

# Isospin dependence of the single-particle potential of the $\Sigma$ hyperon in nuclear matter

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The isospin dependence of the real part of the single-particle potential of a  $\Sigma$  hyperon in nuclear matter,  $\hat{V}_\Sigma$  is investigated. The isospin-dependent part  $V_1$  of  $\hat{V}_\Sigma$  is expressed in terms of an effective  $\Sigma N$  interaction in nuclear matter, which depends on proton and neutron densities. With suitable approximations numerical results for  $V_1$  are obtained for four models of the Nijmegen baryon-baryon interaction. A comparison with recent ( $K^-$ ,  $\pi^\pm$ ) experiments favors model F as a realistic representation of the  $\Sigma N$  interaction. [S0556-2813(99)03008-3]

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## I. INTRODUCTION

The single-particle (s.p.) potential  $V_\Sigma(k_\Sigma)$  of the  $\Sigma^\pm$  and  $\Sigma^0$  hyperons in a spin-saturated nuclear matter with  $Z$  protons and  $N$  neutrons depends on the proton excess parameter  $\alpha = (Z - N)/A$ . Assuming charge independence of the baryon-baryon interaction we have, in the linear approximation in  $\alpha$ ,

$$\begin{aligned} V_{\Sigma^\pm}(k_\Sigma) &= V_0(k_\Sigma) \pm \frac{1}{2} V_1(k_\Sigma) \alpha, \\ V_{\Sigma^0} &= V_0(k_\Sigma). \end{aligned} \quad (1)$$

Similarly as was noticed first by Lane [1] in the case of the nucleon s.p. potential, the s.p. potentials  $V_{\Sigma^\pm}$  and  $V_{\Sigma^0}$  are averaged versions of a more fundamental formula

$$\hat{V}_\Sigma(k_\Sigma) = V_0(k_\Sigma) + V_1(k_\Sigma) \mathbf{t}_\Sigma \mathbf{T}_A / A, \quad (2)$$

where  $\mathbf{t}_\Sigma$  is the  $\Sigma$  isospin ( $t_{\Sigma 3} = \pm 1, 0$  for  $\Sigma^\pm, \Sigma^0$ ) and  $\mathbf{T}_A$  is the nuclear isospin,  $\mathbf{T}_A = \Sigma \mathbf{t}_N$  [ $\mathbf{t}_N$  is the nucleon isospin ( $t_{N3} = \frac{1}{2}$  for protons and  $-\frac{1}{2}$  for neutrons),  $T_{A3} = (Z - N)/2$ ].

The Lane potential  $V_1$  is important for the structure of  $\Sigma$  hypernuclear states. For strong  $V_1$  the states become approximate eigenstates of the isospin. Mixing of the isospin states is caused by the Coulomb interaction, not included into  $\hat{V}_\Sigma$ , and by mass differences. This was first discussed in the case of the  $^{12}_\Sigma\text{C}$  system by Dover, Gal, and Millener [2] and also in [3] (the case of  $^9_\Sigma\text{Be}$  was considered in [4] and [5]). The existence of the only observed  $\Sigma$  hypernuclear bound state of  $^4_\Sigma\text{He}$  is strictly connected with a strong Lane potential  $V_1$  [6,7].

The experimental information on  $V_\Sigma$  comes mainly from the strangeness exchange reactions ( $K^-$ ,  $\pi^\pm$ ). Namely, the observed pion spectrum is sensitive to the final state interaction of the produced  $\Sigma$  hyperons with the nuclear core (see, e.g., [8]). Recently the ( $K^-$ ,  $\pi^\pm$ ) spectra have been measured at BNL [9–12] with an order of magnitude better statistics than reported in the early CERN experiments and with an energy resolution claimed to be reasonable. In the case of the  $^9\text{Be}$  target, a difference was observed between the  $\pi^+$  and  $\pi^-$  spectra. Whereas in the ( $K^-$ ,  $\pi^+$ ) reaction  $\Sigma^-$  hyperons, in the ( $K^-$ ,  $\pi^-$ ) reaction  $\Sigma^0$  and  $\Sigma^+$  are produced. Hence the

difference is related to the difference between  $V_{\Sigma^-}$  and  $V_{\Sigma^0}$ ,  $V_{\Sigma^+}$ , which in turn is determined by  $V_1$  [see Eq. (1)]. Thus an analysis of the ( $K^-$ ,  $\pi^\pm$ ) spectra requires knowledge of  $V_1$ .

