Nucleon form factors in a chiral constituent-quark model

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The electromagnetic form factors of the nucleon have been calculated in a chiral constituent-quark model. The nucleon wave functions are obtained by solving a Schrödinger-type equation for a semirelativistic Hamiltonian with an effective interaction derived from the exchange of mesons belonging to the pseudoscalar octet and singlet and a linear confinement potential. The charge-density current operator has been constructed consistently with the model Hamiltonian in order to preserve gauge invariance and to satisfy the continuity equation. [S0556-2813(99)04008-X]

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I. INTRODUCTION

For a long time constituent-quark models (CQM's) have been proposed to explain spectroscopic properties of hadrons within a nonrelativistic framework [1] (see also Refs. [2,3], and references therein). In these models the effective degrees of freedom are massive quarks moving in a long-range confinement potential with the basic SU(6) spin-flavor symmetry. According to the analysis of Ref. [4] the residual interaction responsible for the SU(6) symmetry breaking is described by the one-gluon-exchange diagram and is identified with the hyperfinelike part of its nonrelativistic reduction. Given the fact that the mass of the three constituent quarks is small, the nonrelativistic approximation is not *a priori* justified. In addition some relativistic corrections are also necessary to account for the observed small spin-orbit effects [5,6]. Relativized versions of CQM's were then discussed [7,8]. Alternatively, relativity is considered from the very beginning in the CQM adopting the light-front formalism [9,10] or using the so-called Bakamjian-Thomas construction [11] to derive the Poincaré invariant formulation of the quark model for baryons [12].

While rather successful in describing the octet and decuplet ground states, these models still face some problems, such as, e.g., the wrong level orderings of positive- and negative-parity excitations, which can be traced back to inadequate symmetry properties of the one-gluon-exchange interaction.

The existence of an increasing number of near-parity doublets in the high-energy sector suggests that the approximate chiral symmetry of quantum chromodynamics (QCD) is realized in the hidden Nambu-Goldstone mode at low excitation and in the explicit Wigner-Weyl mode at high excitation [13]. Thus the spontaneous breaking of chiral symmetry and the associated appearence of the octet of pseudoscalar mesons as the approximate Goldstone bosons induces a chiral interaction between quarks that is mediated by such mesons [14]. Its spin and flavor dependence modifies the symmetry properties of the Hamiltonian and ultimately leads to a correct ordering of the positive- and negative-parity states in the baryon spectra [15].

Various hybrid models have been constructed advocating meson exchanges in addition to sizeable contributions coming from gluon exchanges (see, e.g., Refs. [16–20]).

Recently, a chiral CQM has been proposed whose effective quark-quark interaction is derived from Goldstone-boson exchange alone involving the pseudoscalar meson octet and singlet [21,22]. The model is capable of providing a unified description not only of the nucleon and Δ spectra but also of all strange baryons.

A stringent test of the model would be to probe its eigensolutions in the description of the electromagnetic properties of baryons. In this paper the nucleon electromagnetic form factors are calculated without free parameters starting from the nucleon wave functions obtained with the model of Refs. [21,22] and using a charge-current density operator consistently derived along the lines proposed in Ref. [23]. The model is briefly reviewed in Sec. II, while the expression of the charge-current density operator is given in Sec. III. The results are presented and discussed in Sec. IV.

II. THE MODEL

The chiral model of Refs. [21,22] is semirelativistic in the sense that the kinetic energy operator is taken in the relativistic form:

$$H_0=\sum_{i=1}^{3}\sqrt{\vec{p}_i^{\,2}+m_i^2}, \tag{1}$$

with m_i the masses and $\vec{p}_i$ the three-momenta of the constituent quarks. This form ensures the average quark velocity to be lower than the light velocity, a requirement that is usually not fulfilled by nonrelativistic models. In addition, by the choice (1) one excludes negative-energy states *ab initio* and simply solves a Schrödinger equation for bound states without facing all the complications of a fully covariant treatment of the three-quark system.

