

Probing halo nucleus structure through intermediate energy elastic scattering

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This work addresses the question of precisely what features of few body models of halo nuclei are probed by elastic scattering on protons at high center-of-mass energies. Our treatment is based on a multiple scattering expansion of the proton-projectile transition amplitude in a form which is well adapted to the weakly bound cluster picture of halo nuclei. In the specific case of ^{11}Li scattering from protons at 800 MeV/nucleon we show that because core recoil effects are significant, scattering cross sections cannot, in general, be deduced from knowledge of the total matter density alone. We advocate that the optical potential concept for the scattering of halo nuclei on protons should be avoided and that the multiple scattering series for the full transition amplitude should be used instead. [S0556-2813(99)02409-7]

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I. INTRODUCTION

Models of light halo nuclei have been developed [1–3] where the few body degrees of freedom of a system of loosely bound valence nucleons orbiting around a relatively tightly bound core are properly taken into account. Short range, center of mass, and some Pauli principle effects are often included in these models.

In this work we develop a multiple scattering expansion of the nucleon-projectile transition amplitude for proton scattering from a few body system. When the projectile is composed of weakly bound subsystems a multiple scattering expansion of the transition amplitude in terms of two-body t -matrices describing proton scattering from the projectile subsystems is expected to converge rapidly [4]. The elastic scattering observables may then be derived directly from this expansion. We contrast this with our earlier work [5,6] which is based on a multiple scattering expansion of the optical model operator and therefore treats the ground and excited states of the projectile on a different footing. The present approach is more appropriate for few-body projectiles at high projectile energy.

Our aim in this work is to understand the nuclear structure features that should be incorporated into the reaction mechanism in order to describe elastic scattering of halo nuclei from stable nuclei. In particular it is of considerable interest to examine how far elastic scattering observables probe correlation effects among projectile nucleons [7,8].

II. MULTIPLE SCATTERING EXPANSION

We consider the transition amplitude, T , for scattering of a proton from a many body-system composed of a small number of subsystems. We have in mind, for example, ^{11}Li assumed to be well described by two valence loosely bound

nucleons orbiting around a ^9Li core. T can be written as a multiple scattering expansion in the transition amplitudes $\hat{t}_{\mathcal{I}}$ for proton scattering from each projectile subsystem \mathcal{I} [4]. We ignore explicit reference to excitations of the subsystems, although each $\hat{t}_{\mathcal{I}}$ may implicitly contain effects due to such excitations and will certainly do so if, as we shall assume, they describe elastic proton-subsystem scattering. In other words our model assumes that we only need refer explicitly to excitations of the projectile which involve changes in the relative motion of the subsystems in the projectile. This is consistent with standard few-body treatments of reactions involving halo nuclei [9,10].

The multiple scattering expansion can be written

$$T = \sum_{\mathcal{I}} \hat{t}_{\mathcal{I}} + \sum_{\mathcal{I}} \hat{t}_{\mathcal{I}} G_0 \sum_{\mathcal{J} \neq \mathcal{I}} \hat{t}_{\mathcal{J}} + \dots, \quad (1)$$

where the proton- \mathcal{I} subsystem transition amplitude satisfies

$$\hat{t}_{\mathcal{I}} = v_{\mathcal{I}} + v_{\mathcal{I}} G_0 \hat{t}_{\mathcal{I}}. \quad (2)$$

The propagator G_0 contains the kinetic energy operators of the projectile and all the target subsystems, $G_0 = (E^+ - K)^{-1}$. Here E is the kinetic energy, $E = \hbar^2 k_i^2 / 2\mu_{NA}$ in the overall center of mass frame, and μ_{NA} is the proton-projectile reduced mass. We ignore the interaction between projectile subsystems in G_0 (impulse approximation). We note that in the multiple scattering expansion Eq. (1) both elastic and inelastic excitations of the relative motion of the subsystems in intermediate states are taken into account. For proton scattering from halo nuclei the inelastic channels associated with breakup of the halo nucleus into its subsystems are expected to contribute significantly to the transition amplitude.

In this paper we truncate the series in Eq. (1) at the double scattering terms. We have not evaluated third order terms and we do not claim that they are negligible. They could be handled using the techniques of, for example, Ref. [11]. Our

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purpose here is to assess the applicability of the standard approach for proton scattering on light nuclei. We will show that inadequacies show up even at the second order level.

