πNN decay of a possible d' dibaryon in the $^3P_0$ quark pair creation model

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We study the pionic decay of a possible dibaryon $d' \to \pi + NN$ and pion decay of single baryons. The $^3P_0$ quark-pair-creation model is used for the definition of the effective quark-pion coupling. All necessary coupling constants and vertex form factors both in the baryonic and dibaryonic sectors are calculated in this model using translationally invariant quark-shell-model configurations. A hypothesis for the mass shift of the d' in a nuclear medium is discussed in order to explain the position of the resonance peak observed in double-charge exchange reactions in nuclei and the nonobservation (or at least doubtful observations) of the d' as a free resonance outside of a nuclear medium. In this connection the dependence of the d' decay width on the d' mass is calculated for two alternative models of the final-state NN interaction (one-boson exchange potential and Moscow potential). Off-shell effects in the d' decay are studied on the basis of an algebraic quark-cluster model technique. [S0556-2813(99)04708-1]

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I. INTRODUCTION

A possible d' dibaryon ($T=0$, $J^P=0^-$) proposed earlier [1–3] for the interpretation of the resonancelike behavior of the pionic double-charge exchange (DCX) scattering on nuclei

$$A(z)+\pi^+ \to A(z+2)+\pi^- \qquad (1.1)$$

has been actively studied both experimentally [4–6] and theoretically [7–13] in the last few years. Aside from the exclusive reaction (1.1) where $A(z+2)$ is a double-isobar-analog or ground state, new experiments have been performed for the low-energy process $^4\mathrm{He}+\pi^+ \to ppd' \to pppp\pi^-$ (the CHAOS Collaboration [4]) and the direct production $pp \to d'\pi^+ \to pp\pi^-\pi^+$ (the CELSIUS group [5]) and inclusive $A(\pi^+,\pi^-)X$ (the ITEP group [6]) reactions at intermediate energies. Most of these experiments allow investigations to be made of the d' properties outside a nuclear medium. Contrary to many previous findings [1,2] where a pronounced peak in the cross section of the DCX reaction (1.1) at the pion energy $T_\pi = 40$–60 MeV was rather reliably observed for various targets from ^{7}Li to medium-heavy nuclei, the above experimental groups were unable to confirm the d' signal with a sufficient statistic accuracy. This suggests, among other things, that the sharp d' resonance with the mass and decay width determined earlier

$$M_{d'} \approx 2.065 \ \text{GeV}, \quad \Gamma_{d'} \approx 0.5 \ \text{MeV} \qquad (1.2)$$

does not exist outside a nuclear medium (or at least such a ''free'' d' is too wide to be seen over a large nonresonant background).

On the other hand, recent (more refined) calculations [7,8] of a sequential two-step single-charge-exchange (SCX) mechanism of the DCX reaction (1.1) have shown that the values (1.2) obtained earlier with a simplified version of the background sequential SCX process should be revised. The calculations of Refs. [7,8] take into account pion distortions obtained from a realistic optical model and, as a result, they are able to explain the observed resonancelike behavior of the cross section around $T_\pi = 50$ MeV at least for nuclei with nonclosed shells, e.g., ^{14}C, ^{42,44,46}Ca [8] and ^{128,130}Te [7]. It is possible that when the background is accurately recalculated using the improved sequential mechanism of Refs. [7,8] there will be no sharp resonance left. Without such a recalculation, however, one cannot disprove the existence of some wide dibaryon in the region around the values (1.2). According to Ref. [7] the absolute value of the calculated peak depends on the model of nuclear structure made use of in calculations. That leaves room for a possible d' resonance signal around the values (1.2). By contrast, the authors of Ref. [8] advocate that their results are model independent and the calculated cross section describes the peak in excellent agreement with the data without the necessity of invoking genuine quark degrees of freedom. However, the latter cannot be considered as an unambiguous conclusion in view of the fact that the DCX cross section depends essentially on the short-range NN correlations, but this dependence was not rigorously analyzed in Ref. [8]. It should be also mentioned that the calculations of Refs. [7,8] do not include nuclei with doubly closed shells (^{4}He, ^{16}O, ^{40}Ca). The most pronounced resonancelike data are seen for the ^{40}Ca [12] nucleus.

Due to the great significance of the dibaryon problem all arguments, which could support the dibaryon hypothesis should be carefully considered. Because of the experimental uncertainty in the values of Eq. (1.2) it would be good to have some model evaluations of the d' properties. It is instructive to calculate what values of $M_{d'}$ and $\Gamma_{d'}$ are, in principle, compatible with conventional quark models of baryons. Compared to the known πNN approach [14], the six-quark consideration of the problem offers a clearer view of how to treat on a common ground both the inner structure of all systems of interest (baryons, dibaryons or the NN system at short range) and pion-hadron couplings in all d'-related vertices d'-πNN, N-πN, and N^*-πN (including the vertex form factors as well).

After first evaluations of a low-mass dibaryon with $T(J^P)=0(0^-)$ made in Refs. [15] within the framework of a rotating bag model with nonoverlapping diquark and four-quark clusters, a more realistic consideration of the d' properties has been performed in Refs. [13,16] on a base of the Tuebingen quark model (Tuebingen QM) [17,18]. Such a consideration runs into an obstacle: the value of the d' mass observed in the DCX reaction [see Eq. (1.2)] is too small to be explained with the standard q-q interaction of the nonrelativistic quark model (NRQM) fitted to the baryon spectrum. In Ref. [13] it was proposed that such a small mass of the d' could be explained if for some reason the confinement strength is reduced for six-quark systems. But the mechanism of such a reduction is presently unknown, although a reduction seems to be plausible. At the same time the modification of the d' properties in a nuclear medium could be similar to the well-known modification of baryon properties, e.g., a shift (30–40 MeV) of the Δ isobar mass in nuclei [19,20].

The mechanism of such a modification (the virtual decay $\Delta\rightarrow\pi+N$ and propagation of the virtual pion in the nuclear medium [21]) is applicable to dibaryons, too. In the case of the d' it implies a virtual decay $d'\rightarrow\pi+N+N$. Recall that the quantum numbers of the d' prevent the decay into the NN channel and the only possible decay channel is $\pi+N+N$. As the resonance peak in the DCX scattering on nuclei is only 50 MeV higher than the πNN threshold, the decay width should be strongly dependent on the phase-space volume of the final πNN state. The following scenario could perhaps be responsible for the observation of a narrow d' resonance in the DCX reaction: The nuclear medium lowers the energy of the d' resonance to 2065 MeV just 50 MeV above the $NN\pi$ threshold. Due to the small phase-space integral the decay width is small ($\lesssim 1$ MeV). The free d' has a larger energy. The decay width of the free d' is much larger and the decay of the d' can, therefore, not be observed on top of a large background. This would explain the difficulties in observing the d' as a free resonance outside a nuclear medium. Thus an evaluation of the coupling of the d' to the pion field would be important for a solution of the d' problem.

Our first calculations [10,11,22] of the d' decay width have shown that the $\Gamma_{d'}$ sensitively depends on the nucleon-nucleon dynamics at short range [11] and is very sensitive to the choice of the quark-pion coupling [22]. The latter calls for some comments. As the d' mass is very close to the πNN threshold the behavior of the transition-matrix element in the limit of the zero momentum $\mathbf{k}$ of the emitted pion is very important. For S-wave pions one obtains qualitatively different limits (zero or nonzero) if the pseudoscalar (PS) or pseudovector (PV) quark-pion coupling is used in the lowest order of the v/c expansion (see, e.g., [23]). At the same time the proper relativistic consideration of the PV coupling in Ref. [24] or even the first v/c (recoil) correction to the static PS coupling in Ref. [10] showed that the transition-matrix element is nonzero in the limit $k\rightarrow 0$. Unfortunately our calculations of the d' decay width in Ref. [11] were performed only for PS coupling in the static approximation without the recoil corrections that was criticized in Ref. [24]. Here we give the full analysis for all variants of coupling and compare the new results with the old ones. It will be seen that both transition amplitudes PS corrected and PV are extremely off-shell dependent in the case of S-wave pion emission. Moreover, there are other ambiguities that should be carefully considered.

Another off-shell dependent contribution to the πNN decay width of the d' could be the $q\bar{q}$ substructure of the pion. Usually, in the Tuebingen QM the pion is considered as a structureless (pointlike) Goldstone boson. However, it seems reasonable to say that the pion is not exactly a Goldstone boson. This is not only because of the nonzero mass of the pion, but also because there are processes where the inner $q\bar{q}$ structure of the pion could play an important role, e.g., in the πN decay of the Roper resonance [25]. Note that the empirical pion decay widths of all light hadrons were successfully reproduced in the relativized quark model [26–28] taking into account the inner $q\bar{q}$ pion structure.

For these reasons an extension of our previous calculation of the d' decay width [11,22] to a more realistic quark-pion coupling would be useful for a better understanding of the properties of the d' in and out of the nuclear medium.

In this work, we consider the $\pi d'$ coupling and calculate the d' decay width in the framework of the phenomenologically successful $^3P_0$ quark-pair-creation model (QPCM) [25,30]. This model appears to be very useful in the flux-tube picture of hadrons [26–29]. It will be shown that in the limit of zero $q\bar{q}$ pion radius this model goes to the standard PV coupling. Therefore, by varying the π radius we obtain the corresponding results for the PV coupling too. Moreover, in this limit we can normalize the strength of the (phenomenological) constant of creation of $q\bar{q}$ pairs to the PV pion-nucleon coupling constant $f_{\pi NN}$.

In Sec. II the $^3P_0$ quark-pair-creation model is extended to the description of transition amplitudes in the six-quark system. In this connection the basic ideas and formulas are shortly discussed (including ambiguities of the model) and the translationally invariant six-quark basis is introduced. In Sec. III we discuss two classes of models of NN interaction at short range, the phenomenological models with the repulsive core [e.g., the one-boson exchange potential (OBEP) models [31,32]] and the QCD motivated models taking into account the six-quark configurations instead of the repulsive core (e.g., the potentials of Moscow type [33,34] or the Tuebingen QM [17,18]). Starting from the algebraic consideration we show that these two classes of models lead to different predictions for the d' decay width. The technique of six-quark calculations of transition amplitudes and the procedure of projection of six-quark states onto the NN channel are introduced on the basis of fractional parentage coefficients (f.p.c.) technique [35–37]. The results on the d' decay width in the $^3P_0$ QPCM for different models of short-range NN interaction and different values of the π radius are shown in Sec. IV. Concluding remarks and a summary are given in the last section.