The dynamical part consists of a linear confinement potential,

$$V_{\rm conf}(\vec{r}_{ij})=V_0+Cr_{ij} \tag{2}$$

depending on the interquark distance r_{ij} and the two fitting parameters V_0 and C, and a sum of pseudoscalar meson-exchange potentials:

$$V_\chi^{\rm octet}(\vec{r}_{ij})=\left[\sum_{a=1}^{3} V_\pi(\vec{r}_{ij})\lambda_i^a\lambda_j^a + \sum_{a=4}^{7} V_K(\vec{r}_{ij})\lambda_i^a\lambda_j^a+V_\eta(\vec{r}_{ij})\lambda_i^8\lambda_j^8\right]\vec{\sigma}_i\cdot\vec{\sigma}_j, \tag{3}$$

$$V_\chi^{\rm singlet}(\vec{r}_{ij})=\frac{2}{3}V_{\eta'}(\vec{r}_{ij})\vec{\sigma}_i\cdot\vec{\sigma}_j, \tag{4}$$

where $\vec{\sigma}_i$ and $\vec{\lambda}_i$ are the quark spin and flavor matrices, respectively. In the static approximation used in Refs. [21,22], the meson-exchange potentials are given by

$$V_\gamma(\vec{r}_{ij})=\frac{g_\gamma^2}{4\pi}\frac{1}{12m_im_j}\left[\mu_\gamma^2\frac{e^{-\mu_\gamma r_{ij}}}{r_{ij}}-4\pi\delta(\vec{r}_{ij})\right] \tag{5}$$

with μ_γ being the meson masses and g_γ the meson-quark coupling constants ($\gamma=\pi,K,\eta,\eta'$). In the chiral limit there is only one coupling constant g_8 for all Goldstone bosons. Due to the special character of the singlet η' meson, its coupling constant g_0 was allowed to deviate from g_8.

Since one deals with structured particles (constituent quarks and mesons) of finite extension, one has to smear out the δ function in Eq. (5). In Refs. [21,22] a Yukawa-type smearing was used, i.e.,

$$4\pi\delta(\vec{r}_{ij})\rightarrow\Lambda_\gamma^2\frac{e^{-\Lambda_\gamma r_{ij}}}{r_{ij}}, \tag{6}$$

involving the cutoff parameters Λ_γ, which were assumed to follow a linear scaling with meson masses:

$$\Lambda_\gamma=\Lambda_0+\kappa\mu_\gamma. \tag{7}$$

Once the quark masses are fixed, the model has five fitting parameters, i.e., the depth V_0 and the slope C of the confinement potential, the ratio g_0/g_8 of the singlet to octet meson-quark coupling constants and the two parameters Λ_0 and κ defining Λ_γ.

The specific spin-flavor symmetry inherent in the chiral potential is responsible for the correct level structure. According to the selected values of the quark masses different sets of numerical values for the five free parameters can produce fits to the baryon spectra of comparable quality.

The Schrödinger-type equation for the model is accurately solved in the stochastic variational method [24] using wave functions that are expanded in basis functions involving correlated Gaussians as follows:

$$\Phi^k_{JM,TM_T}(\vec{x}_k,\vec{y}_k)=x_k^\lambda y_k^l\exp(-\beta x_k^2-\delta y_k^2+\gamma\vec{x}_k\cdot\vec{y}_k)\times[\mathcal{Y}^L_{\lambda l}(\hat{x}_k,\hat{y}_k)\otimes\chi^k_{(s_0,1/2)S}]_{JM}\chi^k_{(t_0,1/2)TM_T}, \tag{8}$$

depending on the Jacobi coordinates of partition k,

$$\vec{x}_k=\vec{r}_p-\vec{r}_q,$$

$$\vec{y}_k=\vec{r}_k-\frac{m_p\vec{r}_p+m_q\vec{r}_q}{m_p+m_q}, \tag{9}$$

where $\vec{r}_i$ and m_i ($i=1,2,3$) are the particle coordinates and masses, and (k,p,q) is an even permutation of $(1,2,3)$. The $\mathcal{Y}^L_{\lambda l}$ represent the bipolar spherical harmonics

$$\mathcal{Y}^{LM_L}_{\lambda l}(\hat{x}_k,\hat{y}_k)=[Y_\lambda(\hat{x}_k)\otimes Y_l(\hat{y}_k)]_{LM_L}. \tag{10}$$

