# Analyzing powers for the $\pi^- \vec{p} \rightarrow \pi^0 n$ reaction across the $\Delta(1232)$ resonance

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(Received 22 December 1998; published 30 June 1999)

High quality analyzing powers for the  $\pi^- \vec{p} \rightarrow \pi^0 n$  reaction have been obtained with a polarized proton target over a broad angular range at incident kinetic energies of 98.1, 138.8, 165.9, and 214.4 MeV. This experiment nearly doubled the existing  $\pi N$  single-charge-exchange database for energies ranging from 10 to 230 MeV, with 36 new analyzing powers. The Neutral Meson Spectrometer was used to detect the outgoing neutral pions. The data are well described by recent phase-shift analyses. When combined with high-precision and accurate cross section data at the same energies, the data can provide a good test of the degree of isospin breaking in the region of the  $\Delta(1232)$  resonance. They will also be helpful for constraining the evaluation of the pion-nucleon  $\sigma$  term from the scattering amplitudes. [S0556-2813(99)02408-5]

PACS number(s): 25.80.Gn, 24.70.+s, 13.75.Gx

# **I. INTRODUCTION**

The pion-nucleon interaction has a fundamental role in understanding nuclear reactions, in describing the structure of nuclei, and in providing a window into fundamental issues of the structure of the nucleon. Although there have been many experiments for the  $\pi^- p \rightarrow \pi^0 n$  reaction above the  $\Delta(1232)$  resonance at several different laboratories, the current status of the database and associated phase-shift analyses below the  $\Delta(1232)$  resonance is still very cloudy. Accurate single-charge-exchange (SCX) measurements will significantly help to enhance the  $\pi N$  database, and therefore will contribute to an unambiguous decomposition of the  $\pi N$ amplitudes. Alternatively, they are also needed to determine the degree of isospin-symmetry breaking in the  $\pi N$  system so that the amplitudes can be treated in a fully consistent framework. Polarization measurements are particularly sensitive to the interference between partial waves, and thus help to constrain the partial-wave decompositions of the scattering amplitudes. A solid  $\pi N$  database may be used to determine the  $\pi NN$  coupling constant, test isospin-symmetry breaking, and constrain the estimate of the strange-quark content of the nucleon through extraction of the pionnucleon  $\sigma$  term.

The QCD Lagrangian may be expressed in terms of a part  $\mathcal{L}_0$  which is chirally invariant and depends exclusively on the quark and gluon color fields, and a quark mass term  $\mathcal{L}_{OCD}$  $=\mathcal{L}_0+\Delta\mathcal{L}$ . The quark mass term

$$\Delta \mathcal{L} = -\sum_{q} m_{q} \bar{\psi}_{q}^{i} \psi_{qi} , \qquad (1.1)$$

where  $m_q$  is the quark mass and  $\psi_{qi}$  is the quark field of flavor q and color i, breaks chiral symmetry. At low energies and considering only the two lightest quarks,

$$\Delta \mathcal{L} = -\left(m_u \bar{u} u + m_d \bar{d} d\right) \tag{1.2}$$

$$=\frac{1}{2}(m_u+m_d)(\bar{u}u+\bar{d}d)+\frac{1}{2}(m_u-m_d)(\bar{u}u-\bar{d}d).$$
(1.3)

The first (isoscalar) term of Eq. (1.3) yields the sigma term  $\sigma$ , which is a measure of explicit chiral symmetry breaking. The second (isovector) term breaks isospin invariance. It depends not only on the mass difference of the up and down quarks, but also on the difference in the respective quark condensates.

The  $\pi NN$  coupling constant sets the scale for hadronic interactions. The long-standing historical value  $g^2 = 14.3$ 

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$T_{\pi^-}$ (MeV)	Angular range $\theta_{c.m.}$ (deg)	Number of points	Author
100.00	75.00-130.00	4	Staško [14]
150.00	35.00-141.00	8	Staško [14]
161.00	28.13-73.08	8	Görgen [15]
192.20	32.50-89.30	5	Kim [16]
200.00	35.00-141.00	8	Staško [14]
205.90	30.80-87.60	5	Kim [16]

TABLE I. Analyzing-power database for  $\pi N$  charge exchange below 230 MeV prior to this experiment.

 $\pm 0.2$  was obtained from phase-shift analyses of charged  $\pi^+ p$  and  $\pi^- p$  scattering [1]. However, several recent phase-shift analyses have resulted in lower values in the range 13.0–13.7 [2–5]. An exception is the larger value  $g^2=14.6$  obtained by Ericson *et al.* [6]. The issue is unsettled and remains controversial. Unlike earlier suggestions [7,8], most of the current results on the  $\pi NN$  coupling constant provide no evidence for the breaking of charge independence. On the other hand, other analyses of low-energy data have provided empirical evidence that isospin symmetry is indeed broken in the  $\pi N$  system [9,10]. High-precision polarization data will further constrain the partial-wave analyses, and thereby help determine whether evidence for this extra degree of isospin-symmetry breaking also extends across the  $\Delta(1232)$  resonance.

Although the common picture of nucleons is that they are composed of only up and down quarks, there is considerable current interest as to whether there is also a substantial strange quark content. Relevant evidence can be found in the sigma term, which is renormalized by the strange quarks. One estimate of  $\sigma$  is obtained from the baryon mass spectrum [11]. Another estimate comes from the  $\Sigma_{\pi N}$  term which is obtained via  $\pi N$  phase-shift analyses by extrapolating the isospin-even amplitude to the unphysical Cheng-Dashen point [12]. The consistency of the two estimates is very much an open issue and a subject of continuing discussion. Although the extrapolation is made with isospin-even amplitudes, while the SCX reaction is entirely isospin-odd, SCX data can still play an important role in the determination of  $\Sigma_{\pi N}$ , because  $\pi^{\pm} p$  processes have mixtures of both isospineven and isospin-odd terms. The SCX reaction isolates the isospin-odd terms without the additional complication of coherent Coulomb scattering amplitudes.

The motivation for this experiment is thus twofold. First, if isospin is rigorously good, the additional information provided by the SCX polarization measurements will help to constrain phase-shift analyses at energies near and below the  $\Delta(1232)$  resonance and thus improve our knowledge of the coupling constants as well as the extrapolated value of the  $\Sigma_{\pi N}$  term. On the other hand, if isospin invariance is broken, the SCX reaction is not determined by the  $\pi^{\pm}N$  scattering and it *must* be studied as a separate process. In fact, Weinberg has noted that isospin breaking may make sizeable contributions to the  $\Sigma_{\pi N}$  term, comparable to those from chiral-symmetry breaking [13].

