

## Is there a proton-neutron interacting boson model rule for $M1$ properties?

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We investigate the robustness of the purported correlation between the  $g$ -factor ratios in the  $\gamma$  and ground bands and the corresponding  $E2/M1$  mixing ratios in the proton-neutron interacting boson model. We show that this correlation is dependent on the choice of the Hamiltonian and can be transgressed, when the parameters are chosen appropriately. The recent  $M1$  data in  $^{168}\text{Er}$ , which do not exhibit such a correlation, are analyzed in the light of these results. [S0556-2813(99)05608-3]

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The neutron-proton interacting boson model (IBM-2) [1–3] provides a natural mechanism for description of  $M1$  properties via  $F$ -spin breaking, which is related to the difference between the proton and neutron deformations. Theoretical investigations of  $M1$  properties in the framework of the IBM-2 [4–10] have, in turn, prompted many experiments to measure  $g$ -factors and  $E2/M1$  mixing ratios in rare-earth and transitional nuclei [11–15] (see [16] for a review of literature until 1990). From initial systematic studies [5,6], a correlation between the sense of  $F$ -spin breaking and the sign of the  $E2/M1$  mixing parameter  $\delta$  as well as the  $g$ -factor ratios  $R = g_\gamma/g_g$  in the ground and  $\gamma$  bands has been observed. This prediction, namely  $R > 1$ , if  $\delta > 0$ , or  $R < 1$ , if  $\delta < 0$ , seemed to hold for the available data until recently. The  $g$ -factor measurements of Brandolini *et al.* [14] and mixing ratio measurements of Alfter *et al.* [15] in Er isotopes indicate a violation of this ‘‘rule.’’ The purpose of this article is to investigate in more generality the IBM-2 systematics of  $R$  and  $\delta$  to see if the IBM-2 can accommodate the reported observations.

The IBM-2 Hamiltonian used in detailed fits of deformed nuclei has the form [3]

$$H = \epsilon_\pi \hat{n}_{d\pi} + \epsilon_\nu \hat{n}_{d\nu} + \kappa Q_\pi \cdot Q_\nu + \xi M + \sum_{\rho=\pi,\nu} \left[ \kappa_\rho Q_\rho \cdot Q_\rho + \sum_{L=0,2,4} c_{\rho L} [d_\rho^\dagger d_\rho^\dagger]^{(L)} \cdot [\tilde{d}_\rho \tilde{d}_\rho]^{(L)} \right], \quad (1)$$

where  $\hat{n}_{d\rho}$ ,  $\rho = \pi, \nu$  are the  $d_\rho$ -boson number operators,  $M$  is the Majorana operator in Casimir form, and  $Q_\rho$  are the quadrupole operators given by

$$Q_\rho = [d_\rho^\dagger s_\rho + s_\rho^\dagger \tilde{d}_\rho] + \chi_\rho [d_\rho^\dagger \tilde{d}_\rho]^{(2)}. \quad (2)$$

A simpler form suggested by microscopics is often used in systematic studies, where one omits the  $\kappa_\rho$  and  $c_{\rho L}$  terms in the brackets from the Hamiltonian (1). It will be seen that this microscopic form is behind the IBM-2 ‘‘rule’’ mentioned above, and breaking of this ‘‘rule’’ requires consider-

ation of the full Hamiltonian (1). In discussing  $F$ -spin breaking effects, it is convenient to introduce  $F$ -spin scalar and vector parameters

$$\begin{aligned} \epsilon_s &= (\epsilon_\pi + \epsilon_\nu)/2, & \epsilon_v &= \epsilon_\pi - \epsilon_\nu, \\ \chi_s &= (\chi_\pi + \chi_\nu)/2, & \chi_v &= \chi_\pi - \chi_\nu, \end{aligned} \quad (3)$$

with similar expressions for the  $c_{\rho L}$  parameters. The  $E2$  matrix elements are calculated using the the same quadrupole operator (2) as in the Hamiltonian, with effective charges  $e_\pi = e_\nu = 0.125 e$ . The  $M1$  and magnetic moment operators are given by

