Infinite nuclear matter on the light front: Nucleon-nucleon correlations

G. A. Miller
Department of Physics, Box 351560, University of Washington, Seattle, Washington 98195-1560

R. Machleidt
Department of Physics, University of Idaho, Moscow, Idaho 83844

(Received 2 April 1999; published 27 July 1999)

A relativistic light-front formulation of nuclear dynamics is developed and applied to treating infinite nuclear matter in a method which includes the correlations of pairs of nucleons: this is light-front Brueckner theory. We start with a hadronic meson-baryon Lagrangian that is consistent with chiral symmetry. This is used to obtain a light-front version of a one-boson-exchange nucleon-nucleon potential (OBEP). The accuracy of our description of the nucleon-nucleon (NN) data is good, and similar to that of other relativistic OBEP models. We derive, within the light-front formalism, the Hartree-Fock and Brueckner-Hartree-Fock equations. Applying our light front OBEP, the nuclear matter saturation properties are reasonably well reproduced. We obtain a value of the compressibility 180 MeV that is smaller than that of alternative relativistic approaches to nuclear matter in which the compressibility usually comes out too large. Because the derivation starts from a meson-baryon Lagrangian, we are able to show that replacing the meson degrees of freedom by a NN interaction is a consistent approximation, and the formalism allows one to calculate corrections to this approximation in a well-organized manner. The simplicity of the vacuum in our light-front approach is an important feature in allowing the derivations to proceed. The mesonic Fock space components of the nuclear wave function are obtained also, and aspects of the meson and nucleon plus-momentum distribution functions are computed. We find that there are about 0.05 excess pions per nucleon. [S0556-2813(99)05108-0]

PACS number(s): 21.65.+f, 11.80.−m, 13.75.Cs, 21.30.−x

I. INTRODUCTION

We introduce a light-front formalism for infinite nuclear matter, in which the effects of correlations are taken into account. This is a light-front Brueckner theory of nuclear matter. While the ultimate goal of this and related studies is to provide a fully relativistic treatment of nuclei which includes all previous knowledge about nuclear dynamics, the present work represents one step beyond the previous light-front mean-field calculation [1,2] of the properties of infinite nuclear matter.

Understanding an important class of experiments seems to require that light-front dynamics and the related light cone variables be used. Consider the lepton-nucleus deep inelastic scattering experiments [3] which showed that there is a significant difference between the parton distributions of free nucleons and nucleons in a nucleus. This difference can interpreted as a small (~10%) shift in the momentum distribution of valence quarks towards smaller values of the Bjorken variable x_{Bj}. The Bjorken variable is a ratio of the plus-momentum $k^+=k^0+k^3$ of a quark to that of the target. If one regards the nucleus as a collection of nucleons, $x_{Bj}=p^+/k^+$, where k^+ is the plus momentum of a nucleon bound in the nucleus. If one uses k^0+k^3 as a momentum variable the corresponding canonical spatial variable is $x^-=x^0-x^3$ and the time variable is x^0+x^3 [4]. This is the light-front (LF) approach of Dirac [5]; see the recent reviews [6,7] for more information.

Deep inelastic scattering depends on the light-front momentum distribution which is the probability $f(k^+)$ that a bound nucleon has a momentum k^+. Other nuclear reactions, such as (e,e') and $(p,2p)$ depend also on this very same probability [8–10]. The quantity $f(k^+)$ is simply related to the square of the ground state wave function, computed using light-front dynamics. The usual equal time approach to nuclear dynamics is very successful, and it is natural to use this information to calculate the distribution $f(k^+)$. However, in the standard equal time formulation this quantity is a response function and depends on matrix elements between the ground and all excited states, and therefore can be more difficult to compute.

The use of light-front variables is convenient for interpreting certain experiments, but does not allow one to avoid the necessary task of handling nuclear dynamics. Thus one is faced with the task of computing the ground state nuclear wave function using $x^+=x^0+x^3$ as a time variable. The present effort is a simplification in that the nucleus is taken to be infinite nuclear matter. However, the detailed effects of the interactions between two nucleons are included, so that we are concerned with the relativistic dynamics of a strongly interacting many body system.

Light-front techniques have previously been applied to systems of two hadrons [6,8,9,11–15]. Our emphasis here is in large nuclear systems. The light-front quantization procedure necessary to treat nucleon interactions with scalar and vector mesons was derived by Soper [16], and by Yan and collaborators [17,18].

We next outline our procedure. The necessary Lagrangian, which respects chiral symmetry, and its light-front Hamiltonian is described in Sec. II. Its application to nucleon-nucleon scattering in the one-boson exchange approximation is carried out in Sec. III. A new feature is that the effects of isovector mesons such as the $\boldsymbol{\rho}$ and $\boldsymbol{\delta}$, and the $\boldsymbol{\rho}$-nucleon tensor interaction are included. The NN potential is generated using the one-boson exchange approximation. The Weinberg-type integral equation, which maintains unitarity and boost invariance in one direction, is solved and the results are compared with phase shift data. Section IV is concerned with the many-nucleon problem. Two separate perturbation series are involved. The first step is to eliminate temporarily the meson degrees of freedom in favor of our two nucleon potential. Thus one first proceeds using only nucleon degrees of freedom. The light-front formalism to obtain the nucleonic interacting ground state wave function $|\Phi\rangle$ in terms of a series in which a Brueckner G matrix acts on a best Slater determinant $|\phi\rangle$ is developed. The independent-pair approximation is used. One must find an eigenstate of the P^- operator for which the expectation value of P^+ is equal to the eigenvalue of P^-. This formalism is applied and the results are discussed in Sec. V. The full nuclear wave function $|\Psi\rangle$, including the meson degrees of freedom, is discussed in Sec. VI. The object $|\Psi\rangle$ is related to $|\Phi\rangle$ by a second series involving the difference between the nucleon-nucleon interaction and the meson-nucleon interactions. One finds that the expression for the nuclear mass, evaluated in Sec. V, is valid within our approximation. Furthermore, expressions for the meson and nucleon distribution functions are obtained. We derive a sum rule for obtaining the total number of (nonvector) mesons in the nucleus. A brief discussion of the implications of our results for lepton-nucleus deep inelastic scattering and the nuclear Drell-Yan process is presented in Sec. VII. A brief summary of our results is contained in Sec. VIII. Some of the necessary notation is discussed in an appendix. Some of the present results, but none of the details of the derivation or of our two-nucleon potential have appeared in Ref. [19].

II. LIGHT-FRONT QUANTIZATION: LAGRANGIAN, FIELD EQUATIONS, AND LIGHT-FRONT HAMILTONIAN

The light-front approach is a three-dimensional formalism involving a Hamiltonian which is a P^- operator. One starts with a Lagrangian and derives field equations which allow one to eliminate the appearance of dependent degrees of freedom in the Hamiltonian. Our starting point is a nonlinear chiral model in which the nuclear constituents are nucleons ψ (or ψ'), pions $\boldsymbol{\pi}$, scalar mesons ϕ [20], and vector mesons V^μ. The Lagrangian $\mathcal{L}$ is given by

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi\partial^\mu\phi-m_s^2\phi^2)-\frac{1}{4}V^{\mu\nu}V_{\mu\nu}+\frac{m_v^2}{2}V^\mu V_\mu + \frac{1}{4}f^2\,\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger)+\frac{1}{4}m_\pi^2 f^2\,\mathrm{Tr}(U+U^\dagger-2) + \overline{\psi'}\left[\gamma^\mu\left(\frac{i}{2}\overleftrightarrow{\partial}_\mu-g_v V_\mu\right)-U(M+g_s\phi)\right]\psi', \tag{2.1}$$

where the bare masses of the nucleon, scalar and vector mesons are given by M, m_s, m_v, and $V^{\mu\nu}=\partial^\mu V^\nu-\partial^\nu V^\mu$. The unitary matrix U can be chosen from amongst three forms U_i,

$$U_1\equiv e^{i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f},\quad U_2\equiv\frac{1+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f}{1-i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}/2f}, \quad U_3=\sqrt{1-\boldsymbol{\pi}^2/f^2}+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}/f, \tag{2.2}$$

which correspond to different definitions of the fields. This Lagrangian, based on the linear representations of chiral symmetry used by Gursey [21], is discussed in Ref. [2]. It is approximately $(m_\pi\neq 0)$ invariant under the chiral transformation

$$\psi'\rightarrow e^{i\gamma_5\boldsymbol{\tau}\cdot\mathbf{a}}\psi',\quad U\rightarrow e^{-i\gamma_5\boldsymbol{\tau}\cdot\mathbf{a}}U\,e^{-i\gamma_5\boldsymbol{\tau}\cdot\mathbf{a}}. \tag{2.3}$$

This invariance shows that our scalar meson ϕ is not a chiral partner of the pion. Note the presence of the term $U(M+g_s\phi)$ which was incorrectly given as $MU+g_s\phi$ in Refs. [1,2].

The constant M/f plays the role of the bare pion-nucleon coupling constant. If f is chosen to be the pion decay constant, the Goldberger-Trieman relation says that the axial vector coupling constant $g_A=1$. This is not really a problem because loop effects can make up the needed 25% effect. Corrections of that size are typical of order $(M/f)^3$ effects found in the cloudy bag model [22] for many observables, including g_A. We also note that the Δ is not treated as an explicit degree of freedom in the above Lagrangian.

The present Lagrangian may be thought of as a low energy effective theory for nuclei under normal conditions. A more sophisticated Lagrangian is reviewed in Ref. [23] and used in Ref. [24]; the present one is used to show that light-front techniques can be applied to hadronic theories relevant for nuclear physics. This hadronic model, when evaluated in mean field approximation, gives [25] at least a qualitatively good description of many (but not all) nuclear properties and reactions. There are a variety of problems occurring when higher order terms are included [23]. The aim here is to use a reasonably sophisticated Lagrangian to study the effects that one might obtain by using a light-front formulation.

Reference [2] contains the details of the quantization procedure; we re-state the relevant results here. An essential feature is the quantization of spin-1/2 fermions. Although described by four-component spinors, these fields have only two independent degrees of freedom. The light-front formalism allows a convenient separation of dependent and independent variables via the projection operators $\Lambda_\pm\equiv\gamma^0\gamma^\pm/2$ [16], with $\psi'_\pm\equiv\Lambda_\pm\psi'_\pm$. The independent Fermion degree of freedom is chosen to be ψ'_+. The properties of the projection operators are discussed in the Appendix. One gets coupled equations for $\psi'_\pm$:

$$(i\partial^- - g_v V^-)\psi'_+=[\boldsymbol{\alpha}_\perp\cdot(\mathbf{p}_\perp-g_v\mathbf{V}_\perp)+\beta U(M+g_s\phi)]\psi'_-, \quad (i\partial^+ - g_v V^+)\psi'_-=[\boldsymbol{\alpha}_\perp\cdot(\mathbf{p}_\perp-g_v\mathbf{V}_\perp)+\beta U(M+g_s\phi)]\psi'_+. \tag{2.4}$$

The relation between ψ'_- and ψ'_+ is very complicated unless one may set the plus component of the vector field to zero

[6]. This is immediately obtained in QED and QCD by choosing an appropriate gauge in which the plus-component of the vector potential vanishes. Here the nonzero mass of the vector meson prevents such a choice. Instead, one simplifies the equation for ψ'_- by [16,18] transforming the Fermion field according to

$$\psi' = e^{-ig_v\Lambda(x)}\psi \tag{2.5}$$

with $\partial^+\Lambda = V^+$. This transformation leads to the result [2]

$$(i\partial^- - g_v\overline{V}^-)\psi_+ = [\boldsymbol{\alpha}_\perp\cdot(\mathbf{p}_\perp - g_v\overline{\mathbf{V}}_\perp) + \beta U(M+g_s\phi)]\psi_-,$$

$$i\partial^+\psi_- = [\boldsymbol{\alpha}_\perp\cdot(\mathbf{p}_\perp - g_v\overline{\mathbf{V}}_\perp) + \beta U(M+g_s\phi)]\psi_+, \tag{2.6}$$

where

$$\partial^+\overline{V}^\mu = \partial^+V^\mu - \partial^\mu V^+ = V^{+\mu}. \tag{2.7}$$

The fields V^μ enter the meson field equations, but the fields $\overline{V}^\mu$ enter the fermion field equations. The eigenmode expansion for $\overline{V}^\mu$ is given by

$$\overline{V}^\mu(x) = \int \frac{d^2k_\perp dk^+\theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}}\sum_{\omega=1,3}\overline{\epsilon}^\mu(\mathbf{k},\omega)[a(\mathbf{k},\omega)e^{-ik\cdot x} + a^\dagger(\mathbf{k},\omega)e^{ik\cdot x}], \tag{2.8}$$

where the polarization vectors $\overline{\epsilon}^\mu(\mathbf{k},\omega)$ are given by [18]

$$\overline{\epsilon}^\mu(\mathbf{k},\omega) = \epsilon^\mu(\mathbf{k},\omega) - \frac{k^\mu}{k^+}\epsilon^+(\mathbf{k},\omega), \tag{2.9}$$

with the properties

$$k^\mu\overline{\epsilon}_\mu(\mathbf{k},\omega) = -\frac{m_v^2}{k^+}\epsilon^+(\mathbf{k},\omega),$$

$$\sum_{\omega=1,3}\overline{\epsilon}^\mu(\mathbf{k},\omega)\overline{\epsilon}^\nu(\mathbf{k},\omega) = -\left(g^{\mu\nu} - g^{+\mu}\frac{k^\nu}{k^+} - g^{+\nu}\frac{k^\mu}{k^+}\right). \tag{2.10}$$

The use of the Fermion field equation allows one to obtain the light-front Hamiltonian density

$$T^{+-} = \boldsymbol{\nabla}_\perp\phi\cdot\boldsymbol{\nabla}_\perp\phi + m_\phi^2\phi^2 + \frac{1}{4}(V^{+-})^2 + \frac{1}{2}V^{kl}V^{kl} + m_v^2V^kV^k + (\boldsymbol{\nabla}_\perp\boldsymbol{\pi})^2 + \frac{[(1/2)\boldsymbol{\nabla}_\perp\boldsymbol{\pi}^2]^2}{\boldsymbol{\pi}^2}\left(1 - \frac{f^2}{\boldsymbol{\pi}^2}\sin^2\frac{\boldsymbol{\pi}}{f}\right) + m_\pi^2f^2\sin^2\frac{\boldsymbol{\pi}}{f} + 2\psi_+^\dagger\left(i\frac{1}{2}\overleftrightarrow{\partial}^- - g_v\overline{V}^-\right)\psi_+. \tag{2.11}$$

The expression (2.11) is useful for situations, such as in the mean field approximation, for which a simple expression for ψ_+ is known. This is not always the case, so it is worthwhile to use the Dirac equation to express T^{+-} in an alternate form:

$$T^{+-} = \boldsymbol{\nabla}_\perp\phi\cdot\boldsymbol{\nabla}_\perp\phi + m_\phi^2\phi^2 + \frac{1}{4}(V^{+-})^2 + \frac{1}{2}V^{kl}V^{kl} + m_v^2V^kV^k + (\boldsymbol{\nabla}_\perp\boldsymbol{\pi})^2 + \frac{[(1/2)\boldsymbol{\nabla}_\perp\boldsymbol{\pi}^2]^2}{\boldsymbol{\pi}^2}\left(1 - \frac{f^2}{\boldsymbol{\pi}^2}\sin^2\frac{\boldsymbol{\pi}}{f}\right) + m_\pi^2f^2\sin^2\frac{\boldsymbol{\pi}}{f} + \overline{\psi}[\boldsymbol{\gamma}_\perp\cdot(\mathbf{p}_\perp - g_v\overline{\mathbf{V}}_\perp) + U(M+g_s\phi)]\psi. \tag{2.12}$$

The relationship between ψ and ξ (ξ_- contains no interactions) is discussed in Ref. [2], but we explain it briefly here for the sake of completeness. We follow Refs. [16,26] in expressing ψ_- as a sum of terms, one ξ_- whose relation with ψ_+ is free of interactions, the other η_- containing the interactions. That is, rewrite the second of Eq. (2.6) as [27]

$$\xi_- = \frac{1}{i\partial^+}(\boldsymbol{\alpha}_\perp\cdot\mathbf{p}_\perp + \beta M)\psi_+$$

$$\eta_- = \frac{1}{i\partial^+}[-\mathbf{t}\alpha_\perp\cdot g_v\overline{\mathbf{V}}_\perp + \beta(Ug_s\phi + (U-1)M)]\psi_+. \tag{2.13}$$

Furthermore, define $\xi_+(x) \equiv \psi_+(x)$, so that

$$\psi(x) = \xi(x) + \eta_-(x), \tag{2.14}$$

where $\xi(x) \equiv \xi_-(x) + \xi_+(x)$. This separates the dependent and independent parts of ψ.