For scattering to beam left, the analyzing power is defined as

$$A_{y} = \frac{(d\sigma/d\Omega)_{\uparrow} - (d\sigma/d\Omega)_{\downarrow}}{P_{\downarrow}(d\sigma/d\Omega)_{\uparrow} + P_{\uparrow}(d\sigma/d\Omega)_{\downarrow}}, \qquad (1.4)$$

where  $P_{\uparrow}(P_{\downarrow})$  are the measured up (down) target polarizations. The variables  $(d\sigma/d\Omega)_{\uparrow}$  and  $(d\sigma/d\Omega)_{\downarrow}$  are the respective differential cross sections from the spin-up and spin-down spectra. Compared to the elastic  $\pi N$  database, information for the SCX channel is rather sparse: there were only 38 analyzing-power measurements for energies ranging from 10 to 230 MeV prior to this experiment (see Table I). There is a strong need for more  $\pi N$  analyzing-power SCX data below the  $\Delta(1232)$  resonance. The measurement of analyzing powers, in addition to cross sections, has the advantage that it doubles the number of independent experimental quantities that are used in phase-shift analyses. Furthermore, analyzing powers provide important information on the relative phases between the spin-independent and spindependent terms in the effective  $\pi N$  interaction. The enhanced sensitivity to the terms of the  $\pi N$  interaction is essential for resolving the issues mentioned above.

## **II. EXPERIMENTAL APPARATUS**

Experiment E1178 measured analyzing powers  $A_y$  for the SCX reaction  $\pi^- \vec{p} \rightarrow \pi^0 n$  over a broad angular range at incident pion kinetic energies of 98.1, 138.8, 165.9, and 214.4 MeV. The energies correspond to those where  $\pi^{\pm} p$  analyzing powers [17,18] have been measured. The experiment was carried out at the low energy pion (LEP) channel of the Clinton P. Anderson Meson Physics Facility (also known as the Los Alamos Meson Physics Facility, LAMPF) with a dynamically polarized proton target. The scattered neutral pions were detected by the Neutral Meson Spectrometer [19,20]. A floor layout, with a typical arrangement of the spectrometer and of the magnets, is shown in Fig. 1.

# A. Low energy pion channel

The LEP channel was designed to provide  $\pi^{\pm}$  beams from about 10 MeV through the resonance region (~300 MeV) with variable beam momentum spread  $\Delta p/p$ from  $\pm 0.05$  to  $\pm 4$  %. A detailed description of the LEP channel is provided by Refs. [21,22].

During experiment E1178 the typical values of  $\Delta p/p$  ranged from 0.3 to 3.0%, corresponding to a pion flux of up to about 10<sup>7</sup>  $\pi/s$ . The duty factor was about 6%. The choice of momentum bites was based solely on limiting the singles



FIG. 1. Floor layout of the experiment in the LEP channel. Each crate, which is a detector arm of the NMS, can be rotated independently around the target.

rates in the detectors, particularly in the CsI crystals (see Sec. II C), to a workable range of the data acquisition system.

# 1. Extended beam line

To install the target and its associated apparatus inside the experimental area, an extended beam line that consisted of two quadrupoles and one dipole (Werbecka) magnet was installed at the channel exit, as sketched in Fig. 2. The purpose of the Werbecka dipole magnet was to steer the pion beam away from the standard beam line to compensate for the bending effect of the target polarizing magnet (Zoltan). The field of Werbecka was adjusted until the charged pion beam passed through the center of Zoltan, hitting the target at approximately 0° with respect to its normal. The locations of the additional magnets relative to the last channel quadrupole doublet and the strengths of their field settings were calculated with the code TRANSPORT [23], and the fields were adjusted on-line. The accuracy of the steering was checked by placing polaroid film at the center of the Zoltan field, so as to obtain an image of the beam at the target location.

#### 2. Beam tune and beam monitoring

All of the settings of the magnets of the LEP channel were calculated with the program TRACE. The quadrupoles were then fine-tuned to obtain the smallest beam spot at the target position. For this experiment, the beam spot was usually  $\sim 1 \times 1.5$  cm in diameter (the widest side being oriented horizontally). The optimal beam-collimator openings were determined individually by looking at the beam flux, i.e., the beam envelope at each collimator was cut until the nominal beam intensity was separately reduced by about 10%. The



FIG. 2. Schematic layout of the extended LEP beam line. The upper boxes represent the channel exit and quadrupole magnets. Pion trajectories are bent by the magnetic fields of Werbecka and Zoltan (not to scale).

beam energy is believed to be accurate to better than 1 MeV, a value consistent with some direct time-of-flight measurements that were made.

The incident beam flux was monitored with two independent beam monitors: an ion chamber recording the current produced by the passage of the beam particles just before the Zoltan magnetic field area (Fig. 2), and two toroid coils located on the primary proton beam line. Because the measurement of analyzing powers does not require knowledge of the absolute pion beam flux, it was not necessary to determine the fractions of electrons and muons in the beam, but they were typically at the few percent level or less.

#### **B.** Polarized proton target

The target setup and the nose part of the refrigerator needed for the polarized target are shown in Fig. 3. The protons in the target material were polarized by using the dynamic nuclear polarization (DNP) method [24–27]. A strong, homogeneous magnetic field ( $\Delta B/B \leq 10^{-4}$  over the target volume) was provided by the Zoltan iron-core C magnet. The teflon target holder was placed in a copper can that served as a multimode microwave cavity. The target beads were continuously irradiated by microwaves of about 69–70 GHz to achieve a steady-state polarization. Changing the mi-



FIG. 3. The polarized and background targets in the mixing chamber of the dilution refrigerator between the poles of Zoltan magnet (vertical slice through the center of the cryostat and magnet).

crowave frequency by about 425 MHz reversed the sign of the polarization. No changes in the external magnetic field were required, thus keeping systematic errors to a minimum.

A nuclear magnetic resonance (NMR) system [26,28] was used to monitor and measure the absolute proton polarization of the target material. To obtain a normalized measurement, the enhanced polarization must be calibrated against NMR measurements at thermal equilibrium (TE) by using the area under the peak of the NMR spectrum, which is proportional to the polarization. These calibrations were done about once a week.