$$T(M1) = \sqrt{3/4\pi} \hat{\mu}, \quad \hat{\mu} = g_\pi \mathbf{L}_\pi + g_\nu \mathbf{L}_\nu, \quad (4)$$

where  $\mathbf{L}_\rho$ ,  $\rho = \pi, \nu$ , are the angular momentum operators for proton and neutron bosons and  $g_\rho$  are the respective boson  $g$ -factors. Bare values for the boson  $g$ -factors ( $g_\pi = 1$ ,  $g_\nu = 0$ ) are employed in the  $M1$  operator throughout.

In order to get a perspective on the issues involved, we first present some systematic studies on how  $F$ -spin breaking affects the  $M1$  properties. The  $1/N$  expansion formalism [17] provides a handy tool for this purpose. For example, to leading order in  $1/N$ , the  $g$ -factors for the ground and  $\gamma$  bands are given by [18]

$$\begin{aligned} g_g &= (g_\pi \cos^2 \theta_g + g_\nu \sin^2 \theta_g), \\ g_\gamma &= \frac{g_g}{3} + \frac{2}{3} (g_\pi \cos^2 \theta_\gamma + g_\nu \sin^2 \theta_\gamma). \end{aligned} \quad (5)$$

In Eq. (5)  $\theta_\gamma$  is the  $\pi$ - $\nu$  mixing angle in the  $\gamma$  band, and  $\theta_g$  is related to the ground-band mean fields  $x_{\rho l}$  by

$$\cos \theta_g = \sqrt{\frac{N_\pi x_{\pi 2}}{N x_2}}, \quad \sin \theta_g = \sqrt{\frac{N_\nu x_{\nu 2}}{N x_2}}, \quad (6)$$

where  $N_\pi$ ,  $N_\nu$  denote the proton and neutron boson numbers, respectively;  $N = N_\pi + N_\nu$ ; and  $x_2^2 = (N_\pi x_{\pi 2}^2 + N_\nu x_{\nu 2}^2)/N$  represents the quadrupole deformation averaged over protons and neutrons. (Note that the mean fields are

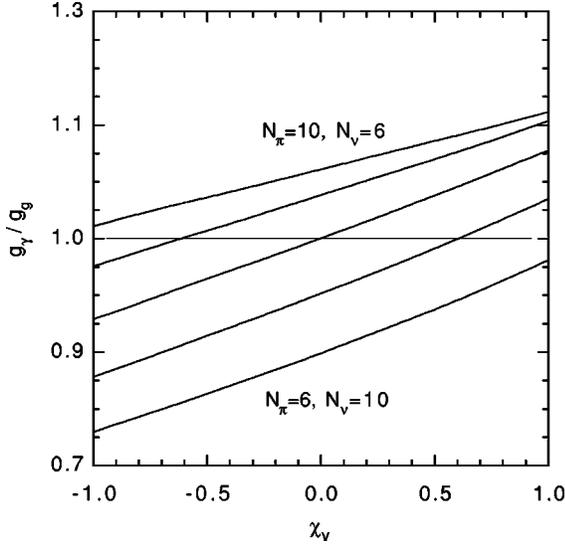


FIG. 1.  $N_\pi$ ,  $N_\nu$  systematics of the  $g$ -factor ratio  $g(2_\gamma)/g(2_g)$  plotted as a function  $\chi_\nu$ . The fixed parameters are  $N=16$ ,  $\epsilon_s=0.2$  MeV,  $\epsilon_\nu=0$ ,  $\chi_s=-0.66$ ,  $\kappa=-0.02$  MeV, and  $\xi=0.100$  MeV. The lines in the middle have  $N_\pi=7,8,9$  and  $N_\nu=9,8,7$ , respectively.

normalized, i.e.,  $x_{\rho 0}^2 + x_{\rho 2}^2 = 1$ .) The mean fields are determined from the variation of the IBM-2 energy surface [17].

