Exchange part of the real α-nucleus potential

M. Ismail, M. M. Osman, and F. Salah*
Physics Department, Faculty of Science, Cairo University, Giza, Egypt
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We improved the accuracy of calculating the exchange α-nucleus real potential by suggesting a simple expression for the nondiagonal density matrix of α-particle $\{\rho_\alpha(\vec{r},\vec{r}')\}$. We found that this expression reduces the error in calculating the total α-nucleus potential by more than 20% compared with using the density matrix expansion method to treat $\rho_\alpha(\vec{r},\vec{r}')$. [S0556-2813(99)04903-1]

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The double folding model is widely used to calculate the real parts of α-nucleus and nucleus-nucleus optical potentials [1,2]. For many scattering processes the heavy-ion (HI) potential is dominated by strong absorption and the elastic scattering data are sensitive to the value of the real optical potential only in the surface and tail regions. However, in some cases of nuclear rainbow scattering observed in α and light HI systems [3] the scattering data is sensitive to the HI optical potential over a wider radial domain. For these cases the simple double folding model in which the finite range exchange nucleon-nucleon (*NN*) force is replaced by a zero range pseudopotential [4] failed to give good results. It was found that the refractive α-scattering data require an accurate treatment of both density dependence of the effective *NN* interaction [5] and the exchange part of the α-nucleus potential which comes from applying the Pauli principle [6]. The exchange part contains the nondiagonal density matrix of the α particle that is usually approximated using the density matrix expansion (DME) method [7].

Recently, we tested the accuracy of calculating the exchange part of the α-nucleus potential [8] using the DME method to simplify the nondiagonal density matrix $\rho(\vec{r},\vec{r}')$. We found that this method produces a large error in the value of the total α-nucleus potential U. The error is mainly due to deriving $\rho(\vec{r},\vec{r}')$ for the α particle $[\rho_\alpha(\vec{r},\vec{r}')]$ from the DME method and using the extended Thomas-Fermi approximation [9] to calculate the α-particle kinetic energy density (τ_α).

In the present Brief Report we show that the error in calculating U can be largely reduced if we calculate $\rho_\alpha(\vec{r},\vec{r}')$ using oscillator model wave functions or even if we use the DME method with τ_α derived from the oscillator model. In the oscillator model the nondiagonal density matrix for the α particle is simply given by

$$\rho_\alpha\left(\vec{R}'+\frac{1}{2}\vec{s},\vec{R}'-\frac{1}{2}\vec{s}\right)=\sum_i \phi_i^*\left(\vec{R}'+\frac{1}{2}\vec{s}\right)\phi_i\left(\vec{R}'-\frac{1}{2}\vec{s}\right)$$
$$=\rho_\alpha(R')\exp\left(-\frac{s^2}{4b^2}\right), \qquad (1)$$

where b is the harmonic oscillator parameter. For a target nucleus with nondiagonal density $\rho(\vec{r},\vec{r}')$, the DME approximation is

$$\rho\left(\vec{R}'+\frac{1}{2}\vec{s},\vec{R}'-\frac{1}{2}\vec{s}\right)=\rho(R')\hat{j}_1[K_{\rm eff}(R')s], \qquad (2)$$

where $\vec{R}'$ and $\vec{s}$ are the center of mass and relative vectors of the two interacting nucleons, respectively. $\hat{j}_1(x)=j_1(x)/x$ and $K_{\rm eff}$ is given by [10]

$$K_{\rm eff}^2(R')=\frac{5}{3\rho(R')}\left[\tau(R')-\frac{1}{4}\nabla^2\rho(R')\right]. \qquad (3)$$

As pointed out in Ref. [10], Eq. (2) is a very good approximation for the α particle if τ is calculated from single particle wave functions and not from the Thomas-Fermi approximation [9].

Proceeding as in Ref. [8], the real part of the α-nucleus potential as a function of the distance between the centers of the two interacting nuclei (R) is given by

$$U(R)=\int \rho_N(\vec{r}_1)\rho_\alpha(\vec{r}_2)v_D(s,\rho)d\vec{r}_1 d\vec{r}_2$$
$$+\int \rho_N(\vec{r}_1,\vec{r}_1+\vec{s})\rho_\alpha(\vec{r}_2,\vec{r}_2-\vec{s})v_{\rm ex}(s,\rho)$$
$$\times\exp\left(\frac{i\vec{K}(R)\cdot\vec{s}}{M}\right)d\vec{r}_1 d\vec{r}_2, \qquad (4)$$

where the first and second terms are the direct (U_D) and exchange ($U_{\rm ex}$) parts of the α-nucleus potential. In Eqs. (4) v_D and $v_{\rm ex}$ are the direct and exchange parts of the effective nucleon-nucleon potential, ρ_N is the matter density distribution of the target nucleus, and *K(R)* is the relative-motion momentum given by

$$K^2(R)=\frac{2mM}{\hbar^2}[E_{\rm c.m.}-U(R)-V_C(R)], \qquad (5)$$

where M and $E_{\rm c.m.}$ are the reduced mass and the relative energy in the center of mass system, respectively, m is the nucleon mass, and $V_C(R)$ is the Coulomb potential.