It is ξ that is expanded in creation and destruction operators according to

$$\xi(x) = \int \frac{d^2k_\perp dk^+\theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}}\sum_{\lambda=+,-}[u(\mathbf{k},\lambda)e^{-ik\cdot x}b(\mathbf{k},\lambda) + v(\mathbf{k},\lambda)e^{+ik\cdot x}d^\dagger(\mathbf{k},\lambda)]. \tag{2.15}$$

The spinors $u(\mathbf{k},\lambda)$ are the usual equal time Dirac spinors, of normalization $\overline{u}u = 2M$. It is legitimate to use these because one is free to choose the representation of the solutions of the Dirac equation in an infinite number of ways. In particular, the correct fermionic anticommutation relation for ξ_+ is obtained with these spinors [17].

The Hamiltonian is a sum of a free $P_0^-(N)$ and interacting terms $P_I^-(N)$:

$$P_0^-(N) = \frac{1}{2}\int d^2x_\perp dx^-\overline{\xi}(\boldsymbol{\gamma}_\perp\cdot\mathbf{p} + M)\xi, \tag{2.16}$$

$$P_I^- = v_1 + v_2 + v_3, \tag{2.17}$$

with

$$v_1 = \int d^2x_\perp dx^- \overline{\xi}(g_v \gamma\cdot \overline{V} + M(U-1) + g_s \phi U)\xi, \tag{2.18}$$

$$v_2 = \int d^2x_\perp dx^- \overline{\xi}(-g_v \gamma\cdot \overline{V} + M(U-1) + g_s \phi U) \times \frac{\gamma^+}{2p^+}(-g_v \gamma\cdot \overline{V} + M(U-1) + g_s \phi U)\xi, \tag{2.19}$$

and

$$v_3 = \frac{g_v^2}{32}\int d^2x_\perp dx^- \int dy_1^- \overline{\xi}(\mathbf{x}_\perp, y_1^-)\gamma^+ \xi(\mathbf{x}_\perp, y_1^-) \times \epsilon(x^- - y_1^-)\int dy_2^- \epsilon(x^- - y_2^-) \times \overline{\xi}(\mathbf{x}_\perp, y_2^-)\gamma^+ \xi(\mathbf{x}_\perp, y_2^-). \tag{2.20}$$

The term v_1 accounts for the emission or absorption of a single vector or scalar meson, as well as the emission or absorption of any number of pions through the operator $U-1$. The term v_2 includes contact terms in which there is propagation of an instantaneous fermion. The term v_3 accounts for the propagation of an instantaneous vector meson.

The component that is related to the plus momentum is T^{++}. The necessary expression is given by

$$T^{++} = V^{ik}V^{ik} + m_v^2 V^+ V^+ + \overline{\psi}\gamma^+ i\partial^+ \psi + \partial^+ \phi \partial^+ \phi + \partial^+ \boldsymbol{\pi}\cdot\partial^+ \boldsymbol{\pi} + \boldsymbol{\pi}\cdot\partial^+ \boldsymbol{\pi}\frac{\boldsymbol{\pi}\cdot\partial^+ \boldsymbol{\pi}}{\boldsymbol{\pi}^2}\left(1 - \frac{f^2}{\boldsymbol{\pi}^2}\sin^2\frac{\boldsymbol{\pi}}{f}\right). \tag{2.21}$$

III. NUCLEON-NUCLEON SCATTERING VIA ONE-BOSON EXCHANGE POTENTIALS

The correlations between nucleons are caused by the nucleon-nucleon interaction. Thus a necessary first step towards a light-front theory of nuclear correlations is the derivation of a light-front theory of the nucleon-nucleon interaction. Previous work [1,2] showed that the light-front version of the Lippmann-Schwinger equation, the Weinberg equation, can be transformed (with one difference remaining [28]) into the Blankenbecler-Sugar equation. Kinematic invariance under boosts in the three-direction is maintained, and we shall obtain a one-boson exchange potential which is in reasonably good agreement with the NN phase shifts.

A. General formalism

It is worthwhile to begin by reviewing how using the light-front Hamiltonian of Eqs. (2.16)–(2.20) leads to the one-boson exchange potential. This derivation is useful in understanding how the full nuclear wave function discussed in Sec. VI is related to the nucleonic truncation of Sec. IV. Consider the scattering process $1+2\rightarrow 3+4$. The use of second-order perturbation theory shows that the lowest-order contribution to the nucleon-nucleon scattering amplitude is given by

$$\langle 3,4|K|1,2\rangle = \langle 3,4|v_1 g(P_{ij}^-)v_1 + v_3|1,2\rangle, \tag{3.1}$$

with

$$g_0(P_{ij}^-) \equiv \frac{1}{P_{ij}^- - P_0^-}, \tag{3.2}$$

where P_{ij}^- is the negative component of the total initial or final momentum which are the same. In constructing the NN potential one uses conservation of four-momentum between the initial and final NN states in constructing the NN potential. The expression (3.1) yields a one-boson exchange approximation to the nucleon-nucleon potential.

It is worthwhile to discuss the energy denominator $P_{ij}^- - P_0^-$ in more detail. To be specific, suppose that $k_1^+ > k_3^+$. Then the emitted meson of mass μ has momentum k with $k^+ = k_1^+ - k_3^+, \mathbf{k}_\perp = \mathbf{k}_{1\perp} - \mathbf{k}_{3\perp}$ and $k^- = (k_\perp^2 + \mu^2)/k^+$. Then

$$P_{ij}^- - P_0^- = P_{12}^- - P_0^- = P_{34}^- - P_0^- = (k_1^- - k_3^-) - \frac{k_\perp^2 + \mu^2}{k_1^+ - k_3^+}. \tag{3.3}$$

The interaction K also contains a factor of k^+ in the denominator, so that the relevant denominator is $D \equiv k^+(P_{ij}^- - P_0^-) = (k_1^+ - k_3^+)(k_1^- - k_3^-) - k_\perp^2 - \mu^2 = q^2 - \mu^2$. This last familiar form involves the four-momentum transfer between nucleons 1 and 3 $(q \equiv k_1 - k_3)$ and leads to the usual Yukawa-type potentials. It is also useful to explore the form of the energy denominator using light-front variables by first defining the plus component P^+ of the initial and final total momentum. We may also define $k_1^+ = xP^+$ and $k_3^+ = x'P^+$ in which x and $x'(x > x')$, as ratios of plus momenta, are invariant under Lorentz transformations in the three direction. Then using Eq. (3.3), we find

$$D = \left(\frac{k_{1\perp}^2 + M^2}{x} - \frac{k_{3\perp}^2 + M^2}{x'}\right)(x - x') - k_\perp^2 - \mu^2. \tag{3.4}$$

This quantity is also invariant under Lorentz transformations in the three direction. This expression is to be used only if $k_1^+ > k_3^+$. If $k_3^+ > k_1^+$, then use a version of expression (2.15) in which x and x' are interchanged.

A straightforward evaluation of Eq. (3.1) using Eqs. (2.18)–(2.20) leads to the result

$$\langle 3,4|K|1,2\rangle = 2\langle 3,4|V|1,2\rangle \frac{M^2\delta^{(2,+)}(P_i-P_f)}{\sqrt{k_1^+k_2^+k_3^+k_4^+}}, \quad (3.5)$$

where $\delta^{(2,+)}(P_i-P_f)\equiv\delta^{(2)}(\mathbf{P}_{i\perp}-\mathbf{P}_{f\perp})\,\delta(P_i^+-P_f^+)$ and V is the standard expression for the sum of the π, ϕ and vector meson exchange potentials:

$$\langle 3,4|V|1,2\rangle = \langle 3,4|V(\phi)+V(\boldsymbol{\pi})+V(V)|1,2\rangle. \quad (3.6)$$

The operator K is twice the usual two-nucleon potential times a factor which includes the light-front phase space factor and a momentum-conserving delta function. Note that $dk^+/k^+ = dk^3/E(k)$ for free nucleons where $k^+=E(k)+k^3$.

For the exchange of scalar and pseudoscalar mesons, only the term $v_1 g_0(P_i^-)v_1$ enters, and one finds

$$\langle 3,4|V(\phi,\pi)|1,2\rangle = \frac{\overline{u}(4)\Gamma u(2)\overline{u}(3)\Gamma u(1)}{4M^2(2\pi)^3(q^2-\mu^2)}, \quad (3.7)$$

in which the momentum transfer q is given by

$$q\equiv k_3-k_1. \quad (3.8)$$

The notation is that $u(i)$ is the Dirac-spinor for a free nucleon of quantum numbers i, and Γ is either of the form g_s or $ig_\pi\gamma_5\boldsymbol{\tau}$. The derivation of the contribution of vector meson exchange proceeds by including the meson exchange $v_1 g_0(P_i^-)v_1$ plus the meson instantaneous term v_3, and the result takes the familiar form

$$\langle 3,4|V(V)|1,2\rangle = -g_v^2\frac{\overline{u}(4)\gamma_\mu u(2)\overline{u}(3)\gamma^\mu u(1)}{4M^2(2\pi)^3(q^2-m_v^2)}. \quad (3.9)$$

The expressions (3.7) and (3.9) represent the usual [29–33] expressions for the chosen one-boson exchange potentials, if no form factor effects are included. The sum of the amplitudes arising from each of the individual one-boson exchange terms gives the invariant amplitude to second order in each of the coupling constants. The factors $1/4M^2$ in Eqs. (3.7) and (3.9) can be thought of as renormalizing the spinors so that $\overline{u}u=1$, and the factors $\sqrt{M/k^+}$ of Eq. (3.5) serve to further change the normalization to $u^\dagger u=1$

These amplitudes are strong, so computing the nucleon-nucleon scattering amplitude and phase shifts requires including higher order terms. One may include a sum which gives unitarity by including all iterations of the two particle irreducible scattering operator K through intermediate two-nucleon states. One first removes kinematic factors by defining a T matrix T using

$$\mathcal{M}\equiv 2T\frac{M^2\delta^{(2,+)}(P_i-P_f)}{\sqrt{k_1^+k_2^+k_3^+k_4^+}}, \quad (3.10)$$

to find that

$$\langle 3,4|T|1,2\rangle = \langle 3,4|V|1,2\rangle + \sum_{\lambda_5,\lambda_6}\int \langle 3,4|V|5,6\rangle \times\frac{2M^2}{p_5^+p_6^+}\frac{d^2p_{5\perp}dp_5^+}{P_i^- -(p_5^- +p_6^-)+i\epsilon}\langle 5,6|T|1,2\rangle. \quad (3.11)$$

One realizes that Eq. (3.11) is of the form of the Weinberg equation [34] (see Ref. [8]) by expressing the plus-momentum variable in terms of a light-front momentum fraction α such that

$$p_5^+ = \alpha P_i^+, \quad (3.12)$$

and using the relative and total momentum variables

$$\mathbf{p}_\perp \equiv (1-\alpha)\mathbf{p}_{5\perp} - \alpha\mathbf{p}_{6\perp},$$

$$\mathbf{P}_{i\perp} = \mathbf{p}_{5\perp}+\mathbf{p}_{6\perp}. \quad (3.13)$$

Then,

$$\langle 3,4|T|1,2\rangle = \langle 3,4|V|1,2\rangle + \int\sum_{\lambda_5,\lambda_6}\langle 3,4|V|5,6\rangle \times\frac{2M^2}{\alpha(1-\alpha)}\frac{d^2p_\perp d\alpha}{P_i^2-(p_\perp^2+M^2)/\alpha(1-\alpha)+i\epsilon} \times\langle 5,6|T|1,2\rangle, \quad (3.14)$$

where P_i^2 is the square of the total initial four-momentum, otherwise known as the invariant energy s and $(p_\perp^2+M^2)/\alpha(1-\alpha)$ is the corresponding quantity for the intermediate state. Because the kernel V is itself invariant under Lorentz transformations in the three direction and the integral involves $p_\perp$ and α the procedure of solving this equation gives T with the same invariance. Note that we use the labels p_i to designate momenta in the intermediate state, and k_i for the initial and final states.

Equation (3.14) can in turn be reexpressed (in the center of mass frame) as the Blankenbecler-Sugar (BbS) equation [35] by using the variable transformation [36]:

$$\alpha = \frac{E(p)+p^3}{2E(p)}, \quad (3.15)$$

with $E(p)\equiv\sqrt{\mathbf{p}\cdot\mathbf{p}+M^2}$ and $P_i^2=4(\mathbf{p}\cdot\mathbf{p}+M^2)$. The result is

$$\langle 3,4|T|1,2\rangle=\langle 3,4|V|1,2\rangle+\int\sum_{\lambda_5,\lambda_6}\langle 3,4|V|5,6\rangle \times\frac{M^2}{E(p)}\frac{d^3p}{\mathbf{p_i^2}-\mathbf{p}^2+i\epsilon}\langle 5,6|T|1,2\rangle, \quad (3.16)$$

which is the desired equation. Rotational invariance is manifestly obeyed. The three-dimensional propagator is exactly that of the BbS equation. There is, one difference between Eq. (3.16) and the standard BbS equation. Our one-boson exchange potentials depend on the square of the four momentum q^2 transferred when a meson is absorbed or emitted by a nucleon. Thus the energy difference between the initial and final on-shell nucleons is included and $q^0\neq 0$. This nonzero value is a consequence of the invariance of D of Eq. (3.4) under Lorentz transformations in the three direction. The usual derivation of the BbS equation from the Bethe-Salpeter equation specifies that $q^0=0$ is used in the meson propagator. Including $q^0\neq 0$ instead of $q^0=0$ increases the range of the potential relative to the usual treatment, and its consequences are explored below. One can convert Eq. (3.16) into the Lippman-Schwinger equation of nonrelativistic scattering theory by removing the factor $M/E(p)$ with a simple transformation [37].

B. Generation of a realistic one-boson exchange potential

The present results are that one can use the light-front technique to derive nucleon-nucleon potentials in the one-boson exchange (OBE) approximation and use these in an appropriate wave equation. Our purpose here is to show that the present procedure yields potentials essentially identical to the Bonn OBEP potentials [30,31] and these potentials lead to a good description of the NN data.

The Bonn one-boson exchange potentials employ six different mesons, namely, $\pi,\eta,\omega,\rho,\sigma$, and the (isovector scalar) δ/a_0 meson. The present formalism can account for the π,η,ω, and σ in an approximately chiral invariant manner. We wish to add in couplings $\overline{\psi}\boldsymbol{\tau}\cdot\boldsymbol{\delta}\psi$ and $\overline{\psi}\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\gamma_\mu\psi$ in a chiral invariant manner. Simply adding such terms to the Lagrangian of Eq. (2.1) would lead to a violation of the approximate symmetry of Eq. (2.3). However, one can redefine the operator U so that the symmetry remains. We replace the operator $\overline{\psi}'U\psi'$ in the Lagrangian (2.1) by $\overline{\psi}'\widetilde{U}\psi'$:

$$\widetilde{U}\equiv e^{(i/2f_\rho)\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\gamma_\mu}e^{(i/2f_\delta)\boldsymbol{\tau}\cdot\boldsymbol{\delta}}Ue^{(i/2f_\delta)\boldsymbol{\tau}\cdot\boldsymbol{\delta}}e^{(i/2f_\rho)\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\gamma_\mu}. \quad (3.17)$$

Then the new Lagrangian is invariant under the transformation

$$\psi'\rightarrow e^{i\gamma_5\boldsymbol{\tau}\cdot\mathbf{a}}\psi', \quad \widetilde{U}\rightarrow e^{-i\gamma_5\boldsymbol{\tau}\cdot\mathbf{a}}Ue^{-i\gamma_5\boldsymbol{\tau}\cdot\mathbf{a}}. \quad (3.18)$$

In the present application we expand the exponential to first order in the meson fields.

The final term we need to include is the tensor $\sigma_{\mu\nu}q^\nu$ part of the ρ-nucleon interaction. The presence of such a tensor interaction makes it difficult to write the equation for ψ_- as $\psi_-=1/p^+\cdots\psi_+$. This is a possible problem because the standard value of the ratio of the tensor to vector ρ-nucleon coupling f_ρ/g_ρ is 6.1, based upon Ref. [38]. Reproducing the observed values of ε_1 and P-wave wave phase shifts requires a large value f_ρ/g_ρ; see Ref. [39]. However our Lagrangian compensates for its lack of a ρ-N interaction with tensor coupling by generating such a term via vertex correction diagrams (which are the origin of the anomalous magnetic moment of the electron in QED). Such diagrams might not generate the phenomenologically required values of the coupling constants, but all that is needed here is that terms of the correct form be produced. This is because the standard procedure is to choose the values of the coupling constants so as to yield a good description of the NN scattering data. Thus we simply add in the necessary tensor terms.