A second aim of our work is to understand the role of various types of correlations in elastic scattering from halo systems. In this paper we make a numerical study of the case of proton scattering from a ^{11}Li projectile at intermediate energies. Our formalism could also be applied to p - ^6He scattering which has been studied extensively elsewhere using methods which do not use a truncated multiple scattering expansion [12,13] but do not lend themselves well to delineating the role of correlations in an explicit way.

We assume that the projectile wave function can be written as the product of the core internal wave function φ_C and the wave function of the two body valence system relative to the core $\varphi_{nn}(\vec{r}, \vec{R})$, where $\vec{r} = \vec{r}_2 - \vec{r}_3$ is the relative position of the two valence bodies 2 and 3, and \vec{R} is the vector from the core center of mass (particle 4) to the center of mass of the valence pair.

For projectile energies in the intermediate energy region the relative momentum between each subsystem pair is small in comparison with the projectile momentum and will be neglected wherever it appears. The elastic transition amplitude to second order in the proton-subsystem transition amplitudes, involves single scattering terms where the projectile scatters from each target subsystem and double scattering terms where the proton scatters from one subsystem and rescatters from another.

A. Single scattering

The contribution to the single scattering term from proton scattering from one of the valence particles, for example particle 2, is given by

$$\langle \vec{k}_f \Phi | \hat{t}_{12} | \vec{k}_i \Phi \rangle = \langle \vec{k}_f \varphi_{nn} | \hat{t}_{12} | \vec{k}_i \varphi_{nn} \rangle = \hat{t}_{12}(\omega_{12}, \vec{\Delta}) \rho_v(\vec{\Delta}), \quad (3)$$

where $\rho_v(\vec{\Delta})$ is defined in terms of the two-body halo density

$$\rho_2(\vec{\Delta}_1, \vec{\Delta}_2) = \int d\vec{Q}_1 d\vec{Q}_2 \varphi_{nn}^*(\vec{Q}_1, \vec{Q}_2) \times \varphi_{nn}(\vec{Q}_1 + \vec{\Delta}_1, \vec{Q}_2 + \vec{\Delta}_2) \quad (4)$$

by

$$\rho_v(\vec{\Delta}) = \rho_2\left(\frac{m_3}{M_{23}} \vec{\Delta}, \frac{m_4}{M_{234}} \vec{\Delta}\right), \quad (5)$$

where $M_{23} = m_2 + m_3$, $M_{234} = m_2 + m_3 + m_4$, etc. In Eq. (4) $\varphi_{nn}(\vec{Q}_1, \vec{Q}_2)$ is the Fourier transform of wave function of the two body valence system relative to the core $\varphi_{nn}(\vec{r}, \vec{R})$. In the case $m_2 = m_3 = m_n$ the quantity $\rho_v(\vec{\Delta})$ is just the Fourier transform of the probability density $\rho(\vec{x})$ of finding a valence neutron at a distance \vec{x} from the center of mass of the projectile as defined by Zhukov *et al.* [1].

The energy parameter ω_{12} in Eq. (3) is given by

$$\omega_{12} = \left[1 - \frac{m_1 M_{234}}{M_{12} M_{234}} \right] E \quad (6)$$

and reduces to $\omega_{12} = E/2$ in the limit of $m_4 \gg m_3, m_2$.

The contribution to the single scattering term from proton scattering from the core is

$$\langle \vec{k}_f \Phi | \hat{t}_{14} | \vec{k}_i \Phi \rangle = \langle \varphi_{\text{core}} | \hat{t}_{14}(\omega_{14}, \vec{\Delta}) | \varphi_{\text{core}} \rangle \rho_2\left(0, \frac{M_{23}}{M_{234}} \vec{\Delta}\right), \quad (7)$$

where ρ_2 is defined in Eq. (4) and the arguments in Eq. (7) mean that what is involved is the density distribution for the motion of the core center of mass, as defined by Zhukov *et al.* [1],

$$\rho_2\left(0, \frac{M_{23}}{M_{234}} \vec{\Delta}\right) = \int d\vec{Q}_1 d\vec{Q}_2 \varphi_{nn}^*(\vec{Q}_1, \vec{Q}_2) \times \varphi_{nn}\left(\vec{Q}_1, \vec{Q}_2 + \frac{M_{23}}{M_{234}} \vec{\Delta}\right) \quad (8)$$

and the energy parameter ω_{14} is given by

$$\omega_{14} = \left[1 - \frac{m_1 M_{23}}{M_{14} M_{234}} \right] E. \quad (9)$$

In the limit of $m_4 \gg 1$, $\omega_{14} = E$ and $\rho_2(0, (M_{23}/M_{234})\vec{\Delta}) \rightarrow \rho_2(0, 0) = 1$ so that Eq. (7) reduces to the expected expression for the proton scattering from subsystem 4.

