)-E_N(-\mathbf{p}-\mathbf{k}_1)-E_\Delta(\mathbf{p}), \tag{3}$$

I give $m^*$ a total width $\Gamma_{\rm tot}$. Consequently

$$G_{12}=\frac{D_a}{D_a^2+\Gamma^2_{\rm tot/2}}+\frac{D_b}{D_b^2+\Gamma^2_{\rm tot/2}}. \tag{4}$$

The Green-function factor $G_{12}^2$ depends on the angles. However, if it is approximated by a suitable angle-averaged value $\langle G_{12}^2\rangle$, and if the baryons in both initial and final dibaryons are in relative orbital $S$ states, the angle integrations shown in Eq. (2) can be performed analytically to give

$$\Gamma\approx\frac{2}{(2\pi)^3}\int\langle G_{12}^2\rangle\langle|(V_1V_2)_{\rm fi}|^2\rangle_{\rm spin,\,angle}E_1E_2E_3dE_1dE_2. \tag{5}$$

Although the baryon-baryon relative wave functions of both $d^*$ and $d$ will eventually be expressed as sums of Gaussians, it is sufficient to show the result for the spin- and angle-averaged squared matrix element in the integrand for single Gaussians such as

$$\psi_{d^*}(\mathbf{p})=(\beta^{*2}/\pi)^{3/4}e^{-p^2/2\beta^{*2}} \tag{6}$$

for $d^*$. The result obtained after the integration over the internal momentum $\mathbf{p}$ shown in Fig. 1 is

 

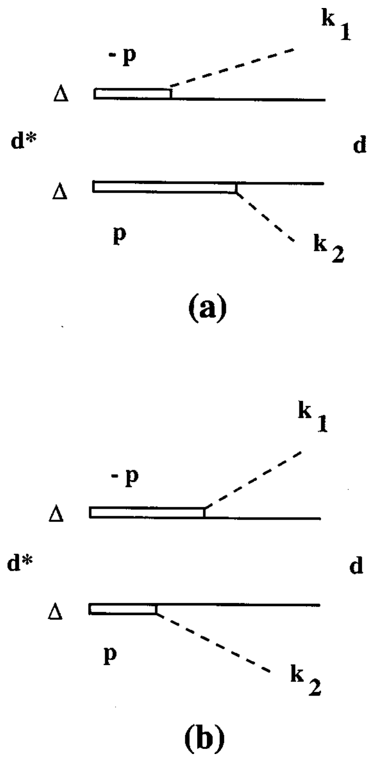


FIG. 1. The two leading-order time-ordered diagrams for the decay $d^* \to d\pi\pi$.

$$\langle |(V_1V_2)_{\rm fi}|^2\rangle_{\text{spin, angle}} = \frac{A^2}{54}\left(\frac{2\beta\beta^*}{B^2}\right)^3 \frac{k_1^2k_2^2}{E_1E_2} \times e^{-\alpha(k_1^2+k_2^2)}\left(j_0(-ix)-\frac{1}{5}j_2(-ix)\right), \tag{7}$$

where

$$A=\Gamma_\Delta\left(\frac{3}{4\pi^2}\right)\frac{M_\Delta}{k^{*3}E_N^*}e^{\alpha_0 k^{*2}},$$

$$B^2=\beta^2+\beta^{*2}, \quad \alpha=\alpha_0+\frac{1}{4B^2}, \quad x=\frac{k_1k_2}{2B^2}. \tag{8}$$

Here $\Gamma_\Delta = 120$ MeV is the $\Delta$ width, $k^*(E_N^*)$ is the momentum of the decay pion (recoiling nucleon) in the rest frame of the decaying $\Delta$, and

$$\alpha_0 = r_p^2/3 = 0.12 \ \ \text{fm}^2 \tag{9}$$

is the parameter, taken to be the same in $N$ and $\Delta$, describing the Gaussian wave functions of quarks inside these baryons. The baryon form factors in the pion-emission vertex give rise to the $\alpha_0$ term in $\alpha$.