In this experiment, some corrections to the scaled polarization values were needed. Large modulations of the base RF level of the NMR circuit can lead to nonlinear NMR signals and asymmetric polarization values. Full details are given elsewhere [29]. The averages of the proton polarization over this experiment were about  $\langle P_{\uparrow} \rangle \approx 90 \pm 4.5\%$  and  $\langle P_{\downarrow} \rangle \approx 80 \pm 4.5\%$  ( $\pm 4.5\%$  is the overall target polarization uncertainty; see Sec. III D).

#### 1. Dilution refrigerator

The low temperatures required for the DNP were achieved with a  ${}^{3}$ He- ${}^{4}$ He dilution refrigerator [30] of the CERN type [31,32]. It was specifically modified for this experiment to reduce the thicknesses of the cryostat walls and for the use of polarized target experiments with the LAMPF beam. It can be operated in a horizontal mode as in this experiment, or vertically. The loading of the target material, which needs to be done at or below liquid nitrogen temperature, can be done directly into the mixing chamber.

TABLE II. Effective thickness of the butanol target chemical components.

Element	Mass fraction (%)	Effective thickness (g/cm <sup>2</sup> )		
Н	12.92	0.107		
С	61.57	0.487		
0	24.95	0.197		

# 2. Target material

The compound 1-butanol CH<sub>3</sub>(CH<sub>2</sub>)<sub>2</sub>CH<sub>2</sub>OH was chosen as target material because of its chemical composition. It is rich in polarizable hydrogen with a minimum of elements that have nonzero nuclear spin, and it also has one of the best ratios of hydrogen to contaminant carbon and oxygen atoms of all suitable materials. Any contributions to the observed analyzing powers from contaminant nuclei were much less than the systematic errors from other sources. Distilled water at 5% by weight was added to the 1-butanol in order to increase the polarization [33]. The target material was doped with  $5 \times 10^{19}$  atoms/cm<sup>3</sup> of EHBA-Cr(V) complex [paramagnetic complex sodium bis(2-ethyl-2-hydroxybutyrato)oxochromate(V)monohydrate], which is soluble in butanol [34]. This complex provides a stable and homogeneous distribution of polarizing centers throughout the target volume.

The target material was prepared in the form of frozen beads of about 1–1.5 mm diameter in order to improve the thermal contact with the <sup>3</sup>He-<sup>4</sup>He bath. The target container was a rectangular perforated teflon basket with dimensions  $H \times L \times T = 2.5 \times 5 \times 1.7$  cm<sup>3</sup>.

Because the hydrogen peak in the energy spectrum of the outgoing  $\pi^0$ 's sits on top of a broad background due to the contaminant materials of the target and of the cryostat, a second target was made to provide data in order to determine the spectral shape of this background. A hydrogen-free target consisting of graphite (C) and of "dry ice" beads (CO<sub>2</sub>) was used for background subtraction. The quantities each of graphite and of "dry ice" were chosen such that the effective thicknesses of carbon and oxygen in the background target were similar to the ones in the butanol target (Tables II and III). Because the geometrical effect of the target cannot be neglected, the background target was built as close as possible to the geometry of the polarized target. It had a teflon basket identical to that of the polarized target, but with a thickness of 0.8 cm instead of 1.7 cm in order to obtain the same effective areal thickness of carbon and oxygen.

# C. Neutral meson spectrometer

The neutral pions from the  $\pi^- \vec{p} \rightarrow \pi^0 n$  reaction were detected in the neutral meson spectrometer (NMS). With a

TABLE III. Effective thickness of the background target chemical component.

Element	Mass fraction (%)	Effective thickness (g/cm <sup>2</sup> )
С	74.03	0.518
0	25.97	0.172



FIG. 4. Schematic diagram of the neutral meson spectrometer. The photons in each detector are detected by two planes of active BGO converters and tracking chambers followed by a total-energy calorimeter.

mean lifetime at rest of  $0.84 \times 10^{-16}$  s, the  $\pi^0$  decays instantaneously within the target in which it is produced. Only the dominant electromagnetic decay mode  $\pi^0 \rightarrow \gamma\gamma$ , which proceeds with a branching ratio of 98.798±0.032% [35], is detected.

# 1. Neutral meson spectrometer design

The presence of a  $\pi^0$  must be inferred from the observation of its decay products. The NMS therefore consists of two identical detectors operating in coincidence mode to detect the two photons from the neutral meson decays. A sketch of the NMS is given in Fig. 4.

The two detectors of the NMS are independent position sensitive, high-energy,  $\gamma$ -ray detectors. The front face of each detector is a plane of fifteen 0.95-cm-thick plastic scintillators which veto any incoming charged particles. This veto plane is followed by two identical conversion systems. Each system consists of an active bismuth germanate (BGO) converter plane followed by a tracking wire-chamber package. Each  $\gamma$  ray can convert in either one of the two active converter planes into an electromagnetic shower. The electrons and positrons exiting the back of each converter plane are detected in the wire chambers, and this information is used to determine the conversion point coordinates of the  $e^+e^-$  vertex and thus the opening angle  $\eta$  of the  $\pi^0$  decay. This vertex position and opening-angle determination is the measurement that is the most critical for achieving good resolution. At the back of each detector is a  $6 \times 10$  array of pure cesium iodide (CsI) crystals which serve as a photon calorimeter. The total shower energy in each arm is determined by adding the signal from the calorimeter to the energy deposited in the active converter planes. The support structure of each detector was designed to permit the independent placement of each detector in the reaction plane. The shape of the structure allowed each detector to be placed very close to the target in order to optimize the count rate.

The  $\gamma$ -ray direction is defined by a line between the conversion point in each detector and the center of the target (see Fig. 4). Thus each detector measures independently the three kinematic variables of one photon: energy  $E_{\gamma}$ , polar angle  $\theta_{\gamma}$ , and azimuthal angle  $\phi_{\gamma}$ . The combined informa-

tion from the two detectors overdetermines the  $\pi^0$  decay kinematics. It is completely described with the following variables: energy sharing parameter *X*, opening angle  $\eta$ ,  $\pi^0$  energy  $E_{\pi^0}$ , and scattering angle  $\theta_{\pi^0}$ , as discussed in the following section.