II. QUARK-PION COUPLING IN THE $^3P_0$ QUARK PAIR CREATION MODEL

A. Transition operator and initial (final) quark configurations

Our starting point for calculating the πNN decay width of the d' dibaryon is the ansatz of the flux-tube breaking model (see, e.g., [28] and references therein), in which the operator T responsible for the transition is

$$T=-\gamma\sum_{\alpha,\bar{\beta}}\int d\mathbf{p}_q d\mathbf{p}_{\bar{q}}\delta(\mathbf{p}_q+\mathbf{p}_{\bar{q}})C_{\alpha\bar{\beta}}F_{\alpha\bar{\beta}}Z(\mathbf{p}_q,\mathbf{p}_{\bar{q}})\times\sum_m(1m1-m|00)\chi^m_{\alpha\bar{\beta}}\mathcal{Y}_1^{-m}(\mathbf{p}_q-\mathbf{p}_{\bar{q}})b^+_\alpha(\mathbf{p}_q)d^+_{\bar{\beta}}(\mathbf{p}_{\bar{q}}). \tag{2.1}$$

Here, $\alpha=\{s_\alpha,f_\alpha,c_\alpha\}$ ($\bar{\beta}=\{s_{\bar{\beta}},f_{\bar{\beta}},c_{\bar{\beta}}\}$) are projections of the spin, flavor, and color of the quark (antiquark); $C_{\alpha\bar{\beta}}$ and $F_{\alpha\bar{\beta}}$ are the color and flavor wave functions of the created $q_\alpha q_{\bar{\beta}}$ pair, both assumed to be singlet,

$$C_{\alpha\bar{\beta}}=\frac{1}{\sqrt{3}}\delta_{c_\alpha c_{\bar{\beta}}},\quad F_{\alpha\bar{\beta}}=\frac{1}{\sqrt{3}}\delta_{f_\alpha f_{\bar{\beta}}}. \tag{2.2}$$

In the nonstrange sector the flavor (f) projections are reduced to the isospin projections t

$$F_{\alpha\bar{\beta}}\rightarrow T_{\alpha\bar{\beta}}=\frac{1}{\sqrt{2}}\delta_{t_\alpha t_{\bar{\beta}}}, \tag{2.3}$$

$\chi^m_{\alpha\bar{\beta}}$ is the spin-triplet wave function of the pair

$$\chi^m_{\alpha\bar{\beta}}=\frac{1}{\sqrt{2}}\{\sigma_m\}_{s_\alpha s_{\bar{\beta}}}. \tag{2.4}$$

On the right-hand side of Eq. (2.4) are the components of the Pauli σ matrix ($m=0,\ \pm 1,\ s_\alpha,s_{\bar{\beta}}=\pm\frac{1}{2}$) as they are the Clebsch-Gordan coefficients in the quark-antiquark spin space. $\mathcal{Y}_1^{-m}(\mathbf{p}_i-\mathbf{p}_j)$ is the vector spherical harmonic indicating that the pair is in a relative P wave.

This form implies that $q\bar{q}$ pairs are created in the $^3P_0$ spin-orbital state with the vacuum quantum numbers $J^P=0^+$, $I^C=0^+$. The amplitude is proportional to the absolute value of the relative momentum $|\mathbf{p}_q-\mathbf{p}_{\bar{q}}|$ of the pair. This hypothesis is justified at least in the limit of the pointlike pion, in which case Eq. (2.1) goes to the standard PV coupling (see below).

$Z(\mathbf{p}_q,\mathbf{p}_{\bar{q}})$ is the flux-tube overlap function, which is usually taken to have ''cigar-shaped'' or spherical contours. However, the analysis of meson and baryon decay widths performed in Refs. [27,28] showed that the inclusion of the function $Z(\mathbf{p}_q,\mathbf{p}_{\bar{q}})$ was unimportant for a successful description of widths as the finite size of the hadron wave functions already effectively restricted the pair creation site to small distances away from the initial hadron. Thus we take $Z=1$ as in the usual version of the $^3P_0$ model (see, e.g., Ref. [25]). In this case the phenomenological constant γ in Eq. (2.1) is the

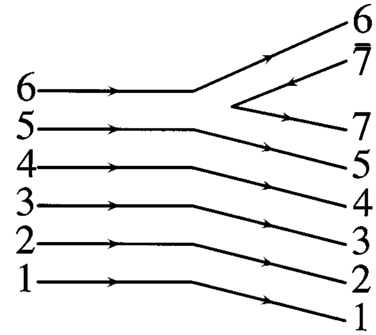

FIG. 1. Diagram of rearrangement of quark lines in the process of the pion formation from the vacuum $q\bar{q}$ pair.

usual $^3P_0$ coupling strength, which in our nonrelativistic approach is normalized to the value of the πNN coupling constant $f_{\pi NN}$.

The transition amplitude is given by

$$M=\langle\pi NN|T|d'\rangle, \tag{2.5}$$

where coordinate parts of quark wave functions of the pion and nucleons are taken in the simplest Gaussian form

$$N\sim\exp\left[-\frac{1}{2b_3^2}\left(\frac{1}{2}\rho_1^2+\frac{2}{3}\rho_2^2\right)\right],$$

$$\pi\sim\exp\left[-\frac{1}{2b_\pi^2}\left(\frac{1}{2}\rho_\pi^2\right)\right]. \tag{2.6}$$

We use the relative quark coordinates

$$\boldsymbol{\rho}_n=\frac{1}{n}(\mathbf{r}_1+\ldots+\mathbf{r}_n)-\mathbf{r}_{n+1},\quad n=1,2,\ldots,5,$$

$$\boldsymbol{\rho}_\pi=\mathbf{r}_6-\mathbf{r}_{\bar{7}},\quad \boldsymbol{\rho}'_5=\frac{1}{5}(\mathbf{r}_1+\ldots+\mathbf{r}_5)-\mathbf{r}_7. \tag{2.7}$$

The labeling of the quarks is illustrated in Fig. 1. The wave function of the d' dibaryon is the lowest six-quark configuration with the quantum numbers of the d'

$$|d'\rangle=|s^5p(b_6)[51]_X[321]_{CS}LST=110\ \ J^P=0^-\rangle. \tag{2.8}$$

The Gaussian part of the translationally invariant coordinate wave function of Eq. (2.8)

$$\exp\left[-\frac{1}{2b_6^2}\left(\frac{1}{2}\rho_1^2+\frac{2}{3}\rho_2^2+\frac{3}{4}\rho_3^2+\frac{4}{5}\rho_4^2+\frac{5}{6}\rho_5^2\right)\right]$$

depends on the radius b_6 of the six-quark harmonic oscillator (h.o.) basis, which should not be equal to the nucleon h.o. radius b_3. In evaluating the d' six-quark wave function in Refs. [16,13] the value of b_6 varied from 0.6 to 0.9 fm, while b_3 was taken to be 0.6 fm. The dependence of the d' decay width on the value b_6/b_3 is not very strong (about 30–50 % as it can be seen from the results of Ref. [11]), and thus here we use $b_6=b_3=0.6$ fm. The radius of the quark content of the pion b_π in the relativized models [27–29] is about 0.26 fm.

In the present work we use the Tuebingen QM, which has the advantage that the center-of-mass motion can be exactly removed. In our calculations we vary the pion radius b_π from 0 up to $b_3=0.6$ fm. The limit $b_\pi \to 0$ is an interesting one as in this limit the $^3P_0$ quark pair creation model (QPCM) goes to the standard PV quark-pion coupling in the leading order of the v/c decomposition (see Sec. II B).

As the initial state of the transition amplitude (2.5) is a six-quark configuration it is reasonable to project the final NN state onto the basis of six-quark configurations too. This can be done by inserting the unit operator

$$I=\sum_{n,f} |n,f\rangle\langle n,f| \tag{2.9}$$

into the matrix element (2.5). Here we use the full basis of six-quark configurations with quantum numbers of the final state

$$|n,f\rangle=|s^{n_s}p^{n_p}[f_X][f_{\rm CS}]LST=001J^P=0^+\rangle. \tag{2.10}$$

$n=\{n_s,n_p\}$ are the numbers of s and p quarks and $f=\{[f_X],[f_{\rm CS}]\}$ are Young schemes in coordinate (X) and color-spin (CS) spaces used for classification of the states. Only configurations

$$s^6[6]_X, \quad (s^4p^2-s^5 2s)[6]_X \text{ and } s^4p^2[42]_X \tag{2.11}$$

with all possible CS Young schemes are important for the transition into the $^1S_0$ final NN channel [11]. As a result the amplitude (2.5) is a finite sum of factorizable terms

$$M=\sum_{n,f} \langle NN|n,f\rangle\langle n,f;\pi|T|d'\rangle, \tag{2.12}$$

where $\langle n,f;\pi|T|d'\rangle$ are transition amplitudes from the configuration (2.8) into configurations (2.11) with emission of the S-wave pion. These amplitudes and the overlap factors $\langle NN|n,f\rangle$ are calculated with the standard technique of fractional parentage coefficients (f.p.c.) described in Ref. [11].

The operator (2.1) can be transformed into the form of the effective quark-pion coupling. As usual (see, e.g., Refs. [25,30]) this can be achieved by recoupling the quarks in the subsystem $q+q\bar{q}$, where q is one of the quarks of the initial six-quark configuration (for definiteness quark q is labeled by the number 6, while the vacuum $q\bar{q}$ pair is labeled by numbers $7\bar{7}$ as in Fig. 1). The pion is considered as a bound state in the $6\bar{7}$ subsystem. In order to calculate the amplitude for pion production, we project the recoupled $6\bar{7}$ pair of Eq. (2.1) onto the pion wave function Ψ_π. As a result we obtain the following expression for the operator for pion emission by the sixth quark (see Appendix A for details):

$$H_\lambda^{(6)}(\mathbf{p}_6,\mathbf{p}_6')=\tilde{\gamma}\tau_{-\lambda}^{(6)}\boldsymbol{\sigma}^{(6)}\cdot\left[\mathbf{k}-\frac{\omega_\pi}{2m_q}(\mathbf{p}_6+\mathbf{p}_6')\right] \times\Psi_\pi\left(\frac{\mathbf{p}_6+\mathbf{p}_6'}{2}\right). \tag{2.13}$$

Here $\mathbf{p}_6'$ is the momentum of the seventh quark in the vacuum $q\bar{q}$ pair (see Fig. 1). Note that quarks 6 and 7 are identical. To satisfy the Pauli exclusion principle we have to antisymmetrize the final $q^7\bar{q}$ system as a whole. As a first step we use the approximation sufficiently separated quark clusters $6\bar{7}$ and 123 457, which implies that quark exchanges between these clusters can be neglected. Note that the full antisymmetrization is no principal problems but we expect that the contribution of antisymmetrization to the amplitude does not dominate the contribution of the final $\pi-6q$ interaction, which is not yet taken into account. Both effects are interfering and cannot be separated. For a rigorous evaluation of the quark-exchange contributions (and of the interaction in the final $\pi+6q$ state as well) a more sophisticated model is needed, e.g., the flux-tube breaking model with a nontrivial parametrization of the function $Z(\mathbf{p}_q,\mathbf{p}_{\bar{q}})$ in Eq. (2.1).

B. Nonlocal coupling in coordinate space: Definition of the coupling constant

In coordinate space the translationally invariant configurations (2.8) and (2.11) are used. Therefore, the transition amplitude (2.13) should be rewritten in terms of operators acting on the relative coordinates (2.7).