The spin (isospin) parts arise from coupling single-particle spins (isospins) following the scheme

$$\chi^k_{(s_0,1/2)SM_S}=[\chi^{(pq)}_{(1/2,1/2)s_0}\otimes\chi^k_{1/2}]_{SM_S},$$

$$\chi^{(pq)}_{(1/2,1/2)s_0m_0}=[\chi^p_{1/2}\otimes\chi^q_{1/2}]_{s_0m_0}. \tag{11}$$

For a given total angular momentum J and isospin T the stochastic variational method selects basis functions according to a set of six discrete parameters (L,λ,l,S,s_0,t_0) and three continuous parameters (β,γ,δ). L is the total orbital angular momentum, λ and l are the orbital angular momenta corresponding to $\vec{x}_k$ and $\vec{y}_k$, S is the total spin, s_0 the spin and t_0 the isospin of the subsystem (pq).

The total wave function is composed of a symmetrized linear combination of basis wave functions of the form (8).

The Schrödinger equation can also be solved in momentum space. The basis functions are of the same form as in Eq. (8) in terms of the corresponding Jacobi conjugate momenta $(\vec{p}_{x_k},\vec{p}_{y_k})$. Assuming the c.m. at rest, the Hamiltonian H_0 in Eq. (1) can be rewritten in terms of the conjugate momenta p_{y_k} alone as

$$H_0=\sum_{k=1}^{3}\sqrt{\vec{p}^{\,2}_{y_k}+m_k^2}, \tag{12}$$

where the sum runs over the three possible partitions. The method has the clear advantage of producing analytical solutions for the total baryon wave function both in momentum and space coordinates.

III. THE CHARGE-CURRENT DENSITY OPERATOR

The relativistic form of the kinetic energy does not permit the use of the traditional one-body current density operator, nor is it necessary to adopt sophisticated procedures to include relativistic effects, such as those proposed, e.g., in Refs. [7,9,25–27]. For each partition the model Hamiltonian has the same structure of the semirelativistic Hamiltonian considered in Ref. [23], i.e.,

$$H_k=\sqrt{\vec{p}_{y_k}^{\,2}+m_k^2}+V_{\rm conf}(\vec{x}_k)+V_\chi^{\rm octet}(\vec{x}_k)+V_\chi^{\rm singlet}(\vec{x}_k). \tag{13}$$

Following the functional derivative formalism proposed in Ref. [23], a gauge-invariant charge-current density operator can be derived consistently. It contains both one- and two-body terms. The one-body contribution includes the charge, the convective- and the spin-current operators. For a particle of charge e and mass m the matrix elements between free particle states are given in momentum space by the following expressions, respectively:

$$\langle\vec{p}\,'|j_0(\vec{x})|\vec{p}\rangle=e\,\frac{1}{(2\pi)^3}\,e^{i(\vec{p}-\vec{p}\,')\cdot\vec{x}}, \tag{14}$$

$$\langle\vec{p}\,'|\vec{j}(\vec{x})|\vec{p}\rangle=e\,\frac{\vec{p}+\vec{p}\,'}{E_{\vec{p}}+E_{\vec{p}\,'}}\,\frac{1}{(2\pi)^3}\,e^{i(\vec{p}-\vec{p}\,')\cdot\vec{x}}, \tag{15}$$

$$\langle\vec{p}\,',s'|\vec{j}^S(\vec{x})|\vec{p},s\rangle=\frac{ie}{E_{\vec{p}}+E_{\vec{p}\,'}}\langle s'|\vec{\sigma}\times(\vec{p}\,'-\vec{p})|s\rangle \times\frac{1}{(2\pi)^3}\,e^{i(\vec{p}-\vec{p}\,')\cdot\vec{x}}, \tag{16}$$

where

$$E_{\vec{p}}=\sqrt{\vec{p}^{\,2}+m^2},\quad E_{\vec{p}\,'}=\sqrt{\vec{p}\,'^{2}+m^2}. \tag{17}$$

