Linked cluster expansion for the calculation of the semi-inclusive $A(e,e'p)X$ processes using correlated Glauber wave functions

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The distorted one-body mixed density matrix, which is the basic nuclear quantity appearing in the definition of the cross section for the semi-inclusive $A(e,e'p)X$ processes, is calculated within a linked cluster expansion based upon correlated wave functions and the Glauber multiple scattering theory to take into account the final state interaction of the ejected nucleon. The nuclear transparency for ^{16}O and ^{40}Ca is calculated using realistic central and noncentral correlations and the important role played by the latter is illustrated.
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I. INTRODUCTION

The accurate calculation of the final state interactions (FSI's) of the ejected nucleons in exclusive and semi-inclusive processes of the type $A(e,e'N)(A-1)$, $A(e,e'N)X$, $A(e,e'NN)X$, etc., induced by medium- and high-energy electrons, is one of the most urgent and important theoretical challenges in the investigation of the properties of hadronic matter. As a matter of fact, the possibilities to get information on basic properties of bound hadrons, such as, for example, their momentum and energy distributions, crucially depend upon the ability to estimate to which extent FSI effects destroy the direct link between the measured cross section and the hadronic properties before interaction with the probe, which is generally provided by approximations, e.g., the impulse approximation (IA), which disregard FSI's (see, e.g., [1]). Another convincing motivation for an accurate treatment of FSI's stems from the expectation that at large Q^2 they should vanish because of color transparency (CT), an effect originally predicted by Brodsky [2] and Mueller [3], and extensively investigated by various authors (for recent reviews on the subject, see, e.g., [4]), according to which the ejectile rescattering amplitudes with elastic and inelastic intermediate states interfere destructively. Since the onset of the phenomenon is expected to show up at large values of Q^2, when FSI effects could be evaluated within the standard Glauber theory, the experimental investigation of CT relies on the detection of possible differences between experimental data and predictions of standard Glauber multiple scattering calculations of FSI's. However, because of the expected small difference, an accurate treatment of nuclear structure effects is a prerequisite in order to get reliable information on CT effects. Among the large variety of nuclear effects, those produced by nucleon-nucleon (NN) correlations, which will be called from now on initial state correlations (ISC's), play a dominant role, for many-body calculations based upon realistic NN interaction models predict a rich correlation structure of the nuclear wave function (see, e.g., [5]). The effect of NN correlations in the calculation of FSI's within the Glauber approach has been considered in various papers [6–14], where, as a result of the difficulty of the problem, various approximations have been introduced either by truncating the Glauber multiple scattering series or by considering oversimplified models of correlations, e.g., by adopting simple phenomenological Jastrow-type wave functions embodying only central correlations.

In this paper a novel approach to the problem is presented, based upon a linked cluster expansion series of the distorted one-body mixed density matrix starting from realistic correlated wave functions and Glauber multiple scattering operators. The expansion is such that, at each order in the correlations, Glauber multiple scattering is included at all order. The expansion is based upon the number conserving approach of [15], properly generalized to take into account Glauber FSI's.

Our paper is organized as follows: the basic elements of the theory, i.e., the concepts of semi-inclusive processes $A(e,e'N)X$, nuclear transparency, and distorted momentum distributions, are reviewed in Sec. II; the formal developments of the linked cluster expansion are illustrated in Sec. III; the basic elements underlying the calculations of the nuclear transparency, i.e., the correlated nuclear wave function and the Glauber multiple scattering operators, are discussed in Sec. IV, where the results of the calculations of the nuclear transparency in the processes $^{16}\mathrm{O}(e,e'p)X$ and $^{40}\mathrm{Ca}(e,e'p)X$ are also presented; finally, the summary and conclusions are given in Sec. V.