When the baryon wave function of the dibaryon $d^*$ is approximated by a single Gaussian, the parameters $\beta^*$ and the mean squared (MS) baryon momentum $\langle \mathbf{p}^2\rangle$ inside the dibaryon are related to the dibaryon radius $r^*$ as

$$\beta^* = \sqrt{3/8}/r^*, \quad \langle \mathbf{p}^2\rangle = \frac{9}{16r^{*2}}. \tag{10}$$

The average baryon energies to be used in the Green-function factor $\langle G_{12}^2\rangle$ are

$$E_\Delta(\mathbf{p}) \approx \sqrt{M_\Delta^2+\langle \mathbf{p}^2\rangle},$$

$$E_N(\mathbf{p}+\mathbf{k}) \approx \sqrt{M_N^2+\langle \mathbf{p}^2\rangle + k^2}. \tag{11}$$

Certain features in our results are worth pointing out: The $\beta$-dependent factor in Eq. (7) can be expressed approximately in terms of baryon radii as

$$\left(\frac{\beta\beta^*}{B^2}\right)^3 \approx \left(\frac{r^*}{r_d}\right)^3. \tag{12}$$

This is the only feature included in the GMSSW vertex factor. Note, however, that if we use $r_d = 2.0$ fm and $r^* = 0.7$ fm, then this volume ratio is smaller than that used by GMSSW by a factor of 3. It is possible that GMSSW have used a larger volume ratio in the expectation that in the quark-delocalization model the quarks from a baryon constituent of the dibaryon are more spread out over the dibaryon than in a conventional baryon-baryon bound state. Unfortunately, this expectation has not been quantified in GMSSW.

A quantitative study of this effective size effect of delocalization on the $d\pi\pi$ decay width is not easy because delocalized quark wave functions [1,6] are so complicated when antisymmetrization and angular momentum projections are made. In addition, delocalized short-distance wave functions will also have to be used in the deuteron for overall consistency. In the more literal approach taken here, this important question will be left as an open problem for future study. On the other hand, certain other features of the problem, such as an increase in the dibaryon size, can be studied rather easily in the simple model used in this paper.

The remaining features of Eq. (7) are not included in the GMSSW vertex factor. They describe the requirements of momentum matchings on pion emissions. Note that the parameter $B^2$ is dominated by $\beta^{*2}$, which is proportional to the MS baryon momentum contained in $d^*$. For $r^* = 0.7$ fm, $B^2 = 0.29$ fm$^2$, which is larger than $\alpha_0$ by a factor 2.4. This shows that the dependence of the parameter $\alpha$ on baryon sizes in the inelastic baryon form factors is significantly weaker than the dependence on $r^*$. If the $\Delta$ radius $r_\Delta$ is different from the nucleon radius $r_p$, the parameter $\alpha_0$ should be replaced by

$$\alpha_0\left(\frac{2r_\Delta^2}{r_p^2+r_\Delta^2}\right) \approx \alpha_0\left(\frac{r_\Delta}{r_p}\right). \tag{13}$$

Since $r_\Delta$ is expected to be larger than $r_p$ by only 10% ($\approx 5\%$) in the Massachusetts Institute of Technology bag model [7] (many potential models [8]), the overall effect in the parameter $\alpha$ is only 3(2)%, which is quite negligible. In contrast, results will be shown below where the parameter $\alpha$