In the present paper, I calculate the Lane potential  $V_1$  in nuclear matter. In particular, I consider the contribution to  $V_1$  arising from the intrinsic dependence of the effective  $\Sigma$ -nucleon interaction in nuclear matter,  $\mathcal{K}$ , on two densities, that of protons  $\rho_p$  and that of neutrons  $\rho_n$ . In the previous simple estimates of  $V_1$  [13], this contribution was disregarded. In Sec. II the expression for  $V_1$  in terms of  $\mathcal{K}$  is derived. In Sec. III the single-density approximation is introduced, and a simplified expression for  $V_1$  is obtained. In Sec. IV, I present and discuss the result for  $V_1$  obtained with  $\mathcal{K}$  calculated in [14] in the low order Brueckner (LOB) theory for four models of the Nijmegen baryon-baryon interaction. The present paper is restricted to the real potential  $V_1$ —the imaginary part of  $\mathcal{K}$  in the isospin- $\frac{1}{2}$  channel (due to the  $\Sigma N \rightarrow \Lambda N$  conversion) is ignored.

## II. EXPRESSION FOR $V_1$

To derive the expression for  $V_1$ , we start from the definition of  $V_1$ , which follows from Eq. (1):

$$V_1(k_\Sigma) = 2[\partial V_{\Sigma^+}(k_\Sigma)/\partial \alpha]_{\alpha=0}. \quad (3)$$

The s.p. potential of  $\Sigma^+$  in spin-saturated nuclear matter,  $V_{\Sigma^+}$ , depends on  $\rho_p$  and  $\rho_n$ , connected with the respective proton and neutron Fermi momenta  $k_F^p \equiv k_F^+$  and  $k_F^n \equiv k_F^-$  by

$$\left. \begin{aligned} k_F^+ &= (3\pi^2 \rho_p)^{1/3} \\ k_F^- &= (3\pi^2 \rho_n)^{1/3} \end{aligned} \right\} = k_F(1 \pm \alpha)^{1/3}, \quad (4)$$

where

$$k_F = [3\pi^2 \rho / 2]^{1/3}, \quad \rho = \rho_p + \rho_n. \quad (5)$$

The dependence of  $V_{\Sigma^+}$  on the two densities, or equivalently on the two Fermi momenta  $k_F^\pm = k_F(1 \pm \alpha)^{1/3}$ , leads to the dependence of  $V_{\Sigma^+}$  on  $\alpha$ , which appears in Eq. (3). To determine this dependence, let us write the expression for  $V_{\Sigma^+}$  in terms of the  $\Sigma N$  effective interaction in spin-saturated nuclear matter with a proton excess  $\mathcal{K}(k_F^+ k_F^-)$ :

$$\begin{aligned}
V_{\Sigma^+}(k_{\Sigma}) &= \frac{1}{2} \sum_{sm_s} \left[ \sum_{\mathbf{k}_N}^{k_N < k_F^+} (\mathbf{k}_N t_{N3} = \frac{1}{2}, \mathbf{k}_{\Sigma} t_{\Sigma 3} = 1) \right. \\
&\quad \times \mathcal{K}(sm_s; k_F^+ k_F^-) | \mathbf{k}_N t_{N3} = \frac{1}{2}, \mathbf{k}_{\Sigma} t_{\Sigma 3} = 1) \\
&\quad + \sum_{\mathbf{k}_N}^{k_N < k_F^-} (\mathbf{k}_N t_{N3} = -\frac{1}{2}, \mathbf{k}_{\Sigma} t_{\Sigma 3} = 1) \\
&\quad \left. \times \mathcal{K}(sm_s; k_F^+ k_F^-) | \mathbf{k}_N t_{N3} = -\frac{1}{2}, \mathbf{k}_{\Sigma} t_{\Sigma 3} = 1) \right], \quad (6)
\end{aligned}$$

where  $sm_s$  denote the total  $\Sigma N$  spin and its  $z$  projection, and  $\mathcal{K}(sm_s; k_F^+ k_F^-)$  is the diagonal matrix element of  $\mathcal{K}$  in the  $sm_s$  representation. Because  $V_{\Sigma^+}$  does not depend on the  $\Sigma$  spin state, the sum over the nucleon spin states may be enlarged by the sum over the two  $\Sigma$  spin states divided by 2, which leads to  $\frac{1}{2} \sum_{sm_s}$ .