Within our model, there are two contributions to the single scattering term. First a valence contribution given by the product of the projectile valence system transition amplitude and $\rho_v(\vec{\Delta})$. Secondly a core contribution in which the nucleon-core transition amplitude is modulated by the form factor $\rho_2(0, (m_{23}/M_{234})\vec{\Delta})$ whose departure from unity arises from the motion of the core center of mass about the projectile center of mass. This modulation differs from standard applications of the multiple scattering expansion of the optical potential operator [6,8] that modulate the core matter density distribution ρ_C by that form factor.

The relevant halo structure information for the single scattering term is thus contained in the matter density form factors $\rho_v(\vec{\Delta})$ and $\rho_2(0, (M_{23}/M_{234})\vec{\Delta})$.

B. Double scattering

We next evaluate the double scattering term in the ^{11}Li case. We distinguish the terms where the proton scatters from the valence neutrons 2 and 3 and the term where the proton scatters once from the core and once from a valence particle. In the former case we find that

$$\begin{aligned}
 \langle \vec{k}_f \Phi | \hat{t}_{12} G_0 \hat{t}_{13} | \vec{k}_i \Phi \rangle &= \langle \vec{k}_f \varphi_{nn} | \hat{t}_{12} G_0 \hat{t}_{13} | \vec{k}_i \varphi_{nn} \rangle \\
 &= \int d\vec{q} \hat{t}_{12} \left(\omega_{12}, \frac{m_2}{M_{23}} \vec{\Delta} + \vec{q} \right) \\
 &\quad \times \hat{t}_{13} \left(\omega_{13}, \frac{m_3}{M_{23}} \vec{\Delta} - \vec{q} \right) \\
 &\quad \times G_0(\vec{q}) \rho_2 \left(\vec{q}, \frac{m_4}{M_{234}} \vec{\Delta} \right), \quad (10)
 \end{aligned}$$

where $\rho_2(\vec{\Delta}_1, \vec{\Delta}_2)$ is defined in Eq. (4) and

$$G_0(\vec{q}) = 2 \frac{\mu_{1(23)}}{\hbar^2} \left[k_i^2 - \left(\frac{(m_3 \vec{k}_f + m_2 \vec{k}_i)}{M_{23}} + \vec{q} \right)^2 + i\epsilon \right]^{-1}. \quad (11)$$

In the case of a heavy core,

$$\begin{aligned}
 \lim_{m_4 \rightarrow \infty} \rho_2 \left(\vec{q}, \frac{m_4}{M_{234}} \vec{\Delta} \right) \\
 &= \int d\vec{Q}_1 d\vec{Q}_2 \varphi_{nn}^*(\vec{Q}_1, \vec{Q}_2) \varphi_{nn}(\vec{Q}_1 + \vec{q}, \vec{Q}_2 + \vec{\Delta}) \\
 &= \int d\vec{r}_2 d\vec{r}_3 e^{i(m_2/M_{23}) \vec{\Delta} + \vec{q} \cdot \vec{r}_2} e^{i(m_3/M_{23}) \vec{\Delta} - \vec{q} \cdot \vec{r}_3} \\
 &\quad \times |\bar{\varphi}_{nn}(\vec{r}_2, \vec{r}_3)|^2, \quad (12)
 \end{aligned}$$

where $\bar{\varphi}_{nn}(\vec{r}_2, \vec{r}_3) = \varphi_{nn}(\vec{r}, \vec{R})$ is the valence wave function expressed in terms of \vec{r}_2 and \vec{r}_3 , the position vectors of the two valence particles relative to the core. Therefore, this density function involves two-body correlations among the valence particles even in the heavy core limit.

The valence system–core double scattering term is given by

$$\begin{aligned}
 \langle \Phi | \hat{t}_{12} G_1 \hat{t}_{14} | \Phi \rangle &= \int d\vec{q} \hat{t}_{12} \left(\omega_{12}, \frac{M_{23}}{M_{234}} \vec{\Delta} + \vec{q} \right) \\
 &\quad \times \left\langle \varphi_{\text{core}} \left| \hat{t}_{41} \left(\omega_{14}, \frac{m_4}{M_{234}} \vec{\Delta} - \vec{q} \right) \right| \varphi_{\text{core}} \right\rangle \\
 &\quad \times G_1(\vec{q}) \rho_2 \left(\frac{m_3}{M_{23}} \vec{q} + \frac{m_3}{M_{234}} \vec{\Delta}, \vec{q} \right), \quad (13)
 \end{aligned}$$

where

$$G_1(\vec{q}) = 2 \frac{\mu_{1(234)}}{\hbar^2} \left[k_i^2 - \left(\frac{(m_4 \vec{k}_f + m_{23} \vec{k}_i)}{M_{234}} + \vec{q} \right)^2 + i\epsilon \right]^{-1}. \quad (14)$$