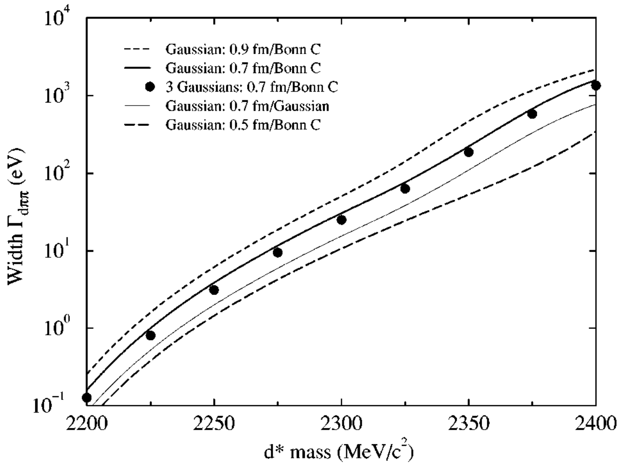


FIG. 2. The $d^* \rightarrow d\pi\pi$ partial decay width $\Gamma_{d\pi\pi}$ as a function of the $d^*$ mass $m^*$ for single-Gaussian and Bonn *C* deuteron *S*-state wave functions, and for single-Gaussian and two-center $d^*$ wave functions. The number shown in the legend is the $r^*$ radius in fm.

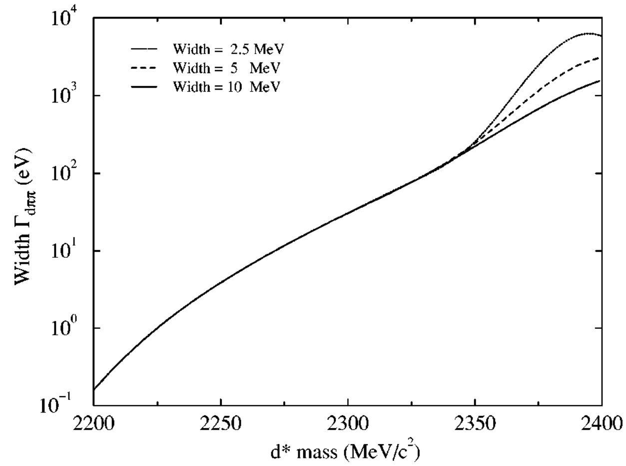


FIG. 3. The $d^* \rightarrow d\pi\pi$ partial decay width $\Gamma_{d\pi\pi}$ as a function of the $d^*$ mass $m^*$ for Bonn *C* deuteron *S*-state wave function and single-Gaussian $d^*$ wave function (with $r^* = 0.7$ fm) when the input total width of $d^*$ is varied.

changes by a factor of almost 2. I therefore conclude that the use of the same baryon size for both $\Delta$ and $N$ is justified both here and in the ''delocalization'' model of [1,6]. Without this simplification of equal baryon sizes, the delocalization model would be even harder to execute technically.

The total width $\Gamma_{\text{tot}}$ of $d^*$ is also needed in the calculation. It is known to increase with increasing $d^*$ mass $m^*$, rising from about 1 MeV at $m^* = 2100$ MeV/$c^2$ to about 10 MeV at $m^* = 2350$ MeV/$c^2$ [2,3]. However, the presence of $\Gamma_{\text{tot}}$ is unimportant at the low end of this mass range, because the energy denominators $D_i$ are large there. Beyond the $\Delta N\pi$ threshold at 2310 MeV, however, the energy denominators could become small; the choice of $\Gamma_{\text{tot}}$ then becomes important. For this reason, I use a nonzero width in the calculation, but choose for simplicity a single constant value of $\Gamma_{\text{tot}} = 10$ MeV appropriate to the high end of the mass range studied here. The effect of using smaller values of $\Gamma_{\text{tot}}$ will be explicitly shown below.

The single *S*-state Gaussian wave function for the deuteron depends on the parameter $\beta = \sqrt{3/8}/r_d$, where $r_d = 1.967$ fm is the radius of the deuteron wave function. The calculated $d\pi\pi$ partial width for $r^* = 0.7$ fm is shown in Fig. 2 as a thin solid curve for a range of $m^*$. The value at $m^* = 2250$ MeV/$c^2$ is 2.0 eV, ten times smaller than the estimate given by GMSSW. Besides the decrease by a factor of 3 already pointed out and discussed previously, there is a remaining reduction by another factor of 3 which should be attributed to the technical improvements made in the present calculation.

