Three-body effects in the $(d,{}^2\mathrm{He})$ charge-exchange reaction

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The importance of an explicit treatment of continuum channels in the $(d,{}^2\mathrm{He})$ charge-exchange reaction is investigated. The continuum channel effects are clarified by a comparison of the full three-body results with calculations which use the distorted waves approximation to the three-body model. Continuum channel effects are shown to reduce the calculated ${}^{12}\mathrm{C}(d,{}^2\mathrm{He}){}^{12}\mathrm{B}$(g.s.) differential cross section at 270 MeV incident deuteron energy. This reduction is consistent with that from an earlier *ad hoc* modification of the absorptive content of an assumed ^{2}He optical potential within distorted waves Born approximation calculations. [S0556-2813(99)05408-4]

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The potential advantages of a pure spin ($\Delta S=1$) and isospin ($\Delta T=1$) selection in the $(d,{}^2\mathrm{He})$ charge-exchange reaction, when populating very low energy two-proton relative motion (${}^1S_0$) configurations, have recently been discussed [1,2]. Differential cross section and also analyzing power measurements, for the $(d,{}^2\mathrm{He})$ reaction on a ^{12}C target at 270 MeV incident deuteron energy, were also reported by Okamura *et al.* [2]. The measurements were made in a kinematical condition which selects, essentially exclusively, the ^{2}He (${}^1S_0, T=1$) channel. Development of the $(d,{}^2\mathrm{He})$ reaction, for spectroscopy, requires a realistic understanding and description of the reaction mechanism in which, potentially, both the continuum coupling (breakup) of the deuteron and an explicit treatment of the unbound three-body final state are necessary.

In Ref. [2] the 270 MeV $(d,{}^2\mathrm{He})$ data were analyzed using the distorted waves Born approximation (DWBA), with transition amplitude

$$\mathcal{T}^{\mathrm{DW}}(\boldsymbol{K},\boldsymbol{K}'\boldsymbol{k})=\langle \chi_{pp}^{(-)}(\boldsymbol{K}')\,\phi_{pp}^{(-)}(\boldsymbol{k})|M^{\mathrm{AB}}|\chi_d^{(+)}(\boldsymbol{K})\,\phi_d\rangle, \tag{1}$$

where $\chi_d^{(+)}$ and $\chi_{pp}^{(-)}$ are distorted waves for the center of mass (c.m.) motions of the deuteron and ^{2}He with asymptotic wave numbers $\boldsymbol{K}$ and $\boldsymbol{K}'$, respectively. ϕ_d is the deuteron ground state wave function and $\phi_{pp}^{(-)}$ is a ${}^1S_0$ pp scattering wave function of relative wave number $\boldsymbol{k}$. Since the measurements detected ^{2}He with internal kinetic energies up to only 1 MeV, ${}^3P_J$ pp configurations can be neglected [2,3]. In Eq. (1)

$$M^{\mathrm{AB}}(\boldsymbol{r},\boldsymbol{R})=\langle \Phi_B|\Delta V|\Phi_A\rangle \tag{2}$$

is the matrix element of ΔV, the sum of the nucleon-nucleon (NN) charge-exchange interactions of the incident (i) and target (j) nucleons, between the target (A) and residual nucleus (B) states. Figure 1 shows the coordinates used. The assumption of an optical potential for the unbound ^{2}He+B system equal to that for the incident deuteron [2] was found to provide a very poor description of the experimental data. Increasing the absorptive part of the final state ^{2}He+B optical potential, by a factor of 3, lead to a much improved description. The changes in the magnitudes of the calculated differential cross sections were about a factor of 2. It was suggested that, qualitatively, the increased absorption was reasonable, given the neglect of explicit three-body final state effects in the DWBA, but that a more explicit treatment of such effects was needed. Here we consider these effects. Related considerations, for distorted waves treatments of unbound three-body final states in the case of the $({}^3\mathrm{He},pp)$ and (p,d^*) single-nucleon transfer reactions, are discussed in Refs. [4] and [5] and references therein.