After a Fourier transform of the transition amplitude (2.13), we obtain the following operator in coordinate representation

$$H_\lambda^{(6)}(\boldsymbol{\rho}_5,\boldsymbol{\rho}_5')=v\,\tau_{-\lambda}^{(6)}e^{i(5/6)\mathbf{k}\cdot\boldsymbol{\rho}_5'}\hat{O}^{(6)}(\boldsymbol{\rho}_5,\boldsymbol{\rho}_5') \times\boldsymbol{\sigma}^{(6)}\cdot\left[\frac{\omega_\pi}{2m_q}\left(\frac{2}{i}\boldsymbol{\nabla}_{\rho_5}+\frac{5}{6}\mathbf{k}\right)+\left(1+\frac{\omega_\pi}{12m_q}\right)\mathbf{k}\right], \tag{2.14}$$

where the nonlocal factor is proportional to the pion wave function

$$\hat{O}^{(6)}(\boldsymbol{\rho}_5,\boldsymbol{\rho}_5')=e^{-i\,1/2\,\mathbf{k}\cdot(\boldsymbol{\rho}_5-\boldsymbol{\rho}_5')}\Psi_\pi\left(\frac{\boldsymbol{\rho}_5-\boldsymbol{\rho}_5'}{\sqrt{2}}\right). \tag{2.15}$$

Here $\boldsymbol{\rho}_5$ and $\boldsymbol{\rho}_5'$ are the relative coordinates of the sixth quark in the initial and final states correspondingly. Further we will omit the subscript 5 in the $\boldsymbol{\rho}_5$ ($\boldsymbol{\rho}_5'$). Note that in the Eq. (2.14) the gradient $\boldsymbol{\nabla}_\rho$ acts only on the wave functions of the initial state, and in the derivation of Eqs. (2.14) and (2.15) we have used the relation

$$\int \Psi_1(\boldsymbol{\rho})e^{i(5/6)\mathbf{k}\cdot\boldsymbol{\rho}}\left(\frac{2}{i}\boldsymbol{\nabla}+\frac{5}{6}\mathbf{k}\right)\Psi_2(\boldsymbol{\rho})d\boldsymbol{\rho} =\int \Psi_1(\boldsymbol{\rho})\left(e^{i(5/6)\mathbf{k}\cdot\boldsymbol{\rho}}\frac{1}{i}\vec{\nabla}-\frac{1}{i}\overleftarrow{\nabla}e^{i(5/6)\mathbf{k}\cdot\boldsymbol{\rho}}\right)\Psi_2(\boldsymbol{\rho})d\boldsymbol{\rho}. \tag{2.16}$$

In Eq. (2.14) we have changed the normalization of the pion wave function

$$\Psi_\pi\left(\frac{\boldsymbol{\rho}-\boldsymbol{\rho}'}{\sqrt{2}}\right)=(4\pi b_\pi^2)^{-3/2}\exp\left[-\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2}{4b_\pi^2}\right], \tag{2.17}$$

so that in the limit $b_\pi \to 0$ it goes to the δ function

$$\Psi_\pi\left(\frac{\boldsymbol{\rho}-\boldsymbol{\rho}'}{\sqrt{2}}\right) \overset{b_\pi\to 0}{\to} \delta(\boldsymbol{\rho}-\boldsymbol{\rho}'). \tag{2.18}$$

In Eq. (2.14) the normalization of the phenomenological constant is also changed in line with changing the normalization of Ψ_π:

$$v=(4\pi b_\pi^2)^{3/4}\tilde{\gamma}. \tag{2.19}$$

It implies that the γ in Eq. (2.1) is a "running constant" which depends on the π radius b_π (if the value of v has been fixed) $\gamma\sim\tilde{\gamma}\sim b_\pi^{-3/2}$. One can see that in the limit of a point-like pion $b_\pi\to 0$ the transition operator (2.14) goes to the standard PV coupling

$$H_\lambda^{(6)}(\boldsymbol{\rho},\boldsymbol{\rho}') \overset{b_\pi\to 0}{\to} H_\lambda^{(6)}(\boldsymbol{\rho}), \tag{2.20}$$

where

$$H_\lambda^{(6)}(\boldsymbol{\rho})=v\,\tau_{-\lambda}^{(6)}\boldsymbol{\sigma}^{(6)}\cdot\left[\frac{\omega_\pi}{2m_q}\left(e^{i(5/6)\mathbf{k}\cdot\boldsymbol{\rho}}\frac{1}{i}\vec{\nabla}-\frac{1}{i}\overleftarrow{\nabla}e^{i(5/6)\mathbf{k}\cdot\boldsymbol{\rho}}\right)+\left(1+\frac{\omega_\pi}{12m_q}\right)\mathbf{k}e^{i(5/6)\mathbf{k}\cdot\boldsymbol{\rho}}\right] \tag{2.21}$$

(we have omitted the trivial integration over $d\boldsymbol{\rho}'$). The phenomenological constant v could be normalized to the value of the PV coupling constant $f_{\pi qq}$ as it is seen from the expression for the PV transition amplitude in the leading order of v/c (see, e.g., [38])

$$v=-i\frac{f_{\pi qq}}{m_\pi}\frac{1}{(2\pi)^{3/2}(2\omega_\pi)^{1/2}}. \tag{2.22}$$

The quark-pion constant $f_{\pi qq}$ is normalized to the πNN constant $f_{\pi NN}$ by the standard procedure of calculating the πNN vertex $\langle\pi,N|H^{(3)}|N\rangle$ on the basis of the same quark model. The transition operator for the third quark $H^{(3)}$ in the three-quark system can be obtained from Eq. (2.21) or (2.14) with the substitution $\rho_5\to\rho_2$ and by replacing the coefficients $\frac{5}{6}$ and $\frac{1}{12}$ in the six-quark formulas to the $\frac{2}{3}$ and $\frac{1}{6}$. In the case of the local operator (2.21) it leads to $f_{\pi qq}=\frac{3}{5}f_{\pi NN}$, but in the case of the nonlocal operator (2.14) the value $f_{\pi qq}$ depends on the pion radius b_π and we renormalize the $f_{\pi qq}$ for each given value of b_π (see next section).

C. Ambiguities of effective quark-pion couplings

As the d' mass is very close to the πNN threshold the standard models of pseudoscalar (PS) or pseudovector (PV) quark-pion couplings lead to very different results on the d' decay width in the lowest order of the v/c expansion. In leading order of the v/c decomposition the PS coupling is proportional to the $\boldsymbol{\sigma}_q\cdot\mathbf{k}$ operator [see the last term in the right side of Eq. (2.21)], that leads to the k^2 dependence of the transition-matrix element for S-wave pion emission [11,24]. In the limit $k\to 0$ the matrix element goes to zero. In the case of the PS coupling we need to use higher-order v/c terms (e.g., the recoil correction as in Ref. [10]) in order to obtain a realistic (nonzero) behavior of the transition amplitude in the low-energy limit $k\to 0$. On the other hand, the PV coupling gives already in leading order of v/c a Galilean-invariant (see comments on this problem below) vertex $\sim-(\omega_\pi/2m_q)\boldsymbol{\sigma}_q\cdot(\mathbf{p}_q+\mathbf{p}_q')+\boldsymbol{\sigma}_q\cdot\mathbf{k}$, and the first (gradient) term of this vertex $\sim(1/2i)\vec{\sigma}_q\cdot(\vec{\nabla}_q-\overleftarrow{\nabla}_{q'})$ gives rise to a nonzero constant at the $k\to 0$ matrix element of the $d'\to\pi NN$ transition. Consideration of the transition amplitude in terms of relativistic bag-model six-quark states [24] leads also to a nonzero constant at the $k\to 0$ matrix element and its value is very close to the matrix element of the gradient term in the NRQM.

The contribution of the gradient term to the transition amplitude is off-shell dependent and it depends strictly on the properties of the wave functions of both the initial and final states. For example, it gives rise to zero amplitudes for other transitions, e.g., for the $\Delta\to\pi N$ decay (at least in the shell-model representation).

Ambiguities of the above results center around the gradient term of Eqs. (2.14) and (2.21), whereas the nongradient part of the coupling operator $(f_{\pi qq}/m_\pi)\boldsymbol{\sigma}\cdot\mathbf{k}$ is fixed (including the value of coupling constant $f_{\pi qq}$) by the observed P-wave transition amplitudes $\Delta\to\pi N$ and $N\to\pi N$. In fact, the gradient part $(f_{\pi qq}/m_\pi)(\omega_\pi/2m_q)\boldsymbol{\sigma}\cdot(1/i)(\vec{\nabla}_q-\overleftarrow{\nabla}_{q'})$ cannot be fixed simultaneously with the nongradient one as it does not contribute to these transitions. However, when the pion momentum $\mathbf{k}$ is close to zero the gradient term has played a decisive role in the S-wave pion emission, which is the case for the d' dibaryon decay. The problem is that the coefficient $\omega_\pi/2m_q$, which defines a relative strength of the gradient term, has been obtained above with such ambiguous approximations as the v/c decomposition of the PV coupling or a by-hand reduction of the QPCM amplitude. Both approximations imply that m_q is the mass of a free Dirac particle, but only confined (off-mass-shell) quarks can take part in low-energy hadron processes, where free constituent quarks are nonobservable. Therefore, the quark mass $m_q\approx\frac{1}{3}m_N$ being an effective parameter must be used with caution in the above approximations. Besides, the parameter $\omega_\pi/2m_q$ is not so small as the analogous parameter $\omega_\pi/2m_N$ of low-energy hadron physics.

We compare matrix elements of the PS and PV couplings in order to evaluate what ambiguity in the value of the parameter $\omega_\pi/2m_q$ can be expected from relativistic and other corrections. Recall that the PS and PV couplings are equivalent for free Dirac states $\overline{\psi}_n^{(0)}(\psi_m^{(0)})$ as there is an exact relationship between matrix elements of the four-dimensional (pseudo-)gradient and (pseudo-)scalar operators

$$\int\overline{\psi}_n^{(0)}\gamma^5\gamma^\mu\psi_m^{(0)}i\partial_\mu\varphi(x)d^3x =-2m\int\overline{\psi}_n^{(0)}\gamma^5\psi_m^{(0)}\varphi(x)d^3x. \tag{2.23}$$

However, this equivalence is lost for bound states $\overline{\psi}_n(\psi_m)$ when the left-hand side of Eq. (2.23) depends sensitively on the form of the wave functions $\overline{\psi}_n(\psi_m)$, but the right-hand side does not. After a nonrelativistic reduction of both sides of Eq. (2.23) made through a v/c expansion the difference between the PS and PV couplings reduces (in the lowest order) to the above gradient term, which must therefore be sensitive to off-shell effects. However, in the next order of the v/c expansion for PS coupling, one also obtains a relativistic (recoil) correction proportional to the gradient term $\omega_\pi/4m_q$, but which is a factor of 2 smaller than the corresponding term for PV coupling (see, e.g., [38]). As is evident from the foregoing, the ambiguity of the coefficient $\omega_\pi/2m_q$ is of the same order as the coefficient itself. In this respect, the relativistic bag models (see, e.g., [39]) with free Dirac states inside the bag (and boundary conditions used instead of interaction potentials) are more appropriate [24]. In bag models current quarks masses (close to zero) are used, and results do not depend sensitively on the light quark masses, which can be taken equal to zero. However, in bag models a new ambiguity appears because of the lack of translational invariance (see, e.g., [40]). The solution of this problem should not be so simple when there is a six-quark bag with the $p_{1/2}$ excitations in the initial state and two interacting three-quark bags in the final state (see, e.g., [41]).

On the other hand, the value of the mass m_q in $\omega_\pi/2m_q$ can be considered as an phenomenological parameter, which effectively accounts for most of the above relativistic and off-mass-shell contributions. A familiar example of such a role of the mass parameter m_q is the successful description of the baryon magnetic moments in terms of Dirac magnetons $e_q/2m_q$ of constituent quarks. From this point of view the above semirelativistic approximations seem to be reasonable (see, e.g., the discussion in Ref. [42]) and then only some constraints on the value of the coefficient $\omega_\pi/2m_q$ should be considered. For these purposes the common symmetry properties of the $d'-\pi NN$ and $B-\pi N$ vertices are usually used.

For low-energy processes, Galilean invariance is the most important symmetry [once the translational invariance has already been provided by using of the translationally invariant quark shell-model basis (2.8)–(2.11)]. However, Galilean invariance is violated for the off-mass-shell πqq vertex

$$\left(\frac{\boldsymbol{p}+\boldsymbol{p}'}{2m_q}-\frac{\boldsymbol{k}}{\omega_\pi}\right)\cdot\boldsymbol{\sigma}, \tag{2.24}$$

if the momentum and energy conservation $\boldsymbol{k}=\boldsymbol{p}-\boldsymbol{p}'$ and $\omega_\pi=E_p-E_{p'}$ are taken into account [for convenience we rewrite the operator part of Eq. (2.13) in the form of Eq. (2.24)]. For example, let the ingoing and outgoing quarks be both on the mass shell, i.e., $E_p=\sqrt{m_q^2+\boldsymbol{p}^2}\approx m_q$ and $E_{p'}=\sqrt{m_q^2+\boldsymbol{p}'^2}\approx m_q$, and thus the emitted pion is off-mass shell $\omega_\pi\neq\sqrt{m_\pi^2+\boldsymbol{k}^2}$. Then in the reference frame moving with the velocity $\boldsymbol{V}$ the on-shell momenta are transformed as $\boldsymbol{p}\to\approx(\boldsymbol{p}-m_q\boldsymbol{V})$ and $\boldsymbol{p}'\to\approx(\boldsymbol{p}'-m_q\boldsymbol{V})$, but the momentum $\boldsymbol{k}=\boldsymbol{p}-\boldsymbol{p}'$ of the off-shell pion remains almost nontransformed ($\boldsymbol{k}\to\approx\boldsymbol{k}$) and the expression (2.24) will depend on the velocity of the reference frame.