With respect to the usual nonrelativistic expressions, in the semirelativistic approach only the spatial components of the charge-current density operator are affected, while the time component is simply given by the charge density. In particular, one does not have a Darwin-Foldy term. This term arises in the nonrelativistic reduction of the Dirac equation which has negative-energy solutions, whereas the semirelativistic Hamiltonian does not have such solutions and the charge-current density operator is here obtained without any nonrelativistic expansion. Incidentally, the energy denominator appearing in the current matrix elements reduces to $2m$ in the low-energy limit recovering the nonrelativistic approximation.

The two-body current operator can be derived directly from the continuity equation consistently with the Hamiltonian of Eq. (13) (see, e.g., Ref. [28]), i.e.,

$$\vec{j}_{\rm ex}(\vec{p}_1,\vec{p}_2)=-ie\sum_\gamma\{[\vec{\tau}^{(1)}\times\vec{\tau}^{(2)}]_3\delta_{\gamma\pi}+(\lambda_4^{(1)}\lambda_5^{(2)}-\lambda_5^{(1)}\lambda_4^{(2)})\delta_{\gamma K}\}\times\left\{\frac{g_\gamma^2\mu_\gamma^2}{12m_1m_2}\,\frac{\vec{p}_2-\vec{p}_1}{(\mu_\gamma^2+p_1^2)(\mu_\gamma^2+p_2^2)}-(\mu_\gamma\to\Lambda_\gamma)\right\}\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}. \tag{18}$$

The $\lambda_{4,5}$ matrices are related to the SU(2) V-spin subgroup contained within SU(3). Together with the exchange character of the isospin dependence of the two-body current they result from the commutator between the quark-quark potential and the charge density that depends on λ_3 and λ_8. Therefore, only pion and kaon exchanges are allowed.

No exchange-current operator can arise from the confining potential used in the model Hamiltonian, because it is neither isospin nor momentum dependent. In a relativistic approach, the explicit expression of the confining interaction depends on its Lorentz structure and its nonrelativistic reduction could give rise to momentum-dependent terms responsible for two-body exchange currents [17]. However, the issue is under debate and different options are available in the literature. Moreover, modifying the assumed confining potential of Eq. (2) would destroy the quality of the baryon spectra obtained in Refs. [21,22].

IV. THE NUCLEON FORM FACTORS

As a first test of the model the electromagnetic form factors of the nucleon have been calculated. In this case only the one-pion exchange part of the two-body interaction, Eq. (3), contributes. The calculation is fully consistent and without free parameters. In the following we give results for two parametrizations of the chiral constituent-quark model with different values for the constituent-quark masses. The first parametrization is from Refs. [21,22] with constituent-quark masses $m_{u,d}=340$ MeV. The second is a modified version with $m_{u,d}=250$ MeV and accordingly readjusted parameters of the model in order to obtain baryon spectra of similar quality.

The electric (G_E) and magnetic (G_M) form factors are plotted in Fig. 1 for the proton and in Fig. 2 for the neutron. The thin (thick) solid lines refer to $m_{u,d}=250$ (340) MeV.

Two remarks can be made on the results. First, the falloff of G_E^p and $G_M^{p,n}$ as a function of Q^2 is lower than observed. This Q^2 dependence reflects the fact that here the constituent quarks are assumed to be pointlike. As in other constituent-quark models this assumption underestimates the electromagnetic radii of the nucleon. Second, the value of $G_M^{p,n}$ at $Q=0$ does not reproduce the nucleon magnetic moment. However, the ratio G_M^p/G_M^n is in good agreement with the corresponding observed ratio of the proton to neutron magnetic moment, a feature common to all nonrelativistic constituent-quark models. The discrepancy at $Q=0$ is due to two effects. (a) Two-body currents do not contribute to the nucleon magnetic form factor because the Hamiltonian of Eqs. (3) and (4) does not contain the full axial dipole-dipole interaction that describes one-pion exchange completely. Therefore, it is not possible to mix different values of the orbital angular momentum with the same parity. (b) The semirelativistic form of the one-body current with an energy-dependent denominator suppresses its contribution with respect to the nonrelativistic case; it would therefore require a