This brings us to the treatment of divergent terms in our procedure. The definition of any effective Lagrangian requires the specification of such a procedure. For the present, it is sufficient to say that we introduce form factors, $F_\alpha(q^2)$ which reduce the strength of the α meson-nucleon coupling for large values of $-q^2$. This is also the procedure of Refs. [30–32]. In principle, calculating the higher order terms using the correct Lagrangian can lead to consistent calculations of these form factors. We use a more phenomenological approach here.

The net result is that the one-boson exchange treatment of the nucleon-nucleon potential and the T-matrix resulting from its use in the BbS equation is essentially the same as the one-boson exchange procedure of Refs. [29–32]. The only difference is the keeping of the retardation effects—the square of the four-vector momentum transfer enters in our potentials.

C. Specific one-boson-exchange amplitudes

The above formalism yields a one-boson-exchange potential (OBEP) which is a sum of one-particle-exchange amplitudes of certain bosons with given mass and coupling. Our explicit expressions are presented here. As noted above, we use the six non-strange bosons with masses below 1 GeV/c^2. Thus,

$$V_{\rm OBEP}=\sum_{\alpha=\pi,\eta,\rho,\omega,a_0,\sigma}V_\alpha^{\rm OBE} \quad (3.19)$$

with π and η pseudoscalar (ps), σ and a_0/δ scalar (s), and ρ and ω vector (v) particles.

The OBE amplitudes (which are the contributions to V of our formalism) in the two-nucleon center-of-mass (c.m.) frame are given by

$$\langle \mathbf{k}'\lambda_3\lambda_4|V_{ps}^{\rm OBE}|\mathbf{k}\lambda_1\lambda_2\rangle = -\frac{g_{ps}^2}{(2\pi)^3 4M^2}\overline{u}(\mathbf{k}',\lambda_3)\gamma^5 u(\mathbf{k},\lambda_1)\overline{u}(-k',\lambda_4)\gamma^5 u(-k,\lambda_2)[q^2-m_{ps}^2]^{-1}, \tag{3.20}$$

$$\langle \mathbf{k}'\lambda_3\lambda_4|V_{s}^{\rm OBE}|\mathbf{k}\lambda_1\lambda_2\rangle = \frac{g_{s}^2}{(2\pi)^3 4M^2}\overline{u}(\mathbf{k}',\lambda_3) u(\mathbf{k},\lambda_1)\overline{u}(-k',\lambda_4) u(-k,\lambda_2)[q^2-m_{s}^2]^{-1}, \tag{3.21}$$

$$\begin{aligned}\langle \mathbf{k}'\lambda_3\lambda_4|V_{v}^{\rm OBE}|\mathbf{k}\lambda_1\lambda_2\rangle =& -\frac{1}{(2\pi)^3 4M^2}\left\{g_v\overline{u}(\mathbf{k}',\lambda_3)\gamma_\mu u(k,\lambda_1)+\frac{f_v}{2M}\overline{u}(\mathbf{k}',\lambda_3)\sigma_{\mu\nu}i(k_3-k_1)^\nu u(\mathbf{k},\lambda_1)\right\}\\ &\times\left\{g_v\overline{u}(-k',\lambda_4)\gamma^\mu u(-k,\lambda_2)+\frac{f_v}{2M}\overline{u}(-k',\lambda_4)\sigma^{\mu\nu}i(k_4-k_2)_\nu u(-k,\lambda_2)\right\}[q^2-m_v^2]^{-1}\\ =& -\frac{1}{(2\pi)^3 4M^2}\left\{(g_v+f_v)\overline{u}(\mathbf{k}',\lambda_3)\gamma_\mu u(\mathbf{k},\lambda_1)-\frac{f_v}{2M}\overline{u}(\mathbf{k}',\lambda_3)(k_3+k_1)_\mu u(\mathbf{k},\lambda_1)\right\}\\ &\times\left\{(g_v+f_v)\overline{u}(-k',\lambda_4)\gamma^\mu u(-k,\lambda_2)-\frac{f_v}{2M}\overline{u}(-k',\lambda_4)(k_4+k_2)^\mu u(-k,\lambda_2)\right\}[q^2-m_v^2]^{-1}.\end{aligned} \tag{3.22}$$

Our notation in the c.m. frame is such that in-coming nucleon 1 carries helicity λ_1 and four-momentum $k_1=k=(E,\mathbf{k})$ with $E\equiv\sqrt{M^2+\mathbf{k}^2}$, and in-coming nucleon 2 carries helicity λ_2 and four-momentum $k_2=(E,-\mathbf{k})$; the out-going nucleons have λ_3, $k_3=k'=(E',\mathbf{k}')$ with $E'\equiv\sqrt{M^2+\mathbf{k}'^2}$, and λ_4, $k_4=(E',-\mathbf{k}')$. The square of the four-momentum transfer between the two nucleons is $q^2=(k_3-k_1)^2=(k'-k)^2=(E'-E)^2-(\mathbf{k}'-\mathbf{k})^2$. The Gordon identity [40] is used in the evaluation of the tensor coupling [41,42]. For the isospin-vector bosons π, a_0, and ρ, the above amplitudes must be multiplied by $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$.

With an eye on the nuclear matter calculations to be conducted later in this paper, we note that in the factor $f_v/2M$ of the tensor coupling the nucleon mass M is used as a scaling mass to make the coupling constant f_v dimensionless. This scaling mass could be anything. Therefore, this M is not to be replaced by M^* in the nuclear medium.

In this subsection, we use Dirac spinors (in helicity representation) given by

$$u(\mathbf{k},\lambda_1)=\sqrt{E+M}\begin{pmatrix}1\\ \dfrac{2\lambda_1|\mathbf{k}|}{E+M}\end{pmatrix}|\lambda_1\rangle, \tag{3.23}$$

$$u(-\mathbf{k},\lambda_2)=\sqrt{E+M}\begin{pmatrix}1\\ \dfrac{2\lambda_2|\mathbf{k}|}{E+M}\end{pmatrix}|\lambda_2\rangle, \tag{3.24}$$

with

$$|\lambda_1\rangle=\chi_{\lambda_1},\quad |\lambda_2\rangle=\chi_{-\lambda_2}, \tag{3.25}$$

where χ denotes the conventional Pauli spinor. The normalization is $\overline{u}(\mathbf{k},\lambda)u(\mathbf{k},\lambda)=2M$.

At each meson-nucleon vertex, a form factor is applied which has the analytic form

$$F_\alpha(q^2)=\left(\frac{\Lambda_\alpha^2-m_\alpha^2}{\Lambda_\alpha^2-q^2}\right)^{n_\alpha}, \tag{3.26}$$

with m_α the mass of the meson involved and Λ_α the so-called cutoff mass; $n_\alpha=1$ for pseudoscalar and scalar mesons and $n_\alpha=2$ for vector mesons. Thus, the OBE amplitudes Eqs. (3.20)–(3.22) are all multiplied by F_α^2.

D. Two-nucleon scattering

In the two-nucleon c.m. frame, the scattering amplitude T is the solution of the integral equation

$$T(\mathbf{k}',\mathbf{k})=V(\mathbf{k}',\mathbf{k})+\int d^3p\,V(\mathbf{k}',\mathbf{p})\frac{M^2}{E_\mathbf{p}}\frac{1}{\mathbf{k}^2-\mathbf{p}^2+i\epsilon}T(\mathbf{p},\mathbf{k}), \tag{3.27}$$

where $\mathbf{k}$, $\mathbf{p}$, and $\mathbf{k}'$ are the initial, intermediate and final relative momenta, respectively, of the two interacting nucleons and $E_\mathbf{p}\equiv\sqrt{M^2+\mathbf{p}^2}$. This is Eq. (3.16) with the spin indices suppressed for the purpose of simplicity. The corresponding equation for the K matrix (which we denote by R) is

$$R(\mathbf{k}',\mathbf{k})=V(\mathbf{k}',\mathbf{k})+\mathcal{P}\int d^3p\,V(\mathbf{k}',\mathbf{p})\frac{M^2}{E_\mathbf{p}}\frac{1}{\mathbf{k}^2-\mathbf{p}^2}R(\mathbf{p},\mathbf{k}), \tag{3.28}$$

where $\mathcal{P}$ denotes the principal value integral.

Using standard techniques [31,41], the potential and the scattering equation are decomposed into partial waves. Numerical solutions are obtained by the matrix inversion method [31,43]. For an uncoupled partial wave, phase shifts are then derived from the on-shell K matrix by

TABLE I. Potential parameters and predictions for the deuteron and low-energy *np* scattering. For the deuteron, the binding energy B_d, the *D*-state probability P_D, the quardrupole moment Q_d, and the asymptotic *D*-state over *S*-state ratio D/S are given. Low-energy *np* scattering is paramertized in terms of a_{np} and r_{np} in ${}^1S_0$ and a_t and r_t in ${}^3S_1$, where *a* denotes the scattering length and *r* the effective range. The nucleon mass is $M = 938.919$ Mev.

	Light-front OBEP	Thompson OBEP[a]	Empirical[b]			
			Meson parameters			
	m_α (Mev)	$g_\alpha^2/4\pi[f/g]$	Λ_α (GeV)	$g_\alpha^2/4\pi[f/g]$	Λ_α (GeV)	$g_\alpha^2/4\pi[f/g]$
π	138.04	14.0	1.2	14.6	1.2	13.5 - 14.6
η	547.5	3	1.5	5	1.5	$\leqslant 5$
ρ	769	0.9 [6.1]	1.85	0.95 [6.1]	1.3	0.6(1) [6.6±1.0]
ω	782	24.5 [0.0]	1.85	20.0 [0.0]	1.5	24±5±7
a_0	983	2.0723	2.0	3.1155	1.5	
σ	550	8.9602	2.0	8.0769	2.0	
			Deuteron			
B_d(Mev)		2.2245		2.2247		2.2245745(9)
P_D (%)		4.53		5.10		
Q_d (fm^2)		0.270[c]		0.278[c]		0.2860(15)
D/S		0.0250		0.0257		0.0256(4)
			Low-energy *np* scattering			
a_{np} (fm)		−23.745		−23.747		−23.748(10)
r_{np} (fm)		2.671		2.664		2.75(5)
a_t (fm)		5.494		5.475		5.424(4)
r_t (fm)		1.856		1.828		1.759(5)

[a]Potential *B* of Brockmann and Machleidt [32].
[b]For more comprehensive information on the empirical data and references, note see Tables 4.1 and 4.2 of Ref [30].
[c]Meson-exchange current contributions not included.

$$\tan \delta^J(T_{\rm lab}) = -\frac{\pi}{2}|\mathbf{k}|\frac{M^2}{E}R^J(|\mathbf{k}|,|\mathbf{k}|) \tag{3.29}$$

with $T_{\rm lab} = 2\mathbf{k}^2/M$ and J the total angular momentum of the partial-wave state. For coupled partial waves and other technical details, see Ref. [31].

E. Results for the two-nucleon system

Following established procedures [30,31], the coupling constants and cutoff masses of the six OBE amplitudes are varied within reasonable limits such as to reproduce the two-nucleon bound state (deuteron) and the two-nucleon scattering data below the inelastic threshold (about 300 MeV laboratory kinetic energy). In Table I, we show the meson parameters for our newly constructed light-front (LF) OBEP together with the predictions for the deuteron as well as low-energy neutron-proton (*np*) scattering. For comparison, we also give the parameters from an OBEP that was previously constructed and applied in the Dirac-Brueckner approach to nuclear matter [30,32]. The latter uses the Thompson formalism [44] which is very similar to the BbS formalism—the propagator in Eq. (3.27) contains an extra factor of M/E_p. Note that the Thompson OBEP uses $n_\alpha = 1$ also for vector meson form factors, which explains the differences in the vector meson cutoff masses between the two OBEP.

Phase shifts for *np* scattering are shown in Fig. 1 for all partial waves with $J \leqslant 2$. Over all, the reproduction of the *NN* data by our LF OBEP is quite satisfactory and certainly as good as by OBEP constructed within alternative relativistic frameworks. Based upon these results, we feel confident in applying this OBEP to the relativistic nuclear many-body problem.

IV. NUCLEONIC TRUNCATION FOR THE MANY-BODY PROBLEM

Now that the light-front treatment of nucleon-nucleon scattering is in hand, we may proceed to the problem of computing the properties of infinite nuclear matter. We derive a light-front Brueckner theory from first principles starting with the field-theoretic light-front Hamiltonian.

The nuclear wave function for the ground state of infinite nuclear matter at rest is defined as $|\Psi\rangle$, and we wish to solve the equation

$$P^-|\Psi\rangle = M_A|\Psi\rangle, \tag{4.1}$$

in which P^- is the light front Hamiltonian of Eqs. (2.11)–(2.20). For a nuclear system at rest we must have also the result that

$$P^+|\Psi\rangle = M_A|\Psi\rangle. \tag{4.2}$$

It is necessary to discuss the light-front Hamiltonian, and to find good approximate solutions of the above equations.

We recall that

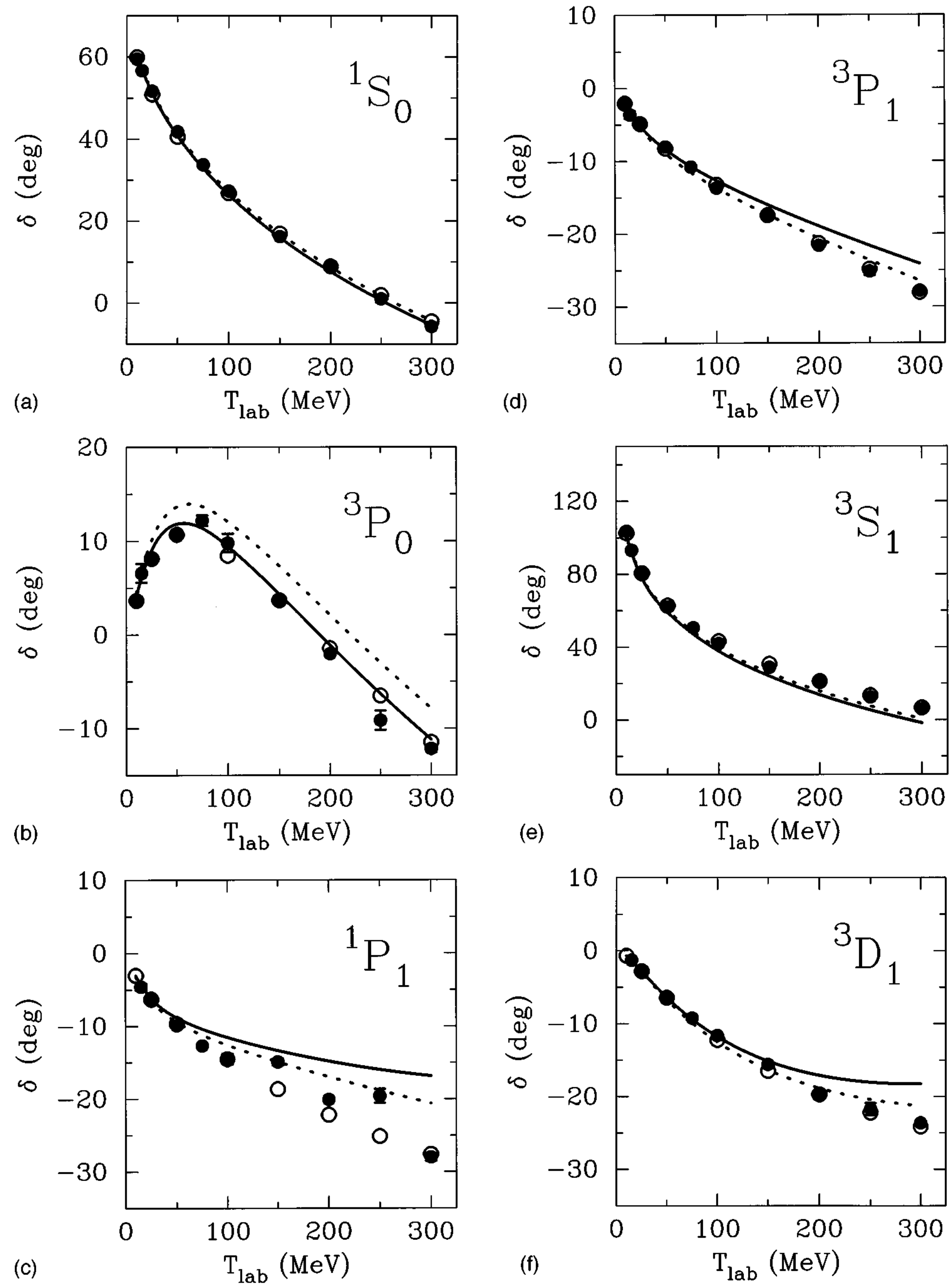

FIG. 1. Phase shifts δ and mixing parameters ϵ of neutron-proton scattering for partial waves with $J \leqslant 2$ and laboratory kinetic energies $T_{\rm lab} \leqslant 300$ MeV. The solid line is the prediction by the LF OBEP presented in Sec. III and the dotted line the one by Potential B of Brockmann and Machleidt [32]. The open circles represent the multi-energy *np* analysis by the Nijmegen group [45] and the solid dots are the VPI analysis SM97 [46].

$$P^- = P_0^-(N) + J \tag{4.3}$$

in which $P_0^-(N)$ is the kinetic contribution to the P^- operator, giving $(p_\perp^2 + m^2)/p^+$ for the minus-momentum of free fermions. The operator J is the sum of three terms of Eqs. (2.18)–(2.20):

$$J \equiv v_1 + v_2 + v_3 . \tag{4.4}$$

The operator v_1 gives all of the single meson-nucleon vertex functions. The operator v_2 accounts for instantaneous fermion exchanges: meson emission followed by instantaneous fermion propagation (propagator is $\gamma^+/2p^+$) followed by another meson emission. The operator v_3 accounts for the instantaneous propagation of vector mesons.