For the nondiagonal density of the target nucleus $\rho_N(\vec{r},\vec{r}')$ we use the DME approximation given by Eq. (2)

*Present address: Physics Department, Faculty of Science, Helwan University, Egypt.

TABLE I. The percentage error $\Delta U_i=(U_{\rm exact}(R)-U_i(R))/U_{\rm exact}(R)\times 100$ for the potentials $U_1(R)$ and $U_2(R)$ calculated for three values of the separation distance R at two different values of the laboratory energy per projectile particle ($E_{\rm lab}/A_p=0.0$ and 20.7 MeV) for α-O^{16} and α-Ca^{40} by using the density- and energy-dependent *NN* forces (BDM3Y) derived in Ref. [6].

Pair	$\frac{E_{\rm lab}}{A_p}$(MeV)	R(fm)	BDM3Y1 $\Delta U1$	BDM3Y1 $\Delta U2$	BDM3Y3 $\Delta U1$	BDM3Y3 $\Delta U2$
		0.0	−0.3	0.7	1.1	1.8
	0.0	3.0	−0.1	4.3	0.2	4.6
		6.0	−0.4	9.5	0.4	9.1
α-^{16}O	20.7	0.0	−0.2	0.6	0.8	1.4
		3.0	−0.0	3.0	0.1	3.6
		6.0	−0.4	6.0	0.4	6.6
		0.0	−0.2	0.2	−1.4	−0.8
	0.0	4.0	0.7	2.2	0.6	2.3
		8.0	−0.3	1.7	−1.9	0.9
α-^{40}Ca		0.0	−0.2	0.2	−1.1	−0.6
	20.7	4.0	0.4	1.6	0.4	1.8
		8.0	0.0	0.5	0.0	0.5

first, then we calculate it exactly by building it from oscillator-model wave functions. In the first case we use Eq. (3) to calculate $K_{\rm eff}$ with τ given by one of the following Thomas-Fermi approximations [9,11]:

$$\tau_N(R')=\frac{3}{5}K_f^2\rho_N(R')+\frac{1}{3}\nabla^2\rho_N(R')+\frac{1}{36}\frac{|\vec{\nabla}\rho_N(R')|^2}{\rho_N(R')}, \tag{6a}$$

$$\tau_N(R')=\frac{3}{5}K_f^2\rho_N(R')+\frac{1}{3}\nabla^2\rho_N(R')+\frac{1}{4}\frac{|\vec{\nabla}\rho_N(R')|^2}{\rho_N(R')}, \tag{6b}$$

where $K_f^2=[\frac{3}{2}\pi^2\rho_N(R')]^{2/3}$.

In the present work we considered the two scattering processes α-^{16}O and α-^{40}Ca at the following three values of the incident energy (in laboratory system) per projectile particle: $E_{\rm lab}/A_P=0$, 20.7, and 46.6 MeV. In all our calculations we used Eq. (1) to calculate $\rho_\alpha(\vec{r},\vec{r}')$. For the nondiagonal density matrix of the target nucleus $\rho_N(\vec{r},\vec{r}')$, we first calculate it using harmonic oscillator wave functions and denote the α-nucleus potential calculated in this way by $U_{\rm exact}(R)$. Second we calculate *U(R)* using the DME method to approximate $\rho_N(\vec{r},\vec{r}')$. We denote by $U_1(R)$ and $U_2(R)$ the potentials calculated using approximations (6a) and (6b) for τ_N of the target nucleus, respectively. The nucleon-nucleon forces considered in the present calculations are the density-dependent M3Y forces (BDM3Y1 and BDM3Y3) proposed in Ref. [9]. These forces reproduced correctly both the features of the normal nuclear matter and the microscopic results of the nucleon optical-model potential. Moreover they generate different values of nuclear matter incompressibility coefficient *K*. Khoa *et al.* [2] have used BDM3Y *NN* forces