II. SEMI-INCLUSIVE PROCESS $A(e,e'p)X$, NUCLEAR TRANSPARENCY, AND DISTORTED MOMENTUM DISTRIBUTIONS

We will consider the process $A(e,e'p)X$ in which an electron with four-momentum $k_1 \equiv \{\mathbf{k}_1, i\epsilon_1\}$ is scattered off a nucleus with four-momentum $P_A \equiv \{\mathbf{0}, iM_A\}$ to a state $k_2 \equiv \{\mathbf{k}_2, i\epsilon_2\}$ and is detected in coincidence with a proton p with four-momentum $k_p \equiv \{\mathbf{k}_p, iE_p\}$; the final $(A-1)$ nuclear system with four-momentum $P_X \equiv \{\mathbf{P}_X, iE_X\}$ is undetected. The cross section describing the process can be written as follows:

$$\frac{d\sigma}{dQ^2 d\nu d\mathbf{k}_p} = K\sigma_{ep} P_D(E_m, \mathbf{k}_m), \tag{1}$$

where K is a kinematical factor, σ_{ep} the off-shell electron-nucleon cross section, and $Q^2 = |\mathbf{q}|^2 - \nu^2$ the four-momentum transfer. The quantity $P_D(E_m, \mathbf{k}_m)$ is the distorted nucleon spectral function which depends upon the observable *missing momentum*

$$\mathbf{k}_m = \mathbf{q} - \mathbf{k}_p \tag{2}$$

and *missing energy*

$$E_m = \nu + M - E_p. \tag{3}$$

The latter equation results from energy conservation

$$\nu + M_A = E_p + \sqrt{M_X^2 + \mathbf{p}_X^2} \tag{4}$$

if the total energy of the system X is approximated by its nonrelativistic expression and the recoil energy is disregarded. The distorted spectral function can be written in the following shorthand form [8]:

$$P_D(E_m, \mathbf{k}_m) = \sum_{f_X} |\langle \mathbf{k}_m, \Psi_{f_X} | \Psi_A \rangle|^2 \delta(E_m - (E_{\min} + E_{f_X})), \tag{5}$$

where $E_{\min} = M + M_{A-1} - M_A$, and

$$\begin{aligned}\langle \mathbf{k}_m, \Psi_{f_X} | \Psi_A \rangle = \int & e^{i\mathbf{k}_m \mathbf{r}_1} S_G^\dagger(\mathbf{r}_1 \cdots \mathbf{r}_A) \Psi_{f_X}^* \\ & \times (\mathbf{r}_2 \cdots \mathbf{r}_A) \Psi_A(\mathbf{r}_1 \cdots \mathbf{r}_A) \\ & \times \delta\left(\sum_{j=1}^{A} \mathbf{r}_j\right) \prod_{i=1}^{A} d\mathbf{r}_i,\end{aligned} \tag{6}$$

with Ψ_A and Ψ_{f_X} being the ground state wave function of the target nucleus and the wave function of the system X in the state f_X, respectively; the quantity S_G is the Glauber operator, which describes the FSI's of the struck proton with the $(A-1)$ system, i.e.,

$$S_G(\mathbf{r}_1 \cdots \mathbf{r}_A) = \prod_{j=2}^{A} G(\mathbf{r}_1, \mathbf{r}_j) \equiv \prod_{j=2}^{A} [1 - \theta(z_j - z_1)\Gamma(\mathbf{b}_1 - \mathbf{b}_j)], \tag{7}$$

where $\mathbf{b}_j$ and z_j are the transverse and the longitudinal components of the nucleon coordinate $\mathbf{r}_j \equiv (\mathbf{b}_j, z_j)$, $\Gamma(\mathbf{b})$ is the Glauber profile function for elastic proton nucleon scattering, and the function $\theta(z_j - z_1)$ takes care of the fact that the struck proton ‘‘1’’ propagates along a straight-path trajectory so that it interacts with nucleon ‘‘j’’ only if $z_j > z_1$. The integral over the missing energy of the distorted spectral function defines the distorted momentum distribution as

$$n_D(\mathbf{k}_m) = \int dE_m P_D(E_m, \mathbf{k}_m). \tag{8}$$

In the impulse approximation [i.e., when the final state interaction is disregarded $(S_G = 1)$], if the system X is assumed to be a $(A-1)$ nucleus in the discrete or continuum states $f_X \equiv f_{A-1}$, the distorted spectral function P_D reduces to the usual spectral function, i.e.,

$$P_D \rightarrow P(k,E) = \sum_{f_{A-1}} |\langle \mathbf{k}, \Psi_{f_{A-1}} | \Psi_A \rangle|^2 \delta(E - (E_{\min} + E_{f_{A-1}})), \tag{9}$$