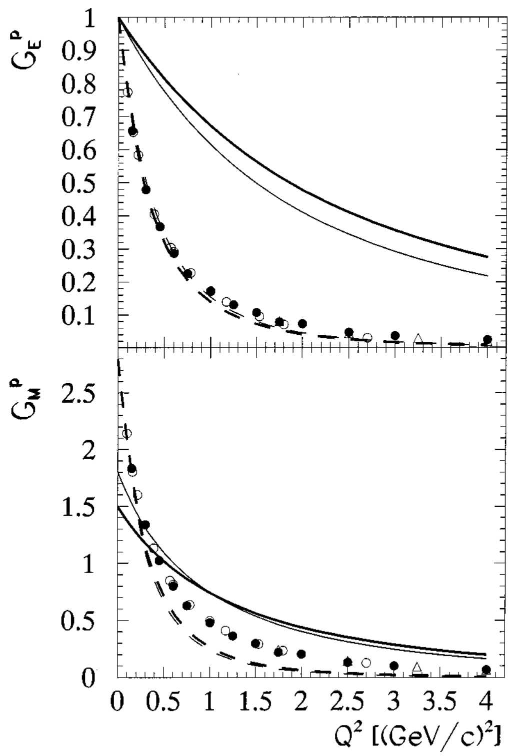

FIG. 1. The electric (G_E^p) and magnetic (G_M^p) form factors of the proton as a function of the four-momentum squared Q^2. The thin (thick) solid lines refer to a quark mass $m_{u,d}=250$ (340) MeV. The thin (thick) dashed lines for $m_{u,d}=250$ (340) MeV include the effects of electromagnetic form factors for quarks (see text). Experimental points are from Ref. [32] (solid circles), Ref. [33] (open circles) and Ref. [34] (triangles).

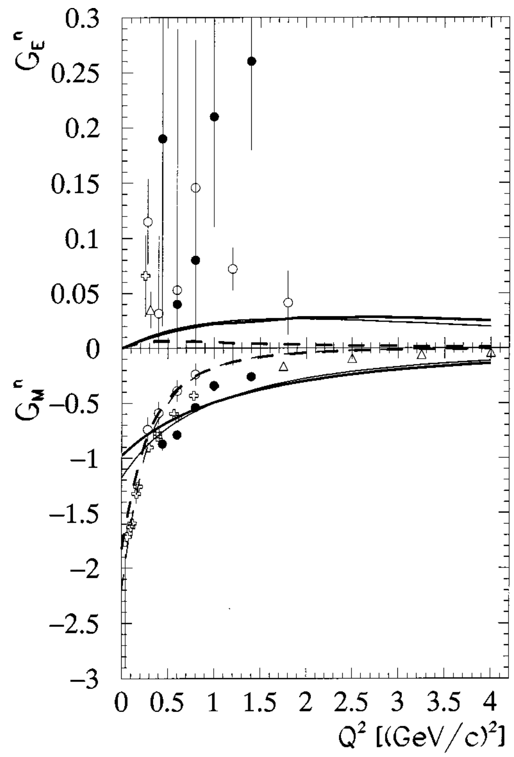

FIG. 2. The same as in Fig. 1, but for the neutron. Experimental points for G_E^n are from Ref. [35] (open circles), Ref. [36] (solid circles), Ref. [37] (cross), Ref. [38] (triangle) and for G_M^n from Ref. [33] (crosses), Ref. [34] (triangles), Ref. [35] (open circles), Ref. [36] (solid circles), respectively.

rather lower value for the constituent-quark mass in order to reproduce the nucleon magnetic moments.