We shall proceed towards an approximate solution of Eq. (4.1), in two stages. We shall first consider the nucleons only part of the Hilbert space. This involves the assumption that

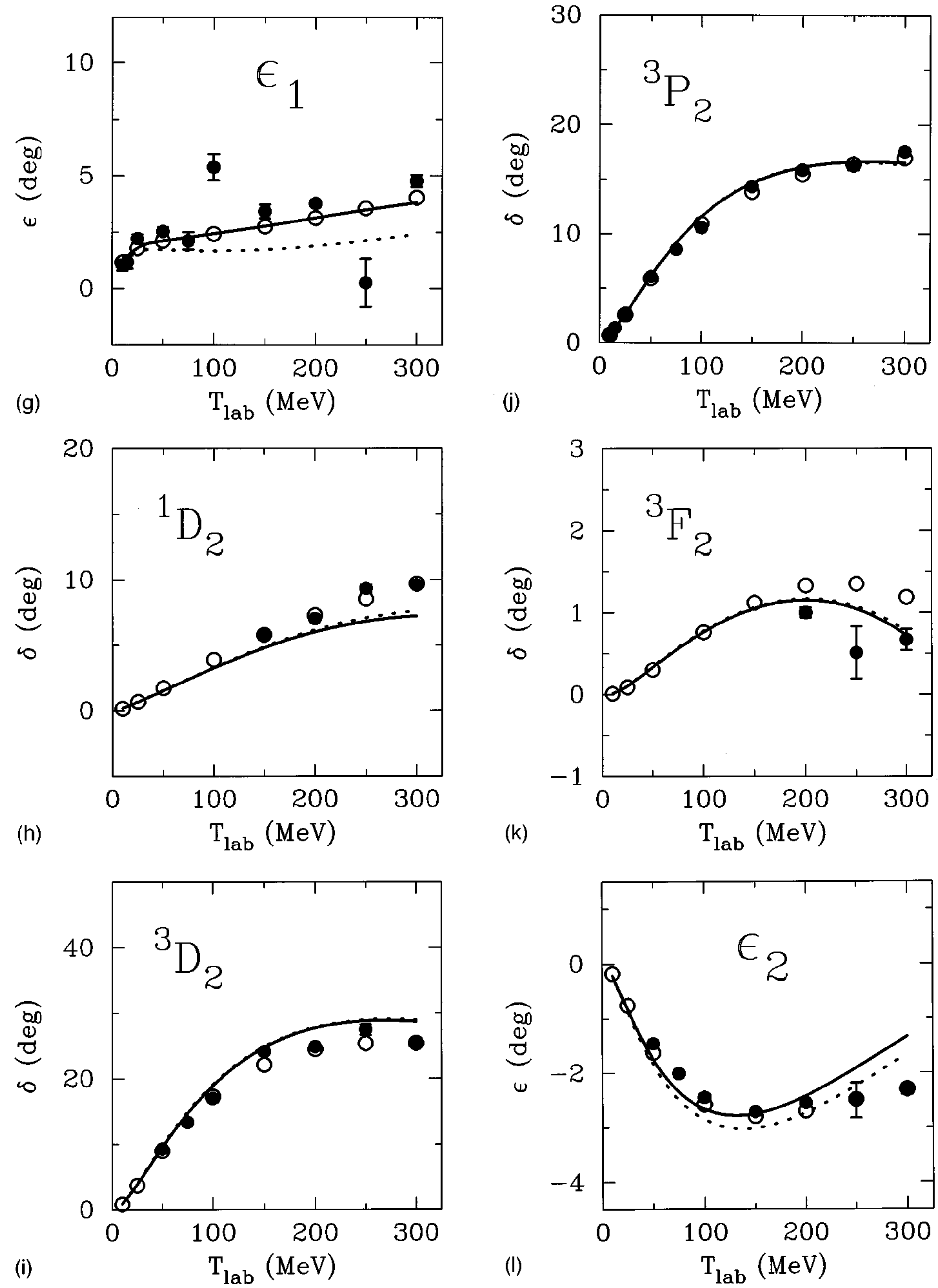

FIG. 1 (*Continued*).

using a nucleon-nucleon interaction K accounts for the meson-nucleon dynamics. This assumption in relaxed in Sec. VI, which displays the formalism necessary to construct the best possible potential and how to include meson degrees of freedom in the wave function. Our Hamiltonian (P^-) contains no terms in which the vacuum can spontaneously emit particles. This simplifying feature causes the derivations to look very similar to those of nonrelativistic theory, even though the treatment is relativistic.

A. Introducing the two-nucleon force

At present all of the interactions are expressed in terms of the meson-nucleon vertex functions and contact terms represented by the operator J. We shall follow the traditional path of using a two-nucleon potential and temporarily eliminate the meson degrees of freedom. One way to accomplish this is to add and subtract the two-nucleon potential to the Hamiltonian and treat terms involving the difference between J and the two-nucleon potential as a perturbation. The use of light-front dynamics mandates that we perform this operation on the Lagrangian because the construction of the Hamiltonian uses the field equations to identify the dynamical degrees of freedom, such as ψ_+ .Therefore we need to study the effective Lagrangian

$$\mathcal{L}_V \equiv \overline{\psi}(i\gamma\cdot\partial - M)\psi - \frac{\mathcal{K}}{2} \tag{4.5}$$

which removes the meson-nucleon interaction term J from the Lagrangian and replaces it with the density $\mathcal{K}$ for the two-nucleon interaction K of the previous section. This means that

$$K=\frac{1}{2}\int d^2x_\perp dx^- \mathcal{K}, \tag{4.6}$$

where K is given in Eq. (3.5) and is twice the nucleon-nucleon potential times a kinematic factor. We recall that one must eliminate, using Eq. (2.5), any components of interaction $\mathcal{K}$ that connect ψ_- to ψ_-.

Given the new Lagrangian $\mathcal{L}_V$ we may construct the corresponding $P^\pm$ operators using the canonical definition

$$T^{\mu\nu}_{\mathcal{V}}=-g^{\mu\nu}\mathcal{L}_{\mathcal{V}}+\sum_r \frac{\partial\mathcal{L}_{\mathcal{V}}}{\partial(\partial_\mu\phi_r)}\partial^\nu\phi_r, \tag{4.7}$$

in which the degrees of freedom $\bar{\psi}$ and ψ are labeled by ϕ_r. The term $\mathcal{V}$ involves only nucleon fields, not their derivatives, so the second term of the energy-momentum tensor do not enter in computing $T^{\mu\nu}_{\mathcal{V}}$. The element $\mu\nu=+-(g^{+-}=2)$ is needed to construct the relevant P-minus operator and we find

$$T^{+-}_{\mathcal{V}}=-2\bar{\psi}(i\gamma\cdot\partial-M)\psi+\mathcal{K}+2\psi_+^\dagger i\partial^-\psi_+. \tag{4.8}$$

The origin of the factor $\frac{1}{2}$ that multiplies $\mathcal{K}$ in the Lagrangian (4.5) is that $\mathcal{K}$ enters here in $T^{+-}_{\mathcal{V}}$. It is worthwhile to define the P-minus operator obtained by using $\mathcal{L}_{\mathcal{V}}$ as P_0^- with

$$P_0^-=\frac{1}{2}\int dx^- d^2x_\perp T^{+-}_{\mathcal{V}}\equiv P_0^-(N)+K. \tag{4.9}$$

The complete P-minus operator is given by

$$P^-=P_0^-+H_1, \tag{4.10}$$

with

$$H_1\equiv J-K+P_0^-(m), \tag{4.11}$$

where $P_0^-(m)$ accounts for the noninteracting mesonic contribution to P^-. The formal problem of choosing the best K by minimizing the effects of $H_1=J-K$ is discussed in the Sec. VI. We shall assume here that the present OBEP is a reasonably satisfactory version of the best interaction, and we shall ignore the influence of the term H_1 in calculations of the energy.

The purely nucleonic part of the full wave function is defined as $|\Phi\rangle$, and is the solution of the light-front Schrodinger equation

$$P_0^-|\Phi\rangle=(P_0^-(N)+K)|\Phi\rangle=M_0|\Phi\rangle. \tag{4.12}$$

The eigenvalue problem stated above is considerably simpler than the initial one, but does contain the full complications of the nuclear many-body problem. We shall next discuss the light-front Hartree-Fock and Brueckner-Hartree-Fock [47] approximations.

B. Light front Hartree-Fock approximation

Equation (4.12) represents a difficult many-body problem. But the similarity between the light-front and equal-time results obtained for the nucleon-nucleon potential indicate that the same physical concepts are relevant, independent of the dynamical scheme. Therefore we use a scheme analogous to that of traditional Brueckner theory. The first step is to introduce a mean-field (MF) potential

$$\begin{aligned}\hat{U}&=\frac{1}{2}\int d^2x_\perp dx^-\bar{\psi}(x)[U_S(x)+\gamma\cdot\bar{U}_V(x)]\psi(x)\\&=\frac{1}{2}\int d^2x_\perp dx^-\bar{\psi}(x)\left(U_S(x)+\frac{\gamma^+}{2}U_V^-(x)\right)\psi(x),\end{aligned} \tag{4.13}$$

a single-nucleon operator, with the second equation true for infinite nuclear matter in which the only nonvanishing component of U_V^μ is $U_V^0=U_V^-=U_V^+=\bar{U}_V^-$. All quantities in the integral of Eq. (4.13) are evaluated at the same value of x^+, chosen to be 0. The operator $\hat{U}$ is to be determined ultimately by the light-front G-matrix defined below. The idea is that $\hat{U}$ can be chosen so as to provide a good representation of the effects of the two-nucleon interaction K. The mean field Lagrangian $\mathcal{L}_{\rm MF}$ is defined by removing the effects of K and replacing these by the effects of $\hat{U}$. Therefore we may specify

$$\mathcal{L}_{\rm MF}\equiv\bar{\psi}(i\gamma_\mu\partial^\mu-M)\psi-\bar{\psi}(x)\left(U_S(x)+\frac{\gamma^+}{2}U_V^-(x)\right)\psi(x). \tag{4.14}$$

This Lagrangian leads to the nucleon field equation

$$\begin{aligned}(i\partial^- - U_V^-)\psi_+&=[\boldsymbol{\alpha}_\perp\cdot\mathbf{p}_\perp+\beta(M+U_S)]\psi_-,\\ i\partial^+\psi_-&=[\boldsymbol{\alpha}_\perp\cdot\mathbf{p}_\perp+\beta(M+U_S)]\psi_+,\end{aligned} \tag{4.15}$$

in which we have made the mean-field version of the transformation (2.5) with

$$\partial^+ g_v\Lambda_{\rm MF}=U_V^+(x). \tag{4.16}$$

The light-front Hamiltonian density $T^{+-}_{\mathcal{V}}$ can now be obtained from Eq. (4.8) using the field equation (4.15) as

$$T^{+-}_{\mathcal{V}}=2\bar{\psi}_+ i\partial^-\psi_+-2\bar{\psi}\left(\frac{\gamma^+}{2}U_V^-+U_S\right)\psi+\mathcal{K}. \tag{4.17}$$

It is also worthwhile to obtain the plus-momentum density $T^{++}_{\mathcal{V}}$ which is

$$T^{++}_{\mathcal{V}}=2\bar{\psi}_+ i\partial^+\psi_+. \tag{4.18}$$

The purpose of introducing the mean field approximation is that the eigenvalues and eigenvectors of light-front mean field Hamiltonian $P^-_{\rm MF}$ are easy to obtain and can be chosen so as best approximate the effects of the two-nucleon interaction. The mean field light front Hamiltonian density $T^{+-}_{\rm MF}$ is obtained from Eq. (4.7) using Eq. (4.14) [or by setting $\mathcal{K}$ to zero in Eq. (4.17)] as

$$T^{+-}_{\rm MF}=2\,\overline{\psi}_+ i\partial^-\psi_+ -2\,\overline{\psi}\left(\frac{\gamma^+}{2}U_V^- + U_S\right)\psi, \tag{4.19}$$

and its volume integral is $P^-_{\rm MF}$:

$$P^-_{\rm MF}=\frac{1}{2}\int d^2x_\perp dx^- T^{+-}_{\rm MF}\,, \tag{4.20}$$

a single-nucleon operator. Setting $\mathcal{K}$ to zero in Eq. (4.17) and using Eq. (4.9) shows that

$$P^-_{\rm MF}=P_0^-(N). \tag{4.21}$$

The ground state eigenvector of this operator is a Slater determinant denoted as $|\phi\rangle$:

$$P^-_{\rm MF}|\phi\rangle=P_0^-(N)|\phi\rangle=m_0|\phi\rangle. \tag{4.22}$$

We shall use both the Hartree-Fock and Bruckner Hartree Fock approximations to obtain expressions for $\hat{U}$. For now we pursue the question: Given a $\hat{U}$, how do we proceed? The first step is to expand the field operator ψ in terms of the eigenfunctions of $i\partial^-$ in the light-front Dirac equation (4.15). The nucleon field operator is constructed as follows:

$$\psi(x)=\int\frac{d^2k_\perp dk^+\,\theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}}\sum_\lambda u(k,\lambda)e^{-ik\cdot x}, \tag{4.23}$$

where $k\cdot x=\frac{1}{2}(k^-x^+ + k^+x^-)-\mathbf{k}_\perp\cdot\mathbf{x}_\perp$. We keep only the nucleon part of $\psi(x)$ as the antinucleon degrees of freedom are not needed here. These spinors $u(k,\lambda)$ are the eigenfunctions of Eq. (4.15), with normalization $\overline{u}(k,\lambda)\gamma^+u(k,\lambda)=2k^+$. For the present treatment of the translationally invariant infinite nuclear matter system, the mean-field potentials U_S and U_V^- are independent of the spatial position $x_\perp, x^-$. The eigenvalues of Eq. (4.15) are given by [1,2]

$$k^-=U_V^- + \frac{\mathbf{k}_\perp^2+(M+U_S)^2}{k^+}, \tag{4.24}$$

in which U_S and U_V depend upon $\mathbf{k}_\perp$ and k^+.

The next step is to better define the Slater determinant $|\phi\rangle$. The occupied states are to fill up a Fermi sea, which is usually defined in terms of a Fermi momentum k_F that is the magnitude of a three vector. This three vector is defined [48] as

$$k^+=\sqrt{(M+U_S)^2+\mathbf{k}\cdot\mathbf{k}}+k^3, \tag{4.25}$$

which implicitly defines k^3. Using Eq. (4.25) allows one to maintain the equivalence between energies computed in the light-front and equal time formulations of scalar field theories [49] and to restore manifest rotational invariance in light-front QED [50].