In contrast, matrix elements of the operator (2.24) over six-quark wave functions for emission of the on-mass-shell pion are approximately Galilean invariant. In that case the quarks are off-mass-shell, but averaging the operator (2.24) over initial and final bound states of quarks in the matrix element restores the (approximate) Galilean invariance. To show this we rewrite expression (2.24) using the relative (Jacobi) momenta of the sixth quark in the six-quark system $\boldsymbol{\pi}_5$, $\boldsymbol{\pi}_5'$ and the total momenta $\boldsymbol{P}$, $\boldsymbol{P}'$

$$\left[\left(\frac{\boldsymbol{P}+\boldsymbol{P}'}{12m_q}-\frac{\boldsymbol{k}}{\omega_\pi}\right)+\frac{5}{12}\left(\frac{\boldsymbol{\pi}_5+\boldsymbol{\pi}_5'}{m_q}\right)\right]\cdot\boldsymbol{\sigma}, \tag{2.25}$$

where $\boldsymbol{P}=\boldsymbol{p}_1+\boldsymbol{p}_2+\cdots+\boldsymbol{p}_6$, $\boldsymbol{P}'=\boldsymbol{p}_1+\cdots+\boldsymbol{p}_5+\boldsymbol{p}_6'$, $\boldsymbol{p}_6\equiv\boldsymbol{p}$, $\boldsymbol{p}_6'\equiv\boldsymbol{p}'$, $\boldsymbol{k}=\boldsymbol{P}-\boldsymbol{P}'=\boldsymbol{p}_6-\boldsymbol{p}_6'$, $\boldsymbol{\pi}_5=\frac{1}{6}(\boldsymbol{p}_1+\cdots+\boldsymbol{p}_5)-\frac{5}{6}\boldsymbol{p}_6$, and $\boldsymbol{\pi}_5'=\frac{1}{6}(\boldsymbol{p}_1+\cdots+\boldsymbol{p}_5)-\frac{5}{6}\boldsymbol{p}_6'$.

For translationally invariant basis wave functions (2.8)–(2.11) dependent only on relative Jacobi coordinates, the matrix element of the last term in the square brackets of Eq. (2.25) does not depend on the velocity of the reference frame. For the S-wave pion emission this matrix element goes to a nonzero constant at $\boldsymbol{k}\to 0$ (see the next section).

The first term in the square brackets of Eq. (2.25) is an approximately Galilean-invariant expression. It follows from the fact that in first order of the small parameter $\omega_\pi/6m_q\approx 1/10$ this expression can be rewritten in the explicitly invariant form

$$\frac{\boldsymbol{P}+\boldsymbol{P}'}{12m_q}-\frac{\boldsymbol{k}}{\omega_\pi}\cong\frac{\boldsymbol{P}}{2M_{d'}}+\frac{\boldsymbol{P}'}{2M_{NN}}-\frac{\boldsymbol{k}}{\omega_\pi}+\mathcal{O}\left(\frac{\omega_\pi^{\max}}{6m_q}\right), \tag{2.26}$$

where $M_{NN}=\sqrt{(M_{d'}-\omega_\pi)^2-\boldsymbol{P}'^2}\approx M_{d'}-\omega_\pi$ is the mass of the outgoing NN system. Equation (2.26) follows from equations

$$M_{d'}\cong 2m_N+\omega_\pi^{\max}\cong 6m_q+\omega_\pi^{\max}$$
$$M_{NN}\cong M_{d'}-\omega_\pi\cong 6m_q+(\omega_\pi^{\max}-\omega_\pi), \tag{2.27}$$

where $\omega_\pi^{\max}\approx 200$ MeV and we have taken into account that $m_N\approx 3m_q$. Here, all particles (d', $N+N$, and π) are on their mass shell. In the moving reference frame, all the on-mass-shell momenta of the vertex $d'-\pi NN$ transform as $\boldsymbol{P}\to\boldsymbol{P}-M_{d'}\boldsymbol{V}$, $\boldsymbol{P}'\to\boldsymbol{P}'-M_{NN}\boldsymbol{V}$, and $\boldsymbol{k}\to\boldsymbol{k}-\omega_{d'}\boldsymbol{V}$ which conserves Galilean invariance of the expression on the right-hand side of Eq. (2.26). This demonstrates approximate Galilean invariance of the resulting on-mass-shell matrix elements of the effective quark-pion coupling operator (2.24). Therefore, the coefficient $\omega_\pi/2m_q$ in Eqs. (2.14) and (2.21) is an optimal one to satisfy the Galilean invariance of matrix elements.

It should be noted that the operator $[(\boldsymbol{P}+\boldsymbol{P}')/12m_q-\boldsymbol{k}/\omega_\pi]\cdot\boldsymbol{\sigma}$ does not affect the limiting value of the matrix element of the operator (2.25) for emission of the S-wave pion at $k\to 0$, as this limit is a constant defined by the second term $[(\boldsymbol{\pi}_5+\boldsymbol{\pi}_5')/m_q]\cdot\boldsymbol{\sigma}$. The matrix element of the first term at $k\to 0$ will behave as $\sim\boldsymbol{k}\cdot[(\boldsymbol{P}+\boldsymbol{P}')/12m_q-\boldsymbol{k}/\omega_\pi]$, i.e., it is zero at $k=0$. Such behavior of the S-wave pion emission amplitude of an arbitrary vertex $\sim\boldsymbol{n}\cdot\boldsymbol{\sigma}$ in the limit $k\to 0$ originates from the dominance of centrifugal potential barriers on outgoing (or ingoing) P-wave quark states (see, e.g., [23]). In our case such behavior can be seen in the following matrix elements for basis states (2.8)–(2.11):

$$\langle s^5pJ^P=0^-|e^{i(5/6)\boldsymbol{k}\cdot\boldsymbol{\rho}_5}\boldsymbol{\sigma}^{(6)}\cdot\boldsymbol{n}|s^6J^P=0^+\rangle\sim\boldsymbol{n}\cdot\boldsymbol{k},$$
$$\langle s^5pJ^P=0^-|e^{i(5/6)\boldsymbol{k}\cdot\boldsymbol{\rho}_5}\boldsymbol{\sigma}^{(6)}\cdot\boldsymbol{n}|s^4p^2J^P=0^+\rangle\sim\boldsymbol{n}\cdot\boldsymbol{k},\quad k\to 0, \tag{2.28}$$

where n is an arbitrary constant vector. Note that the expressions (2.14) and (2.21) are written in the rest frame where $\boldsymbol{P}=0$ and $\boldsymbol{P}+\boldsymbol{P}'=-\boldsymbol{k}$, so that the term $(\boldsymbol{P}+\boldsymbol{P}')/12m_q - \boldsymbol{k}/\omega_\pi$ reduces to $(-\boldsymbol{k}/12m_q-\boldsymbol{k}/\omega_\pi)$ which gives rise to the operator $(1+\omega_\pi/12m_q)\boldsymbol{\sigma}\cdot\boldsymbol{k}$ in both Eqs. (2.14) and (2.21). In line with Eqs. (2.28) the matrix element of the S-wave pion emission for the vertex operator $\boldsymbol{k}\cdot\boldsymbol{\sigma}$ behaves as $\sim k^2$ at $k\rightarrow 0$ (see also Refs. [11,23]).

In summary, it may be said that (i) at small values of k, $\omega_\pi \ll 6m_q$ the vertex of the effective quark-pion coupling proportional to the operator (2.25) leads to (approximately) Galilean-invariant amplitudes for emission of S-wave pions; (ii) in the limit $k\rightarrow 0$ these amplitudes are nonzero constants the values of which are defined by the second term in the square brackets of Eq. (2.25).

III. QUARK SHELL-MODEL APPROACH TO THE d' DECAY AND THE NUCLEON-NUCLEON INTERACTION AT SHORT RANGE

The dibaryon decay width should be very sensitive to the short-range behavior of the nucleon-nucleon wave function of the final state. The initial state of the decay is a six-quark configuration of a characteristic hadron radius $b_6 \lesssim 1$ fm. Thus the transition-matrix element can only depend on the short-range part of the final-state wave function. Therefore the dibaryon decay could be used as a tool for studying the NN interaction at short range. In this section this is illustrated by the example of the d' decay. As in Ref. [11], we project the wave function of the final NN ($^1S_0$) state onto the large basis of six-quark configurations (2.11) and we use the methods of the translationally invariant shell model for calculating transition-matrix elements within the Tuebingen QM [13,16–18].

A. Models of nucleon-nucleon interaction at short range

There are two qualitatively different approaches for the description of the NN interaction at short range. In the first one only hadronic degrees of freedom are taken into account to obtain an effective NN potential, e.g., the one-boson exchange potential (OBEP) [31,32], which at short distances is very close to the old phenomenological potentials with a repulsive core [43]. Another approach is based on a microscopic picture of the NN overlap region. A qualitative analysis of the overlap problem performed in Refs. [36,44,45,17] points out that in low partial waves $L=0$ and 1 there are two possibilities to reproduce a characteristic ''repulsive-core'' behavior of the NN phase shifts at low and intermediate energies. (We mean the negative and approximately constant slope of the energy dependence of S and P phase shifts.) For the S wave they are

(i) Destructive interference of lowest six-quark configurations $s^6[6]_X$ and $s^4p^2[42,44]$, which are dominant at short distances. As a consequence the full wave function dies out at short range and it is very similar to the behavior of the NN wave function in a phenomenological repulsive-core potential. Note that a 50-50 ratio of probabilities of s^6 and s^4p^2 is necessary for its mutual cancellation at $r \lesssim b_6$.

(ii) s^4p^2 dominates over s^6 in a wide interval of energy. (This is possible [36,45] if the mean value of the quark-quark interaction is attractive for the nontrivial Young scheme $[42]_X$ and repulsive for the symmetrical one $[6]_X$; see however, an alternative point of view in Ref. [47].) Then the projection of the six-quark configurations onto the NN channel is a nodal wave function at short distances. A stable node position at distances $r_c \approx b_6 \approx 0.6$ fm plays the same role in NN scattering as a repulsive core of radius r_c.

In the P-wave channel there is a similar situation. The short-range behavior is dominated by two six-quark configurations of different permutational symmetry $s^5p[51]_X$ and $s^3p^3[3^2]_X$. The configuration of the nontrivial symmetry $s^3p^3[3^2]_X$ has a nodal projection onto the NN channel. If the probability of s^3p^3 dominates over s^5p at intermediate energies it could provide an alternative explanation of the negative slope of the energy dependence of P-wave phase shifts at $E \gtrsim 0.3$–0.5 GeV [46]. In higher partial waves there is no strict necessity for the repulsive core phenomenology, and incidentally there are no suitable candidates for configurations of a nontrivial symmetry which could lead to a nodal NN wave function at short range.

In line with this analysis, the phenomenological Moscow NN potential (MP) was proposed [33,34,46]. It was postulated that in low partial waves $L=0,1$ the NN wave function is orthogonal to the ''baglike'' six-quark configurations $s^6[6]_X$ and $s^5p[51]_X$. (The orthogonality is provided by introducing a special projector into the NN potential that implies a nonlocal interaction model, which is close to the phenomenological model of Tabakin [48,49].) As a consequence the S- and P-wave functions of the MP model have a node at $r \approx 0.5$–0.6 fm. The last version of the MP [34] describes all nucleon-nucleon phase shifts on the same level of precision as the new versions of the NN repulsive-core potentials such as that of Argonne and Nijmegen. Hence they are phase equivalent but off-shell different.