With the help of the transformation to states of total  $\Sigma N$  isospin  $T$  and its third component  $T_3$ ,

$$\begin{aligned}
|t_{N3} = \frac{1}{2} t_{\Sigma 3} = 1) &= |T = \frac{3}{2} T_3 = \frac{3}{2}), \\
|t_{N3} = -\frac{1}{2} t_{\Sigma 3} = 1) &= \sqrt{\frac{1}{3}} |T = \frac{3}{2} T_3 = \frac{1}{2}) - \sqrt{\frac{2}{3}} |T = \frac{1}{2} T_3 = \frac{1}{2}), \quad (7)
\end{aligned}$$

we may rewrite Eq. (6) in terms of the diagonal matrix elements of  $\mathcal{K}$  in the  $TT_3$  representation:

$$\begin{aligned}
V_{\Sigma^+}(k_{\Sigma}) &= \frac{1}{2} \left[ \sum_{\mathbf{k}_N}^{k_N < k_F^+} (\mathbf{k}_N \mathbf{k}_{\Sigma} | \mathcal{K}(T = \frac{3}{2} T_3 = \frac{3}{2}; k_F^+ k_F^-) | \mathbf{k}_N \mathbf{k}_{\Sigma}) \right. \\
&\quad + \sum_{\mathbf{k}_N}^{k_N < k_F^-} (\mathbf{k}_N \mathbf{k}_{\Sigma} | \frac{1}{3} \mathcal{K}(T = \frac{3}{2} T_3 = \frac{1}{2}; k_F^+ k_F^-) \\
&\quad \left. + \frac{2}{3} \mathcal{K}(T = \frac{1}{2} T_3 = \frac{1}{2}; k_F^+ k_F^-) | \mathbf{k}_N \mathbf{k}_{\Sigma}) \right], \quad (8)
\end{aligned}$$

where

$$\mathcal{K}(TT_3; k_F^+ k_F^-) = \sum_{sm_s} \mathcal{K}(sm_s TT_3; k_F^+ k_F^-). \quad (9)$$

To obtain the expression for  $V_1$ , we have to calculate—according to Eq. (3)—the derivative of  $V_{\Sigma^+}$  with respect to  $\alpha$ , at  $\alpha=0$ , taking into account the dependence of  $k_F^{\pm}$  on  $\alpha$ , as given by Eq. (4).

The derivative consists of two parts.

The first part comes from the dependence on  $\alpha$  of the limits of the sums in Eq. (8). Its contribution to  $V_1$ , which we denote by  $V_1^{(0)}$ , can be easily calculated, and the result is

$$\begin{aligned}
V_1^{(0)}(k_{\Sigma}) &= \frac{1}{6} A \left[ \int \frac{d\hat{k}_N}{4\pi} (\mathbf{k}_N \mathbf{k}_{\Sigma} | \mathcal{K}(T = \frac{3}{2}; k_F) \right. \\
&\quad \left. - \mathcal{K}(T = \frac{1}{2}; k_F) | \mathbf{k}_N \mathbf{k}_{\Sigma}) \right]_{k_N = k_F}, \quad (10)
\end{aligned}$$

where  $\mathcal{K}(T; k_F) = \mathcal{K}(TT_3; k_F k_F)$  is defined for  $Z=N=A/2$  and does not depend on  $T_3$ .