In this Brief Report we present calculations of the differential cross section for the ${}^{12}\mathrm{C}(d,{}^2\mathrm{He}){}^{12}\mathrm{B}$(g.s.) reaction based on a two nucleon+target three-body description. Here we include a consistent treatment of these three-body degrees of freedom in both the entrance and exit channels, and so avoid reference to an artificial ^{2}He+B final state distorting interaction. Our calculations are formulated within a few-body model based on straight line trajectories (eikonal approximation) which has been used with success in deuteron elastic scattering analyses [6] at energies similar to those of

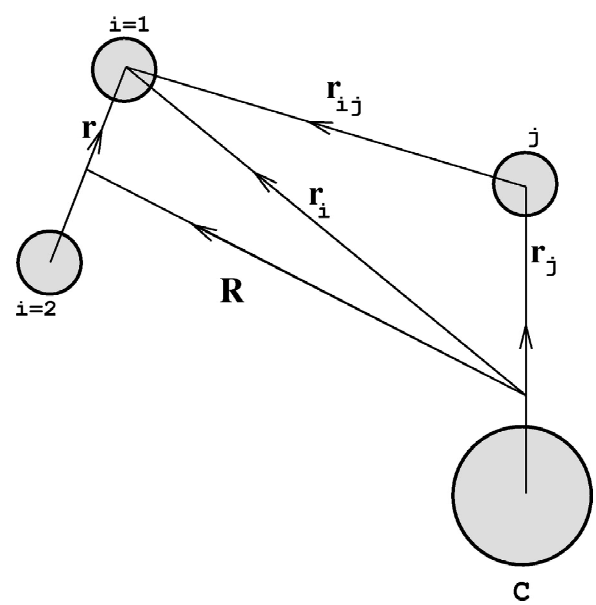

FIG. 1. Coordinates for the two-nucleon+target system. The projectile nucleons ($i=1,2$) charge exchange with a target nucleon (j) bound with respect to the remaining core (C) of target nucleons.

interest here. To clarify the role of the three-body continuum channels both the full three-body charge-exchange transition amplitude and its distorted waves limit are evaluated.

The exact post-form transition amplitude for the $A(d,{}^2\mathrm{He})B$ reaction is

$$\mathcal{T}^{\mathrm{CE}}(\boldsymbol{K},\boldsymbol{K}'\boldsymbol{k})=\langle \boldsymbol{K}'\,\phi_{pp}^{(-)}(\boldsymbol{k})\Phi_B|V|\Psi_{dA}^{(+)}(\boldsymbol{K})\rangle, \tag{3}$$

where $\Psi_{dA}^{(+)}$ is the exact many-body wave function with deuteron incident wave boundary conditions on target A and V is the full many-body interaction of the incident and target nucleons. $\boldsymbol{K}'$ denotes a plane wave state for the pp center of mass in the final state. With an effective three-body description in mind, we introduce an interaction U, which we will take to be the sum of the optical potentials for the incident nucleons $(i=1,2)$ with the target. Thus, $\Delta V\ (=V-U)$ is assumed a perturbation, responsible for the charge-exchange process, and $U(\boldsymbol{R},\boldsymbol{r})\ [=U_1(\boldsymbol{r}_1)+U_2(\boldsymbol{r}_2)]$ is responsible for the distortion of each nucleon, including breakup effects. The charge-exchange process, and hence ΔV, is treated only to first order. The three-body transition amplitude can then be written

$$\mathcal{T}^{\mathrm{CE}}(\boldsymbol{K},\boldsymbol{K}'\boldsymbol{k})=\langle \mathcal{X}_{pp}^{(-)}(\boldsymbol{K}',\boldsymbol{k})|M^{\mathrm{AB}}|\mathcal{X}_{d}^{(+)}(\boldsymbol{K})\rangle. \tag{4}$$