It turns out that the most pronounced off-shell difference can be seen in the d' decay. This was shown in Ref. [11] for a calculation of the d' decay width using PS quark-pion coupling. Here, we demonstrate it in a more general framework of the $^3P_0$ quark-pair-creation model and for the PV quark-pion coupling as well.

B. Two modes of decay in terms of six-quark configurations

The results of this section have been obtained with standard shell-model techniques. Apart from trivial harmonic-oscillator matrix elements all calculations are performed algebraically.

In the quark shell-model approach to the d' decay [11] we can restrict ourselves to two modes of the S-wave pion emission pictured in Fig. 2. They correspond to the one-quantum (P-wave) excitation or de-excitation of the configuration $s^5p[51]_X$. At the last stage of the process both modes give rise to the final $^1S_0$ NN state. To calculate the transition amplitude we have to project the final six-quark configurations onto the NN state as it is shown in Eq. (2.12).

In the S wave, which is of interest in the d' decay, the NN wave function of the Moscow potential has a zero projection

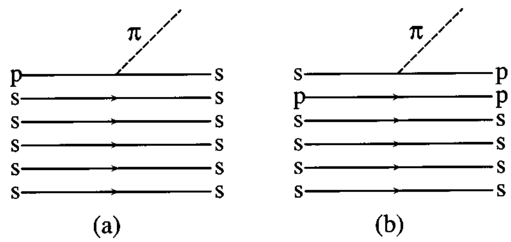

FIG. 2. Excitation (b) and de-excitation (a) modes of the S-wave pion emission in the d' decay.

onto the configuration $s^6[6]_X$, because at short ranges the MP wave function is a superposition of all possible (i.e., satisfying the Pauli exclusion principle) color-spin (CS) states in the configurations $s^4p^2[42]_X$ and $(s^52s-s^4p^2)\times[6]_X$. In contrast to this, the standard OBEP wave function has approximately equal projections onto the s^6 and s^4p^2 configurations. This qualitatively explains the origin of a considerable off-shell effect in the d' decay calculated for these alternative models of the NN interaction. The point is that the transition amplitudes for $s^6[6]_X$ and $s^4p^2[42]_X$ final states are of quite different values. The probability for the d' decaying into the $s^6[6]_X$ configuration is more than four times larger than for a decay into the superposition of $s^4p^2[42]_X$ and $(s^52s-s^4p^2)[6]_X$ configurations.

Both modes of the d' decay are generated by a one-particle operator of Eq. (2.14), and thus we need only two harmonic oscillator (h.o.) matrix elements for the single-quark transitions $0S\leftrightarrow 1P$ and $1P\rightarrow 2S$. For the gradient and nongradient parts of the operator (2.14) they are of the form

$$\langle 0S|\hat{O}_6(\mathbf{k},b_\pi)\frac{\omega_\pi}{2m_q}\boldsymbol{\sigma}\cdot\left(\frac{2}{i}\boldsymbol{\nabla}+\frac{5}{6}\mathbf{k}\right)|1P,m\rangle$$
$$=-i\frac{\omega_\pi}{m_qb_6}\frac{1}{2}\sqrt{\frac{5}{3}}\frac{1}{N_6}\left(1+\frac{x_6^2}{6}\right)F_6(k^2)\sigma_m,$$

$$\langle 0S|\hat{O}_6(\mathbf{k},b_\pi)\boldsymbol{\sigma}\cdot\mathbf{k}|1P,m\rangle$$
$$=-ik\frac{1}{2}\sqrt{\frac{5}{3N_6}}\frac{1}{}kb_6\left(1+\frac{x_6^2}{6}\right)F_6(k^2)\{(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}\}_1^m,$$

$$\langle 2S|\hat{O}_6(\mathbf{k},b_\pi)\frac{\omega_\pi}{2m_q}\boldsymbol{\sigma}\cdot\left(\frac{2}{i}\boldsymbol{\nabla}+\frac{5}{6}\mathbf{k}\right)|1P,m\rangle$$
$$=-i\frac{\omega_\pi}{m_qb_6}\frac{1}{3}\sqrt{\frac{5}{2N_6}}F_6(k^2)[A(k^2,x_6^2)\sigma_m$$
$$+k^2b_6^2B(k^2,x_6^2)\{(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}\}_1^m],$$

$$\langle 2S|\hat{O}_6(\mathbf{k},b_\pi)\boldsymbol{\sigma}\cdot\mathbf{k}|1P,m\rangle$$
$$=-ik\frac{1}{3}\sqrt{\frac{5}{2N_6}}kb_6C(k^2,x_6^2)F_6(k^2)$$
$$\times\{(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})\hat{\mathbf{k}}\}_1^m, \tag{3.1}$$

where m is the projection of the angular momentum of the h.o. $1P$ state. $A(k^2,x_6^2)$, $B(k^2,x_6^2)$ and $C(k^2,x_6^2)$ are polynomials [see Appendix B, Eqs. (B1)–(B3)]. The operator $\hat{O}_6$ on the left-hand side of Eqs. (3.1) combines the standard plane-wave exponent with the nonlocal kernel (2.15)

$$\hat{O}_6\equiv\hat{O}_6(\mathbf{k},b_\pi)$$
$$=e^{i(5/6)\mathbf{k}\cdot\boldsymbol{\rho}}e^{-(i/2)\mathbf{k}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}(4\pi b_\pi^2)^{-3/2}$$
$$\times\exp\left[-\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2}{4b_\pi^2}\right]. \tag{3.2}$$

The function $F_6(k^2)$ in Eqs. (3.1) is the standard Gaussian form factor of the h.o. model

$$F_6(k^2)=\exp\left[-\frac{5}{24}k^2b_6^2\left(1+\frac{1}{30}x_6^2\right)\right]. \tag{3.3}$$

In Eqs. (3.1)–(3.3) the parameters

$$x_6^2=\frac{b_\pi^2}{b_6^2}\left(1+\frac{5}{6}\frac{b_\pi^2}{b_6^2}\right)^{-1},\quad N_6=\left(1+\frac{5}{6}\frac{b_\pi^2}{b_6^2}\right)^{3/2} \tag{3.4}$$

are used.

C. Shell-model calculations of pion-baryon coupling

To check this simplified decay model we have calculated the baryon vertices $B\rightarrow\pi N$ for all nonstrange baryons with $n=0,1$ and 2 h.o. excitation quanta using for the $B=N$, Δ, $N^*(1535)$, and $N^{**}(1440)$ baryons the simplest shell-model configurations of the ST classification scheme

$$N=\left|s^3[3]_XL=0,\quad ST=\frac{1}{2}\frac{1}{2}[3]_{ST}J^P=\frac{1}{2}^+\right\rangle,$$
$$\Delta=\left|s^3[3]_XL=0,\quad ST=\frac{3}{2}\frac{3}{2}[3]_{ST}J^P=\frac{3}{2}^+\right\rangle,$$
$$N^*(1535)=\left|s^2p[21]_XL=1,\quad ST=\frac{1}{2}\frac{1}{2}[21]_{ST}J^P=\frac{1}{2}^-\right\rangle, \tag{3.5}$$
$$N^{**}(1440)=\left|sp^2[3]_XL=0,\quad ST=\frac{1}{2}\frac{1}{2}[3]_{ST}J^P=\frac{1}{2}^+\right\rangle.$$

In the description of the decay of these baryons two additional h.o. matrix elements $0S\rightarrow 0S$ and $2S\rightarrow 0S$ for P-wave pion emission

$$\langle 0S|\hat{O}_3(\mathbf{k},b_\pi)\frac{\omega_\pi}{2m_q}\boldsymbol{\sigma}\cdot\left(\frac{2}{i}\boldsymbol{\nabla}+\frac{2}{3}\mathbf{k}\right)|0S\rangle=0, \tag{3.6}$$

$$\langle 0S|\hat{O}_3(\mathbf{k},b_\pi)(\boldsymbol{\sigma}\cdot\mathbf{k})|0S\rangle=\frac{1}{N_3}F_3(k^2)(\boldsymbol{\sigma}\cdot\mathbf{k}),$$

$$\langle 0S|\hat{O}_3(\mathbf{k},b_\pi)\frac{\omega_\pi}{2m_q}\boldsymbol{\sigma}\cdot\left(\frac{2}{i}\boldsymbol{\nabla}+\frac{2}{3}\mathbf{k}\right)|2S\rangle$$
$$=-\frac{\omega_\pi}{m_q}\frac{1}{3}\sqrt{\frac{2}{3}}\frac{1}{N_3}a(k^2,x_3^2)F_3(k^2)(\boldsymbol{\sigma}\cdot\mathbf{k}),$$
$$\langle 0S|\hat{O}_3(\mathbf{k},b_\pi)(\boldsymbol{\sigma}\cdot\mathbf{k})|2S\rangle$$
$$=\sqrt{\frac{3}{2}}\frac{1}{N_3}b(k^2,x_3^2)F_3(k^2)(\boldsymbol{\sigma}\cdot\mathbf{k})$$

should be taken into account. Here, $a(k^2,x_3^2)$ and $b(k^2,x_3^2)$ are polynomials [see Appendix B, Eqs. (B4) and (B5)], the parameters N_3, x_3^2, the form factor F_3, and the operator $\hat{O}_3$ are similar to N_6, x_6^2, F_6, and $\hat{O}_6$ of Eqs. (3.2)–(3.4) and differ only in the coefficients in front of b_π^2/b_3^2 and $\mathbf{k}$

$$\hat{O}_3\equiv\hat{O}_3(\mathbf{k},b_\pi)=e^{i(2/3)\mathbf{k}\cdot\boldsymbol{\rho}}e^{-i/2\,\mathbf{k}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}(4\pi b_\pi^2)^{-3/2}$$
$$\times\exp\left[-\frac{(\boldsymbol{\rho}-\boldsymbol{\rho}')^2}{4b_\pi^2}\right],$$
$$F_3(k^2)=\exp\left[-\frac{1}{6}k^2b_3^2\left(1+\frac{1}{6}x_3^2\right)\right],$$
$$x_3^2=\frac{b_\pi^2}{b_3^2}\left(1+\frac{2}{3}\frac{b_\pi^2}{b_3^2}\right)^{-1},\quad N_3=\left(1+\frac{2}{3}\frac{b_\pi^2}{b_3^2}\right)^{3/2}. \quad (3.7)$$

From the first line of Eq. (3.6) it is seen that the gradient term gives no contribution to the $N\to\pi N$ and $\Delta\to\pi N$ vertices (in the h.o. approximation).

The advantages of the quark approach to the baryon (dibaryon) decay over the standard hadron phenomenology are the smaller numbers of free parameters (only one coupling constant $f_{\pi qq}$ and three phenomenological parameters m_q, b_3, and b_π are needed) and a more reasonable treatment of vertex form factors. To establish a link between the standard hadron phenomenology (coupling constants, form factors, etc.) and the Tuebingen QM transition amplitudes, one can use the formal procedure of analytical continuation of transition amplitudes to zero radius of quark configurations. It is readily calculated from the analytical expressions for the amplitudes obtained with h.o. matrix elements of Eqs. (3.1) and (3.6). The transition to the pointlike pion is a trivial limit $b_\pi\to 0$ [all nontrivial elements of this limit are in the transition to the ''running constant'' γ in Eqs. (2.1) and (2.19)], but the transition to the pointlike baryon (dibaryon) cannot simply be obtained by taking the limit $b_3\to 0$ ($b_6\to 0$). The point is that the contribution of the gradient part of the effective quark-pion interaction (2.14) to the transition amplitude does not only depend on the model-independent parameter k/m_π, but also on the parameter $(m_qb_3)^{-1}$, which goes to infinity in the limit $b_3\to 0$. If the quark mass m_q remains constant in the limit $b_3\to 0$ it would not correspond to the physical picture of going to a pointlike three-quark system. For this reason we fix first the parameter $m_qb_3\approx 1$ (this value is the most natural from a relativistic point of view, in our model $m_qb_3=0.95$) and after that we go to the limit $b_3\to 0$ making use of the ''running'' quark mass $m_q\sim b_3^{-1}$ in the analytical expressions of transition amplitudes. In this limit the transition amplitudes can be written in the form

$$\langle N(b_3)m_ft_f|3H_\lambda^{(3)}|B(b_3)m_it_i\rangle \xrightarrow{b_3\to 0} f_{\pi NB}K_{\pi NB}^{(3)}(k;\lambda,m_ft_f,m_it_i), \quad (3.8)$$

TABLE I. Kinematical part of baryon transition amplitudes from nucleons to different baryons. Matrix elements for baryons as elementary particles are in brackets.