Bismuth germanate converters. Each converter plane is composed of 28 strips of BGO crystals (arranged in a 2  $\times$ 14 array) with each strip coupled to a light guide and a photomultiplier tube. The vertex resolution is optimized by choosing a material with short radiation length such as BGO. The emitted light (480 nm) was transmitted to a photocathodes through lead-glass light guides.

The dimensions of the total active area are 40.64 cm  $\times$ 71.12 cm $\times$ 0.635 cm. The dimensions of the converter planes were chosen so as to cover the wire chambers. The overall conversion efficiency was increased by using two converter planes, separated by tracking wire chambers. The conversion efficiency is defined as the probability that an incident  $\gamma$ -ray results in at least one charged particle that exits the back face of the converter and is tracked in the wire chambers. The conversion efficiency depends on the thickness of the converters. The thickness of 0.5 radiation lengths was optimized with Monte Carlo studies (a converter too thick will prevent the charged particles to emerge, whereas a too thin one will reduce the light collection).

Tracking wire chambers. The tracking chamber system was designed to provide excellent position resolution for the conversion vertex reconstruction, and also to handle multiple tracks so as to use the maximum available beam flux. The cathode-strip readout chambers in the NMS have an intrinsic resolution on the order of 100  $\mu$ m, thus giving submillimeter vertex resolution.

The requirements of high resolution and multiple hits per event were achieved by associating two X-Y pairs of anode wire planes and cathode-readout chamber planes with each converter plane. A complete tracking chamber system consists of four pairs of anode-cathode planes. In each pair, the direction of the anode plane is perpendicular to that of the cathode plane. Each anode-cathode pair is separated from the others by a mylar "inactive cathode" plane to avoid cross talk between the adjacent pairs.

The active window of the cathode-strip chambers is  $35.56 \text{ cm} \times 71.12 \text{ cm}$ . This area was determined so that it matched the fiducial area of the NMS calorimeter corresponding to a  $4 \times 8$  array of crystals, thus excluding the crystals along the edges. The total thickness of the chamber system (3.97 cm) was minimized because the acceptance of the spectrometer decreases rapidly with the target-to-detector distance (as  $R^3$ ). Moreover, the *X*-*Y* planes were placed as close as possible to the converter for better position resolution (0.16 cm).

The anode planes are of an alternating-gradient construction: every other wire is a ground wire. The anode wires are gold-plated tungsten 20  $\mu$ m in diameter, placed 5.556 mm apart, which are run at positive high voltage. Centered between the anode wires are 76- $\mu$ m, gold-plated, copper-clad, aluminum cathode wires which are run at ground potential [36,37]. Cathode wires are spaced 1.389 mm apart in the cathode plane. Adjacent wires in the cathode plane are connected together to form strips with 2.778-mm separation.

The complete NMS spectrometer with two detectors, two converters per detector, and two X-Y pairs of chambers per converter requires in principle nearly 50 000 channels of amplifiers and ADCs. The number of amplifiers, and therefore of ADCs needed to transmit the pulse information from the wires in the cathode plane, was reduced to 512 channels by multiplexing the cathode channels. More details on this multiplexing scheme for cathode-strip readout chambers can be found elsewhere [38].

Calorimeter. Following each of the two convertertracking packages is a calorimeter. Each calorimeter is composed of a  $6 \times 10$  array of pure CsI crystals, each of which is optically isolated from its neighbors and coupled to a photomultiplier tube. The 60 crystals are assembled so that the array is a mechanically rigid device which can be deployed in various spatial orientations. The length of each crystal is 30.48 cm so that 97% of photons, having an energy up to 800 MeV, can be detected. The dimensions of the back face of each crystal are 10.16 cm×10.16 cm. Each crystal is tapered on two sides by 1°, thus making the front-face dimensions 10.16 cm×9.096 cm. The stability of this arched configuration prevents any possibility of a load being applied to the entrance window of the calorimeter. More than 97% of the deposited energy is contained in a  $3 \times 3$  cluster of crystals for  $\gamma$  rays incident at the center of the central crystal in the cluster. The signals of all photomultiplier tubes are summed to give a measure of the total energy of the photon.

# 2. Concept of $\pi^0$ detection

The NMS measures six quantities: the polar and azimuthal angles  $(\theta_1, \theta_2, \phi_1, \phi_2)$  and the energies  $(E_1, E_2)$  of the two  $\gamma$  rays from this  $\pi^0$  decay mode. Under these conditions, a good  $\pi^0$  energy resolution can be achieved by measuring precisely the angles of the two photons even with a rough measurement of their energies. The total energy  $E_{\pi^0}$ and the laboratory scattering angle  $\theta_{\pi^0}$  of a neutral meson are reconstructed in terms of these measured quantities, as shown below.

Considering the decay of the  $\pi^0$  with four-momentum  $p_{\pi^0}$  into two photons  $(p_1 \text{ and } p_2)$ , energy and momentum conservation lead to the result that the total  $\pi^0$  energy is

$$E_{\pi^0} = E_1 + E_2 = m_{\pi^0} c^2 \sqrt{\frac{2}{(1 - X^2)(1 - \cos \eta)}}, \quad (2.1)$$

where the energy sharing parameter between the two photons is

$$X = \frac{E_1 - E_2}{E_1 + E_2},\tag{2.2}$$

and  $\eta$  is the opening angle between the two decay photons in the laboratory frame.

In a similar way, the scattering angle is given by

$$\cos \theta_{\pi^0} = \frac{E_1 \cos \theta_1 + E_2 \cos \theta_2}{\sqrt{E_1^2 + E_2^2 + 2E_1 E_2 \cos \eta}}.$$
 (2.3)

The opening angle  $\eta$  is the most important quantity to measure and it can be determined very precisely by means of tracking chambers. The photon energies  $E_1$  and  $E_2$  are measured with the calorimeters.

#### **III. DATA ACQUISITION AND ANALYSIS**

To minimize systematic uncertainties between spin-up and spin-down data, the target polarization was flipped systematically for each kinematic set. The sequence used was  $\uparrow \downarrow \downarrow \uparrow$ , with some background runs inserted. In practice, each of these four settings contained multiple data runs of about two hours duration. Data for a full kinematic set usually required several days.