The second part comes from the intrinsic dependence of  $\mathcal{K}$  on  $k_F^+$  and  $k_F^-$ . Its contribution to  $V_1$ , which we denote by  $V_1^{(I)}$ , is

$$\begin{aligned}
V_1^{(I)}(k_{\Sigma}) &= \frac{1}{3} \sum_{\mathbf{k}_N}^{k_N < k_F} \left( \mathbf{k}_N \mathbf{k}_{\Sigma} \left[ k_F \left[ \left( \frac{\partial}{\partial \kappa} - \frac{\partial}{\partial \lambda} \right) \{ \mathcal{K}(\frac{3}{2} \frac{3}{2}; \kappa \lambda) \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{1}{3} \mathcal{K}(\frac{3}{2} \frac{1}{2}; \kappa \lambda) + \frac{2}{3} \mathcal{K}(\frac{1}{2} \frac{1}{2}; \kappa \lambda) \right] \right]_{\kappa = \lambda = k_F} \right) | \mathbf{k}_N \mathbf{k}_{\Sigma} \right), \quad (11)
\end{aligned}$$

where we use the notation defined in Eq. (9).

Equations (10) and (11) present our final result for  $V_1 = V_1^{(0)} + V_1^{(I)}$ .

Notice that plane wave states in Eqs (10) and (11) are normalized in the periodicity box of volume  $\Omega$ , i.e.,  $(\mathbf{r} | \mathbf{k}) = \Omega^{-1/2} \langle \mathbf{r} | \mathbf{k} \rangle = \Omega^{-1/2} \exp(i\mathbf{k}\mathbf{r})$ . Thus we have  $(\mathbf{k}_N \mathbf{k}_{\Sigma} | \mathcal{K} | \mathbf{k}_N \mathbf{k}_{\Sigma}) = \Omega^{-1} \langle \mathbf{k}_{N\Sigma} | \mathcal{K} | \mathbf{k}_{N\Sigma} \rangle$ , where  $\mathbf{k}_{N\Sigma}$  is the relative  $N\Sigma$  momentum. This factor  $\Omega^{-1}$  in Eq. (10)—together with  $A$ —leads to a factor  $\rho$ , and in Eq. (11) is canceled by the factor  $\Omega$  in the relation  $\Sigma_{\mathbf{k}} = \Omega / (2\pi)^3 \int d\mathbf{k}$ .

### III. SINGLE-DENSITY APPROXIMATION

Whereas  $V_1^{(0)}$  is expressed by  $\mathcal{K}(T; k_F)$ , the effective  $\Sigma N$  interaction in nuclear matter with  $Z=N$ , to determine  $V_1^{(I)}$  we should know  $\mathcal{K}(TT_3; k_F^+ k_F^-)$ , the effective  $\Sigma N$  interaction in a two component nuclear matter with  $Z \neq N$ . A calculation of  $\mathcal{K}(TT_3; k_F^+ k_F^-)$  starting from a realistic  $\Sigma N$  interaction would be very tedious and so far has not been performed. On the other hand, results for the simpler effective interaction  $\mathcal{K}(T, k_F)$ , obtained with the Brueckner theory, are available in the literature. To be able to use the results obtained for  $\mathcal{K}(T, k_F)$  in our theory of the Lane potential  $V_1$ , we introduce the single-density (or single-Fermi-momentum) approximation, applied originally in the problem of spin symmetry energy of liquid  ${}^3\text{He}$  [15] and later in the problem of the nuclear isospin [16] and spin [17] symmetry energy. Namely, we introduce the following simplifying assumptions:

$$\mathcal{K}(TT_3 = \frac{3}{2}; k_F^+ k_F^-) \approx \mathcal{K}(T; k_F^+), \quad (12)$$

$$\mathcal{K}(TT_3 = \frac{1}{2}; k_F^+ k_F^-) \approx \mathcal{K}(T; k_F^-). \quad (13)$$

Approximation (12) says that the effect of the proton excess on the  $\Sigma^+ p$  effective interaction in nuclear matter is determined primarily by the shift in Fermi momentum of protons. This assumption seems to be physically plausible and it corresponds exactly to the way in which the action of the exclusion principle is altered by the proton excess. The motivation of approximation (13), which applies to the  $\Sigma^+ n$  interaction, is analogous.