B	$(2\pi)^{3/2}(2\omega_\pi)^{1/2}K_{\pi NB}^{(3)}(k;\lambda,m_ft_fm_it_i)$
N	$-i\langle Nm_ft_f\vert\frac{k}{m_\pi}(\boldsymbol{\sigma}^{(N)}\cdot\hat{\mathbf{k}})\tau_{-\lambda}^{(N)}\vert Nm_it_i\rangle$
Δ	$-i\langle Nm_ft_f\vert\frac{k}{m_\pi}(\mathbf{S}^{(N\Delta)}\cdot\hat{\mathbf{k}})T_{-\lambda}^{(N\Delta)}\vert\Delta m_it_i\rangle$
$N^*(1535)$	$\frac{1}{m_qb_3}\langle Nm_ft_f\vert\frac{\omega_\pi}{m_\pi}\delta_{m_fm_i}\tau_{-\lambda}^{(N)}\vert N^*m_it_i\rangle$
$N^{**}(1440)$	$-\frac{i}{m_qb_3}\langle Nm_ft_f\vert\frac{k}{m_\pi}(\boldsymbol{\sigma}^{(N)}\cdot\hat{\mathbf{k}})\tau_{-\lambda}^{(N)}\vert Nm_it_i\rangle$

where $K_{\pi NB}^{(3)}(k;\lambda,m_ft_f,m_it_i)$ is ''a kinematical part'' of the vertex given in Table I. It depends only on the standard normalization of the pion field and is proportional to the nonrelativistic transition operators for baryons as elementary particles ($\sigma^{(N)}\tau^{(N)}$, $S^{(N\Delta)}T^{(N\Delta)}$, etc.). For excited nucleons $N^*(1535)$ and $N^{**}(1535)$ we also include in the $K_{\pi NB}^{(3)}$ the kinematical factor $(m_qb_3)^{-1}$, which depends on the quark mass (but for the ''running'' mass $m_q\sim b_3^{-1}$ this factor is a constant).

In our model we define the coupling constants $f_{\pi NB}$ as factors which appear in front of the above kinematical part on the right-hand side of Eq. (3.8) in the limit $b_3\to 0$. They are listed in Table II. One can see that they are combinations of algebraic coefficients originating from the color-spin-isospin part of the baryon wave functions, shell-model fractional parentage coefficients (f.p.c.), and the effective quark-pion coupling constant $f_{\pi qq}$. Note that the limit $b_3\to 0$ can also be calculated imposing the condition $b_\pi/b_3=\text{const}\neq 0$. These more complex expressions for coupling constants $f_{\pi NB}$, which depend on the ratio b_π/b_3 are listed in the third column of Table II.

It is interesting that the right-hand side of Eq. (3.8), coincides with the full transition amplitude taken in the limit $k\to 0$

$$\langle N(b_3)m_ft_f|3H_\lambda^{(3)}|B(b_3)m_it_i\rangle \xrightarrow{k\to 0} f_{\pi NB}K_{\pi NB}^{(3)}(k=0;\lambda,m_ft_f,m_it_i), \quad (3.9)$$

if it is not evaluated in the limit $b_3\to 0$, but for physical values of the parameters b_3 and m_q.

TABLE II. Baryon coupling constants and decay widths. Here b_π is the oscillator length in the pion wave function (2.17) and b_3 for the three quark baryon wave function, which can be different from the oscillator length b_6 of a dibaryon. Parameters x_3 and N_3 are combinations of b_π and b_3 and are defined in Eq. (3.7).

	$f_{\pi NB}/f_{\pi qq}$		$\Gamma_{\pi NB}$ (MeV)			
B	$\frac{b_\pi}{b_3}=0$	$\frac{b_\pi}{b_3}\neq 0$	$\frac{b_\pi}{b_3}=0$	$\frac{b_\pi}{b_3}=0.5$	$\frac{b_\pi}{b_3}=1$	Expt.
N	$\frac{5}{3}\left(1+\frac{m_\pi}{6m_q}\right)$	$\frac{5}{3N_3}\left(1+\frac{m_\pi}{6m_q}\right)$				
Δ	$2\sqrt{2}\left(1+\frac{m_\pi}{6m_q}\right)$	$\frac{2\sqrt{2}}{N_3}\left(1+\frac{m_\pi}{6m_q}\right)$	69	68.7	68	115
$N^*(1535)$	$\frac{2\sqrt{2}}{3\sqrt{3}}$	$\frac{2\sqrt{2}}{3\sqrt{3}N_3}\left(1-\frac{1}{6}x_3^2\right)$	58.5	26.2	8.1	(0.35–0.55) $\Gamma^{\text{tot}}_{N^*}$ [a]
$N^{**}(1440)$	$\frac{5}{9\sqrt{3}}m_\pi b_3$	$\frac{5}{9\sqrt{3}N_3}[m_\pi b_3 P(x_3^2)+3m_q b_3 x_3^2]$ [b]	4.3	25.3	118	(0.6–0.7) $\Gamma^{\text{tot}}_{N^{**}}$ [a]

[a] $\Gamma^{\text{tot}}_{N^*}=100$–$250$ MeV, $\Gamma^{\text{tot}}_{N^{**}}=250$–$450$ MeV, from Ref. [50].
[b] $P(x_3^2)=1+\frac{1}{6}x_3^2+\frac{5}{9}x_3^4$.

Therefore, we can write the full transition amplitude in the form

$$\langle N(b_3)m_f t_f|3H_\lambda^{(3)}|B(b_3)m_i t_i\rangle = f_{\pi NB}F_{\pi NB}(k^2)K_{\pi NB}^{(3)}(k;\lambda,m_f t_f,m_i t_i) \qquad (3.10)$$

where $F_{\pi NB}(k^2)$ is the vertex form factor normalized by Eqs. (3.9) and (3.10) as $F_{\pi NB}(0)=1$. The analytical expressions for the form factors $F_{\pi NB}(k^2)$ are given in Appendix B.

The πN decay widths of baryons $\Gamma_{\pi NB}$ have been calculated using the standard formula

$$\Gamma_{\pi NB}=2\pi\int d\mathbf{k}\,\delta\left(M_B-\sqrt{M_N^2+k^2}-\sqrt{m_\pi^2+k^2}\right) \times\frac{1}{2J_i+1}\sum_{m_i}\sum_{m_f t_f}|\langle N m_f t_f|3H_\lambda^{(3)}|B m_i t_i\rangle|^2 \qquad (3.11)$$

with the transition amplitude parametrized by Eq. (3.10). The results for three values of the ratio $b_\pi/b_3=0$, 0.5, and 1. are listed in Table II.

The calculated values of $\Gamma_{\pi NB}$ for $N^*(1535)$ and $N^{**}(1440)$ are very sensitive to the ratio b_π/b_3 and a value of about $b_\pi/b_3\approx 0.5$ would be the best one, if both resonances should be described with the same value of the parameter b_π/b_3. The origin of the high sensitivity of $\Gamma_{\pi NN^{**}}$ to the hadron size may be clarified by the last line of Table II, where the analytical dependence of the coupling constant $f_{\pi NN^{**}}$ on the baryon size b_3 is shown. One can see that in the limit $b_3\to 0$ $(m_q\to\infty)$ the coupling constant for $N^{**}(1440)$ goes to zero (if $b_\pi=0$), while for other baryons it is stable. Apart from this nonstability, the transition amplitudes for both baryons $N^*(1535)$ and $N^{**}(1440)$ depend on the π radius [see Eqs. (B7) and (B8) in Appendix B] because of the spatial orthogonality of the final and initial states in the transitions $N^*\to\pi+N$ and $N^{**}\to\pi+N$ when $b_\pi\to 0$. For these two reasons it is in fact impossible to accurately fit the decay widths of $N^*(1535)$ and $N^{**}(1440)$ in the framework of the Tuebingen QM. More sophisticated calculations are needed, e.g., a relativistic model [28].

The main conclusion drawn from the baryon width calculation is that our simplified model can only be used for a crude estimate of the transition amplitudes (only the order of magnitude of the decay width is guaranteed).

D. Shell-model calculations of pion-dibaryon coupling

In the calculation of the d' decay width we follow the same scheme as in the case of baryons. First, we define the coupling constants $f_{\pi d_0 d'}$, $f_{\pi d_2 d'}$ and $f_{\pi d_f d'}$ and form factors $F_{\pi d_0 d'}(k^2)$, $F_{\pi d_2 d'}(k^2)$, and $F_{\pi d_f d'}(k^2)$ for transitions from the initial six-quark configuration (2.8) to the final ones Eqs. (2.10) and (2.11). The latter we will shortly label as d_0, d_2 and d_f, where

$$|d_0\rangle=|s^6(b_6)[6]_X[2^3]_{CT}LST=001J^P=0^+\rangle,$$

$$|d_2\rangle=|(s^4p^2-s^5 2s)(b_6)[6]_X[2^3]_{CT}LST=001J^P=0^+\rangle,$$

$$|d_f\rangle=|s^4p^2(b_6)[42]_X[f_{CT}]LST=001J^P=0^+\rangle. \qquad (3.12)$$

Here, we use the color-isospin classification of the states (see Ref. [11] for details) and thus in the last line of Eq. (3.12) the following Young schemes are implied:

$$[f_{CT}]=[42],\ [321],\ [2^3],\ [31^3],\ [21^4]. \qquad (3.13)$$

The coupling constants are defined by going to the limit of pointlike dibaryons $b_6\to 0$ at $m_q b_6=\text{const}$ and $b_\pi=0$

TABLE III. Dibaryon coupling constants and algebraic coefficients $U^{NN}_{\{f\}}$ defined in Eq. (3.17) for projection of d_j configurations {*f*} onto the *NN* channel. b_π and b_6 are the oscillator lengths of the pion and the dibaryon wave functions. Parameters x_6 and N_6 are defined in Eq. (3.4).