For a more realistic *S*-state wave function, I use the three-term Gaussian fit to the Bonn *C* deuteron *S*-wave wave function obtained in [2]. The fitted *S*-state probability of 94.34% (versus 94.39% for the original Bonn *C* wave function) has been renormalized back to 100% for a pure *S* state. The resulting $d\pi\pi$ partial width is shown in Fig. 2 as a solid (dashed, long-dashed) curve for $r^* = 0.7$ (0.9,0.5) fm.

These curves show that the decay width for the realistic Bonn *C* wave function is larger than that for the single Gaussian wave function rather uniformly over the mass range. At $m^* = 2250$ MeV, the increase is by a factor of 1.9 at $r^* = 0.7$ fm. Since the deuteron radius is essentially the same in these calculations, the change must have come entirely from the increase in the high-momentum components in the Bonn *C* wave function.

Figure 2 also shows results calculated with the same Bonn *C* wave function but different $d^*$ radii. In the simple volume scaling model of GMSSW, the decay width for $r^* = 0.7$ fm would be increased (decreased) a factor of 2.2 (2.7) when $r_d^*$ is increased to 0.9 fm (decreased to 0.5 fm). The actual calculated factor at $m^* = 2250$ MeV turns out to be 1.6 (2.7) for the Bonn *C* deuteron, and 1.9 (3.3) for the single-Gaussian deuteron. Thus the overall volume scaling effect proposed by GMSSW appears to be present, but in a more complicated form that also depends on other details of the wave function. The main reason for the complication is that the parameter $\alpha$ also changes drastically with $r^*$—from 0.40 fm$^2$ for $r^* = 0.7$ fm to 0.28 (0.58) fm$^2$ for $r^* = 0.5$ (0.9) fm.

The additional sensitivity to the high-momentum components of the deuteron wave function shown in the present calculation suggests that the deuteron *D*-state component, though constituting only 5% of the deuteron, might have a disproportionate effect on these decay widths. Its inclusion is easy to visualize, but tedious to execute. I believe that its inclusion will not change the order of magnitude of the partial widths estimated here — after all, the effect of changing the *S*-state wave function from the smooth single Gaussian to the realistic Bonn *C* wave function is an increase by ''only'' a factor of 2.

I now turn to the effect of using smaller total decay widths in the calculation. The results for $\Gamma_{\text{tot}} = 5$ (2.5) MeV are shown in Fig. 3 as a dashed (dotted) curve, and compared with the corresponding result for $\Gamma_{\text{tot}} = 10$ MeV from Fig. 2, reproduced here as a solid curve. The calculated width can be seen to develop significant dependence on the total decay width only well above the $\Delta N\pi$ threshold of 2310 MeV.

The single-Gaussian wave function for $d^*$ is of course a rather crude approximation to the model of ''delocalized'' quarks in $d^*$ [1,6]. The least improvement one could make is to use a two-center Gaussian wave function for each *baryon* proportional to $\exp[-(\mathbf{x}-\mathbf{s})^2/2]+\exp[-(\mathbf{x}+\mathbf{s})^2/2]$, where $\mathbf{x}$ is the (dimensionless) relative baryon-baryon coordinate and $2\mathbf{s}$ is the separation between the two centers. The projected $S$-state component of this relative wave function has the form

$$\psi_0(x,s)=N_0\frac{1}{x}[e^{-(x-s)^2/2}-e^{-(x+s)^2/2}]. \tag{14}$$

The lowest-order effect of $s$ can be included in the single-Gaussian wave function by matching the MS radius

$$\langle x^2\rangle=\frac{1}{2}+\frac{s^2}{1-e^{-s^2}}\approx\frac{3}{2}+\frac{s^2}{2}. \tag{15}$$

For small $s$, one sees the separate contributions from the zero-point motion of the baryons in the two harmonic oscillator potentials and from the separation $2s$ of these potentials. If one starts with quark wave functions described by a size constant $b=0.6$ fm, the relative baryon-baryon motion for the six-quark state will be described by the size constant $b_r=b/\sqrt{6}$. Hence $r^*=b_r\langle x^2\rangle^{1/2}$. For a potential separation $2S=1.40$ fm calculated for the $d^*$ [1,6], $s=S/b_r=2.86$, and hence $r^*=0.72$ fm. This is close to the middle of the three values used in Fig. 2.

