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Neutron-proton ratio of collective excitations

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Using a harmonic oscillator model, it is shown that the isovector giant quadrupole resonance carries little isoscalar quadrupole strength while the isoscalar giant quadrupole resonance contains an isovector strength with $B(IV)/B(IS) \approx ((N\text{-}Z)/A)^2$, when a consistency condition is fulfilled. Taking into account the consistency between the one-particle excitation spectra and the vibrational fields is of vital importance in the proper estimate of the ratio of the contribution by neutrons to that by protons to collective excitations of nuclei with $N \neq Z$. The ratio may not be properly estimated in the models such as traditional shell models, in which the Coulomb potential (or interaction) is simulated by using the parameters $\omega_p < \omega_n$ for nuclei with a neutron excess. [S0556-2813(99)50609-2]

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Because of the recent development of radioactive nuclear ion beams, the experimental study of the structure of nuclei far away from β stability lines has become possible. Examples of interesting exotic structure of those nuclei are the presence of nucleons with very small binding energies and a large difference between the separation energy of neutrons and that of protons. In the present Rapid Communication we discuss the neutron-proton ratio in the collective shape oscillations of nuclei with neutron excess. Though our present discussion is also applicable to β stable nuclei, such as, $^{208}_{82}\mathrm{Pb}_{126}$ in which the neutron excess is already appreciable, the ratio $(N\text{-}Z)/A$ can be much larger in nuclei on the neutron-rich side of the β stability line where the consequence of the present discussion can be more serious. In $N = Z$ nuclei the isoscalar ($\tau = 0$) excitation modes do not mix with the isovector ($\tau = 1$) modes to the extent that the ground state of $N = Z$ nuclei has isospin $T = T_z = 0$. Then, it is trivial that in those nuclei the isoscalar excitation modes do not carry isovector strength, and vice versa.

In the self-consistent random-phase approximation, in which the Hartree-Fock (HF) and random-phase approximation (RPA) calculation are performed using the same effective interaction, it is pointed out [1,2] that the isoscalar giant resonance contains the isovector ($\tau = 1$) strength of the approximate ratio $((N\text{-}Z)/A)^2$, while the isovector giant resonance carries little of the isoscalar ($\tau = 0$) strength. In Fig. 1 we show the result [1] of a self-consistent calculation of the quadrupole strength of the nucleus $^{28}_{8}\mathrm{O}_{20}$, in which the RPA response function is calculated in the coordinate space so as to take properly into account the continuum effect. The light doubly magic nucleus with a large neutron excess is chosen in illustration, in which the peak height of the isoscalar giant resonance is not too high to be compared with that of the isovector giant resonance. Whether or not the nucleus $^{28}_{8}\mathrm{O}_{20}$ lies in reality inside the neutron drip line does not matter for the present purpose. The quadrupole strength for $E_x \lesssim 14$ MeV seen in Fig. 1 is the so-called threshold strength, which is not of collective character and comes from the excitations of excess neutrons with small binding energies. It is shown that the isoscalar giant quadrupole resonance (GQR) peaking at $E_x = 17.85$ MeV carries an appreciable isovector strength, while the isovector GQR, of which the strength is split into a few peaks in the region of $26 \lesssim E_x \lesssim 35$ MeV, contains little of the isoscalar strength. In particular, it is shown that in the energy region of $26 \lesssim E_x \lesssim 35$ MeV in Fig. 1 the isoscalar RPA strength becomes extremely small at every peak energy of the isovector RPA strength. The absence of the isoscalar strength at the energy of the isovector GQR is also demonstrated cleanly in the self-consistent but discrete calculation of the continuum spectra, namely, by expanding the continuum wave functions in terms of harmonic oscillator bases [2].