d_j	$\{f\}$	$f_{\pi d_j d'}/f_{\pi qq}$ $\frac{b_\pi}{b_6}=0$	$f_{\pi d_j d'}/f_{\pi qq}$ $\frac{b_\pi}{b_6}\neq 0$	$U^{NN}_{\{f\}}$
d_0	$\{[6]_X,[2^3]_{CT}\}$	$\frac{5\sqrt{2}}{3\sqrt{3}}$	$\frac{5\sqrt{2}}{3\sqrt{3}N_6}\left(1-\frac{5}{6}x_6^2\right)$	$\frac{1}{3}$
d_2	$\{[6]_X,[2^3]_{CT}\}$	$\frac{2\sqrt{5}}{9}$	$\frac{2\sqrt{5}}{9N_6}\left[1-\frac{5}{12}x_6^2\left(7-\frac{25}{6}x_6^2\right)\right]$	$\frac{1}{3}$
	$\{[42]_X,[42]_{CT}\}$	0	0	$-\frac{3}{2\sqrt{5}}$
	$\{[42]_X,[321]_{CT}\}$	$-\frac{5}{9}$	$-\frac{5}{9N_6}\left(1-\frac{5}{6}x_6^2\right)$	$\frac{4}{3\sqrt{5}}$
d_f	$\{[42]_X,[2^3]_{CT}\}$	$\frac{4\sqrt{5}}{9}$	$\frac{4\sqrt{5}}{9N_6}\left(1-\frac{5}{6}x_6^2\right)$	$\frac{1}{6}$
	$\{[42]_X,[31^3]_{CT}\}$	$\frac{\sqrt{10}}{9}$	$\frac{\sqrt{10}}{9N_6}\left(1-\frac{5}{6}x_6^2\right)$	$-\frac{1}{3\sqrt{2}}$
	$\{[42]_X,[21^4]_{CT}\}$	$-\frac{2\sqrt{5}}{9}$	$-\frac{2\sqrt{5}}{9N_6}\left(1-\frac{5}{6}x_6^2\right)$	0

$$\langle d_j(b_6)t_f|6H^{(6)}_\lambda|d'(b_6)\rangle \xrightarrow{b_6\to 0} f_{\pi d_j d'}K^{(6)}_{\pi d_j d'}(k;\lambda,t_f), \tag{3.14}$$

where $d_j=d_0,\ d_2,\ d_f$ and the kinematical factor $K^{(6)}_{\pi d_j d'}$ is of a universal form for all dibaryon states d_j

$$K^{(6)}_{\pi d_j d'}(k;\lambda,t_f)=\frac{1}{m_q b_6}\frac{\omega_\pi}{m_\pi}\frac{\delta_{t_f,-\lambda}}{(2\pi)^{3/2}(2\omega_\pi)^{1/2}}. \tag{3.15}$$

Form factors by definition have to describe the nontrivial k^2 dependence of the transition amplitudes arising when the size of the dibaryon b_6 goes to its physical value. For dibaryons we use the same definition of the form factor as for baryons in Eq. (3.10)

$$\langle d_j(b_6)t_f|6H^{(6)}_\lambda|d'(b_6)\rangle=f_{\pi d_j d'}F_{\pi d_j d'}(k^2)K^{(6)}_{\pi d_j d'}(k;\lambda,t_f). \tag{3.16}$$

The form factors are normalized as usual by Eq. $F_{\pi d_j d'}(0)=1$. The coupling constants $f_{\pi d_j d'}$ are listed in the third $(b_\pi/b_6=0)$ and fourth $(b_\pi/b_6\neq 0)$ columns of Table III. In the fifth column we list the algebraic coefficients $U^{NN}_{\{f\}}$ for the projection of the six-quark configurations (3.12) onto the *NN* channel (see Ref. [11] for details). The form factors $F_{\pi d_j d'}(k^2)$ are given in Appendix B.

E. The full expression for the transition amplitude in the plane-wave approximation

The techniques developed here make an algebraic calculation of the d' decay amplitude possible. It is instructive to perform such calculations with the optimal coupling constants and vertex form factors, in order to obtain a qualitative estimate for the relative probabilities of the two transition modes (see Fig. 2). These are (i) deexcitation $d'\to d_0$, or (ii) excitation $d'\to d_2$, of the initial d' configuration. Here, the index $\{f\}$ on d_f implies the full set of the Young schemes $\{f\}=\{[f_X],[f_{CT}]\}$ used for the classification of final six-quark configurations. For d_0 and d_2 we use the following notation for indices {f}: $\{f_0\}=\{f_2\}=\{[6]_X,[2^3]_{CS}\}$.

For the plane-wave final state $|NN(b_3)\mathbf{q},t_f\rangle_{pw}$ the overlap integrals ${}_{pw}\langle NN(b_3)\mathbf{q},t_f|d_j(b_6),t_f\rangle$ of the decomposition of the amplitude in Eq. (2.12) have a simple analytical form that allows us to write the final result in the following compact form (for more details, see Ref. [11]):

$$\begin{aligned}\sum_{d_j}\ {}_{pw}\langle NN(b_3)\mathbf{q},t_f|d_j(b_6),t_f\rangle\langle d_j,t_f|6H^{(6)}_\lambda|d'\rangle \\ =\Phi_{NN}(q^2)K^{(6)}_{\pi d_j d'}(k;\lambda,t_f)\Bigg\{F_{\pi d_0 d'}(k^2)f_{\pi d_0 d'}U^{NN}_{\{f_0\}} \\ -\sqrt{\frac{3}{10}}\left(1-\frac{4}{9}q^2b_6^2\right)\Bigg[F_{\pi d_2 d'}(k^2)f_{\pi d_2 d'}U^{NN}_{\{f_2\}} \\ +\frac{2}{3}F_{\pi d_f d'}(k^2)\sum_{\{f\}}f_{\pi d_f d'}U^{NN}_{\{f\}}\Bigg]\Bigg\}.\end{aligned} \tag{3.17}$$

Here, $U_{\{f\}}^{NN}$ are algebraic coefficients for the projection of six-quark states onto the NN channel in CST space; the q-dependent functions, including the function

$$\Phi_{NN}(q^2)=\sqrt{10}\frac{2^{3/4}b_6^{3/2}}{3^{1/4}\pi^{3/4}}\left(\frac{2b_6/b_3}{1+b_6^2/b_3^2}\right)^6 e^{-q^2b_6^2/3}, \quad (3.18)$$

are from the projection of the coordinate part of the six-quark configurations onto the NN plane wave. The first term in the curly brackets in Eq. (3.17) is the contribution of the de-excitation mode $d'\to d_0$ and the remaining part on the right-hand side is the contribution of the excitation mode $d'\to d_2$, $d_{\{f\}}$. The latter is considerably smaller than the former because of a destructive interference of the amplitudes in the square brackets (see Table III for coupling constants $f_{\pi d_j d'}$ and coefficients $U_{\{f\}}^{NN}$). Note that the destructive interference has been established here on an algebraic level. In the d' decay it leads to a predominance (for probabilities) of the de-excitation mode $d'\to d_0$ over the excitation one, i.e., the probability for the de-excitation mode is four times larger than for the excitation mode.

IV. RESULTS FOR THE d' DIBARYON WIDTH

The total d' decay width has been calculated by integration of the transition amplitude (3.17) over the phase-space volume

$$\Gamma_{d'}^{\text{tot}}=2\pi\int d\mathbf{q}\int d\mathbf{k}_\pi\delta\left(M_{d'}-2M_N-\frac{k_\pi^2}{4M_N}-\omega_\pi\right)\times\sum_{t_f}\left|\sum_{d_j}\langle NN(b_3)\mathbf{q},t_f|d_j(b_6),t_f\rangle\langle d_j,t_f|6H_\lambda^{(6)}|d'\rangle\right|^2. \quad (4.1)$$

In Eq. (4.1) the NN state $|NN(b_3)\mathbf{q},t_f\rangle$ takes into account the final-state interaction (f.s.i.) in distinction to Eq. (3.17), in which the state $|NN(b_3)\mathbf{q},t_f\rangle_{pw}$ is a plane wave. We use the final NN state $|NN(b_3)\mathbf{q},t_f\rangle$ from two different potential models for the NN interaction: (i) the OBEP of Ref. [32] and (ii) the Moscow potential of Ref. [34,48]. The results on the d' width $\Gamma_{d'}^{\text{tot}}$ as function of the d' mass $M_{d'}$ are shown in Fig. 3 for two different values of the π radius, $b_\pi=0$ (the curves labeled by PV) and $b_\pi=0.5b_3=0.3$ fm (the curves labeled by QPCM). The results of the OBEP are given as solid lines, while the results of the Moscow potential are given by dashed lines. The results for the PS coupling of Refs. [11] are also shown (the curve labeled by PS).

In Fig. 3 the d' mass $M_{d'}$ varies from the πNN threshold 2.02 GeV up to the value 2.3 GeV, which is characteristic of the Tuebingen QM calculations of $M_{d'}$ in Refs. [16,13]. One can see that in all the interesting intervals of variation of $M_{d'}$, there is a considerable larger d' width $\Gamma_{d'}$ calculated for the OBEP model of the NN interaction than the results for the Moscow potential model. It means that in both cases the f.s.i. does not destroy the destructive interference of the amplitudes for the transition into excited configurations $s^4p^2[42]_X$ and $(s^4p^2-s^52s)[6]_X$, which we have algebraically calculated at the end of the previous section. As a consequence the probability for a d' decay into the configuration $s^6[6]_X$ prevails. For this reason we have obtained anomalously small decay widths for the model, in which an orthogonality condition to the configuration $s^6[6]_X$ (Moscow potential [34]) is imposed. Introduction of a nonzero π radius b_π in the framework of the QPCM softens the orthogonality condition, but as it is seen from Fig. 3 this does not qualitatively change the results.

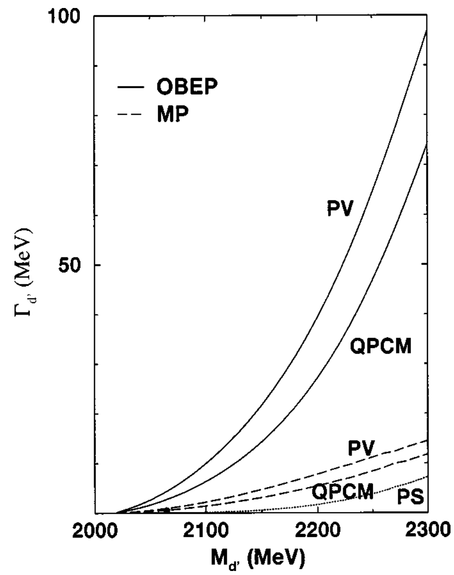

FIG. 3. The d' decay width $\Gamma_{d'}^{\text{tot}}$ as a function of the d' mass $M_{d'}$ for two different models of the NN interaction (the Moscow potential from Ref. [34] dashed lines and the OBEP interaction from Ref. [32] solid lines) and for two different values of the pion q-$\bar{q}$ radius b_π. The curves for $b_\pi=0$ are labeled by PV and for $b_\pi=0.3$ fm are labeled by QPCM (quark pair creation model). For comparison the results of Ref. [11] for the pseudoscalar coupling are also shown (the curve labeled by PS).

The experimental values $\Gamma_{d'}=0.5$ MeV and $M_{d'}=2.065$ GeV observed in DCX reactions on nuclei (the fat point in Fig. 3) are compatible with the theoretical results for $\Gamma_{d'}$ for the Moscow NN potential (MP) and the quark pair creation model (QPCM) with finite $b_\pi=0.5b_3=0.3$ fm. The results for the OBEP model are four times larger. At larger values of $M_{d'}$ the difference between MP and OBEP results increases and at $M_{d'}\approx 2.3$ GeV, this difference reaches an order of magnitude. At $M_{d'}\gtrsim 2.3$ GeV the value of $\Gamma_{d'}$ for the OBEP f.s.i. exceeds 100 MeV.

V. CONCLUDING REMARKS AND SUMMARY

From Fig. 3 it is evident that the experimental value of the d' width $\Gamma_{d'}\approx 0.5$ MeV is compatible with the proposed six-quark nature of the d' dibaryon if the d' mass is close to 2.065 GeV as observed in DCX reactions on nuclei. At the same time the theoretical value of the free d' mass (outside the nuclear medium) obtained in the Tuebingen QM in Refs. [16,13] with a confinement parameter determined in the three-quark system $M_{d'}\approx 2.3$–2.5 GeV is considerably larger. The decay width of such a "free" dibaryon could be about 100 MeV or even larger depending on the NN interaction at short range, see Fig. 3. Therefore, a free d' dibaryon could not be seen as a narrow resonance outside the nuclear medium due to its large width.