# A. Calibration of NMS

Before taking data for the reaction  $\pi^- p \rightarrow \pi^0 n$ , runs of cosmic rays were taken to balance the high voltages and to match the hardware gains of the CsI and the BGO crystals. The high voltage for each crystal was adjusted so that the ADC channel number was ten times the expected energy loss in MeV of the cosmic-ray muons. The hardware gains were set such that the energy deposited over all the crystals was uniform. With the detectors oriented with their long axes vertical and the crystals horizontal, events that fired at least five CsI crystals in a vertical column, without firing any CsI crystals in the adjacent columns, were considered CsI cosmic-ray events. During data acquisition, cosmic-ray data were recorded between beam gates along with the data from the  $\pi^0$  decay. These cosmic-ray events were used during replay to determine any gain shifts of the crystals during the experiment, and to correct for them. The centroids of cosmic-ray events for the CsI crystals can be determined to about 300 keV, and those for the BGO crystals to about 100 keV, if there are no significant backgrounds.

The second part of the calibration process was a  $\pi$ -stop measurement. A  $\pi^-$  beam was stopped in a pure hydrogen target so as to provide monoenergetic 129.4-MeV photons from the  $\pi_{stopped}^- p \rightarrow \gamma n$  reaction. The data could then be used to optimize the software gains to achieve a high resolution. The beam channel was tuned to produce a  $\pi^-$  beam of 54 MeV. The target was a bottle of pure hydrogen gas with a 5.1-cm-thick beryllium absorber placed on the outside of the bottle to stop the  $\pi^-$  beam within the gas.

A good calibration is necessary to obtain data with good resolution, because even small relative gain changes can have a large effect on the  $\pi^0$  mass reconstruction. A typical invariant  $\pi^0$  mass reconstruction is shown in Fig. 5. This quantity is computed in the analyzer according to Eq. (2.1), with  $E_1 + E_2$  given by the sum of the energies from the BGO and CsI crystals.



FIG. 5. Invariant  $\pi^0$  mass reconstruction.

# B. Good $\pi^0$ event

Several tests are necessary to ensure that the data used in the analysis are from good  $\pi^0$  events. The first selection of good events is done by the hardware. However, this selection only ensures that the events recorded are coincidence events with the right energy threshold, and that they were not vetoed. Therefore more selections are needed in the software, such as the following.

(1) At least one of the wire chamber systems must fire in each arm of the spectrometer. A chamber system is good when all of the four anode planes fire.

(2) The photons from the  $\pi^0$  decay must be within the chosen fiducial areas.

(3) The photons from the  $\pi^0$  decay must be within angular limits which will exclude events that do not come from the target.

(4) The events must have an invariant  $\pi^0$  mass reconstruction within acceptable resolution limits.

#### C. Extraction of asymmetries

#### 1. Definition

The experimental differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{YJ}{N_{\pi} - N_{p} \Delta \Omega \varepsilon_{\rm WC} f_{\gamma\gamma} \tau}.$$
(3.1)

Here, Y is the yield, J the Jacobian of the transformation of cross section from the laboratory to the center-of-mass frame,  $N_{\pi^-}$  the number of  $\pi^-$  particles in the incident beam,  $N_p$  the number of protons in the target per cm<sup>2</sup>,  $\Delta\Omega$  the effective solid angle,  $\varepsilon_{\rm WC}$  the overall wire chamber efficiency,  $f_{\gamma\gamma}$  the  $\pi^0 \rightarrow \gamma\gamma$  branching ratio, and  $\tau$  the experimental live-time. The analyzing power is given by Eq. (1.4).

When extracting the analyzing power from the data, the yields are used instead of the differential cross sections because quantities such as the absolute scale of the beam flux, target thickness, solid angle, and the branching ratio will cancel. These factors are assumed to be unchanged between



FIG. 6. Spin-up, spin-down, and background spectra as a function of the missing mass (relative to the neutron mass) at several different center-of-mass scattering angles  $\theta_{c.m.}$  at 138.8 MeV. The spectrum with higher counts is the spin-up spectrum.

polarization runs, and they are independent of the scattering angle. The quantities that can vary from run to run include the relative pion beam flux  $\Phi_{IC}$  as measured by the ion chamber, the wire-chamber efficiencies, and the live-time  $\tau$ . Combining these quantities into a normalization factor

$$N = \frac{1}{\Phi_{\rm IC} \,\varepsilon_{\rm WC} \,\tau},\tag{3.2}$$

and substituting into the expression for  $A_{y}$ , one obtains

$$A_{y} = \frac{Y_{\uparrow}^{n} - Y_{\downarrow}^{n}}{P_{\downarrow}Y_{\uparrow}^{n} + P_{\uparrow}Y_{\downarrow}^{n}},$$
(3.3)

where  $Y_{\uparrow}^{n}(Y_{\downarrow}^{n})$  is the normalized yield (Y/N) for spin-up (spin-down), with backgrounds subtracted.

Typical spin-up, spin-down, and background spectra are shown in Fig. 6. The small peak in the background spectrum under the hydrogen peak is due to the <sup>3</sup>He in the dilution refrigerator. More details on the background can be found in Sec. III C 4.

### 2. Adding runs with different polarization

For statistical reasons, it is better to sum the events in the spectra (after background subtraction) over all of the multiple runs for a particular kinematic setting and target-spin orientation rather than to obtain the yields for each individual run with a peak-fitting program [39]. However, each of the

individual runs had a different value of the target polarization. The data for the multiple runs had to be combined properly in order to extract the yields of the spin-up, spin-down, and background spectra. One thus finds that the expression of the analyzing power becomes

$$A_{y} = \frac{J\sum_{i} Z_{\uparrow i}^{n} - I\sum_{j} Z_{\downarrow j}^{n}}{\sum_{i} Z_{\uparrow i}^{n} \sum_{j} P_{\downarrow j} + \sum_{j} Z_{\downarrow j}^{n} \sum_{i} P_{\uparrow i} - Bg}, \quad (3.4)$$

where

$$Bg = B^n \left( I \sum_j P_{\downarrow j} + J \sum_i P_{\uparrow i} \right).$$
(3.5)

Here *i* (*j*) labels the individual spin-up (spin-down) runs, *I* (*J*) is the total number of runs with spin-up (spin-down),  $Z^n$  is the normalized yield of the observed spectra, and  $B^n$  is the normalized yield of the background that needs to be subtracted ( $Z^n = Y^n + B^n$ ).