To limit the number of parameters, the microscopic form of the Hamiltonian [i.e., the top line in Eq. (1)] is used in the systematic studies. We first consider the effect of the boson numbers  $N_\pi$  and  $N_\nu$  on the relative  $g$ -factors of the ground and  $\gamma$  bands, which does not appear to be well appreciated in the literature. This point is illustrated in Fig. 1, where the ratio  $g_\gamma/g_g$ , calculated from Eq. (5), is plotted against  $\chi_\nu$  for various values of  $N_\pi$  and  $N_\nu$ . Because the Er isotopes are the main focus of this work, the choice of parameters are skewed towards  $^{168}\text{Er}$ , but the main features should also hold for other deformed nuclei. It is clear from Fig. 1 that  $g_\gamma/g_g=1$  at  $\chi_\nu=0$  only when  $N_\pi=N_\nu$ , in which case  $F$ -spin symmetry remains almost intact. In other cases, difference in boson numbers is sufficient to break the  $F$ -spin symmetry and generate differences between the  $g$ -factors without recourse to the  $F$ -spin vector parameters. The apparent correlation found in earlier systematic studies between the sign of  $\chi_\nu$  and  $g_\gamma/g_g$  is due to the specific choice of the Hamiltonian, which has  $F$ -spin symmetry, when  $\chi_\nu=0$ . The increasing trend of the  $g$ -factor ratio with  $\chi_\nu$  has been noted earlier [7]. This is a general feature of the microscopic Hamiltonian and can be traced, via the mixing angles in Eq. (5), back to the mean fields. It simply results from the fact that  $F$ -spin breaking is more effective in the  $\gamma$  band compared to the ground band [17].

A similar study of the  $E2/M1$  mixing ratio using the  $1/N$  expansion is, unfortunately, not possible as the band-mixing contributions to the  $M1$  ( $\gamma \rightarrow g$ ) matrix elements have not yet been calculated. We resort to numerical diagonalization for this purpose, limiting ourselves to the case  $N_\pi=7$  and  $N_\nu=9$ , relevant for  $^{168}\text{Er}$ . The results for  $g_\gamma/g_g$  and  $1/\delta$  are plotted against  $\chi_\nu$  for three values of  $\epsilon_\nu$  in Fig. 2. Clearly

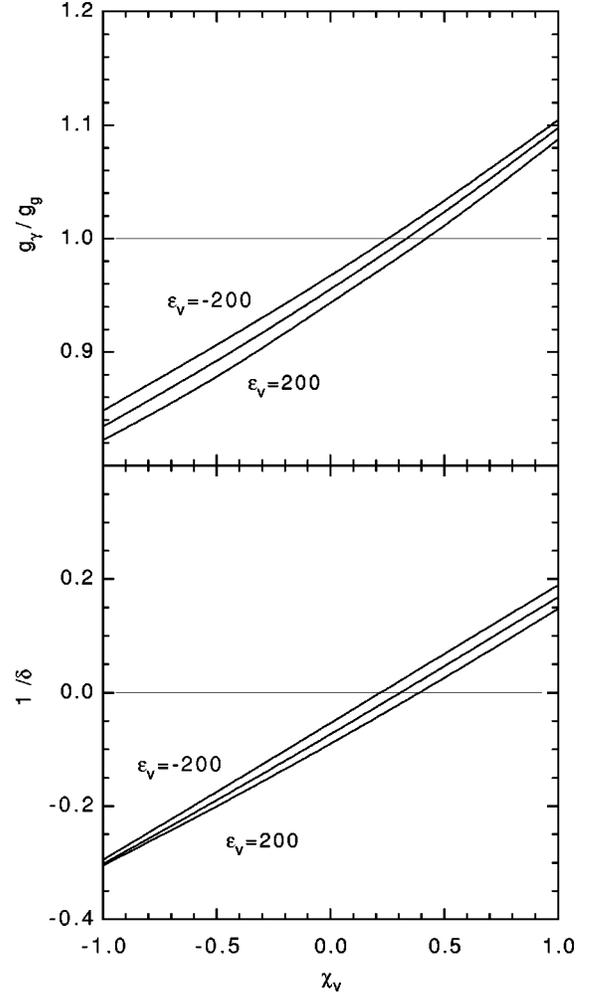


FIG. 2. The  $\epsilon_\nu$  dependence of the  $g$ -factor ratio and the mixing parameter  $1/\delta$  plotted as a function  $\chi_\nu$ . Here and in Fig. 3 the units of  $\epsilon_\nu$  are keV. The fixed parameters are  $N_\pi=7$ ,  $N_\nu=9$ , and the rest are as in Fig. 1. The line in the middle is for  $\epsilon_\nu=0$ .