It is easy to go beyond this rough approximation and actually fit the momentum wave function of the projected two-center Gaussian, namely, $\sim\exp[-(b_r p)^2/2]j_0(pS)$, as a sum of three Gaussians

$$\psi_{2c}(p)\approx\sum_{i=1}^{3}c_i\psi_i(p), \tag{16}$$

where $\psi_i(p)$ is a normalized Gaussian wave function. In order to emphasize the stronger high-momentum components introduced by the oscillating factor $j_0(pS)$, the range parameters are obtained by minimizing the *percentage* MS deviation. The fitted result is

$$\boldsymbol{\gamma}=(\gamma_1,\gamma_2,\gamma_3)=0.9961(1,1.25,1.46)/b_r^2, \tag{17}$$

where $\gamma_i=2\beta_i^2$. The expansion coefficients, renormalized from the fitted normalization of 1.0019 back to 1, are

$$\mathbf{c}=(c_1,c_2,c_3)=(14.8311,-27.4124,13.3505). \tag{18}$$

The $d\pi\pi$ partial decay width can now be calculated with this improved $d^*$ wave function but with everything else treated in the same way as the single-Gaussian case with $r^*=0.7$ fm. The results, shown as solid circles in Fig. 2, are about 20% smaller than those for the single-Gaussian $d^*$ wave function. The radius of the fitted $d^*$ wave function, at 0.72 fm, is actually marginally larger, but the decrease in the calculated decay width shows that it is the additional high-momentum components in the $d^*$ wave function that dominate the result.

Although the present calculation is a significant improvement over that of GMSSW, it is also not a quantitative calculation. Too many approximations have to be made to reduce the problem to a manageable form. The most important features that should be included in a realistic calculation include the following: (1) higher-order Feynman diagrams, especially those involving intermediate- and final-state interactions; (2) quark antisymmetrization between baryons and quark-delocalization effects in both $d^*$ and $d$; (3) deuteron $D$ state and perhaps better short-distance wave functions for both $d$ and $d^*$, in both $NN$ and exotic channels; (4) improved treatment of the energy denominator, requiring the use of full angle integrations; (5) better treatment of the $\Delta N\pi$ vertex; (6) a better choice of the full decay width, but only when the dibaryon mass is above the $\Delta N\pi$ threshold. It is clear, however, that these improvements will require much more extensive calculations than those undertaken here.

I would like to thank Terry Goldman for urging me to improve on the single-Gaussian approximation for $d^*$ and Fan Wang for providing the parameters of the two-center Gaussian wave function.

[1] T. Goldman, K. Maltman, G.T. Stephenson, Jr., K.E. Schmidt, and F. Wang, Phys. Rev. C **39**, 1889 (1989).
[2] C.W. Wong, Phys. Rev. C **57**, 1962 (1998).
[3] C.W. Wong, Phys. Rev. C **58**, 2414 (1998).
[4] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987); R. Machleide, in *Advances in Nuclear Physics*, edited by J.W. Negele and E. Vogt (Plenum Press, New York, 1990), Vol. 19, p. 189.
[5] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[6] F. Wang, G.H. Wu, L.J. Teng, and T. Goldman, Phys. Rev. Lett. **69**, 2901 (1992); F. Wang, J.L. Ping, G.H. Wu, L.J. Teng, and T. Goldman, Phys. Rev. C **51**, 3411 (1995).
[7] T. DeGrand, R.L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2060 (1975).
[8] C.W. Wong, Prog. Part. Nucl. Phys. **8**, 223 (1982).