# 3. "Out-of-plane" correction

Due to the large acceptance area of the NMS spectrometer, the detected  $\pi^{0}$ 's can have scattering planes that are rotated out of the horizontal plane which is perpendicular to the target polarization. These "out-of-plane" events dilute the observed analyzing power because

$$d\sigma_{\uparrow/\downarrow} = d\sigma [1 + A_y \mathbf{P}_{\uparrow/\downarrow} \cdot \hat{\mathbf{n}}], \qquad (3.6)$$

where  $d\sigma$  is the differential cross section for an unpolarized target,  $\hat{\mathbf{n}}$  is a unit vector normal to the scattering plane, and  $\mathbf{P}_i$  is a positive real quantity which has a magnitude equal to the polarization of the target. The angle between the normal to the particle's scattering plane  $\hat{\mathbf{n}}$ , and the target polarization vector  $\mathbf{P}$  is defined by  $\phi$ . The value of  $\cos \phi$  for each event is directly extracted from the data during the replay according to

$$\cos\phi = \frac{\sqrt{p_{\pi^0,y}^2 + p_{\pi^0,z}^2}}{\sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\eta}}.$$
 (3.7)

The value of  $\phi$  was less than 10° for every kinematic set.

Correcting for the effects of the "out-of-plane" angles, the linear combination of sums of yields over all runs and all events for spin-up and spin-down gives

$$A_{y} = \frac{JJ'\sum_{i} Z_{\uparrow i}^{n} - II'\sum_{j} Z_{\downarrow j}^{n}}{\sum_{i} Z_{\uparrow i}^{n}\sum_{j} P_{\downarrow j}\sum_{j'} \cos \phi_{jj'} + \sum_{j} Z_{\uparrow j}^{n}\sum_{i} P_{\downarrow i}\sum_{i'} \cos \phi_{ii'} - Bgd},$$
(3.8)

where

$$Bgd = B^{n} \left( II' \sum_{j} P_{\downarrow j} \sum_{j'} \cos \phi_{jj'} + JJ' \sum_{i} P_{\uparrow i} \sum_{i'} \cos \phi_{ii'} \right).$$
(3.9)

The symbols are the same as for Eq. (3.4) with the addition that i'(j') labels the events within runs i(j), and I'(J') is the total number of events with spin-up (spin-down). The polarizations were not corrected on an event-by-event basis, but scaled by the sum of the  $\cos \phi$  values for all of the events in the run.

#### 4. Background subtraction

For the method of summing to be consistent, it is required to have a good background subtraction and to know the boundaries of the "pure" hydrogen peak. The analyzing powers are very sensitive to the quality of background subtraction, and an inexact subtraction could introduce a false asymmetry.

Ideally, background data would be taken with the complete target but with the polarizable hydrogen removed. However, even if a strong effort were made to build a background target as identical as possible to a butanol target without hydrogen, as explained in Sec. II B 2, some discrepancies still exist. It is not possible to create a target with the correct molecular combination of  $(C_4O)$ , or even with pure carbon or oxygen at the same density as the one in the butanol target. From Tables II and III, it is clear that the effective thicknesses of the elements in both targets are not exactly identical. Moreover, the background target is half the physical thickness of the butanol target (see Sec. II B 2), which increases the quantity of <sup>3</sup>He in the path of the incident pion beam. These discrepancies are listed in Table IV. Nevertheless, it is safe to assume that the background due to the dilution refrigerator itself is perfectly subtracted because the background target and the butanol target were both enclosed within the same refrigerator.

In the case of perfect background subtraction, one would expect the analyzing power to be constant within the hydrogen peak and to be zero outside it. When the analyzing powers were plotted for each missing-mass channel, it was obvious that the background was not uniform in the tails of the hydrogen peak, as shown in Fig. 7. However, it can be seen



FIG. 7. A representative spectral distribution of experimental analyzing powers for each missing-mass channel.

that within the interval  $[Q_1, Q_2]$ , the analyzing power is constant. This interval corresponds to the hydrogen peak. Therefore a cut on the missing mass was made to suppress the events that could create a false asymmetry. Only the data within the interval  $[Q_1, Q_2]$  were kept. This method was applied to each angular bin of the kinematic sets because the positions of the <sup>3</sup>He, C, and O peaks in the missing-mass spectra are strongly dependent on the beam momentum and the scattering angle.

# **D.** Uncertainties in $A_v$

The fact that the data consisted of multiple runs for each kinematic set could be used to estimate the systematic errors of the analyzing powers. It was assumed that the statistical uncertainties for the non-normalized yield of the observed spectra could be evaluated by using Poisson statistics  $\sigma_{Z_{\uparrow/\downarrow}} = \sqrt{Z_{\uparrow/\downarrow}}$ .

Recalling that the data-acquisition polarization sequence used was  $\uparrow \downarrow \downarrow \uparrow$ , the analyzing power for the first and fourth (or second and third group) can be evaluated. An even better method for finding time-dependent factors was to evaluate analyzing powers for random combinations of the individual runs within each group. From these results, which ideally would have  $A_y = 0$ , it was possible to determine which combinations were inconsistent with the expected value and then to infer those individual runs that were responsible for such inconsistency. Some inconsistencies arose, for example, from computer failures before the end of a run, or failures of the wire-chamber high voltage system. Some of these issues could be resolved from the remaining data for a run.

Once the inconsistencies were removed, the set of analyzing powers  $A_i$  for random combinations of runs with identical spin orientation were extracted, as illustrated in Fig. 8. The standard deviation for this distribution, which measures the fluctuations of  $A_i$  about zero, is  $\sigma_{A_i} = 0.031$ . The variations of these analyzing powers can arise from fluctuations in the scaler counters, fluctuations in the steering of the beam, and uncertainties in the determination of the target polarization. If the fluctuations of the analyzing powers  $A_i$  about zero



FIG. 8. Analyzing powers for combinations of runs with the same kinematic conditions and spin direction. The error bars represent the statistical uncertainties.

were due only to statistical uncertainties, one would expect that the standard deviation of the distribution  $A_i/\Delta A_i$  to be equal to unity, i.e.,

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i} \left(\frac{A_{i}}{\Delta A_{i}} - \mu\right)^{2}} = 1 = \sigma^{\text{stat}}, \quad (3.10)$$

where *N* is the total number of random combinations, the  $A_i$  are the analyzing-power values of each combination, and the mean  $\mu$  is given by

$$\mu = \frac{1}{N} \sum_{i} \frac{A_i}{\Delta A_i}.$$
 (3.11)