$F$ -spin breaking induced by  $\epsilon_\nu$  plays a marginal role and is not expected to make much difference in fitting the  $M1$  data. A more significant observation is the apparent correlation between the  $g$ -factor and mixing ratios. To make this point clearer, we plot one against the other in Fig. 3, as has been done for the data in Ref. [15]. The slight deviation of the lines from zero crossing is due to the difference of the boson numbers  $N_\pi$  and  $N_\nu$ . This deviation is negligible for all practical purposes, and, moreover, demonstrates the robustness of the IBM-2 ‘‘rule’’ correlating the  $g$ -factor and mixing ratios against the boson number variations, cf. Fig. 1, where the ratio  $g_\gamma/g_g$  exhibits considerable dependence on  $|N_\pi - N_\nu|$  when plotted against  $\chi_\nu$ . We note that the recent experiments [14,15] place  $^{166-168}\text{Er}$  in the second quadrant, and the microscopic form of the IBM-2 Hamiltonian clearly cannot accommodate these results, as seen in Fig. 3.

In detailed studies of specific nuclei, the terms with the parameters  $\kappa_\rho$  and  $c_{\rho L}$ ,  $\rho=\pi,\nu$ ,  $L=0,2,4$  are often included in the IBM-2 Hamiltonian (1). For example, earlier IBM-2 fits to the W isotopes also employed variations in the  $c_{\rho L}$  parameters in order to describe the  $M1$  properties [10].

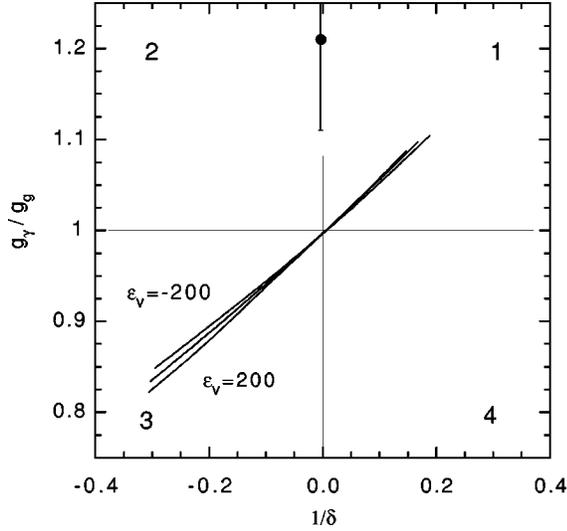


FIG. 3. Correlation diagram for the  $g$ -factor ratios and the mixing parameters in Fig. 2. The black dot with error bars indicates the current experimental value for  $^{168}\text{Er}$ , Refs. [14,15].

These theoretical calculations would place the  $M1$  results for  $^{182}\text{W}$  in the fourth quadrant in Fig. 3. It is of interest to see if a similar parameter set can explain the recent  $M1$  observations in  $^{168}\text{Er}$ . For this purpose, we have first fitted the excitation energies and  $E2$  transitions in  $^{168}\text{Er}$  using the  $F$ -spin scalar parameters of the Hamiltonian and then introduced  $F$ -spin breaking via the vector parameters to describe the  $M1$  data. The results for the excitation energies and the electromagnetic properties are compared to the data in Table I. A good agreement is obtained for all the low-lying observables. Most importantly, a negative  $1/\delta$  is associated with  $g_\gamma/g_g > 1$ , in accordance with the recent measurements of these quantities [14,15].