However, these conclusions interfere with an unsolved problem: the theoretical value of the mass shift of the d' dibaryon in a nuclear medium is unknown. For further evaluation of the mass shift in different approaches the developed models of quark-pion, baryon-pion, and dibaryon-pion couplings would be very useful, but it is evident that in any approach the mass shift could not be as large as really needed 200–400 MeV. In our opinion for solving the problem of the d' dibaryon two possibilities should be considered:

(i) The resonancelike behavior of the DCX pion scattering on nuclei at $T_\pi \approx 50$ MeV does not correspond to the excitation of a six-quark dibaryon. Alternative mechanisms of resonancelike behavior of DCX scattering have been considered in the literature (see, e.g., Refs. [7,8]). In this case, the presence of a six-quark state with quantum numbers of the d' dibaryon and a mass 2.3–2.5 GeV, as obtained in the Tuebingen QM, would not contradict the experimental data because its decay width would be too large for the resonance to be seen over the large background.

(ii) A six-quark dibaryon d' is excited in nuclei, but the dynamics of the six-quark system in the nuclear medium differs from the dynamics of a ‘‘free’’ six-quark bag (see, e.g., [13]) and the d' mass in nuclei is $M_{d'} \approx 2.065$ GeV.

In both cases new experiments sensitive to d' production outside of nuclei (the electro- or photoproduction off the deuteron [9], in the pp collisions [5], etc.) are desirable because a narrow resonance signal ($\Gamma_{d'} \approx 10$ MeV) at a large value of the ‘‘free’’ d' mass ≈ 2.3–2.5 GeV is possible (see Fig. 3), if the Moscow potential is an adequate model of the NN interaction at short range.

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APPENDIX A: EFFECTIVE QUARK-PION COUPLING

Here we transform the transition operator (2.1) into the form of the effective quark-pion coupling (see, e.g., Ref. [25]). It involves several steps.

First, we substitute the pion wave function

$$|\pi^\lambda\rangle = \Psi_\pi(\mathbf{p}_\pi)\,\phi^\lambda_{\delta\bar\sigma} S_{\delta\bar\sigma} b^+_\delta(\mathbf{p}_6)\, d^+_{\bar\sigma}(\mathbf{p}_{\bar 7})|0\rangle, \qquad \text{(A1)}$$

into the matrix element $\langle n,f;\pi|T|d'\rangle$ and multiply the transition operator T by the unit operator

$$I^{(6)} = \sum_\gamma b^+_\gamma(\mathbf{p}_6)\, b_\gamma(\mathbf{p}_6) \qquad \text{(A2)}$$

for the sixth quark. Here, the quark (antiquark) momentum $\mathbf{p}_6$ ($\mathbf{p}_{\bar 7}$) is labeled by index 6 ($\bar 7$) following Fig. 1 and $\mathbf{p}_\pi = \frac{1}{2}(\mathbf{p}_6 - \mathbf{p}_{\bar 7})$ is used. In Eq. (A1) the isospin triplet and spin singlet quark-antiquark wave functions are of the same form as in Eqs. (2.3) and (2.4)

$$\phi^\lambda_{\delta\bar\sigma} = \frac{1}{\sqrt{2}}\{\tau_\lambda\}_{t_\delta t_{\bar\sigma}}, \qquad S_{\delta\bar\sigma} = \frac{1}{\sqrt{2}}\,\delta_{s_\delta s_{\bar\sigma}}. \qquad \text{(A3)}$$

Second, we make use of the identity of quarks to recouple and rename quark lines in diagrams following from Eqs. (2.1), (A1), and (A2) for the process pictured in Fig. 1. It gives rise to the following expression for the amplitude of the pion emission associated with the sixth quark $M^{(6)}_\lambda(\mathbf{p}_6,\mathbf{p}'_6)$ (see Ref. [25] for details)

$$-\tilde\gamma \int d\mathbf{p}_{\bar 7}\,\delta(\mathbf{p}'_6 + \mathbf{p}_{\bar 7})\,\tau^{(6)}_{-\lambda}\,\boldsymbol{\sigma}^{(6)}\cdot(\mathbf{p}'_6 - \mathbf{p}_{\bar 7})\,\Psi_\pi(\mathbf{p}_\pi). \quad \text{(A4)}$$

One can see that we have renamed momentum $\mathbf{p}_7$ of the seventh quark of the created $q_7\bar q_{\bar 7}$ pair to the momentum $\mathbf{p}'_6$ (see Fig. 1). We also have dropped the trivial factor of momentum conservation for the other five quarks spectators

$$\prod_{i=1}^{5} \delta(\mathbf{p}_i - \mathbf{p}'_i). \qquad \text{(A5)}$$

Note that instead of a summation over angular momentum projections m in the initial expression (2.1) we use the equivalent scalar product form $\boldsymbol{\sigma}\cdot(\mathbf{p}_q - \mathbf{p}_{\bar q})$. A new constant $\tilde\gamma$ is introduced. It combines the old γ and factors $1/\sqrt{2}$, $1/\sqrt{3}$, $1/\sqrt{4\pi}$, etc. from Eqs. (2.2)–(2.4) and from vector spherical harmonics and Clebsch-Gordan coefficients $(1m1-m|00)\mathcal{Y}_1^{-m}$.

Third, before integration over $d\mathbf{p}_{\bar 7}$ in Eq. (A4) we do the usual substitution [25]

$$(\mathbf{p}'_6 - \mathbf{p}_{\bar 7})\,\delta(\mathbf{p}'_6 + \mathbf{p}_{\bar 7}) = [(\mathbf{p}_6 + \mathbf{p}'_6) - (\mathbf{p}_6 - \mathbf{p}'_6)]\,\delta(\mathbf{p}'_6 + \mathbf{p}_{\bar 7}) \qquad \text{(A6)}$$

and obtain in the integrand the known expression for the quark-pion coupling of the ‘‘naive’’ $^3P_0$ model [30]

$$-\boldsymbol{\sigma}\cdot[\mathbf{k} - (\mathbf{p} + \mathbf{p}')]\,\tau_{-\lambda} \qquad \text{(A7)}$$

Finally, the factor $\omega_\pi/2m_q$ in front of the second term in Eq. (A7) has to be inserted to restore Galilean invariance [25,30] which gives rise to the final expression for the quark-pion emission amplitude in the momentum representation

$$M^{(6)}_\lambda = \tilde\gamma\,\tau^{(6)}_{-\lambda}\,\boldsymbol{\sigma}^{(6)}\cdot\left[\mathbf{k} - \frac{\omega_\pi}{2m_q}(\mathbf{p}_6 + \mathbf{p}'_6)\right]\Psi_\pi(\mathbf{p}_\pi). \quad \text{(A8)}$$

Here $\mathbf{p}_\pi = \frac{1}{2}(\mathbf{p}_6 + \mathbf{p}'_6)$.

APPENDIX B: ANALYTICAL EXPRESSIONS FOR SHELL-MODEL MATRIX ELEMENTS AND FORM FACTORS

$$A(k^2,x_6^2)=\left(1+\frac{5}{24}k^2b_6^2\right)-\frac{5}{12}\left(7-\frac{25}{6}x_6^2\right)-\frac{5}{144}k^2b_6^2x_6^2\left(7-\frac{11}{6}x_6^2+\frac{5}{36}x_6^4\right), \tag{B1}$$

$$B(k^2,x_6^2)=-\frac{5}{12}+\frac{5}{36}x_6^2\left(1+\frac{1}{12}x_6^2-\frac{35}{144}x_6^4\right)+\frac{25}{1728}x_6^2k^2b_6^2\left(1-\frac{1}{6}x_6^2-\frac{1}{36}x_6^4+\frac{1}{216}x_6^6\right). \tag{B2}$$

$$C(k^2,x_6^2)=\left(1-\frac{5}{24}k^2b_6^2\right)-\frac{1}{12}x_6^2\left(1-\frac{25}{6}x_6^2\right)+\frac{5}{144}k^2b_6^2x_6^2\left(1+\frac{1}{6}x_6^2-\frac{1}{36}x_6^4\right), \tag{B3}$$

$$a(k^2,x_3^2)=1-\frac{1}{3}x_3^2+\frac{5}{9}x_3^4+\frac{1}{9}k^2b_3^2x_3^2\left(1+\frac{2}{3}x_3^2+\frac{1}{9}x_3^4\right), \tag{B4}$$

$$b(k^2,x_3^2)=-\frac{2}{3}x_3^2+\frac{1}{9}k^2b_3^2\left(1+\frac{2}{3}x_3^2+\frac{1}{9}x_3^4\right), \tag{B5}$$

$$F_{\pi NN}(k^2)=F_{\pi N\Delta}(k^2)=\exp\left[-\frac{k^2b_3^2}{6}\left(1+\frac{x_3^2}{6}\right)\right], \tag{B6}$$

$$F_{\pi NN*}(k^2)=\left[1-\frac{k^2b_3^2}{18}\frac{[1-(4/3)x_3^2][1+(1/3)x_3^2]}{[1-(2/3)x_3^2]}-\frac{m_q}{\omega_\pi}\frac{k^2b_3^2}{3}\frac{[1+(1/3)x_3^2]}{[1-(2/3)x_3^2]}\right]F_{\pi NN}(k^2), \tag{B7}$$

$$F_{\pi NN**}(k^2)=\left[\frac{m_\pi}{m_q}\left(1+\frac{1}{6}x_3^2+\frac{5}{9}x_3^4\right)+3x_3^2\right]^{-1}\left\{\frac{\omega_\pi}{m_q}\left[1+\frac{1}{6}x_3^2+\frac{5}{9}x_3^4-\frac{k^2b_3^2}{12}\left(1-\frac{4}{3}x_3^2\right)\left(1+\frac{2}{3}x_3^2+\frac{1}{9}x_3^4\right)\right]\right.$$
$$\left.+3x_3^2-\frac{k^2b_3^2}{2}\left(1+\frac{2}{3}x_3^2+\frac{1}{9}x_3^4\right)\right\}F_{\pi NN}(k^2), \tag{B8}$$

$$F_{\pi d_0 d'}(k^2)=F_{\pi d_f d'}(k^2)=\left[1+\frac{5}{216}k^2b_6^2x_6^2\frac{1+(1/6)x_6^2}{1-(5/6)x_6^2}-\frac{m_q}{\omega_\pi}\left(1+\frac{\omega_\pi}{12m_q}\right)\frac{k^2b_6^2}{3}\left(\frac{1+(1/6)x_6^2}{1-(5/6)x_6^2}\right)\right]\exp\left[-\frac{5}{24}k^2b_6^2\left(1+\frac{x_6^2}{30}\right)\right], \tag{B9}$$

$$F_{\pi d_2 d'}(k^2)=\left\{1+\left[1-\frac{5}{12}x_6^2\left(7-\frac{25}{6}x_6^2\right)\right]^{-1}\left[\frac{5}{72}k^2b_6^2+\frac{5}{432}k^2b_6^2x_6^2\left(17-\frac{35}{6}x_6^2+\frac{25}{18x_6^4}\right)+\frac{25}{5184}k^4b_6^4x_6^2\right.\right.$$
$$\times\left.\left(1-\frac{x_6^2}{6}-\frac{x_6^4}{36}+\frac{x_6^6}{216}\right)\right]+\frac{m_q}{\omega_\pi}\left(1+\frac{\omega_\pi}{12m_q}\right)\left[1-\frac{5}{12}x_6^2\left(7-\frac{25}{6}x_6^2\right)\right]^{-1}\left[\frac{k^2b_6^2}{3}\left(1-\frac{5}{24}k^2b_6^2\right)\right.$$
$$\left.\left.-\frac{k^2b_6^2}{36}x_6^2\left(1-\frac{25}{6}x_6^2\right)+\frac{5}{216}k^4b_6^4x_6^2\left(1+\frac{x_6^2}{6}-\frac{x_6^4}{36}\right)\right]\right\}\exp\left[-\frac{5}{24}k^2b_6^2\left(1+\frac{x_6^2}{30}\right)\right]. \tag{B10}$$

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