The computation of the energy and plus momentum distribution proceeds from taking the appropriate expectation values of the energy momentum tensor $T^{\mu\nu}_{\mathcal{V}}$.

$$P^\mu_{\mathcal{V}}=\frac{1}{2}\int d^2x_\perp dx^-\langle\phi|T^{+\mu}_{\mathcal{V}}|\phi\rangle. \tag{4.26}$$

We are concerned with the light-front energy P^- and momentum P^+. The relevant components of $T^{\mu\nu}_{\mathcal{V}}$ are presented in Eqs. (4.17) and (4.18). Taking the nuclear matter expectation value of $T^{+-}_{\mathcal{V}}$ and $T^{++}_{\mathcal{V}}$ and performing the spatial integral of Eq. (4.26) leads to the result

$$\frac{P^-_{\mathcal{V}}}{\Omega}=\frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+\left\{\frac{\mathbf{k}_\perp^2+(M+U_S)^2}{k^+} -2\,\frac{1}{2}\sum_\lambda\frac{\overline{u}(k,\lambda)}{\sqrt{2k^+}}U_S\frac{u(k,\lambda)}{\sqrt{2k^+}}\right\}+\langle\phi|K|\phi\rangle, \tag{4.27}$$

$$\frac{P^+_{\mathcal{V}}}{\Omega}=\frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+k^+, \tag{4.28}$$

where Ω is the volume of the system $\Omega\equiv\frac{1}{2}\int d^2x_\perp dx^-$. The subscript F denotes that $|\vec{k}|<k_F$ with k^3 defined by the relation (4.25). The integral involving $\overline{u}(k,\lambda)U_Su(k,\lambda)$ may also be expressed as

$$\langle\phi|U_S|\phi\rangle\equiv\frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+\frac{1}{2}\sum_\lambda\frac{\overline{u}(k,\lambda)}{\sqrt{2k^+}}U_S\frac{u(k,\lambda)}{\sqrt{2k^+}}. \tag{4.29}$$

Equations (4.27) and (4.28) along with the expression for k^+, Eq. (4.25), allow an evaluation of P^- and P^+. We shall obtain the mass M_0 of the A-nucleon system as $M_0=\frac{1}{2}(P^+_{\mathcal{V}}+P^-_{\mathcal{V}})$ and then minimizing M_0 per nucleon. For a nuclear system at rest, the exact eigenvalues of the plus and minus momentum operators must be the same. In light-front work this is usually achieved by constraining each component of the Fock space to have the same value. This cannot be done for our infinite system. But minimizing $\frac{1}{2}(P^+_{\mathcal{V}}+P^-_{\mathcal{V}})$ is the same as minimizing $P^-_{\mathcal{V}}$, subject to the constraint that $P^+_{\mathcal{V}}=P^-_{\mathcal{V}}$ [2]. Using standard Lagrange multiplier techniques for the constrained minimization justifies this procedure. Consider the quantity $P^-_{\mathcal{V}}-\lambda(P^-_{\mathcal{V}}+P^+_{\mathcal{V}}=(1-\lambda)M_A^2/P^+_{\mathcal{V}}+P^+_{\mathcal{V}}$. Setting the derivative with respect to $P^+_{\mathcal{V}}$ to zero gives $2\lambda=M_A^2/(P^+_{\mathcal{V}})^2=P^-_{\mathcal{V}}/P^+_{\mathcal{V}}=1$, so that $\lambda=1/2$ and one is minimizing $\frac{1}{2}(P^+_{\mathcal{V}}+P^-_{\mathcal{V}})$.

Summing equations (4.27) and (4.28) and dividing by a factor of 2 leads to

$$\frac{M_0}{\Omega}=\frac{4}{(2\pi)^3}\frac{1}{2}\int_F d^2k_\perp dk^+\left(\frac{\mathbf{k}_\perp^2+(M+U_S)^2}{k^+}+k^+\right) -\langle\phi|U_S|\phi\rangle+\left\langle\phi\left|\frac{K}{2}\right|\phi\right\rangle. \tag{4.30}$$

Then replace the integration over k^+ by one over k^3 using Eq. (4.25) and the definition

$$E^*_{\mathbf{k}} \equiv \sqrt{\mathbf{k}\cdot\mathbf{k}+(M+U_S)^2}, \tag{4.31}$$

so that Eq. (4.30) takes the form

$$\frac{M_0}{\Omega}=\frac{4}{(2\pi)^3}\int d^3k\,\theta(k_F-k)E^*_{\mathbf{k}}-\langle\phi|U_S|\phi\rangle+\left\langle\phi\left|\frac{K}{2}\right|\phi\right\rangle \tag{4.32}$$

with

$$\langle\phi|U_S|\phi\rangle=\frac{4}{(2\pi)^3}\int d^3k\,\theta(k_F-k)\frac{1}{2}\times\sum_\lambda\frac{\overline{u}(k,\lambda)}{\sqrt{2E^*_{\mathbf{k}}}}U_S\frac{u(k,\lambda)}{\sqrt{2E^*_{\mathbf{k}}}}=\frac{4}{(2\pi)^3}\int d^3k\,\theta(k_F-k)\frac{2(M+U_S)}{2E^*_{\mathbf{k}}}U_S. \tag{4.33}$$

One obtains a formalism that looks more conventional by using a discrete representation of the single nucleon states. We define a set of spinors $|\alpha\rangle$, with α representing the quantum numbers $\mathbf{k}$ and λ such that

$$\langle x|\alpha\rangle\equiv\frac{e^{-ik\cdot x}}{\sqrt{\Omega}}u(k,\lambda)\sqrt{\frac{1}{2E^*_{\mathbf{k}}}},\quad\langle\overline{\alpha}|\equiv\langle\alpha|\gamma^0,$$

$$1=\frac{1}{2}\int d^2x_\perp dx^-\langle\alpha|x\rangle\langle x|\alpha\rangle. \tag{4.34}$$

The difference between $\langle x|\alpha\rangle$ and a usual equal time (ET) spinor $\langle x|\alpha\rangle_{\rm ET}$ of the same quantum numbers, energy $\epsilon(k)=E^*_{\mathbf{k}}+U^0_V$, and normalization can be determined by considering the phase factor, using Eq. (4.24):

$$k\cdot x=\frac{k^-x^+}{2}+\frac{k^+x^-}{2}-\mathbf{k}_\perp\cdot\mathbf{x}_\perp=\left(U^-_V+\frac{\mathbf{k}^2_\perp+(M+U_s)^2}{k^+}\right)\frac{(t+z)}{2}+k^+\frac{(t-z)}{2}-\mathbf{k}_\perp\cdot\mathbf{x}_\perp=\epsilon(k)t-\vec{k}\cdot\vec{r}-i\frac{U^0_Vx^-}{2}. \tag{4.35}$$

The last factor is the consequence of using the barred form of the vector potential according to Eq. (4.16). The only difference between the light-front spinors and those of the equal time form is due to this phase factor. The consequences of this phase factor for computations of the light-front momentum density are that nucleons carry only 65% of the nuclear plus momentum in the mean field calculation [1,2]. A similar result is to be found below. Furthermore this phase factor has the desirable feature of suppressing the number of nuclear antinucleons [51].

For now, we consider the effects of using light-front spinors on the calculation of the energy. We express the energy M_0 of Eq. (4.32) in terms of the spinors $|\alpha\rangle$ of Eq. (4.34) as

$$M_0=\sum_{\alpha<F}E_\alpha-\sum_{\alpha<F}\langle\overline{\alpha}|U_S|\alpha\rangle+\frac{1}{2}\sum_{\alpha,\beta<F}\langle\overline{\alpha}\overline{\beta}|V|\alpha\beta\rangle_a, \tag{4.36}$$

in which $|\alpha\beta\rangle_a\equiv|\alpha\beta\rangle-|\beta\alpha\rangle$. In this notation

$$E_\alpha=\epsilon_\alpha-\langle\overline{\alpha}|\gamma^0U^0_V|\alpha\rangle, \tag{4.37}$$

so that we obtain

$$M_0=\sum_{\alpha<F}\epsilon_\alpha-\sum_{\alpha<F}\langle\overline{\alpha}|U_S+\gamma^0U^0_V|\alpha\rangle+\frac{1}{2}\sum_{\alpha,\beta<F}\langle\overline{\alpha}\overline{\beta}|V|\alpha\beta\rangle_a. \tag{4.38}$$

This result gives the value of the nuclear energy in terms of the eigenvalues of the light-front Dirac equation and in terms of light-front spinors. The term $K/2$ has been replaced by V according to Eq. (3.5) and using the normalization of Eq. (4.34).

It is necessary, however, to consider the effects of the phase factor $-i(U^0_Vx^-/2)$ of Eq. (4.35) which accounts for the difference between light-front and equal time spinors. This factor and its complex conjugate multiply to unity in the calculation of the matrix elements $\langle\overline{\alpha}|U_S+\gamma^0U^0_V|\alpha\rangle$ and in the matrix element $\langle\overline{\alpha}\overline{\beta}|V|\alpha,\beta\rangle$. We need to consider also the matrix element $\langle\overline{\alpha}\overline{\beta}|V|\beta\alpha\rangle$ for which the effects of the phase factor do not automatically cancel. In principle, U^0_V is a function of the momentum denoted by the quantum numbers α,β. In practice, this dependence is weak and can be ignored in calculations of the energy, provided one evaluates the potential at a reasonably chosen average value. Furthermore, in evaluating the matrix element $\langle\overline{\alpha}\overline{\beta}|V|\beta\alpha\rangle$ both of the states $\beta\alpha$ are below the Fermi sea and have a momentum separated by an amount small compared to the scale of the momentum dependence. Thus the phase factor does not enter in present calculations of the energy (but does in the evaluation of the plus-momentum distribution). Our calculations of the energy in the light-front and equal time formulations yield the same results. However calculations of the plus-momentum distributions can only be done using the light-front formalism.

Let us determine the mean field, and the corresponding value of M_0 in terms of V. The light-front Hartree-Fock (HF) approximation is defined by taking the mean field to be calculated from the average potential according to

$$\langle\overline{\alpha}|(U_S+\gamma^0U^0_V)^{\rm HF}|\alpha\rangle\equiv U(\alpha)=\sum_{\beta<F}\langle\overline{\alpha}\overline{\beta}|V|\alpha\beta\rangle_a. \tag{4.39}$$

Summing over the occupied orbitals gives

$$\sum_{\alpha<F} U(\alpha)=2\langle\phi|V|\phi\rangle. \tag{4.40}$$

In this case, the use of the above equation in the expression (4.38) for M_0, leads to the HF approximation for the nuclear energy:

$$M_0^{\rm HF}=\sum_{\alpha<F}\epsilon_\alpha-\frac{1}{2}\sum_{\alpha,\beta<F}\langle\overline{\alpha}\overline{\beta}|V|\alpha\beta\rangle_a, \tag{4.41}$$

which has the same form as the expression for the energy in the usual equal time Hartree-Fock expression.

We shall also need to obtain m_0, the eigenvalue of $P^-_{\rm MF}$. This expression will be used in obtaining the Brueckner-Hartree-Fock approximation. According to Eqs. (4.17) and (4.22), the difference between M_0 and m_0 is the expectation value of the potential V. Thus

$$m_0=\sum_{\alpha<F}\epsilon_\alpha-\sum_{\alpha<F}\langle\overline{\alpha}|U_S+\gamma^0U_V^0|\alpha\rangle. \tag{4.42}$$

C. Light front Brueckner-Hartree-Fock approximation

The interaction K between two nucleons is strong and the scattering amplitude is obtained, as discussed in Sec. III, by solving the Weinberg equation—the light-front version of the Lippmann-Schwinger equation. Thus we need to go beyond the Hartree-Fock approximation. This shall be accomplished by treating the interaction between two nucleons to all orders in K.

The idea is that we wish to find the Slater determinant $|\phi\rangle$, recall Eq. (4.22), that leads to the best approximation for the energy M_0 of the full nucleonic wave function $|\Phi\rangle$, recall Eq. (4.12). Both of the states $|\phi\rangle$ and $|\Phi\rangle$ are eigenstates of a P-minus operator, and both are eigenstates of the operator $P_0^+(N)$. We shall use standard techniques to derive a perturbation theory in K to obtain an expression for the state $|\Phi\rangle$ in terms of $|\phi\rangle$. Thus we write

$$|\Phi\rangle=|\phi\rangle+\Lambda|\Phi\rangle \tag{4.43}$$

with

$$\Lambda\equiv1-|\phi\rangle\langle\phi|. \tag{4.44}$$

Then use Eq. (4.43) in Eq. (4.12) and multiply the result on the left by Λ to obtain

$$\Lambda|\Phi\rangle=\frac{1}{M_0-\Lambda(P_0^-(N)+K)\Lambda}\Lambda K|\phi\rangle, \tag{4.45}$$

so that

$$|\Phi\rangle=|\phi\rangle+\frac{1}{M_0-\Lambda(P_0^-(N)+K)\Lambda}\Lambda K|\phi\rangle. \tag{4.46}$$

We can obtain a useful expression for M_0 by acting with the operator $\langle\phi|[P_0^-(N)+K]$ on the left of Eq. (4.46) and using the result $\langle\phi|\Phi\rangle=1$, which follows from Eq. (4.46). Then we find

$$M_0=\langle\phi|P_0^-+K|\phi\rangle+\langle\phi|K\Lambda\frac{1}{M_0-\Lambda[P_0^-(N)+K]\Lambda}\Lambda K|\phi\rangle. \tag{4.47}$$

Using Eq. (4.22) in Eq. (4.47) leads to

$$M_0-m_0=\langle\phi|K|\phi\rangle+\langle\phi|K\Lambda\frac{1}{M_0-\Lambda[P_0^-(N)+K]\Lambda}\Lambda K|\phi\rangle, \tag{4.48}$$

which can be restated as

$$M_0-m_0=\langle\phi|X|\phi\rangle, \tag{4.49}$$

where

$$X=K+K\Lambda\frac{1}{M_0-\Lambda P_0^-(N)\Lambda}\Lambda X. \tag{4.50}$$

The operator X is a many-body operator acting on all nucleons via the iterations of the two-nucleon interaction K. We shall make the independent pair approximation of including only pair-wise interactions. Thus we approximate

$$\langle\phi|X|\phi\rangle\approx\left\langle\phi\left|\frac{1}{2}\sum_{i,j}\Gamma_{i,j}(P_{ij}^-)\right|\phi\right\rangle\equiv\langle\phi|\Gamma|\phi\rangle, \tag{4.51}$$

where $\Gamma_{i,j}$ is a two-nucleon operator which is a solution of the integral equation

$$\Gamma_{i,j}(P_{ij}^-)=K_{ij}+K_{ij}\frac{\Lambda}{P_{ij}^--\Lambda P_0^-(N)\Lambda}\Gamma_{i,j}(P_{ij}^-). \tag{4.52}$$

The notation i,j refers to a pair of particles. The relevant matrix element is expressed using the eigenstates of Eq. (4.15) as

$$\langle3,4|\Gamma(P_{1,2}^-)|1,2\rangle=\langle3,4|K|1,2\rangle+\sum_{\lambda_5,\lambda_6}\int\langle3,4|K|5,6\rangle\frac{2M^{*2}}{p_5^+p_6^+}\times\frac{d^2p_5dp_5^+Q}{P_{1,2}^--(p_5^-+p_6^-)+i\epsilon}\langle5,6|\Gamma|1,2\rangle, \tag{4.53}$$

in which we define

$$M^*=M+U_s. \tag{4.54}$$

The operator Q, to be specified below, is the two-body version of Λ and projects the momenta p_5 and p_6 above the Fermi sea. The factor $M^{*2}\delta^{2,+)}(P_i-P_f)/\sqrt{k_1^+k_2^+k_3^+k_4^+}$ appears in each of the terms of Eq. (4.53), so it is worthwhile to define a Brueckner G-matrix G using

$$\Gamma = 2G\frac{M^{*2}\delta^{(2,+)}(P_i - P_f)}{\sqrt{k_1^+ k_2^+ k_3^+ k_4^+}}. \quad (4.55)$$

To follow the steps of Sec. III in converting Eq. (4.53) into one of a more familiar form, in which rotational invariance is manifest, one needs to know the values of

$$P_{1,2}^- = k_1^- + k_2^-, \quad (4.56)$$

which for the case of relevance here in computing the nuclear expectation value in the independent pair approximation, is the same as $p_5^- + p_6^-$. The single-particle minus-momentum eigenvalues are given according to Eq. (4.24) as

$$k_i^- = \frac{\mathbf{k}_{i\perp}^2 + (M+U_S)^2}{k_i^+} + U_V. \quad (4.57)$$

Our approximation is that U_V is independent of orbital i. Thus this potential cancels out in computing the difference $P_i^- - (p_5^- + p_6^-)$ and the energy denominator is as in the free space considerations of Sec. III, except that the mass of the nucleon is replaced by $M+U_S$. Thus the previous derivation of an equivalent three-dimensional integral equation that is manifestly covariant and rotationally invariant proceeds as before.

One expresses the plus-momentum variable in terms of a light-front momentum fraction α of Eqs. (3.15) and (3.13) so that one obtains

$$\langle 3,4|G|1,2\rangle = \langle 3,4|V|1,2\rangle + \int \sum_{\lambda_5,\lambda_6} \langle 3,4|V|5,6\rangle \times \frac{2M^{*2}}{\alpha(1-\alpha)} \frac{d^2p_\perp d\alpha Q}{P_i^2 - (p_\perp^2 + M^{*2})/\alpha(1-\alpha)} \times \langle 5,6|G|1,2\rangle, \quad (4.58)$$

where P_i^2 is square of the total initial four-momentum, computed using $M+U_S$ for the nucleon mass. Equation (4.58) can in turn be reexpressed as a medium-modified Blankenbecler-Sugar (BbS) equation [35] by using the medium-modified version of the variable transformation [36]:

$$\alpha = \frac{E_{\mathbf{p}}^* + p^3}{2E_{\mathbf{p}}^*}, \quad (4.59)$$

with $E_{\mathbf{p}}^*$ given in Eq. (4.31). The result is

$$\langle 3,4|G|1,2\rangle = \langle 3,4|V|1,2\rangle + \int \sum_{\lambda_5,\lambda_6} \langle 3,4|V|5,6\rangle \times \frac{M^{*2}}{E_{\mathbf{p}}^*} \frac{d^3pQ}{\mathbf{p}_i^2 - \mathbf{p}^2} \langle 5,6|G|1,2\rangle, \quad (4.60)$$

which is the desired equation (with Dirac spinors normalized as in Sec. III).