While the current results for  $^{168}\text{Er}$  and those in Ref. [10] for  $^{182}\text{W}$  indicate that the IBM-2 can, indeed, accommodate  $M1$  properties that lie in the second and fourth quadrants of Fig. 3, respectively, this is achieved at the expense of introducing many more parameters in the model. The parameters in the microscopic Hamiltonian have been well-studied over the years and, as a result, there is an intuitive understanding of how they relate to spectroscopy. This is not the case for the  $c_{\rho L}$  parameters, and, provided the recent observations withstand a closer scrutiny, their effect on the  $M1$  observables need to be understood better. In this respect we note that the  $g$ -factor measurement of the  $2_\gamma$  state in  $^{166}\text{Er}$  [14] is in conflict with an earlier measurement [11], which gives  $g_\gamma/g_g < 1$ . The same group obtained  $g_\gamma/g_g \sim 1$  in  $^{168}\text{Er}$  [12]. In a similar vein, two values  $[-0.003(8)$  and  $+0.031(13)]$ ,

TABLE I. Comparison of the IBM-2 calculations for the excitation energies and  $E2$  and  $M1$  matrix elements to the experimental data in  $^{168}\text{Er}$ . Energies are in keV and  $E2$  matrix elements are in  $e$  b. The data are from Refs. [14,15,19–21]. The parameters of the IBM-2 Hamiltonian are  $\kappa = -82.5$ ,  $\kappa_\pi = \kappa_\nu = 0$ ,  $\epsilon_\pi = \epsilon_\nu = 235$ ,  $\xi = 100$ ,  $c_{\pi 0} = -252$ ,  $c_{\nu 0} = -112$ ,  $c_{\pi 2} = -245$ ,  $c_{\nu 2} = -105$ ,  $c_{\pi 4} = 164$ ,  $c_{\nu 4} = 24$ , all in keV, and  $N_\pi = 7$ ,  $N_\nu = 9$ ,  $\chi_\pi = -0.015$ ,  $\chi_\nu = -0.815$ . The boson effective charges are  $e_\pi = e_\nu = 0.125 e$  b and the boson  $g$  factors are  $g_\pi = 1$ ,  $g_\nu = 0$ .

|  | Theory | Expt.     |
|--|--------|-----------|
| $E(2_g)$                                   | 80     | 80        |
| $E(4_g)$                                   | 265    | 264       |
| $E(6_g)$                                   | 554    | 549       |
| $E(2_\gamma)$                              | 814    | 821       |
| $E(3_\gamma)$                              | 897    | 896       |
| $E(4_\gamma)$                              | 1007   | 995       |
| $E(0_\beta)$                               | 1195   | 1217      |
| $E(2_\beta)$                               | 1284   | 1276      |
| $E(4_\beta)$                               | 1490   | 1411      |
| $E(1_1^+)$                                 | 3174   | 3206      |
| $\langle 2_g    T(E2)    0_g \rangle$      | 2.40   | 2.40(2)   |
| $\langle 2_\gamma    T(E2)    0_g \rangle$ | 0.36   | 0.36(1)   |
| $\langle 2_\gamma    T(E2)    2_g \rangle$ | 0.47   | 0.48(1)   |
| $\langle 2_\beta    T(E2)    0_g \rangle$  | 0.02   | $< 0.03$  |
| $g(2_g)$                                   | 0.350  | 0.321(6)  |
| $g(2_\gamma)$                              | 0.428  | 0.387(30) |
| $g(2_\gamma)/g(2_g)$                       | 1.22   | 1.21(10)  |
| $1/\delta(2_\gamma \rightarrow 2_g)$       | -0.02  | -0.003(8) |

are quoted for the mixing ratio  $1/\delta$  in  $^{168}\text{Er}$  in Ref. [15], though the authors prefer the negative value as more reliable. Thus the current experimental situation seems far from clear, and more precise  $M1$  measurements in the Er isotopes would be desirable before investing heavily on a theoretical understanding of the breaking of the IBM-2 ‘‘rule.’’

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