In order to illustrate the role of self-consistency, we use a simple harmonic oscillator model for $N \neq Z$ nuclei, which is worked out in Ref. [3]. In general, the harmonic oscillator frequency is different for neutrons and protons and, thus, the potential is described as

$$V(r) = \tfrac{1}{2} M \omega_p^2 r^2 \tfrac{1}{2}(1 - \tau_z) + \tfrac{1}{2} M \omega_n^2 r^2 \tfrac{1}{2}(1 + \tau_z). \qquad (1)$$

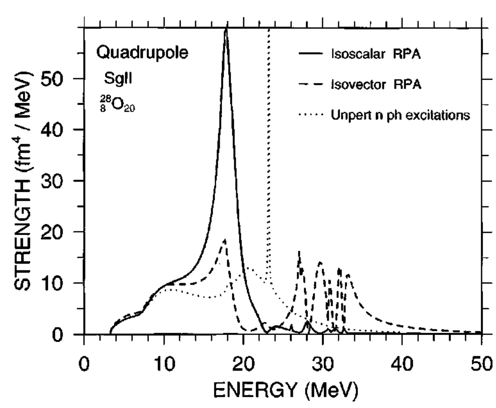

FIG. 1. Isoscalar and isovector RPA quadrupole strength function of "a theoretical nucleus" $^{28}_{8}\mathrm{O}_{20}$ as a function of excitation energy. The SgII interaction is used in both the HF and RPA calculation. The strength of unperturbed neutron p-h excitations is denoted by dotted curves for reference. All unperturbed proton p-h excitations lie in the region of $26 \lesssim E_x \lesssim 32.5$ MeV, which are the excitations either to the bound one-particle levels or to the one-particle resonant levels.

The quadrupole field interaction consists of the isoscalar and isovector part as

$$X_0 \sum_\mu F_{2\mu}^{(0)} F_{2\mu}^{(0)\dagger} + X_1 \sum_\mu F_{2\mu}^{(1)} F_{2\mu}^{(1)\dagger}, \tag{2}$$

where

$$\left.\begin{array}{l} F_{2\mu}^{(0)} = r^2 Y_{2\mu} \\ F_{2\mu}^{(1)} = \tau_z r^2 Y_{2\mu} \end{array}\right\}. \tag{3}$$

In Eq. (6–78) of Ref. [3] the self-consistent isoscalar quadrupole coupling constant is obtained as

$$X_{0,\mathrm{sc}} = -\frac{4\pi}{5} \frac{M\omega_0^2}{A\langle r^2 \rangle}, \tag{4}$$

where we will use $\langle r^2 \rangle = 0.87 A^{2/3}\,\mathrm{fm}^2$. On the other hand, in Ref. [3] [see Eq. (6–127)], the expression

$$X_1^{BM} = \frac{\pi V_1}{A\langle r^4 \rangle} \tag{5}$$

is obtained for the isovector the coupling constant, and the strength V_1 is taken from the isovector part of the phenomenological Woods-Saxon potential. Equation (5) is obtained from the consideration that the isovector density variations give rise to corresponding variations in the isovector potential, which can be estimated from the isovector component in the static nuclear potential,

$$\delta V = \tfrac{1}{4} V_1 \tau_z \frac{\delta\rho_1}{\rho_0}. \tag{6}$$

See p. 378 of Ref. [3].

If we take $\omega_n = \omega_p = \omega_0$ for $N \neq Z$ nuclei, the unperturbed one-particle quadrupole response function contains only one excitation frequency, $2\hbar\omega_0$. Then, the collective isoscalar RPA solution for $X_1 = 0$ carries the isovector strength with $B(IV)/B(IS) = ((N\text{-}Z)/A)^2$, while the collective isovector RPA solution for $X_0 = 0$ contains the isoscalar strength with $B(IS)/B(IV) = ((N\text{-}Z)/A)^2$. By $B(IS)$ and $B(IV)$ we express the calculated reduced isoscalar and isovector quadrupole transition probability, respectively. Though those RPA solutions have the simple intuitive structure, they are not self-consistent solutions.