The Brueckner light front Hartree-Fock (BHF) approximation is defined by taking the mean field to be calculated from the average G matrix according to

$$U(\alpha) = \langle\overline{\alpha}|(U_S + \gamma^0 U_V^0)^{\mathrm{BHF}}|\alpha\rangle = \sum_{\beta<F} \langle\overline{\alpha}\overline{\beta}|G|\alpha\beta\rangle_a, \quad (4.61)$$

where Dirac spinors are normalized as in Eq. (4.34). The sum over occupied orbitals gives

$$\sum_{\alpha<F} U(\alpha) = 2\langle\phi|G|\phi\rangle. \quad (4.62)$$

We use this BHF mean field to determine the value of m_0 via Eq. (4.42). Then the use of Eq. (4.51) in Eq. (4.49) determines the value of M_0 as the eigenvalue of $P_{\mathcal{V}}^-$. But M_0 is also the eigenvalue (or in this case the expectation value) of $P_{\mathcal{V}}^+$. The minimization of $P_{\mathcal{V}}^-$ subject to the constraint that the expectation value of $P_{\mathcal{V}}^+$ is the value of $P_{\mathcal{V}}^-$ leads to the BHF version of Eqs. (4.27) and (4.28):

$$\frac{P_{\mathcal{V}}^-}{\Omega} = \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ \left\{ \frac{\mathbf{k}_\perp^2 + (M+U_S)^2}{k^+} - 2\frac{1}{2}\sum_\lambda \frac{\overline{u}(k,\lambda)}{\sqrt{2k^+}} U_S \frac{u(k,\lambda)}{\sqrt{2k^+}} \right\} + \langle\phi|\Gamma|\phi\rangle, \quad (4.63)$$

$$\frac{P_{\mathcal{V}}^+}{\Omega} = \frac{4}{(2\pi)^3}\int_F d^2k_\perp dk^+ k^+. \quad (4.64)$$

Note that the quantity k^+ is defined in Eq. (4.25).

Taking the average of equations (4.63) and (4.64), and using the basis of Eq. (4.34) leads to our result for the BHF version of the nuclear mass

$$M_0 = \sum_{\alpha<F} \epsilon_\alpha - \frac{1}{2}\sum_{\alpha,\beta<F} \langle\overline{\alpha}\overline{\beta}|G|\alpha\beta\rangle_a. \quad (4.65)$$

This is equivalent to the usual expression of Refs. [30,32], see Eqs. (5.10) and (5.11) below.

V. LIGHT-FRONT BRUECKNER THEORY OF NUCLEAR MATTER

A. Summary

The formalism of the previous section can be summarized using the notation of Refs. [30,32]. In that work, single-nucleon motion in nuclear matter is described by the Dirac equation

$$(\not{k} - M - U)u^*(\mathbf{k},s) = 0 \quad (5.1)$$

or in Hamiltonian form

$$(\boldsymbol{\alpha}\cdot\mathbf{k}+\beta M+\beta U)u^*(\mathbf{k},s)=\epsilon_k u^*(\mathbf{k},s) \tag{5.2}$$

with

$$U=U_S+\gamma^0 U_V^0, \tag{5.3}$$

where we use the usual notation [40] $\beta\equiv\gamma^0$ and $\boldsymbol{\alpha}\equiv\gamma^0\boldsymbol{\gamma}$. The solution of Eq. (5.1) is

$$u^*(\mathbf{k},s)=\sqrt{\frac{E^*_{\mathbf{k}}+M^*}{2E^*_{\mathbf{k}}}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{E^*_{\mathbf{k}}+M^*}\end{pmatrix}\chi_s \tag{5.4}$$

with M^* defined in Eq. (4.54). $E^*_{\mathbf{k}}$ given by Eq. (4.31), and χ_s a Pauli spinor. The normalization is $\overline{u}^*(\mathbf{k},s)u^*(\mathbf{k},s)=E^*_{\mathbf{k}}/M^*$ and $u^{*\dagger}(\mathbf{k},s)u^*(\mathbf{k},s)=1$, as in Eq. (4.34). Notice that the in-medium Dirac spinor Eq. (5.4) is obtained from the free Dirac spinor by simply replacing M by M^*. The difference between the equal time spinors used in Refs. [30,32] and light-front spinors is the phase factor discussed above in Sec. IV C. As noted, this factor cancels out of the matrix elements when using the approximation that the vector potential is independent of momentum.

The nuclear matter G matrix is the solution of the integral equation (4.60) without the explicit spin indices

$$G(\mathbf{k}',\mathbf{k};|\mathbf{P}|,k_F)=V^*(\mathbf{k}',\mathbf{k})+\int\frac{d^3p}{(2\pi)^3}V^*(\mathbf{k}',\mathbf{p})\times\frac{M^{*2}}{E^*_{(1/2)\mathbf{P}+\mathbf{p}}}\frac{\overline{Q}(|\mathbf{p}|,|\mathbf{P}|,k_F)}{\mathbf{k}^2-\mathbf{p}^2+i\epsilon}G(\mathbf{p},\mathbf{k};|\mathbf{P}|,k_F), \tag{5.5}$$

where $\mathbf{P}$ is the total three-momentum of the two nucleons in the nuclear matter rest frame and $\mathbf{k}$, $\mathbf{p}$, and $\mathbf{k}'$ are the initial, intermediate, and final relative momenta, respectively, of the two nucleons interacting in nuclear matter. Note that the third components of the various momenta are obtained by using the appropriate versions of Eq. (4.25). k_F denotes the magnitude of the Fermi momentum corresponding to the nuclear matter density under consideration. The Pauli operator Q, for which we use the angle average $\overline{Q}$ [43], projects onto unoccupied states. In the derivation, we have used the angle averages $(\frac{1}{2}\mathbf{P}\pm\mathbf{k})^2\approx\frac{1}{4}\mathbf{P}^2+\mathbf{k}^2$ and $(\frac{1}{2}\mathbf{P}\pm\mathbf{p})^2\approx\frac{1}{4}\mathbf{P}^2+\mathbf{p}^2$. The latter implies $E^*_{(1/2)\mathbf{P}+\mathbf{p}}\approx\sqrt{M^{*2}+\frac{1}{4}\mathbf{P}^2+\mathbf{p}^2}$.

The essential difference with standard Brueckner theory is the use of the potential V^* in Eq. (5.5). As indicated by the asterix, the OBE potential of Eqs. (3.20)–(3.22) is evaluated by using the in-medium spinors Eq. (5.4) instead of the free ones.

It is necessary to discuss a technical point concerning the retardation effects in the form factors Eq. (3.26) that enter in evaluating V^*. Consider the matrix element of V^* for which our formalism says to use

$$q^0=E^*_\alpha-E^*_\beta=[E^*_\alpha+U_V(\alpha)]-[E^*_\beta+U_V(\alpha)]. \tag{5.6}$$

We have consistently ignored the state dependence of U_V. This is a good approximation for energy differences governed by the Fermi energy. However, the relevant energy scale in the form factors are the parameters Λ of Table I, which are on the order of a GeV or more. Thus for these terms it would be better to use

$$q^0=[E^*_\alpha+U_V(\alpha)]-[E^*_\beta+U_V(\beta)]. \tag{5.7}$$

The quantity $E^*_\alpha+U_V(\alpha)$ for occupied orbitals α is close to the nucleon mass. For high energy orbitals β, $U(\beta)$ is small. Thus the use of Eq. (5.7) is numerically very similar to ignoring the effects of the medium modifications in evaluating the retardation effects in the form factors, which is what we do.

The single-particle potential for nucleons in nuclear matter is

$$U(\alpha)=\langle\overline{\alpha}|U|\alpha\rangle=\langle\overline{\alpha}|U_S+\gamma^0U_V^0|\alpha\rangle=\frac{M^*}{E^*_\alpha}U_S+U_V^0; \tag{5.8}$$

which is calculated from the G matrix by

$$U(\alpha)=\mathrm{Re}\sum_{\beta<F}\langle\overline{\alpha}\overline{\beta}|G|\alpha\beta\rangle_a, \tag{5.9}$$

where α denotes a state below or above the Fermi surface (continuous choice).

Using the notation of the previous section, we define ''the energy per nucleon in nuclear matter'' by

$$\frac{\mathcal{E}}{A}=\frac{M_0}{A}-M, \tag{5.10}$$

which can also be written as

$$\frac{\mathcal{E}}{A}=\frac{1}{A}\sum_{\alpha<F}\langle\overline{\alpha}|\boldsymbol{\gamma}\cdot\mathbf{k}_\alpha+M|\alpha\rangle+\frac{1}{2A}\sum_{\alpha,\beta<F}\langle\overline{\alpha}\overline{\beta}|G|\alpha\beta\rangle_a-M. \tag{5.11}$$

Note that this equation depends on the Fermi momentum k_F and, thus, on the density of nuclear matter. It is useful to have the following summary of formulas concerning the single particle energy ϵ_α:

$$\epsilon_\alpha=\langle\overline{\alpha}|\boldsymbol{\gamma}\cdot\mathbf{k}_\alpha+M+U|\alpha\rangle \tag{5.12}$$

$$=\langle\overline{\alpha}|\boldsymbol{\gamma}\cdot\mathbf{k}_\alpha+M|\alpha\rangle+U(\alpha) \tag{5.13}$$

$$=\frac{MM^*+\mathbf{k}_\alpha^2}{E^*_\alpha}+\frac{M^*}{E^*_\alpha}U_S+U_V^0 \tag{5.14}$$

$$=E^*_\alpha+U_V^0, \tag{5.15}$$

with $E^*_\alpha\equiv\sqrt{M^{*2}+\mathbf{k}_\alpha^2}$ and $M^*=M+U_S$.

The calculation of the nuclear matter G matrix involves a self-consistency, since the solution of Eq. (5.5) for G re-

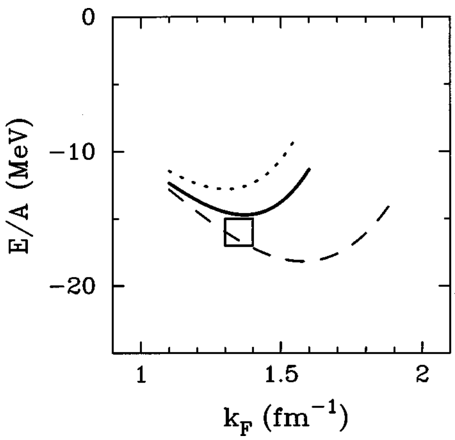

FIG. 2. Energy per nucleon in nuclear matter E/A (in units of MeV), as a function of density expressed by the Fermi momentum k_F (in units of fm^{-1}). The solid line is our prediction using light-front Brueckner theory. The dotted curve is obtained when the medium effect on meson propagation is omitted. The dashed line is the result from conventional (nonrelativistic) Brueckner theory. The box describes the area in which nuclear saturation is expected to occur empirically.

quires knowledge of M^* which, in turn, is determined from G via Eqs. (5.8) and (5.9). In practice, one starts out with an educated guess for M^*, solves Eq. (5.5) for G and uses this G to calculate a new M^* from Eqs. (5.8) and (5.9). The procedure is then repeated starting with the new M^*. This is reiterated until the calculated M^* reproduces accurately the starting M^*.

B. Results

The formalism of the previous section is used to calculate the energy per nucleon in nuclear matter as a function of density, Eq. (5.11). Our result is plotted in Fig. 2 by the solid line. The curve saturates at $\mathcal{E}/A=-14.71\,\mathrm{MeV}$ and $k_F=1.37\,\mathrm{fm}^{-1}$, and predicts an incompressibility of $K=180\,\mathrm{MeV}$ at the minimum. These predictions agree well with the empirical values $\mathcal{E}/A=-16\pm1$ MeV, $k_F=1.35\pm0.05\,\mathrm{fm}^{-1}$, and $K=210\pm30\,\mathrm{MeV}$ [52].

To get a better idea of the quality of our predictions, it is useful to compare with the results from alternative relativistic approaches. Brockmann and Machleidt [32] predict $\mathcal{E}/A=-13.6\,\mathrm{MeV}$, $k_F=1.37\,\mathrm{fm}^{-1}$, and $K=250\,\mathrm{MeV}$ at saturation, using the equal-time formalism and their ''Potential B.'' The greatest difference occurs for the incompressibility which is predicted smaller by the LF Brueckner theory implying a softer equation of state. This can be partially attributed to the medium effect that comes from the meson propagators in the LF approach and that is absent in the equal time (ET) approach. Recall that in the LF formalism the momentum transfer between two nucleons exchanging a meson is

$$q=(q_0,\mathbf{q})=(E'-E,\mathbf{k}'-\mathbf{k}), \tag{5.16}$$

where E and E' are nucleon on-mass-shell energies [more explanations can be found below Eq (3.22)], implying the meson propagators

$$\frac{i}{q^2-m_\alpha^2}=\frac{i}{(E'-E)^2-(\mathbf{k}'-\mathbf{k})^2-m_\alpha^2}, \tag{5.17}$$

while in the ET formalism no energy is transfered, thus,

$$q=(0,\mathbf{q})=(0,\mathbf{k}'-\mathbf{k}), \tag{5.18}$$

and the propagators are

$$\frac{i}{-\mathbf{q}^2-m_\alpha^2}=\frac{i}{-(\mathbf{k}'-\mathbf{k})^2-m_\alpha^2}. \tag{5.19}$$

In nuclear matter, the free-space LF meson propagators Eq. (5.17) are replaced by

$$\frac{i}{(E'^*-E^*)^2-(\mathbf{k}'-\mathbf{k})^2-m_\alpha^2}, \tag{5.20}$$

while the ET propagators undergo no changes. The medium effect on the LF meson propagators enhances them off-shell which leads to more binding energy. This is demonstrated in Fig. 2 where the difference between the dotted and solid curve is generated by the medium effect on the meson propagators.

There is another difference that arises from a technical issue in the solution of the transcendental equation for the G matrix. We obtain new values of the mean fields $M^*(k_F)=M_N+U_S=718\,\mathrm{MeV}$ and $U_V=165\,\mathrm{MeV}$. The mean field potentials obtained here from the G matrix are considerably smaller than those of mean field theory in which the potential is used. We discuss the implications for nuclear deep inelastic scattering in Sec. VII.

The most important medium effect in relativistic approaches to nuclear matter comes from the use of in-medium Dirac spinors representing the nucleons in nuclear matter (''Dirac effect''). This effect (and the medium effect on meson propagators) is absent in the conventional (nonrelativistic) Brueckner calculation which yields the dashed curve in Fig. 2. Characteristic for all predictions using conventional Brueckner theory is that the saturation density is predicted too high and, thus, they all fail to explain nuclear saturation correctly.

The effect that is generated by the in-medium Dirac spinors is strongly density dependent (due to the density dependence of M^*) shifting the saturation curve towards lower densities such that nuclear saturation is predicted at the correct density (solid and dotted curves in Fig. 2). The effect from the in-medium Dirac spinors is, of course, largest for σ and ω exchange for which the LF and ET formalisms predict essentially the same. However, π and ρ also contribute to the medium effect and, here, we have differences between LF and ET. The general underlying reason for this difference is that for derivative coupling, implying a momentum dependence of the meson-nucleon vertex, the difference in the momentum transfer (=meson momentum) between LF and ET [Eqs. (5.16) and (5.18), above] creates a difference in the vertices (and OBE amplitudes)—besides the one in the meson propagators. The ρ includes the tensor coupling $i(f_\rho/2M)\,\sigma_{\mu\nu}q^\nu$ and the LF OBE amplitude is given in Eq.

(3.22). However, in the ET formalism, the ρ amplitude consists of Eq. (3.22) plus additional terms that contribute only off-shell, as discussed [41]. In nuclear matter, the medium effect generated by these off-shell terms essentially cancels the medium effect that comes from the main part of the amplitude, Eq. (3.22). Therefore, ET ρ exchange produces a much weaker medium effect than LF.