For $m_4 \gg m_2, m_3$

$$\begin{aligned}
 \lim_{m_4 \rightarrow \infty} \rho_2 \left(\frac{m_3}{M_{23}} \vec{q} + \frac{m_3}{M_{234}} \vec{\Delta}, \vec{q} \right) \\
 &= \int d\vec{r}_2 e^{i(\vec{q} \cdot \vec{r}_2)} \int d\vec{r}_3 |\bar{\varphi}_{nn}(\vec{r}_2, \vec{r}_3)|^2. \quad (15)
 \end{aligned}$$

In contrast to the double scattering term arising from the two valence particles, the particular elements of ρ_2 which enters in the heavy core limit is just the one body density of subsystem 2 in the halo.

C. Numerical results for ^{11}Li scattering at 800 MeV/nucleon

In order to obtain some quantitative idea of the various terms we have identified, we have evaluated the multiple scattering expansion for the specific case of proton scattering at 800 MeV/nucleon from a three-body model of ^{11}Li . For the purposes of the estimate, only the central components of the transition amplitudes were taken into account. Coulomb interaction effects were not included.

For the description of ^{11}Li we take the Faddeev wave functions of Thompson and Zhukov [3] referred in that work as the P3 model. In describing the ^9Li ground state matter density distribution we consider a simplified structure model of a Gaussian distribution with a range chosen to reproduce the rms radius [6]. The first and second order terms were evaluated using a NN transition amplitude derived from the Paris potential [14,15] evaluated at the appropriate fixed energy parameter with finite mass effects properly taken into account. The transition amplitude for proton scattering from ^9Li was generated by an optical potential calculated in the single scattering approximation appropriate for intermediate energy elastic scattering [5].

In the evaluation of the second order terms, the propagators were evaluated using the eikonal approximation and the principal value term was neglected. For example we use

$$G_1(\vec{q}) = - \frac{\mu_{1(234)}}{\hbar^2} \left(\frac{1}{\vec{k}_i \cdot \vec{q} + i\epsilon} \right). \quad (16)$$

An explicit evaluation using Gaussian functional forms for the transition matrices and densities involved shows that for small scattering angles the ratio of the principal value and delta function terms in Eq. (16) is less than $1/k_i R$, where R is a measure of the halo size. This ratio is very small in the cases we consider.

In Fig. 1 we show the differential cross section for ^{11}Li scattering from a proton target at 800 MeV/nucleon for center-of-mass in the range we expect to be covered by experiments (e.g. [8]). The dashed curve was evaluated from the single scattering contributions, Eqs. (3) and (7). The solid curve includes in addition the double scattering contributions valence-valence Eq. (10) and valence-core Eq. (13). We emphasise that in the present context ‘‘double scattering’’ means second order in the proton-subsystem t matrix. Terms of all orders in the p -subsystem potentials are included in our first order terms. The other curves in the figure are obtained by taking $\rho_2(0, (M_{23}/M_{234})\vec{\Delta}) = 1$ for all $\vec{\Delta}$ in Eq. (7). This limit corresponds to ignoring the relative motion of the core and projectile centres of mass. The dotted-dashed and dotted curves correspond to single and double scattering calculated cross section, respectively, and clearly show that the inclusion of the relative motion of the core and projectile centers of mass has a significant effect in the calculated differential cross section.

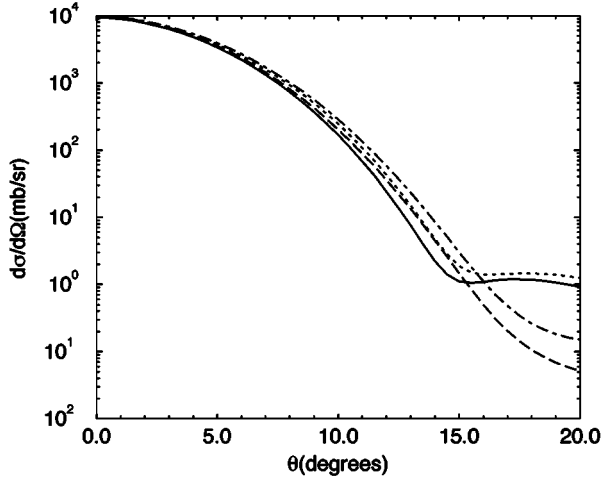


FIG. 1. Differential cross section for proton scattering from ^{11}Li at 800 MeV/nucleon. The dashed curve was evaluated from the single scattering contributions. The solid curve includes in addition the double scattering contributions. The other curves in the figure are obtained by ignoring the relative motion of the core and projectile centers of mass in the single scattering term and the dotted-dashed and dotted curves are cross sections calculated without and with double scattering terms, respectively.