Writing

$$\left.\begin{array}{l} \omega_n = \omega_0\left(1 + a\,\dfrac{N-Z}{A}\right) \\ \omega_p = \omega_0\left(1 - a\,\dfrac{N-Z}{A}\right) \end{array}\right\}, \tag{7}$$

we solve the RPA equation for a given value of $X_1/X_{0,\mathrm{sc}}$ as a function of the parameter a. The unperturbed quadrupole response consists of two excitations with frequencies $2\hbar\omega_n$ and $2\hbar\omega_p$, which carry the quadrupole strength proportional to N/ω_n and Z/ω_p, respectively. The latter is obtained from the energy-weighted sum rules, which are proportional to $N\langle r^2 \rangle_n$ and $Z\langle r^2 \rangle_p$, divided by $2\hbar\omega_n$ and $2\hbar\omega_p$, respectively, assuming $\langle r^2 \rangle_p = \langle r^2 \rangle_n$. The relation $\langle r^2 \rangle_p \approx \langle r^2 \rangle_n$ is valid for β-stable nuclei, while it is obtained also for the core part of neutron-rich nuclei towards the drip line. The harmonic oscillator potential does not have two independent parameters, which express the depth and the radius. In the present model the oscillator frequencies ω_n and ω_p are regarded as those expressing the depth of the nuclear potential or the frequency of one-particle excitation spectra, while the radius of the density distribution is supposed to be influenced by some other origin such as the Coulomb interaction. Two RPA solutions are obtained, of which the higher frequency solution we call as the IVGQR mode and the lower one as the ISGQR mode. Since the isoscalar (isovector) field interaction is attractive (repulsive), the ISGQR mode is the collective isoscalar mode, while the IVGQR mode is the collective isovector mode. In the present work we are interested in the isoscalar strength carried by the IVGQR mode and the isovector strength by the ISGQR mode. In Fig. 2 we show a numerical example for the parameters $Z = 20$, $N = 40$, and $X_1/X_{0,\mathrm{sc}} = -2.0$. The dotted line in Fig. 2 expresses the value of $((N\text{-}Z)/A)^2$. It is shown that at $a = -0.50$ the $B(IS)/B(IV)$ value for the IVGQR mode vanishes while the $B(IV)/B(IS)$ value for the ISGQR mode becomes equal to $((N\text{-}Z)/A)^2$. It is also interesting to observe that at $a = +0.25$ the $B(IS)/B(IV)$ value for the IVGQR mode becomes equal to $((N\text{-}Z)/A)^2$ while the $B(IV)/B(IS)$ value for the ISGQR mode vanishes.

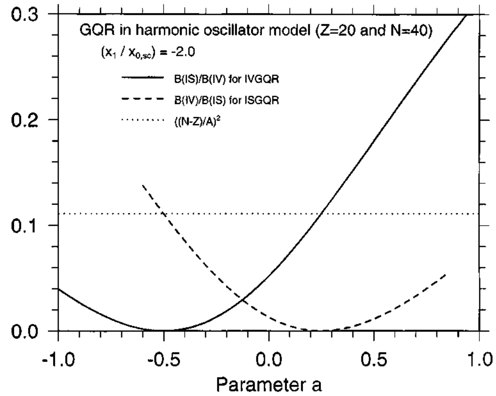

FIG. 2. The strength ratio $B(IS)/B(IV)$ carried by the IVGQR and $B(IV)/B(IS)$ contained in the ISGQR for the $Z = 20$ and $N = 40$ system as a function of the parameter a defined by Eq. (7). The solid and dashed curves are calculated for $X_1/X_{0,\mathrm{sc}} = -2.0$. The dotted line shows the value of $((N\text{-}Z)/A)^2 = 0.111$ in the present example. See the text for details.

Now, in order to obtain an estimate of V_1 in Eq. (6) we replace the isoscalar part of the harmonic oscillator potential for $a = 0, \frac{1}{2} M\omega_0^2 r^2$, by

$$\tfrac{1}{2} M\omega_0^2 r^2 - (N_F + 2)\hbar\omega_0 \Rightarrow \tfrac{1}{2} M\omega_0^2 r^2 - (\tfrac{3}{2}A)^{1/3}\hbar\omega_0 \quad \text{for} \quad N_F \gg 1, \tag{8}$$

so that the potential is attractive inside the nucleus. We define the average depth of the potential by

$$V_0 \equiv \frac{\frac{1}{2}M\omega_0^2\langle r^4\rangle - (\frac{3}{2}A)^{1/3}\hbar\omega_0\langle r^2\rangle}{\langle r^2\rangle} < 0. \tag{9}$$