By applying approximations (12) and (13) in Eq. (11), we get the following approximate expression for  $V_1^{(I)}$ :

TABLE I. Different components (in MeV) of  $V_\Sigma(k_\Sigma=0)$  calculated at  $k_F=1.35 \text{ fm}^{-1}$  with the YNG interaction obtained from the indicated models of the  $\Sigma N$  interaction.

Model	$V_0$	$V_1^{(0)}$ ( $\tilde{V}_1^{(0)}$ )	$V_1^{(I)}$	$V_1$
D	-13.1	51.0 (51.3)	4.1	55.1
F	23.5	67.4 (70.2)	13.1	80.4
SC	-9.6	13.3 (14.4)	17.7	31.0
NSC	-16.6	-26.3 (-25.8)	-10.4	-36.7

$$V_1^{(I)}(k_\Sigma) \approx \frac{2}{9} \sum_{\mathbf{k}_N}^{k_N < k_F} \left( \mathbf{k}_N \mathbf{k}_\Sigma \left| k_F \frac{d}{dk_F} \{ \mathcal{K}(\frac{3}{2}; k_F) - \mathcal{K}(\frac{1}{2}; k_F) \} \right| \mathbf{k}_N \mathbf{k}_\Sigma \right). \quad (14)$$

#### IV. RESULTS FOR THE YNG INTERACTION AND DISCUSSION

In calculating  $V_1^{(0)}$ , Eq. (10), and  $V_1^{(I)}$ , Eq. (14), I have used the YNG effective  $\Sigma N$  interaction of Yamamoto *et al.* [14]. The YNG interaction is the configuration space representation of the  $G$  matrix calculated in the LOB approximation from model D [18], model F [19], and the soft-core (SC) model [20] of the Nijmegen baryon-baryon interaction. I use also the YNG interaction obtained from the new soft-core (NSC) model of Rijken, Stoks, and Yamamoto [21].

The results for  $V_1^{(0)}$ ,  $V_1^{(I)}$ , and  $V_1 = V_1^{(0)} + V_1^{(I)}$ , calculated at  $k_F=1.35 \text{ fm}^{-1}$  for  $k_\Sigma=0$  are shown in Table I which contains also  $V_0$  calculated according to the expression

$$V_0(k_\Sigma) = \sum_T \frac{2T+1}{6} \sum_{\mathbf{k}_N}^{k_N < k_F} (\mathbf{k}_N \mathbf{k}_\Sigma | \mathcal{K}(T; k_F) | \mathbf{k}_N \mathbf{k}_\Sigma) = \sum_T V_0(T, k_\Sigma), \quad (15)$$

which follows from expression (6). Furthermore, Table I contains also approximate values of

$$V_1^{(0)}(k_\Sigma) \approx \tilde{V}_1^{(0)}(k_\Sigma) = V_0(\frac{3}{2}, k_\Sigma) - 2V_0(\frac{1}{2}, k_\Sigma). \quad (16)$$

To obtain this approximate relation between  $V_1^{(0)}(k_\Sigma)$  and the parts  $V_0(T, k_\Sigma)$  of  $V_0(k_\Sigma)$  produced by the effective interaction in the  $\Sigma N$  states with isospin  $T$ , one approximates  $V_1^{(0)}$ , Eq. (10), by its value averaged over the nucleon momenta  $k_N$  in nuclear matter. That means, one introduces on the right hand side of expression (10) the sum  $4 \sum_{\mathbf{k}_N} / A$ , which leads immediately to expression (16). As is seen from Table I, approximation (16) is quite accurate.