Concerning the pion, the pseudovector (pv) coupling (or gradient coupling) $i(f_{\pi NN}/m_\pi)\gamma_5\gamma_\mu q^\mu$ has been generally used, in the ET formalism [32]. The resulting one-pion-exchange (OPE) amplitude can be cast into a form that consists of the amplitude Eq. (3.20) plus off-shell terms [53,54]. In the LF formalism, no matter if the pv or ps coupling is used, the OPE amplitude always comes out to be Eq. (3.20), and there are no additional terms [54]. In nuclear matter, the medium effect from the off-shell terms of the ET formalism damp the medium effect from the main OPE amplitude, Eq. (3.20), similar to what happens with the ρ. Therefore, LF π exchange produces a stronger (more repulsive) medium effect than ET.

In summary, the Dirac effect comes out more repulsive in the LF formalism as compared to ET due to off-shell differences in the π and ρ exchange amplitudes. On the other hand, the LF formalism generates an attractive meson propagator effect that is absent in ET. As it turns out, these two effects cancel to a large extent, leading to a nontrivial similarity between the LF and ET results.

VI. FULL WAVE FUNCTION Ψ AND MESON DEGREES OF FREEDOM

The nucleonic wave function $|\Phi\rangle$ has been determined in Eq. (4.46). This gives us the purely nucleonic part of the Fock space, in which the effects of the mesons have been replaced by the two-nucleon interaction K. However, the full wave function is $|\Psi\rangle$ of Eq. (4.1). We need to assess whether $|\Phi\rangle$ is a good approximation to $|\Psi\rangle$ and we also need to determine the mesonic plus-momentum distributions.

We recall the relation between the full P^- operator and the one of Eq. (4.12) $[P_0^-=P_0^-(N)+K]$ corresponding to the nucleonic wave function $|\Phi\rangle$:

$$P^-=P_0^-+J-K+P_0^-(m). \tag{6.1}$$

Using this in Eqs. (4.1) and (4.12) allows us to obtain

$$|\Psi\rangle=|\Phi\rangle+\frac{1}{M_A-\Lambda_\Phi P^-\Lambda_\Phi}\Lambda_\Phi(J-K)|\Phi\rangle, \tag{6.2}$$

where $\Lambda_\Phi=1-|\Phi\rangle\langle\Phi|$.

An expression for the nuclear mass M_A can be obtained by multiplying Eq. (6.2) by $\langle\Phi|P^-$ using $\langle\Phi|\Psi\rangle=1$ to obtain the result

$$M_A=\langle\Phi|P^-|\Phi\rangle+\langle\Phi|(J-K)\Lambda_\Phi\frac{1}{M_A-\Lambda_\Phi P^-\Lambda_\Phi}\times\Lambda_\Phi(J-K)|\Phi\rangle. \tag{6.3}$$

For the purely nucleonic wave function $|\Phi\rangle$ we have

$$\langle\Phi|P^-|\Phi\rangle=M_0+\langle\Phi|J-K|\Phi\rangle, \tag{6.4}$$

so that

$$M_A=M_0+\langle\Phi|J-K|\Phi\rangle+\langle\Phi|(J-K)\times\Lambda_\Phi\frac{1}{M_A-\Lambda_\Phi P^-\Lambda_\Phi}\Lambda_\Phi(J-K)|\Phi\rangle. \tag{6.5}$$

The difference between M_A and M_0 is the expectation value of the operator O:

$$O\equiv J-K+(J-K)\Lambda_\Phi\times\frac{1}{M_A-\Lambda_\Phi P_0^-\Lambda_\Phi-\Lambda_\Phi(J-K)\Lambda_\Phi}\Lambda_\Phi(J-K), \tag{6.6}$$

and a bit of algebra shows that O satisfies the integral equation

$$O=J-K+(J-K)\Lambda_\Phi G_0(M_A)\Lambda_\Phi O, \tag{6.7}$$

where

$$G_0(M_A)\equiv\Lambda_\Phi\frac{1}{M_A-\Lambda_\Phi P_0^-\Lambda_\Phi}\Lambda_\Phi. \tag{6.8}$$

The lowest relevant order of Eq. (6.7) is given by

$$O\approx J-K+(J-K)\Lambda_\Phi G_0(M_A)(J-K).$$

The one-boson exchange interaction K is given by Eq. (3.1) (which now includes also the effects of Sec. III B) and we may determine if the expectation value $\langle\Phi|O|\Phi\rangle$ is reasonably small. If this is true, the quantity M_0 would be a good approximation to the true eigenvalue of the P^- operator M_A. In the one-boson exchange approximation

$$\langle\Phi|O|\Phi\rangle\approx\langle\Phi|v_3-K|\Phi\rangle+\langle\Phi|JG_0(M_A)J|\Phi\rangle \tag{6.9}$$

$$=\langle\Phi|v_3-K|\Phi\rangle+\langle\Phi|v_1G_0(M_A)v_1|\Phi\rangle. \tag{6.10}$$

The use of Eq. (3.1) yields

$$\langle\Phi|O|\Phi\rangle\approx\langle\Phi|v_1[G_0(M_A)-g_0(P_{ij}^-)]v_1|\Phi\rangle, \tag{6.11}$$

where the term $P_{i,j}^-$ is specified in Eqs. (4.53)–(4.57). But within the independent pair approximation [in which one includes only the energy (minus-momentum) differences for a chosen pair of nucleons]

$$G_0(M_A)\equiv g_0(P_{ij}^-), \tag{6.12}$$

and the expectation value of O vanishes.

Thus within our approximations, it is consistent to say that the exact nuclear mass M_A is well approximated by M_0. This means that we have shown that it is acceptable to remove the explicit mesons for calculations of the nuclear mass. One could evaluate the presumably small corrections by going beyond the independent pair approximation. The simplicity of the derivation of this result is made possible by the dynamical simplicity of the vacuum, which is one of the defining features of light-front field theory.

A. Momentum distributions

We can now compute the $+$ components of momentum. Look at T^{++} as given by Eq. (2.21) The plus momentum carried by the scalar meson is given by

$$P^+(\phi)=\int d^2k_\perp dk^+k^+a^\dagger(k)a(k), \tag{6.13}$$

while that of the pion is given by

$$P^+(\pi)=\int d^2k_\perp dk^+k^+\mathbf{a}^\dagger(k)\cdot\mathbf{a}(k), \tag{6.14}$$

and that of the vector meson is given by

$$P^+(\omega)=\sum_{\omega=1,3}\int d^2k_\perp dk^+k^+a^\dagger(\mathbf{k},\omega)a(\mathbf{k},\omega). \tag{6.15}$$

We shall handle the scalar term first. The evaluation of Eq. (6.13) using Eq. (6.2) leads to

$$\begin{aligned}P^+(\phi)&=\int d^2k_\perp dk^+k^+\langle\Phi|j(\mathbf{k})\frac{1}{(M_A-\Lambda P^-\Lambda)^2}j(\mathbf{k})|\Phi\rangle\\&\approx\int d^2k_\perp dk^+k^+\langle\Phi|j(\mathbf{k})\frac{1}{(M_A-\Lambda P_0^-\Lambda)^2}j(\mathbf{k})|\Phi\rangle,\end{aligned} \tag{6.16}$$

in which the approximation is motivated by the near equality of M_A and M_0 and the universal expectation that the impulse approximation evaluation of the meson exchange potential is valid. The term $j(k)$ is defined via the contribution of the scalar mesons to v_1:

$$v_1(\phi)=\int d^2k_\perp dk^+[j(\mathbf{k})a(\mathbf{k})+\text{H.c.}], \tag{6.17}$$

in which $j(\mathbf{k})$ can be obtained from Eq. (2.18) and is a nucleonic operator that depends on $\mathbf{k}_\perp$ and k^+.

The operator to be evaluated in the above equation has one and two body pieces. The one-body terms are related to a shift in the self energy of the nucleon caused by the medium. In infinite nuclear matter the ratio of pairs to single nucleons is infinite so that the number density is well approximated by the two nucleon terms of Eq. (6.16). The evaluation is simplified by the use of Eq. (6.12) and noting that the relevant matrix element is the same as occuring in the one-boson exchange operator K except that the denominator is squared. Thus the momentum density $n_\phi(\mathbf{k})$, defined by

$$P^+(\phi)\equiv\int d^2k_\perp dk^+k^+n_\phi(\mathbf{k}), \tag{6.18}$$

is given as a derivative of the scalar-meson exchange contribution to the nucleon-nucleon potential

$$n_\phi(\mathbf{k})\approx-2\langle\Phi|\frac{\partial V_\phi(P_{ij}^-,\mathbf{k})}{\partial P_{ij}^-}|\Phi\rangle \tag{6.19}$$

with

$$\frac{\partial V_\phi(P_{ij}^-,\mathbf{k})}{\partial P_{ij}^-}\equiv\left[j_i(\mathbf{k})\frac{1}{(M_A-\Lambda P^-\Lambda)^2}j_j(\mathbf{k})\right] \tag{6.20}$$

in which the notation i,j specifies that only two-nucleon contributions are included. Note that $|\Phi\rangle$ is the correlated ground state. Note that the expectation value is taken using the single particle basis specified by Eq. (4.34). We recall Eqs. (3.1) and (3.2), and use [2]

$$k^+(P_{ij}^--P_0^-)=q^2-m_\phi^2. \tag{6.21}$$

Note that the momentum of the exchanged meson is k, and

$$k^+=q^+,\mathbf{k}_\perp=\mathbf{q}_\perp,k^-=\frac{\mathbf{k}_\perp^2+m_\phi^2}{k^+}, \tag{6.22}$$

where q is the nucleon momentum transfer. Thus one may obtain the result that

$$-\frac{\partial}{\partial P_{ij}^-}\frac{1}{(P_{ij}^--P_0^-)}=\frac{k^+}{(q^2-m_\phi^2)^2}. \tag{6.23}$$

This means that evaluating the plus-momentum distribution for scalar mesons is the same as evaluating the expression for the scalar meson contribution to the nuclear potential energy, except that the potential $V_\phi(\mathbf{k})$ (the dependence on $P_{i,j}^-$ is supressed) is multiplied by the factor $-k^+/(q^2-m_\phi^2)$. The net result is that

$$n_\phi(\mathbf{k})=\sum_{\alpha,\beta<F}\langle\alpha\beta|\Omega_{\alpha\beta}^\dagger V_\phi(\mathbf{k})\frac{(-k^+)}{(q^2-m_\phi^2)}\Omega_{\alpha\beta}|\alpha\beta\rangle_a, \tag{6.24}$$

where $\Omega_{\alpha,\beta}$ is the Moeller scattering operator for the two-nucleon state $\alpha\beta$. One finds a similiar expression for the pionic density with

$$n_\pi(\mathbf{k})=\sum_{\alpha,\beta<F}\langle\alpha\beta|\Omega_{\alpha\beta}^\dagger V_\pi(\mathbf{k})\frac{(-k^+)}{(q^2-m_\pi^2)}\Omega_{\alpha\beta}|\alpha\beta\rangle_a. \tag{6.25}$$

The evaluation of the vector meson density $n_\omega(\mathbf{k})$ requires more steps because the meson-exchange potential has a contribution from the instantaneous meson exchange term v_3. This instantaneous term does not lead to a meson ''in the air'' and therefore does not contribute to $n_\omega(\mathbf{k})$. One finds that

$$n_\omega(\mathbf{k})=\sum_{\alpha,\beta<F}\langle\alpha\beta|\Omega^\dagger_{\alpha\beta}\tilde{V}_\omega(\mathbf{k})\frac{(-k^+)}{(q^2-m_\omega^2)}\Omega_{\alpha\beta}|\alpha\beta\rangle_a, \tag{6.26}$$

where

$$\tilde{V}_\omega(\mathbf{k})=g_\omega^2/(2\pi)^3\frac{F_\omega^2(q^2)}{(q^2-m_v^2)}\left[\frac{k_\perp^2+m_v^2}{k^{+2}}\right]. \tag{6.27}$$

It is also necessary to discuss the nucleonic plus-momentum distribution. This is determined in Ref. [2]. Here the nucleon-nucleon correlations cause the momentum density to have contributions from above the Fermi sea. We have

$$\frac{P_N^+}{A}=\frac{4}{\rho_B(2\pi)^3}\int d^2k_\perp dk^+k^+N(k_\perp,k^+), \tag{6.28}$$

where ρ_B is the nuclear baryon density and $N(k_\perp,k^+)$ is the occupation number for a nucleon of momentum $(k_\perp,k^+)$. Recall that the variable k^+ is defined in Eq. (4.25). Since the integral gives the total plus-momentum carried by nucleons, the integrand (over k^+) which multiplies the factor k^+ is the probability $f(k^+)$ that a nucleon has momentum k^+ distribution. Thus:

$$\frac{P_N^+}{A}=\int dk^+k^+f(k^+), \tag{6.29}$$

where

$$f(k^+)=\frac{4}{\rho_B(2\pi)^3}\int d^2k_\perp N(k_\perp,k^+). \tag{6.30}$$

A function $f(y)$ can be obtained by replacing k^+ by the dimensionless variable y using $y\equiv k^+/\bar{M}$ with $\bar{M}\equiv M-14.71\,\mathrm{MeV}$.

The relation to experiments is obtained by recalling that the nuclear structure function F_{2A} can be obtained from the light front distribution function $f(y)$ (which gives the probability for a nucleon to have a plus momentum fraction y) and the nucleon structure function F_{2N} using the relation [55].

$$\frac{F_{2A}(x)}{A}=\int dy f(y)F_{2N}(x/y), \tag{6.31}$$

where y is A times the fraction of the nuclear plus-momentum carried by the nucleon, and x is the Bjorken variable computed using the nuclear mass minus the binding energy. This formula is the expression of the usual convolution model, with validity determined by a number of assumptions. Our formalism enables us to calculate the function $f(y)$ from the integrand of Eq. (4.28).

B. Computing the total number of mesons

We can get the total number of each kind of meson (except the ω) using a sum rule. Consider the schematic form of the equation for the G matrix

$$G(P_{ij}^-)=V(P_{ij}^-)+V(P_{ij}^-)\frac{Q}{\Delta E}G(P_{ij}^-), \tag{6.32}$$

where $Q/\Delta E$ is a schematic representation of the propagator of Eq. (4.53). Differentiating with respect to P_{ij}^- yields the result

$$\frac{\partial G(P_{ij}^-)}{\partial P_{ij}^-}=\left(1+G\frac{Q}{\Delta E}\right)\frac{\partial V(P_{ij}^-)}{\partial P_{ij}^-}\left(1+\frac{Q}{\Delta E}G\right). \tag{6.33}$$

The Moeller operators appear to the right and left of the derivative of the potential. Furthermore, in our one boson exchange approximation, the total nucleon-nucleon potential is the sum of the contributions due to individual bosons. Thus we may define

$$\frac{\partial G^m(P_{ij}^-)}{\partial P_{ij}^-}\equiv\left(1+G\frac{Q}{\Delta E}\right)\frac{\partial V^m(P_{ij}^-)}{\partial P_{ij}^-}\left(1+\frac{Q}{\Delta E}G\right), \tag{6.34}$$

in which the label m refers to the type of meson. But the potential V^m appearing in Eq. (6.32) is simply the Fourier transform of the potential $V^m(\mathbf{q})$. Thus an examination of Eqs. (6.25) and (6.24) shows that, considering the pion for example,

$$N_\pi\equiv\int d^3q\,n_\pi(\mathbf{q})=\sum_{\alpha,\beta<F}\langle\alpha,\beta|\frac{\partial G^\pi(P_{ij}^-)}{\partial P_{ij}^-}|\alpha,\beta\rangle_A. \tag{6.35}$$

Numerical evaluation of Eq. (6.35) leads to the result that $N_\pi/A=0.05$. This is smaller than the 18% of Friman *et al.* [56] because we use scalar mesons instead of intermediate Δ states to provide the bulk of the attractive force. The expression for the density of vector mesons involves the removal of the effects of the instantaneous exchange and one must use explicit light-front variables.

VII. IMPLICATIONS FOR LEPTON-NUCLEUS DEEP INELASTIC SCATTERING AND THE NUCLEAR DRELL-YAN PROCESS

The values of U_S, U_V^-, and N_π allow us to assess the deep inelastic scattering of leptons from our version of the ground state of nuclear matter. Using $M^*(k_F)=744\,\mathrm{MeV}$ [57] and neglecting the influence of two-particle-two-hole states to approximate $f(k^+)$ [1,2,58] shows that nucleons carry 81% (as opposed to the 65% of mean field theory [1]) of the nuclear plus momentum. This represents a vast improvement in the description of nuclear deep inelastic scattering. The minimum value of the ratio F_{2A}/F_{2N}, obtained from the convolution formula (6.31) is increased by a factor of 20 towards the data as extrapolated in Ref. [59]. But this calculation provides only a lower limit of the nucleon contribution because of the neglect of effects of the two-particle-two hole states [60].

Turn now to the experimental information about the nuclear pionic content. The Drell-Yan experiment on nuclear targets [61] showed no enhancement of nuclear pions within an error of about 5–10 % for their heaviest target. No substantial pionic enhancement is found in (p,n) reactions [62]. Understanding this result is an important challenge to the understanding of nuclear dynamics [63]. Here we have a good description of nuclear dynamics, and our 5% enhancement is consistent [64], within errors, with the Drell-Yan data.