III. DISCUSSION

In the context of nucleon scattering from conventional stable heavy nuclei one usually associates two-body correlation effects with the double scattering terms which in the present case would mean through the two-body density $\rho_2(\vec{q}, (m_4/M_{234})\vec{\Delta})$ in Eq. (10). The contribution from this to the second order term is very small here, and only valence-core double scattering contributions remain relevant. However, that does not mean that the scattering is sensitive only to the projectile density $\rho_v(\vec{\Delta})$ of Eq. (5), which we might reasonably call “the halo density.” The scattering involves the halo wave function in several other distinct ways: First, through the two-body density $\rho_2(0, (M_{23}/M_{234})\vec{\Delta})$ of Eq. (8). In the limit of a infinite massive core, $\lim_{m_4 \rightarrow \infty} \rho_2(0, (M_{23}/M_{234})\vec{\Delta}) = 1$, but this is a poor approximation in the cases considered. Secondly, the halo wave function is involved through the two-body density $\rho_2((m_3/M_{23})\vec{q} + (m_3/M_{234})\vec{\Delta}, \vec{q})$ in Eq. (13). In the limit of a infinite massive core, $\lim_{m_4 \rightarrow \infty} \rho_2((m_3/M_{23})\vec{q} + m_3/M_{234})\vec{\Delta}, \vec{q}) = \rho_v(\vec{\Delta})$, and this limit was found to be a good approximation here.

There are several consequences which flow from our analysis. We have shown that core recoil effects are important. The same claim has been made by others but within the framework of formalisms which differ from ours. Reference [16] corrects the projectile matter density as a whole for recoil effects. One of our points is that we find no justification for describing the scattering of protons from a light system such as ^{11}Li in terms of an optical potential expressed as

a nucleon-nucleon transition amplitude, t_{NN} , and a total matter density given by the sum of the valence density and a core density modulated by a center-of-mass factor $\rho_2(0, (M_{23}/M_{234})\vec{\Delta})$. In the first place even in the first order term it is the N -core transition amplitude which is modulated in this way. Secondly center-of-mass corrections to the second order terms do not have the structure that would arise from iterating the first order term as would be expected in a “ $t\rho$ ” type optical model theory. Equations (3), (7), and (13) can be made to have this structure if the following three assumptions are made.

- (i) t_{14} is approximated by its “ $t\rho$ ” limit.
- (ii) The average t_{NN} matrices for the core and halo nucleons are assumed equal.
- (iii) The limit $m_4 \rightarrow \infty$ for the core mass is assumed.

In our calculations we can find no justification for (i) and the inadequacy of (ii) was shown very clearly in [5,11]. We have shown here that (iii) is a poor approximation in Eq. (8). In [17] core recoil effects are taken into account in a way which is consistent with a few-body model of the reaction and without making a multiple scattering expansion. It is, however, not as transparent as in our formalism how the halo density functions contribute to the scattering. An advantage of our approach is that reaction mechanism and structure effects are clearly delineated.

IV. CONCLUSIONS

We have seen in this work that two-body correlation effects associated with the double scattering term are small in the case of ^{11}Li scattering from a proton target at 800 MeV/nucleon. The density distribution of the core center of mass, $\rho_2(0, (M_{23}/M_{234})\vec{\Delta})$, does, however, have a large effect on the calculated cross section.

We have shown that the halo structure information associated with $\rho_v(\Delta)$ and $\rho_2(0, (M_{23}/M_{234})\vec{\Delta})$ does not contribute to the scattering simply combined as a total matter density. Thus, a proper treatment of the reaction mechanism for halo nuclei elastic scattering needs necessarily to incorporate structure features that go beyond knowledge of the total halo matter density distribution alone.

In summary, we advocate that in microscopic theories of proton scattering from light nuclei such as halo nuclei, at intermediate and high energies the multiple scattering series for the full transition amplitude should be used and that the optical potential concept should be avoided.

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