However, the standard deviation obtained for the distribution is  $\sigma = 1.65$ . It represents the fluctuations due to both statistics and systematics. Therefore, with the assumption that the errors combine in quadrature, the standard deviation for the systematic error is  $\sigma^{\text{syst}} = \sqrt{1.65^2 - 1^2} = 1.31$ . Converting back to the  $A_i$  distribution, the standard deviation due to the systematic uncertainties is

$$\sigma_{A_i}^{\text{syst}} = \frac{\sigma^{\text{syst}}\sigma_{A_i}}{\sigma} = 0.025.$$
(3.12)

Therefore, the "total" uncertainty for the experimental analyzing powers

$$\sigma_{A_y} = \sqrt{(\sigma_{A_y}^{\text{stat}})^2 + (\sigma_{A_y}^{\text{syst}})^2}$$
(3.13)

TABLE IV. Background discrepancies.

Element	Effective thickness (g/cm <sup>2</sup> )		$\frac{\text{Backgd} - \text{Butanal}}{\text{Backgd}}$	
	Background target	Butanol target	(%)	
<sup>3</sup> He	0.187	0.149	+20	
С	0.518	0.487	+6	
0	0.172	0.197	-14.5	



FIG. 9. Analyzing powers for the  $\pi^- \vec{p} \rightarrow \pi^0 n$  reaction (solid circles). The error bars are only statistical, and do not include the target polarization uncertainty of 4.5%. The solid and dashed lines are the SM95 and the KH80 phase-shift solutions, respectively. Data at nearby energies from Refs. [14–16] are also shown.

includes the statistical errors and an empirical estimate of the systematic errors.

The overall polarization uncertainty only includes the systematic errors because the statistical errors are very small. The primary sources of the polarization error arise from the accuracy of the temperature measurements for the thermalequilibrium data, and the ability to fit the spectra for the enhanced-polarization measurements. The overall polarization uncertainty was estimated to be 4.5% [29].

#### **IV. RESULTS**

#### A. Pion beam energy and scattering angle

The energy and direction of the beam at the interaction vertex must be evaluated in order to determine the energy of the reaction and the scattering angles in the final tabulations, and for use in the replay of the data.

The energy loss of the pion was evaluated for all materials between the exit window of the channel and the butanol target, including kapton at the end of the beam pipe, the ion chamber, air volume between the ion chamber and the refrigerator, and all refrigerator and target materials. About 95% of the energy loss occurred in the cryostat. The energy-loss calculations were based on the expressions of Barkas and Berger [40].

As mentioned earlier in Sec. II A, the pion beam deviates from the direct beam line because of the magnetic field for the polarized target. However, the beam was suitably steered by the magnet Werbecka so that it passed through the target at the center of the magnetic field. Therefore the effect of the Werbecka magnet and the magnetic field of the target gives a  $0^{\circ}$  exit angle different from the standard  $0^{\circ}$  beam line (see Fig. 2). The accurate determination of the angle  $\theta_{exit}$  ( $\theta$  in Fig. 2) between the new and the standard  $0^{\circ}$  exit angle is very important for determinating the scattering angle  $\theta_{scat}$ . Calculations of  $\theta_{exit}$  with different algorithms from maps of the Zoltan magnetic field profile agreed to better than  $0.1^{\circ}$ .

To determine the scattering angles, the positions of the two detectors also need to be known accurately. These positions were determined from theodolite measurements. Estimates of the uncertainties of the scattering angles are more difficult to make, but they are unlikely to exceed  $0.5^{\circ}$ .

# B. Consistency checks

# 1. Fiducial areas

As mentioned previously, the NMS detectors were designed and built for a main fiducial area of  $4 \times 8$  CsI crystals. It is also possible to choose smaller fiducial areas as determined by a given number of radiation lengths into the  $4 \times 8$  group. The fiducial area used for this spectrometer followed a conic projection back to the target. The main requirement in the selection of the fiducial area is that the limits must be sufficiently far from the edges of the wirechamber planes because events near the edges could produce showers that might exit the chamber window, resulting in too low an energy.

The check for consistency was to define different fiducial areas in the BGO planes and to require that the two decay photons fall within these windows. During replay, analyzing powers were calculated for different fiducial areas. It was found that the analyzing powers were independent of the chosen fiducial area, as expected, which proved their consistency even within the small statistical error bars.

#### 2. CsI crystal temperature

The CsI crystals and/or the phototubes are known to be very sensitive to temperature. To prevent any effects from variations of the ambient temperature, a cooling system was installed on the detectors. The gains of the CsI crystals are inversely correlated with the detector temperature and therefore to the room temperature. The centroids of the energy distributions of cosmic-ray events are a sensitive measure of the CsI crystal gains. These centroids were plotted with respect to time and they showed that any effects from ambient temperature variations on the detector temperatures and the CsI gains were very small and well within the uncertainties from other sources.

# 3. Overall wire chamber efficiency and live-time

The overall wire-chamber efficiency and the live-time variables have a direct impact on the analyzing power through the normalization factor, Eq. (3.2). These two parameters were evaluated and plotted for every run. Both the overall wire-chamber efficiency and the live-time were very stable within most of the kinematic sets. However, in some cases, the live-time changed by more than 1% from one polarization direction to the other, hence the importance of including it in the normalization factor. The overall wire-chamber efficiencies differed by well less than 0.5%, typically less than 0.1%, from one polarization direction to the other, which shows the high stability of the NMS over the course of this experiment ( $\sim 2$  months).

# C. Determination of the angular bins

The NMS angular coverage was approximately 20°. For better angular resolution, the data were divided into three angular bins during replay. The boundaries of each angular bin were determined as follows.

(1) A cut was applied on the missing mass to reject the events due to background.

(2) The number of events left in the missing-mass spectrum was extracted.

(3) The histogram of the scattering angle for these events was then divided into three bins of approximately equal statistics. The scattering angle for each angular bin is the average of the distribution of the scattering angles of the events within that bin. The distributions were not uniform across the bins.