Then, using the expression (7) the variation of the isovector field is written as

$$\delta V = V_0 2a\tau_z \frac{\delta\rho_1}{\rho_0}. \tag{10}$$

Comparing expression (10) with Eq. (6), we obtain the correspondence

$$V_1 \text{ in Eq (6)} \Leftrightarrow 8aV_0. \tag{11}$$

Then, from Eqs. (4), (5), and (11) the ratio of the self-consistent isovector quadrupole coupling constant to $X_{0,\mathrm{sc}}$ is written as

$$\frac{X_{1,\mathrm{sc}}}{X_{0,\mathrm{sc}}} = -5a\left(1 - \frac{(\frac{3}{2}A)^{1/3}\hbar\omega_0}{\frac{1}{2}M\omega_0^2\langle r^2\rangle}\frac{\langle r^2\rangle^2}{\langle r^4\rangle}\right) = 5.0a \tag{12}$$

where we have used $\langle r^2\rangle^2/\langle r^4\rangle = 0.75$ obtained for $N_F \gg 1$.

From expression (12) we obtain $a = -0.4$ for $X_{1,\mathrm{sc}}/X_{0,\mathrm{sc}} = -2.0$. The value of $a = -0.4$ is compared with the value of $a = -0.5$ in Fig. 2, for which the $B(IS)/B(IV)$ value for the IVGQR mode vanishes while the $B(IV)/B(IS)$ value for the IVGQR mode becomes equal to $((N\text{-}Z)/A)^2$. Considering that relation (12) is estimated for $N_F \gg 1$, the similar values of the above two a go very well with the observation that in the realistic self-consistent RPA calculation the isoscalar GR carries the isovector strength with $B(IV)/B(IS) \approx ((N\text{-}Z)/A)^2$, while the isovector GR contains little of the isoscalar strength.

In the present harmonic oscillator model the negative value of $X_{1,\mathrm{sc}}/X_{0,\mathrm{sc}}$ corresponds to $\omega_p > \omega_n$ for the $N > Z$ case. This is consistent with the fact that for $N > Z$ nuclei the depth of the nuclear potential in HF calculations is deeper for protons than for neutrons. It is consistent also with the fact that the average energy of the unperturbed particle-hole (p-h) quadrupole (the so-called $\Delta N = 2$) excitations in HF calculations is lower for neutrons than for protons even in the $N > Z$ nuclei along the β stability line. In contrast, in the traditional shell model calculation in which harmonic oscillator wave functions are used, the Coulomb interaction (or Coulomb potential) is often not included. The presence of the Coulomb potential is simulated by adjusting the parameters ω_p and ω_n, since the Coulomb potential has an r^2 dependence inside nuclei assuming a constant charge distribution. Then, in the presence of neutron excess the values used traditionally are $\omega_p < \omega_n$, which corresponds to $a > 0$. For example, $a = +1/3$ is used in order to get approximately equal neutron and proton rms radii in β stable nuclei with neutron excess [4]. Then, it is unlikely that in such shell model calculations the ratio of the contribution by protons to that by neutrons in collective excitations can be properly estimated.

In conclusion, using a harmonic oscillator model, we have shown that the isovector GQR carries little of the isoscalar quadrupole strength while the isoscalar GQR contains an isovector strength with $B(IV)/B(IS) \approx ((N\text{-}Z)/A)^2$, if the consistency condition is fulfilled between the parameters of the one-particle excitation spectra and the vibrational field. What is shown is exactly what is exhibited in realistic self-consistent RPA calculations, of which the Hamiltonian contains also the Coulomb interaction. The ratio of the contribution by neutrons to that by protons to collective excitations of nuclei with $N \neq Z$ can hardly be properly estimated in the model, in which the consistency condition is not taken into account. In particular, the shell model with parameters $\omega_p < \omega_n$ for nuclei with $N > Z$ may be in trouble.

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