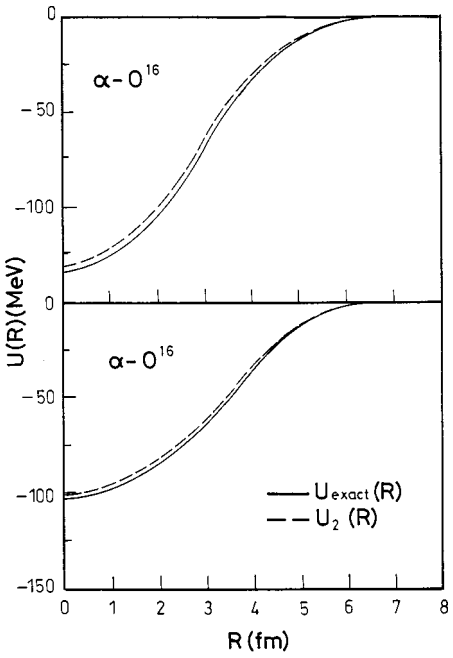

FIG. 1. The real α-O^{16} interaction potential at energy $E_{\rm lab}/A_p=0.0$ MeV calculated from Eq. (4) for the two *NN* forces BDM3Y1 (upper part), BDM3Y3 (lower part). The solid line and dashed lines represent $U_{\rm exact}(R)$ and $U_2(R)$, respectively.

to derive the α-nucleus potential whose internal region is sensitive to the value of *K*. They used them to fit the data for refractive α-nucleus scattering which is also sensitive to the value of the real potential at small distances. This permits the determination of the incompressibility coefficient for cold

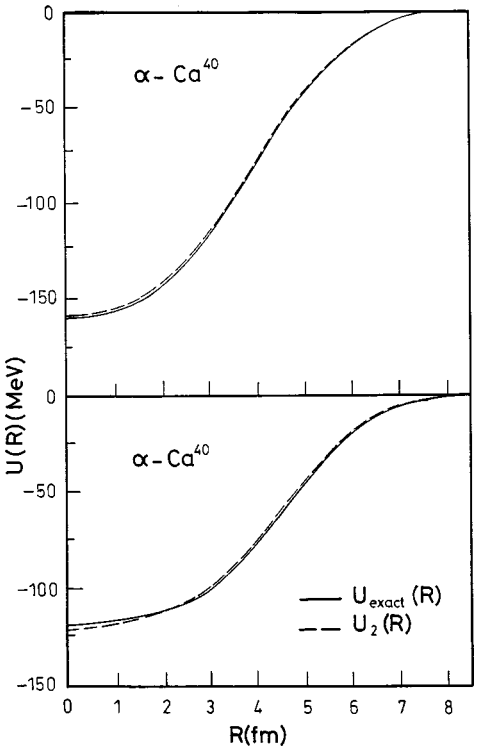

FIG. 2. The same as Fig. 1 but for α-Ca^{40}.

nuclear matter. This recent application of α-nucleus potential needs a correct real potential at all distances.

Our results are shown in Table I and Figs. 1 and 2. The figures show two examples of our calculations where the two density-dependent nucleon-nucleon interactions, namely, BDM3Y1 and BDM3Y3 of Ref. [9] have been used to calculate the α-^{16}O and α-^{40}Ca potentials at $E_{\rm lab}/A_P = 0.0$ MeV.

The figures and the table indicate that the exchange part of the real α-nucleus optical potential can be calculated correctly if Eq. (1) is used for $\rho_\alpha(\vec{r},\vec{r}')$ and Eqs. (2), (3), (6a) are used to approximate the density matrix of the target nucleus. In this case the maximum error in the potential U_1 for the two considered α-nucleus scattering pairs is less than 2% for BDM3Y3 force and 0.7% for BDM3Y1 force. If $\rho_\alpha(\vec{r},\vec{r}')$ is calculated from Eq. (6a) the maximum error in U_1 jumps to more than 25% for α-^{16}O [8]. Equation (6b) produces an error in tail region of about 10% for α-^{16}O scattering pair. As the mass number of the target nucleus increase the modified Thomas-Fermi approximation for τ [Eq. (6)], becomes more correct and the maximum errors U_1 and U_2 are reduced to less than 2% and 3%, respectively, for α-^{40}Ca scattering pair. Table I shows that the error in calculating both U_1 and U_2 is reduced by increasing the incident energy. Moreover the error corresponding to the force BDM3Y3 (K=275 MeV is usually larger than that for BDM3Y1 NN force (K=232 MeV). The reason for this is the difference in the shape of the density dependent functions of the two *NN* forces.

At the end of this Brief Report we can say that the α-nucleus real potential can be calculated with very good accuracy using the very simple expression for $\rho_\alpha(\vec{r},\vec{r}')$ given by Eq. (1) together with the DME approximation given by Eq. (2), (3), (6a) for the target nucleus nondiagonal density $\rho_N(\vec{r},\vec{r}')$.

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