#### **D.** Results

The results from the analysis of the data are shown in Fig. 9 along with the VPI SM95 [3] and Karlsruhe-Helsinki KH80 [41,42] phase-shift solutions and existing data at nearby energies. The error bars are only statistical. The analyzing powers are plotted against the center-of-mass scattering angles for each of the four energies. The data for several

TABLE V. Analyzing powers at 98.1, 138.8, 165.9, and 214.4 MeV. The total uncertainty  $\sigma_{A_y}$  is defined in Sec. III D. The errors do not include the overall target polarization uncertainty of 4.5%.

$T_{\pi^{-}}$	Scatterin	g angle	A <sub>v</sub>	$\sigma_A^{\mathrm{stat}}$	$\sigma_{A_{}}$
(MeV)	c.m. (deg.)	lab (deg.)		- y	y
98.1	8.02	6.51	0.054	0.019	0.031
	16.61	13.51	0.108	0.022	0.033
	28.06	22.91	0.223	0.021	0.033
	54.12	44.92	0.491	0.019	0.031
	71.90	60.70	0.499	0.019	0.031
	86.05	73.89	0.349	0.021	0.033
138.8	10.87	8.59	0.076	0.024	0.035
	20.00	15.84	0.169	0.025	0.035
	28.60	22.72	0.200	0.013	0.028
	30.94	24.61	0.231	0.016	0.030
	41.94	33.59	0.314	0.015	0.029
	51.19	41.31	0.426	0.021	0.033
	55.06	44.59	0.450	0.021	0.033
	65.89	53.97	0.556	0.019	0.031
	78.64	65.42	0.5545	0.031	0.040
	78.76	65.54	0.553	0.026	0.036
	91.81	77.85	0.428	0.025	0.035
	103.05	89.00	0.3125	0.027	0.037
165.9	14.13	10.97	0.109	0.014	0.029
	24.90	19.41	0.187	0.015	0.029
	29.04	22.68	0.193	0.013	0.028
	36.38	28.54	0.285	0.016	0.030
	41.86	32.97	0.322	0.013	0.028
	53.99	42.98	0.423	0.016	0.030
	56.59	45.17	0.461	0.013	0.028
	69.95	56.70	0.681	0.018	0.031
	79.10	64.93	0.708	0.038	0.045
	82.41	67.97	0.664	0.019	0.031
	91.95	77.00	0.639	0.031	0.040
	102.52	87.45	0.454	0.027	0.037
214.4	21.91	16.55	0.150	0.012	0.028
	32.60	24.77	0.248	0.014	0.029
	42.82	32.80	0.326	0.018	0.031
	49.62	38.25	0.4315	0.014	0.029
	61.27	47.85	0.5595	0.020	0.032
	73.32	57.34	0.663	0.022	0.032

sets of very similar angles were taken with adjacent and overlapping kinematic positionings of the NMS. It is important to note the good agreement of the data values for these sets. The data for these plots are given in Table V, along with their respective statistical and systematic errors. However, the overall polarization uncertainty of 4.5% is omitted because the analyzing-power values scale with a shift in polarization values.

## V. DISCUSSION

Experiment E1178 measured analyzing powers  $A_y$  for the SCX reaction  $\pi^- \vec{p} \rightarrow \pi^0 n$  over a broad angular range at four

incident pion kinetic energies between 98.1 and 214.4 MeV. The database for  $\pi^- p \rightarrow \pi^0 n$  analyzing powers across the  $\Delta(1232)$  resonance has nearly doubled with the addition of 36 new values. Therefore, an important gap in polarization observables for the SCX channel has been filled. The data are well described by the SM95 solution of the VPI phase-shift analysis [3].

In principle, the final  $A_y$  results can be combined with cross-section measurements from the  $\pi^- p \rightarrow \pi^0 n$  reaction to form spin-up and spin-down "transversity" cross sections [16]. These cross sections are defined by the relations

$$d\Sigma_{\uparrow} = d\sigma (1 + A_y), \qquad (5.1)$$

$$d\Sigma_{\perp} = d\sigma (1 - A_{\nu}). \tag{5.2}$$

As a result of the triangular inequality relationships arising from isospin invariance in the  $\pi N$  system, the transversity cross sections must be within limits given by combinations of the amplitudes from the  $\pi^+ p$  and  $\pi^- p$  elastic-scattering reactions. Thus,

$$\frac{1}{2}(\sqrt{d\Sigma_{\uparrow}^{+}} - \sqrt{d\Sigma_{\uparrow}^{-}})^2 \leqslant d\Sigma_{\uparrow}^{0} \leqslant \frac{1}{2}(\sqrt{d\Sigma_{\uparrow}^{+}} + \sqrt{d\Sigma_{\uparrow}^{-}})^2,$$
(5.3)

where  $d\Sigma^+ = d\Sigma(\pi^+ p \rightarrow \pi^+ p)$ ,  $d\Sigma^- = d\Sigma(\pi^- p \rightarrow \pi^- p)$ and  $d\Sigma^0 = d\Sigma(\pi^- p \rightarrow \pi^0 n)$ , with a similar expression for spin down.

Because the region of the  $\Delta(1232)$  is dominated by a single resonance, the SCX transversity cross sections are ex-

pected to lie very close to the lower limit of the elasticscattering combination. This situation is ideal for investigating isospin breaking in the  $\pi N$  system. Even relatively small isospin breaking might push the transversity cross sections significantly outside the good-isospin limits. Unfortunately, however, SCX cross-section data of sufficient accuracy are not currently available for such a test. Hope is expressed that this situation can be remedied soon. It is very important to know whether the unusually large isospin breaking that has been observed at pion energies below 100 MeV [9,10] also extends into the  $\Delta(1232)$  region. A solid understanding of such possible isospin breaking is essential to refine the extraction of the important  $\Sigma_{\pi N}$  term.

# ACKNOWLEDGMENTS

The authors wish to thank the management of LAMPF for coordinating resources for this experiment at a difficult time and for ensuring that it could be completed. The accelerator and the technical staff provided outstanding support for the preparation and execution of the experiment. Special thanks are also extended to John Zumbro for expert assistance with the surveys, and to John O'Donnell for very helpful assistance in the analysis of the data. This work was supported in part by the National Science Foundation under Grants No. PHY92-000435 and PHY95-04847; the U.S. Department of Energy, Division of Nuclear Physics under Grants No. W-31-109-ENG-38, DE-FG03-94ER40860, DE-FG03-94ER40847, and others; and the German Bundesministerium für Bildung and Forschung (BMFT) under Grant No. 06Tue669.

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