The YNG interaction is constructed so that it simulates the  $G$  matrix in the ground states of the system of a  $\Sigma$  hyperon in nuclear matter, and it is best suited for the case of  $k_\Sigma=0$ , presented in Table I. Calculations for  $k_\Sigma>0$  show

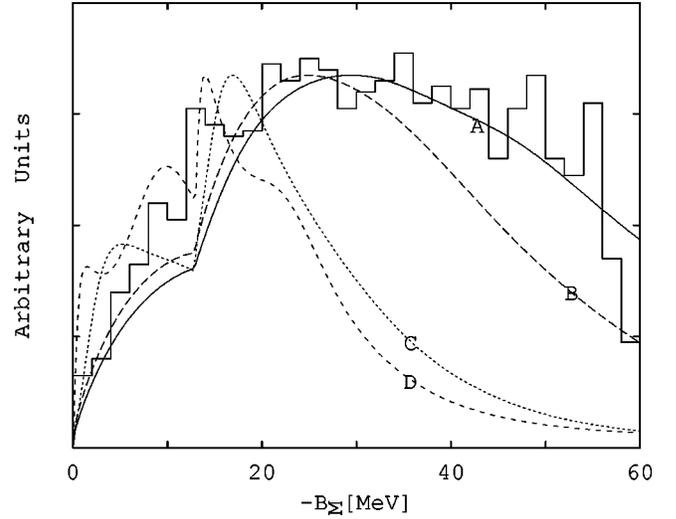


FIG. 1. Pion spectrum from  $(K^-, \pi^+)$  reaction on  ${}^9\text{Be}$  at  $p_K = 600 \text{ MeV}/c$ .  $B_\Sigma$  is the  $\Sigma^-$  binding energy.

that the dependence of  $V_1(k_\Sigma)$  on  $k_\Sigma$  is weak [e.g.,  $V_1(k_\Sigma = 1 \text{ fm}^{-1})$  differs from  $V_1(k_\Sigma = 0)$  by less than 2%].

The contribution  $V_1^{(I)}$  increases the magnitude of  $V_1$ . Its importance depends on the interaction model. In the case of the SC model  $V_1^{(I)}$  is even bigger than  $V_1^{(0)}$ . In the important case of model F,  $V_1^{(I)}$  is also relatively important—it increases  $V_1$  by about 20%.

Because of the LOB approximation applied in [14] and [21], the accuracy of our results may appear uncertain. In the LOB approximation pure kinetic energies are used in the intermediate states of the  $G$ -matrix equation, and the spectrum of these states has a gap at the Fermi momentum. As is well known, this approximation seriously affects  $V_0$  (it shifts it towards more positive values—see, e.g., [22]). However, one may hope that the LOB approximation affects  $V_1$  to a much lesser degree, i.e., that  $V_1$  is much less sensitive than  $V_0$  to the choice of the spectrum of the intermediate states in the  $G$ -matrix equation. This may be demonstrated in the case of model D. For this model, the  $G$ -matrix calculation at  $k_F = 1.35 \text{ fm}^{-1}$  performed with a continuous spectrum led to  $V_0 = -36.2 \text{ MeV}$  [23] which differs essentially from the LOB value of  $-13.1 \text{ MeV}$  in Table I. On the other hand, the results of Ref. [23] lead to  $\tilde{V}_1^{(0)} = 60.5 \text{ MeV}$  which is reasonably close to the value of  $51.3 \text{ MeV}$  in Table I.

The most striking feature which distinguishes model F is that it leads to an repulsive  $V_0$  whereas all the remaining  $\Sigma N$  interaction models lead to an attractive  $V_0$ . Furthermore, model F leads to  $V_1$  of a much larger magnitude than the remaining models.

In comparing our results with experimental data, we restrict ourselves to the recent  $(K^-, \pi^+)$  experiments at BNL (at  $p_K = 600 \text{ MeV}/c$ ) because of their superior statistics. First of all, we want to discuss the experiments on the  ${}^9\text{Be}$  target [9–11].

Let us start with the simpler case of the  $(K^-, \pi^+)$  reaction in which only one direct elementary strangeness exchange process  $K^- p \rightarrow \pi^+ \Sigma^-$  occurs. The observed  $\pi^+$  spectrum is

compared in Fig. 1 with the spectrum calculated in [8] in the plane wave impulse approximation with the final state  $\Sigma^-$  s.p. square well potential with the depth  $V_{\Sigma 0}$  (positive for an attractive potential). Four values of  $V_{\Sigma 0}$  were used:  $-20$ ,  $-10$ ,  $10$ , and  $20$  MeV (curves A, B, C, and D, respectively). We see that curve A (obtained with a repulsive potential) shows the best agreement with the experimental data in contradistinction to the C and D curves (obtained with attractive potentials) which fail completely in reproducing the data at higher  $-B_{\Sigma}$ . Our clear conclusion is that the interaction of the produced  $\Sigma^-$  hyperon with the nuclear core is repulsive.<sup>1</sup> This obviously favors model F as a realistic representation of the  $\Sigma N$  interaction, and excludes the remaining models.