Here $\mathcal{X}_{pp}^{(-)}$ and $\mathcal{X}_{d}^{(+)}$ are *not* distorted waves, but are three-body wave functions, with the exit and entrance channel boundary conditions, respectively, for the two interacting nucleons moving in the presence of the potential U. Treating each of these three-body wave functions in the eikonal (or Glauber) model, e.g., [6,7], we obtain the eikonal approximation to the three-body charge-exchange transition amplitude

$$\mathcal{T}^{\mathrm{E}}(\boldsymbol{K},\boldsymbol{K}'\boldsymbol{k})=\int d\boldsymbol{R}\ \exp(i\boldsymbol{q}\cdot\boldsymbol{b})\times\langle \phi_{pp}^{(-)}(\boldsymbol{k})|M^{\mathrm{AB}}\ \exp[i\chi_U(\boldsymbol{b},\boldsymbol{s})]|\phi_d\rangle, \tag{5}$$

where the bra-ket now denotes integration over the two nucleon spin and relative motion ($\boldsymbol{r}$) coordinates only. Here $\boldsymbol{q}=\boldsymbol{K}-\boldsymbol{K}'$ is the momentum transfer and $\boldsymbol{b}$ is the two-nucleon's c.m. impact parameter, the component of vector $\boldsymbol{R}$ in the plane perpendicular to the beam direction. χ_U is the eikonal phase-shift function for the potential U, at c.m. impact parameter $\boldsymbol{b}$,

$$\chi_U(\boldsymbol{b},\boldsymbol{s})=-\frac{\mu_i}{\hbar^2K}\int_{-\infty}^{\infty}dR_3'U(\boldsymbol{R}',\boldsymbol{r}), \tag{6}$$

and depends also on $\boldsymbol{s}$, the component of $\boldsymbol{r}$ in the plane perpendicular to $\boldsymbol{K}$. μ_i is the initial state reduced mass and R_3' is the z component of $\boldsymbol{R}'=(\boldsymbol{b},R_3')$, with the z-axis chosen along the incident momentum $\boldsymbol{K}$. The integral over R_3 in Eq. (5) now involves only the structure matrix element M^{AB}. The sudden/adiabatic and eikonal approximations involved in writing this amplitude are expected to be very reliable given the range of excitation energies of interest in the 270 MeV charge exchange data. In combining the three-body eikonal phases from the initial and final states in Eq. (5), to form χ_U, we have however assumed that U is spin and isospin independent. This has implications for the treatment of the Coulomb interaction, which is discussed below.

Since the potential U is the sum of the nucleon-target optical potentials, $\mathcal{T}^{\mathrm{E}}$ retains three-body effects to all orders. Furthermore, we see these three-body effects arise as a result of χ_U entering the integral over $\boldsymbol{r}$. In addition to the explicit inclusion of the three-body nature of the $pp+B$ final state $\mathcal{T}^{\mathrm{E}}$ also includes new and interfering paths to the final state from the continuum of the np system. In the limit that U acts only on the center of mass of the two nucleons, i.e., is replaced by an optical potential $U_{\mathrm{opt}}(\boldsymbol{R})$, then we obtain the distorted waves limit of the eikonal amplitude

$$\mathcal{T}^{\mathrm{E,DW}}(\boldsymbol{K},\boldsymbol{K}'\boldsymbol{k})=\int d\boldsymbol{R}\ \exp(i\boldsymbol{q}\cdot\boldsymbol{b})\times\langle \phi_{pp}^{(-)}(\boldsymbol{k})|M^{\mathrm{AB}}|\phi_d\rangle\exp[i\chi_{\mathrm{opt}}(\boldsymbol{b})]. \tag{7}$$

The c.m. distortion now enters as the $\boldsymbol{r}$ independent eikonal phase

$$\chi_{\mathrm{opt}}(\boldsymbol{b})=-\frac{\mu_i}{\hbar^2K}\int_{-\infty}^{\infty}dR_3'U_{\mathrm{opt}}(\boldsymbol{R}'). \tag{8}$$