In order to improve the quality of the results without destroying the agreement with the observed baryon spectra one has to consider that constituent quarks are effective degrees of freedom with some spatial extension [29,9]. As such, a charge form factor $f(Q^2)$ could be appended to the charge-density operator (14) and the convective part of the current density operator (15) as well as a magnetic form factor $g(Q^2)$ to the spin part of the current density operator (16). This will modify the Q^2 dependence. In fact, a rather good agreement with data can already be obtained for $G_M^{p,n}$ at $Q^2>0.5$ $(\mathrm{GeV}/c)^2$ assuming a simple dipole form factor

$$f(Q^2)=\frac{1}{[1+aQ^2]^2} \tag{19}$$

common to all (u and d) quarks. This is achieved with a rather small quark charge radius, i.e., $r_c=0.35$ fm, almost independently of its mass. Due to the small radius r_c the Q^2 dependence of G_E^p, although largely improved, could not yet be reproduced.

On the other hand, once constituent quarks are treated as extended objects, it is not unreasonable to introduce an anomalous magnetic moment κ in their electromagnetic form factor. Thus, besides a dipole form for $f(Q^2)$, the following form for $g(Q^2)$ has been considered:

$$g(Q^2)=f(Q^2)+\kappa\frac{1}{[1+bQ^2]^3}. \tag{20}$$

The actual value of κ has been fixed in order to obtain the experimental value of the proton magnetic moment. For a quark mass $m=340$ (250) MeV one obtains $\kappa=0.867$ (0.549). Correspondingly, the neutron magnetic moment turns out to be -1.828 (-1.812) n.m. in good agreement with experiment. The other two parameters a and b in Eqs. (19) and (20) are then fixed by fitting the Q^2 dependence of G_M^p. The resulting values for the quark charge and magnetic radius are $r_c=0.691$ fm and $r_m=1.050$ (0.935) fm with a quark mass $m=340$ (250) MeV. It is worth noting that the extracted value of the quark charge radius is significantly close to the value required by the assumption of vector-meson dominance. Without any free parameter one can then calculate the other nucleon form factors. The results are shown in Figs. 1 and 2 by the thin (thick) dashed lines for $m=250$ (340) MeV. A rather satisfactory agreement is obtained, especially for G_E^p.

The electric form factor of the neutron turns out to be too small in all cases. This is due to the deficiency of the charge-density operator (14) derived in Ref. [23]. In order to be consistent with the semirelativistic Hamiltonian no other contributions involving spin-dependent terms, like, e.g., the Darwin-Foldy correction, are possible and the quark charges add up to a total vanishing neutron charge.

V. CONCLUSIONS

A completely consistent calculation of the nucleon electromagnetic form factors has been performed within the chiral constituent-quark model proposed in Refs. [21,22]. Considering pointlike quarks is not sufficient to reproduce the observed form factors. In particular, the model eigenfunctions do not permit contributions from two-body currents arising from pion exchanges and the semirelativistic one-body current alone fails to produce the correct values of the nucleon magnetic moments. However, with the inclusion of suitable electromagnetic form factors for quarks and considering an anomalous quark magnetic moment a rather satisfactory agreement with data is obtained.

Possible improvements of the dynamic model are obviously under discussion. The pseudoscalar tensor term has been neglected in the model Hamiltonian of Refs. [21,22]. In Ref. [30] its effect on the light-baryon spectrum was estimated together with that of the Thomas-Fermi precession spin-orbit contribution arising from the scalar confining interaction. The result was that the agreement with the observed spectra was destroyed. The inclusion of vector-(ρ,ω,ϕ,K^*) and scalar-meson (σ) exchanges was considered in Ref. [31]. The tensor forces of vector- and pseudoscalar-meson-exchange interactions have opposite signs and largely cancel each other. The effects of the spin-orbit forces from the vector- and scalar-meson-exchange interactions are rather weak. The problem of the spin-orbit force from the Thomas-Fermi precession (which was not taken into account in Ref. [31]) remains, however. In any case, pseudoscalar tensor and vector-meson exchange contributions modify the model eigenfunctions, so that one can expect that the two-body currents will contribute even in the simplest case of the nucleon electromagnetic form factors. Work along these lines is in progress.

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