The  $\pi^-$  spectrum observed in the BNL ( $K^-, \pi^-$ ) experiments indicates that the final state interaction of the  $\Sigma$  hyperon is less repulsive than in the ( $K^-, \pi^+$ ) reaction or possibly even attractive [10]. Here, two elementary processes may occur: (A)  $K^- p \rightarrow \pi^- \Sigma^+$  and (B)  $K^- n \rightarrow \pi^- \Sigma^0$ . The difference between the final state interaction in case (A) and that in the ( $K^-, \pi^+$ ) reaction is [see Eq. (1)]  $\Delta V^{(A)} \equiv V_{\Sigma^+} - V_{\Sigma^-} = \alpha V_1 = -\frac{1}{4} V_1$  ( $Z=3, N=5$  in the nuclear core in the final state). For the same difference in case (B), we have  $\Delta V^{(B)} \equiv V_{\Sigma^0} - V_{\Sigma^-} = \frac{1}{2} \alpha V_1 = \frac{1}{2} \Delta V^{(A)}$ .<sup>2</sup> If we use for  $V_1$  in the nuclear core our nuclear matter results, Table I, we get  $\Delta V^{(A)} \simeq -20$  MeV and  $\Delta V^{(B)} \simeq -10$  MeV for model F. This are sizable decreases in the repulsion, required by the comparison of the  $\pi^+$  and  $\pi^-$  BNL spectra. The effect is

smaller for model D and SC, and model NSC leads to an opposite effect:  $\Delta V^{(A)} \simeq +9$  MeV,  $\Delta V^{(B)} \simeq +4.5$  MeV; i.e., it represents an increase in the repulsion (or a decrease in the attraction). Thus model NSC appears to be incompatible with the BNL data.

Finally, let us mention the ( $K^-, \pi^\pm$ ) experiments at BNL on the  ${}^4\text{He}$  target [11,12]. They confirmed the existence of a bound state of  ${}^4_\Sigma\text{He}$ , originally reported in the  ${}^4\text{He}$  (stopped  $K^-, \pi^-$ ) reaction [24]. The existence of this bound state was predicted by Harada *et al.* [6]. In their theoretical description of this state, Harada and Akaishi [7] apply phenomenological  $\Sigma N$  interactions, in particular the interaction SAP-F simulating at low energies the Nijmegen model F interaction. With this phenomenological interaction, they calculate the  $\Sigma$  s.p. potential in the  $A=3$  nuclear core, which turns out to have a strong Lane component  $V_1$  which at the center of the nuclear core is equal to 78.7 MeV. This is close to the corresponding value of 80.4 MeV in Table I, although the  $A=3$  system can hardly be considered as a piece of nuclear matter.

We conclude that model F of the Nijmegen baryon-baryon interaction leads to the s.p. potential  $\hat{V}_\Sigma$  of a  $\Sigma$  hyperon in nuclear matter which has a repulsive isoscalar part  $V_0$  and a sizable positive Lane potential  $V_1$ , and is the only model of the Nijmegen interaction which is compatible with the recent BNL ( $K^-, \pi^\pm$ ) data.

Let us notice that our conclusion is consistent with the analysis of the energy levels of  $\Sigma^-$  atoms which also indicates that the  $\Sigma$ -nucleus interaction is repulsive [25].

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<sup>1</sup>The same conclusion is drawn by Shimizu [10] from the discrepancy between the observed pion spectrum and that calculated with the quasifree model.

<sup>2</sup>Whereas the pion spectra in case (A) and in the ( $K^-, \pi^+$ ) reaction depend on the s.p. states of the target protons, the pion spectrum in case (B) depends on the s.p. states of the target neutrons. The effect of this difference requires a more detailed investigation.

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