Here we calculate the ${}^{12}\mathrm{C}(d,{}^2\mathrm{He}){}^{12}\mathrm{B}$(g.s.) reaction differential cross section as a function of the emerging ^{2}He c.m. angle, integrated over the range (0–1 MeV) of detected ^{2}He relative energies ε and all relative solid angles, i.e.,

$$\frac{d\sigma}{d\Omega_{K'}}=\frac{1}{2}\int_0^{1\ \mathrm{MeV}}d\varepsilon\int d\Omega_k\frac{d^3\sigma}{d\varepsilon d\Omega_{K'}d\Omega_k}. \tag{9}$$

The $\frac{1}{2}$ factor arises from the identity of the two protons [2] in the final state. The triple differential cross section is

$$\frac{d^3\sigma}{d\varepsilon d\Omega_{K'}d\Omega_k}=\frac{\mu_i\mu_f}{(2\pi\hbar^2)^2}\frac{K'}{K}\rho(\varepsilon)\times\frac{1}{(2S_d+1)(2J_A+1)}\sum_{\sigma\sigma'MM'}|\mathcal{T}|^2, \tag{10}$$

where S_d and J_A are the deuteron and target spins and the notation implies the averaging over initial state and sums over final state spin projections. The density of states factor is $\rho(\varepsilon)=\mu_{pp}\hbar k/(2\pi\hbar)^3$, and μ_{pp} is the pp and μ_f the ${}^2\mathrm{He}+\mathrm{B}$ reduced mass.

The Hulthén wave function [8] is used for the deuteron ground state and the ${}^1S_0$ ^{2}He scattering states are generated using Reid's soft-core potential [9]. Here, in ΔV, we include only the central $t_{\sigma\tau}$ effective interaction (at 140 MeV) as tabulated by Franey and Love [10,11]. It was shown in [2] that contributions from the tensor term are small for the excitation to the ^{12}B ground state. Single-nucleon knock-on exchange effects are included via the short-range prescription, discussed in Refs. [10,11]. The nucleon bound state single-particle radial wave functions for the target nucleons

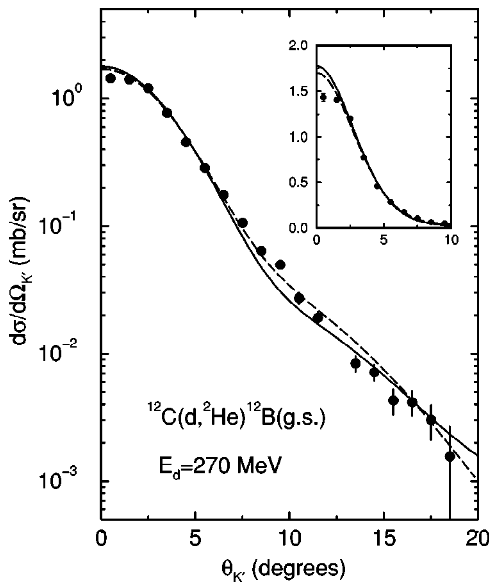

FIG. 2. Differential cross sections for three-body model calculations of the $^{12}C(d,{}^2He)^{12}B$(g.s.) reaction at 270 MeV in the presence (solid lines) and the absence (dashed lines) of the proton's Coulomb interaction. The inset shows the same calculations on a linear scale. All calculations include a normalization factor of 0.6. The data are from Ref. [2].

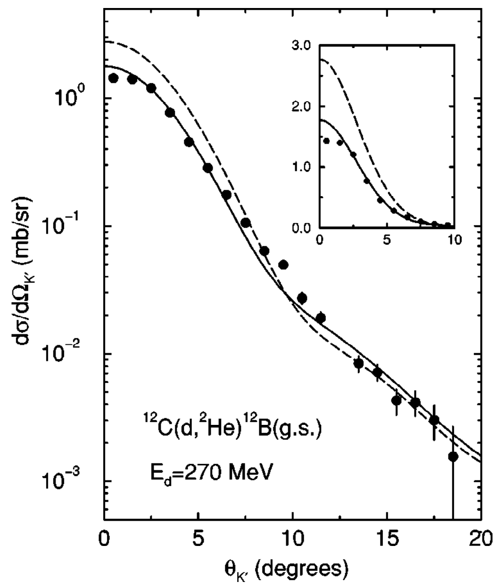

FIG. 3. Differential cross sections for three-body (solid line) and distorted wave (dashed line) model calculations of the $^{12}C(d,{}^2He)^{12}B$(g.s.) reaction at 270 MeV. The inset shows the same calculations on a linear scale. All calculations include a normalization factor of 0.6. The data are from Ref. [2].