VIII. SUMMARY AND DISCUSSION

This paper contains a new relativistic light-front theory of nuclear matter. Light-front quantization is used to obtain a nucleon-nucleon potential which yields phase shifts in good agreement with data. We use this as input in a light-front many body theory. A straightforward derivation leads to a theory in which the effective interaction is the light-front G matrix. We obtain a good description of the binding energy, density, and incompressibility of nuclear matter. The binding energy per nucleon is 14.7 MeV and $k_F = 1.37\,\mathrm{fm}^{-1}$. The compressibility is 180 MeV.

The use of a meson-nucleon Lagrangian enables us to also compute the mesonic content of the wave function using a consistent approach represented by Eqs. (4.1), (4.12), and (6.2). The results are not in conflict with extrapolations of deep inelastic scattering and Drell-Yan data to nuclear matter.

The omega and rho mesonic content are still to be evaluated. We believe that the possible nuclear enhancement of vector mesons is a promising avenue for future theoretical and experimental research.

ACKNOWLEDGMENTS

We thank D.R. Phillips for a useful discussion. This work was supported in part by the U.S. National Science Foundation under Grant No. PHY-9603097 and by the U.S. DOE.

APPENDIX: NOTATION, CONVENTIONS, AND USEFUL RELATIONS

This is patterned after the review of Harindranath [7]. The light-front variables are defined by

$$x^+ = x^0 + x^3, \quad x^- = x^0 - x^3, \tag{A1}$$

so the four-vector x^μ is denoted

$$x^\mu = (x^+, x^-, \mathbf{x}^\perp). \tag{A2}$$

With this notation the scalar product is denoted by

$$x \cdot y = \frac{1}{2} x^+ y^- + \frac{1}{2} x^- y^+ - \mathbf{x}^\perp \cdot \mathbf{y}^\perp. \tag{A3}$$

The metric tensor $g^{\mu\nu}$ with $\mu = (+,-,1,2)$ is obtained from the usual one by using Eq. (A1) (i.e., $g^{0\mu} = g^{0\mu} + g^{3\mu}$). Then $g^{+-} = g^{-+} = 2$, $g^{ij} = -1$, with the other elements vanishing. The term $g_{\mu\nu}$ is obtained from the condition that $g^{\alpha\beta} g_{\beta\gamma} = \delta_{\alpha\gamma}$. Its elements are the same as those of $g^{\mu\nu}$ except for $g_{-+} = g_{+-} = 1/2$. Thus

$$x_- = \frac{1}{2} x^+, \quad x_+ = \frac{1}{2} x^-, \tag{A4}$$

and the partial derivatives are similarly given by

$$\partial^+ = 2\partial_- = 2\frac{\partial}{\partial x^-}, \quad \partial^- = 2\partial_+ = 2\frac{\partial}{\partial x^+}. \tag{A5}$$

The Bjorken and Drell [40] convention for gamma matrices is used and

$$\gamma^\pm \equiv \gamma^0 \pm \gamma^3. \tag{A6}$$

The relations

$$\gamma^\pm \gamma^\pm = 0, \quad \gamma^+ \gamma^- \gamma^+ = 4\gamma^+, \quad \gamma^- \gamma^+ \gamma^- = 4\gamma^- \tag{A7}$$

can be used to simplify various computations. The Hermitian projection operators $\Lambda_\pm$ are given by

$$\Lambda_\pm = \frac{1}{4} \gamma^\mp \gamma^\pm = \frac{1}{2} \gamma^0 \gamma^\pm = \frac{1}{2}(I \pm \alpha^3) \tag{A8}$$

and obey the following relations:

$$(\Lambda_\pm)^2 = \Lambda_\pm, \quad \gamma^\perp \Lambda_\pm = \Lambda_\pm \gamma^\perp, \tag{A9}$$

$$\gamma^0 \Lambda \pm = \Lambda_\mp \gamma^0, \quad \alpha^\perp \Lambda_\pm = \Lambda_\mp \alpha^\perp, \tag{A10}$$

$$\gamma^5 \Lambda_\pm = \Lambda_\pm \gamma^5, \quad \gamma^\mp = 2\Lambda_\pm \gamma^0 = \gamma^\mp \Lambda_\mp, \tag{A11}$$

$$\gamma^i \Lambda_\mp = \frac{1}{2} \gamma^i \pm i \frac{1}{2} \epsilon^{ij} \gamma^j \gamma^5, \tag{A12}$$

$$\alpha^j \gamma^i \Lambda_+ = \frac{i}{2} \epsilon^{ij} \gamma^+ \gamma^5 \tag{A13}$$

[1] G. A. Miller, Phys. Rev. C **56**, R8 (1997).
[2] G. A. Miller, Phys. Rev. C **56**, 2789 (1997).
[3] J. Aubert *et al.*, Phys. Lett. **123B**, 275 (1982); R. G. Arnold *et al.*, Phys. Rev. Lett. **52**, 727 (1984); A. Bodek *et al.*, *ibid.* **51**, 534 (1983); R. L. Jaffe, in *Relativistic Dynamics and Quark-Nuclear Physics*, edited by M. B. Johnson and A. Picklesimer (Wiley, New York, 1986); M. Arneodo, Phys. Rep. **240**, 301 (1994); D. F. Geesaman, K. Saito, and A. W. Thomas, Annu. Rev. Nucl. Part. Sci. **45**, 337 (1995).
[4] Our notation is that a four vector A^μ is defined by the plus,

minus and perpendicular components as $(A^0+A^3, A^0-A^3, \mathbf{A}_\perp)$ and $A\cdot B=\frac{1}{2}A^+B^-+\frac{1}{2}A^-B^+-\mathbf{A}_\perp\cdot\mathbf{B}_\perp$. See the Appendix for more on this.

[5] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[6] S. J. Brodsky, H. C. Pauli, and S. S. Pinsky, Phys. Rep. **301**, 299 (1998); M. Burkardt Adv. Nucl. Phys. **23**, 1 (1993); S. J. Brodsky and G. P. Lepage, in *Perturbative Quantum Chromodynamics*, edited by A. Mueller (World Scientific, Singapore, 1989); X. D. Ji, Comments Nucl. Part. Phys. **21**, 123 (1992); W.-M. Zhang, Chin. J. Phys. **32**, 717 (1994); in *Theory of Hadrons and Light-Front QCD*, edited by S. D. Glazek (World Scientific, Singapore, 1994).

[7] A. Harindranath, hep-ph/9612244.

[8] L. L. Frankfurt and M. I. Strikman, Phys. Rep. **76**, 215 (1981).

[9] L. L. Frankfurt and M. I. Strikman, Phys. Rep. **160**, 235 (1988).

[10] X. Ji and B. W. Filippone, Phys. Rev. C **42**, R2279 (1990).

[11] F. Coester, Helv. Phys. Acta **38**, 7 (1965); in *Constraint's Theory and Relativistic Dynamics*, edited by G. Longhi and L. Lusanna (World Scientific Singapore, 1987), p. 159, in *The Three Body Force in the Three-Nucleon System*, edited by B. L. Berman and B. F. Gibson (Springer, New York, 1986).

[12] B. D. Keister and W. N. Polyzou, in *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1991), Vol. 20, p. 1.

[13] V. A. Karmanov and A. V. Smirnov, Nucl. Phys. **A575**, 691 (1994); J. Carbonell, V. A. Karmanov, B. Desplanques, V. A. Karmanov, and J. F. Mathiot, *ibid.* **A589**, 697 (1995); Phys. Rep. **300**, 215 (1998).

[14] M. G. Fuda and Y. Zhang, Phys. Rev. C **51**, 23 (1995); **54**, 495 (1996).

[15] N. C. J. Schoonderwoerd, B. L. G. Bakker, and V. A. Karmanov, Phys. Rev. C **58**, 3093 (1998); N. C. J. Schoonderwoerd and B. L. G. Bakker, Phys. Rev. D **57**, 4965 (1998); **58**, 025013 (1998); N. E. Ligterink and B. L. G. Bakker, *ibid.* **52**, 5917 (1995); **52**, 5954 (1995).

[16] D. E. Soper, Report No. SLAC pub-137, 1971; Phys. Rev. D **4**, 1620 (1971); J. B. Kogut and D. E. Soper, *ibid.* **1**, 2901 (1971).

[17] S. J. Chang, R. G. Root, and T-M Yan, Phys. Rev. D **7**, 1133 (1973); **7**, 1147 (1973).

[18] T.-M. Yan, Phys. Rev. D **7**, 1760 (1974); **7**, 1780 (1974).

[19] G. A. Miller and R. Machleidt, Phys. Lett. B **455**, 19 (1999).

[20] The use of scalar mesons is meant as a simple representation of part of the two pion exchange potential which causes much of the medium range attraction between nucleons, as well as the effects of a possible fundamental scalar meson.

[21] F. Gursey, Nuovo Cimento **16**, 230 (1960); Ann. Phys. (N.Y.) **12**, 91 (1961); P. Chang and F. Gursey, Phys. Rev. **164**, 1752 (1967).

[22] S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D **22**, 2838 (1980); **23**, 2106 (1981); A. W. Thomas, S. Théberge, and G. A. Miller, *ibid.* **24**, 216 (1981); A. W. Thomas, Adv. Nucl. Phys. **13**, 1 (1984); G. A. Miller, Int. Rev. Nucl. Phys. **2**, 190 (1984).

[23] B. D. Serot and J. D. Walecka, Report No. IU-NTC-96-17, 1997, nucl-th/9701058.

[24] R. J. Furnstahl, B. D. Serot, and H.-B. Tang, nucl-th/9608035; Nucl. Phys. **A615**, 441 (1997).

[25] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

[26] D. Mustaki, S. Pinsky, J. Shigemitsu, and K. Wilson, Phys. Rev. D **43**, 3411 (1991).

[27] The determination of the inverse of the operator $p^+=i\partial^+$ is discussed in the original literature [16] and in other places, and recently in A. Harindranath and W-M Zhang, Phys. Rev. D **48**, 4868 (1993); **48**, 4881 (1993); **48**, 4903 (1993).

[28] If one rewrites the Weinberg-type equation in terms of equal time variables, the result is the same as the Blancebecler-Sugar equation except for the appearance in the meson propagator of the energy difference between on-shell initial and final nucleons.

[29] R. Machleidt, K. Holinde, and Ch. Elster, Phys. Rep. **149**, 1 (1987).

[30] R. Machleidt, Adv. Nucl. Phys. **19**, 189 (1989).

[31] R. Machleidt, in *Computational Nuclear Physics 2—Nuclear Reactions*, edited by K. Langanke, J. A. Maruhn, and S. E. Koonin (Springer, New York, 1993), Chap. 1, p. 1.

[32] R. Brockmann and R. Machleidt, Phys. Lett. **149B**, 283 (1984); Phys. Rev. C **42**, 1965 (1990); in *Open Problems in Nuclear Matter*, edited by M. Baldo (World Scientific, Singapore, 1997), Chap. 3; nucl-th/9612004.

[33] G. E. Brown and A. D. Jackson, *Nucleon-Nucleon Interaction* (North-Holland, Amsterdam, 1976); T. Ericson and W. Weise, *Pions and Nuclei* (Oxford Science Publications, New York, 1987).

[34] S. Weinberg, Phys. Rev. **150**, 1313 (1966).

[35] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966).

[36] M. V. Terent'ev, Yad. Fiz. **24**, 207 (1976) [Sov. J. Nucl. Phys. **24**, 106 (1976)].

[37] M. -H. Partovi and E. L. Lomon, Phys. Rev. D **2**, 1999 (1970).

[38] G. Hoehler and H. Pietarinen, Nucl. Phys. **B95**, 210 (1975).

[39] G. E. Brown and R. Machleidt, Phys. Rev. C **50**, 1731 (1994).

[40] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964); J. D. Bjorken and S.D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965); C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).

[41] Note that the explicit expression for the tensor coupling OBE amplitude in the LF formalism differs from the one used in the BbS formalism [compare Eq. (A.9) of Ref. [30] or Eq. (1.61) of Ref. [31] with our Eq. (3.22)]. The reason for this difference is that in the BbS formalism $q_0=0$ while in the LF formalism $q_0=E'-E$. As a consequence of this, the partial-wave decomposed tensor expressions as given in Ref. [31] do not apply here. The expressions to be used here can be found in Ref. [42] (Appendix A, case 1, therein). Moreover, all meson propagators $[m_\alpha^2+(\mathbf{k}'-\mathbf{k})^2]^{-1}$ in Ref. [31] must be replaced by $[m_\alpha^2-(E'-E)^2+(\mathbf{k}'-\mathbf{k})^2]^{-1}$ and a similar replacement must be done in the denominators of the cutoffs.

[42] K. Holinde and R. Machleidt, Nucl. Phys. **A247**, 495 (1975).

[43] M. I. Haftel and F. Tabakin, Nucl. Phys. **A158**, 1 (1970).

[44] R. H. Thompson, Phys. Rev. D **1**, 110 (1970).

[45] V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester, and J. J. de Swart, Phys. Rev. C **48**, 792 (1993).

[46] R. A. Arndt, Virginia Polytechnic Institute (VPI) and State University, Interactive Dial-In Program SAID, Solution SM97.

[47] The original nonrelativistic Brueckner theory is reviewed in M. A. Preston and R. K. Bhaduri, *Structure of the Nucleus*,

(Addison-Wesley, Reading, 1975). The technical discussion therein relies heavily on P. Grangé and M. A. Preston, Nucl. Phys. **A204**, 1 (1973); **A219**, 266 (1974).

[48] St. Glazek and C. M. Shakin, Phys. Rev. C **44**, 1012 (1991).

[49] E. A. Bartnik and St. Glazek, Phys. Rev. D **39**, 1249 (1989).

[50] M. Brisudová and R. Perry, Phys. Rev. D **54**, 6453 (1996).

[51] P. Blunden, M. Burkardt, and G.A. Miller, Phys. Rev. C **59**, R2998 (1999); 1999 UW Report, Phys. Rev. C (submitted), nucl-th/9906012.

[52] J. P. Blaizot, Phys. Rep. **65**, 171 (1980).

[53] The explicit expression for the pv OPE amplitude as derived in the ET formalism is given in Eq. (A.7) of Ref. [30] and Eq. (1.59) of Ref. [31].

[54] When comparing OPE amplitudes using different couplings, the coupling constants must be related properly. The relationship is

$$g_\pi = \left(\frac{2M}{m_\pi}\right) f_{\pi NN}$$

in free space and

$$g^*_\pi = \left(\frac{2M^*}{m_\pi}\right) f_{\pi NN}$$

in the nuclear medium. The latter is related to the fact that, based upon our Lagrangian Eq. (2.1), the ps pion coupling constant in the medium is

$$g^*_\pi = \frac{M^*}{f} = \frac{M^*}{M}\frac{M}{f} = \frac{M^*}{M} g_\pi$$

with $g_\pi = M/f$ the free-space coupling constant (f is the pion decay constant).

[55] Eq. (6.31) is essentially Eq. (5.2) of Frankfurt and Strikman [9] with a correspondence between our $Af(y)$ and their $\int d^2p_\perp \rho^N_A(\alpha,p_\perp)/\alpha$.

[56] B. Friman, V. R. Panharipande, and R. B. Wiringa, Phys. Rev. Lett. **51**, 763 (1983)

[57] U depends on momentum, so to use a constant value is to take an average. Equations (4.60) and (5.8) are nonlinear, and the solutions have a spread of values. Indeed, $M^*(k_F) = 744$ also leads to good saturation properties.

[58] We use the Slater determinant $|\phi\rangle$ to evaluate the expectation value of the nucleon number operator. This means that the occupation number N of Eq. (6.28) is either unity or zero.

[59] I. Sick and D. Day, Phys. Lett. B **274**, 16 (1992).

[60] We estimate that nucleons with momentum greater than k_F would substantially increase the computed ratio F_{2A}/F_{2N} because $F_{2N}(x)$ decreases very rapidly with increasing values of x and because M^* would increase at high momenta.

[61] D. M. Alde *et al.*, Phys. Rev. Lett. **64**, 2479 (1990).

[62] L. B. Rees *et al.*, Phys. Rev. C **34**, 627 (1986); J. B. McClelland *et al.*, Phys. Rev. Lett. **69**, 582 (1992); X. Y. Chen *et al.*, Phys. Rev. C **47**, 2159 (1993); T. N. Taddeucci *et al.*, Phys. Rev. Lett. **73**, 3516 (1994).

[63] G. F. Bertsch, L. Frankfurt, and M. Strikman, Science **259**, 773 (1993).

[64] H. Jung and G. A. Miller, Phys. Rev. C **41**, 659 (1990).