are calculated using a Woods-Saxon potential well, the depth of which is adjusted to reproduce the experimental separation energy. The parameters assumed are those used in [2], a radius $r_0 = 1.25$ fm, a diffuseness $a = 0.65$ fm and a spin-orbit potential strength $V_{LS} = 6$ MeV. The one-body density matrices for the target structure are taken from the analysis of Brady *et al.* [12].

In the calculations of $\mathcal{T}^{\mathrm{E}}$ the proton- and neutron-target optical potentials used are the Schrödinger equivalent potentials to the Dirac optical potential global parametrization of data of Hama *et al.* [13]. For the calculation of $\mathcal{T}^{\mathrm{E,DW}}$ these same interactions are averaged (folded) over the deuteron ground state wave function to produce U_{opt}. This is the appropriate interaction for the no breakup limit of the entrance channel. As was mentioned earlier, in writing Eq. (5) we assume isospin independence of U, and so we also assume that the nucleon optical potentials are the same in both the entrance and exit channels. It follows that the differences in the Coulomb interactions acting in the initial (np) and final (pp) states are not treated exactly.

We present calculations in Fig. 2, together with the data [2], which show that the effects of the Coulomb interaction on the cross section are small for the present energy and target. We compare the differential cross-sections for the $^{12}C(d,{}^2He)^{12}B$(g.s.) reaction at 270 MeV showing the full eikonal three-body model calculations in the complete absence (dashed lines) and the presence (solid lines) of the Coulomb interaction acting on a single proton in the entrance and exit channels. The error involved in not treating the Coulomb interactions exactly is clearly rather small. Our treatment of the Coulomb interaction in the eikonal amplitude follows the conventional screening procedure, such as can be found in [6]. The inset in Fig. 2 shows the small angle cross section on a linear scale. All calculations shown include an overall normalization factor of 0.6. This scaling was also required, even after renormalization of the final state absorption, in the DWBA calculations of Ref. [2] and its origin remains to be clarified. Our principal interest here is to assess the importance of the three-body effects, as determined by calculations based on $\mathcal{T}^{\mathrm{E}}$ and on $\mathcal{T}^{\mathrm{E,DW}}$.

The calculated differential cross sections using the three-body (solid curves) and distorted waves (dashed curves) transition amplitudes are shown in Fig. 3, together with the data [2]. The inset shows the cross sections on a linear scale. The differences in these calculations should provide a good indication of the importance of three-body effects, from both the initial and final states, within the reaction mechanism. The differences are significant. The three-body effects reduce the magnitude of the differential cross section by almost a factor of two. This magnitude of effect is the same as was obtained in the DWBA analysis of Ref. [2], when the absorption of the assumed exit channel optical potentials was increased by a factor of 3 to better reproduce the cross section and analyzing power data. Whether, in the present calculations, this reduction arises principally from our three-body treatment of the final state, or from destructive interference between new pathways opened up to the final state from the entrance channel continuum, is not delineated. The level of absorption required phenomenologically does appear however to arise quite naturally within the three-body description when using realistic nucleon-target interactions and absorption.

In summary, we have presented a three-body eikonal description of the $(d,{}^2He)$ charge-exchange reaction, as an alternative to the DWBA, and which retains two-nucleon +target three-body continuum effects to all orders. We show, by comparing the full three-body calculations with their distorted waves (no breakup) limit, that these three-body effects are very significant for calculations of the reaction differential cross sections. Comparisons with earlier DWBA calculations suggest that these three-body effects may explain the necessity for *ad hoc* increases in the final state absorption in those effective two-body calculations.

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