where E is the nucleon removal energy, i.e., the energy required to remove a nucleon from the target, leaving the $A-1$ nucleus with excitation energy $E_{f_{A-1}}$, and $-\mathbf{k} = \mathbf{k}_m = \mathbf{q} - \mathbf{k}_p$ is the nucleon momentum before interaction. The integral of the spectral function over the E defines the (undistorted) momentum distributions

$$n(\mathbf{k}) = \int dE P(E, \mathbf{k}). \tag{10}$$

In this paper we will consider the effect of FSI's $(S_G \neq 1)$ on the semi-inclusive $A(e,e'p)X$ process, i.e., the cross section (1) integrated over the missing energy E_m, at fixed value of $\mathbf{p}_m$. Owing to

$$\sum_{f_X} \Psi_{f_X}^*(\mathbf{r}_2' \cdots \mathbf{r}_A') \Psi_{f_X}(\mathbf{r}_2 \cdots \mathbf{r}_A) = \prod_{j=2}^{A} \delta(\mathbf{r}_j - \mathbf{r}_j'), \tag{11}$$

the cross section (1) becomes directly proportional to the distorted momentum distributions (8), i.e.,

$$n_D(\mathbf{k}_m) = (2\pi)^{-3} \int e^{i\mathbf{k}_m(\mathbf{r}-\mathbf{r}')} \rho_D(\mathbf{r}, \mathbf{r}') d\mathbf{r} d\mathbf{r}', \tag{12}$$

where

$$\rho_D(\mathbf{r}, \mathbf{r}') = \frac{\langle \Psi_A S_G^\dagger \hat{O}(\mathbf{r}, \mathbf{r}') S_G' \Psi_A' \rangle}{\langle \Psi_A \Psi_A \rangle} \tag{13}$$

is the one-body mixed density matrix, and

$$\hat{O}(\mathbf{r}, \mathbf{r}') = \sum_i \delta(\mathbf{r}_i - \mathbf{r}) \delta(\mathbf{r}_i' - \mathbf{r}') \prod_{j \neq i} \delta(\mathbf{r}_j - \mathbf{r}_j') \tag{14}$$

the one-body density operator. In Eq. (13) and in the rest of the paper, the primed quantities have to be evaluated at $\mathbf{r}'$ with $i = 1, \ldots, A$. By integrating $n_D(\mathbf{k}_m)$ the nuclear transparency T is obtained, which is defined as follows:

$$T=\int n_D(\mathbf{k}_m)d\mathbf{k}_m$$

$$=(2\pi)^{-3}\int \rho_D(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}'\int e^{i\mathbf{k}_m(\mathbf{r}-\mathbf{r}')}d\mathbf{k}_m=\int \rho_D(\mathbf{r})d\mathbf{r}, \tag{15}$$

i.e.,

$$T=\int \rho_D(\mathbf{r})d\mathbf{r}=1+\Delta T, \tag{16}$$

where ΔT originates from FSI's. The nuclear momentum distributions and the one-body density are normalized as follows:

$$\int n(\mathbf{p})d\mathbf{p}=\int \rho(\mathbf{r})d\mathbf{r}=1. \tag{17}$$

III. ONE-BODY MIXED DENSITY MATRIX AND NUCLEAR TRANSPARENCY WITHIN A LINKED CLUSTER EXPANSION FOR GLAUBER CORRELATED WAVE FUNCTIONS

We have evaluated the one-body density matrix (13) using for S_G the form (7) and for the nuclear wave function Ψ_A the following form:

$$\Psi_A=\hat{S}\left[\prod_{i<j}\hat{f}(ij)\right]\Psi_0, \tag{18}$$

where

$$\hat{f}(ij)=\sum_n f_n(r_{ij})\hat{O}_n(ij), \tag{19}$$

$\hat{S}$ is the symmetrization operator, Ψ_0 the Slater determinant describing the nucleon-independent particle motion, and $f_n(r_{ij})$ the correlation function associated with the operator $\hat{O}_n(ij)$ [if $\hat{O}_n(ij)=0$ for $n>1$, the usual Jastrow wave function is recovered]. If Glauber FSI's and nucleon-nucleon correlations are both absent ($S_G=1$, $f_1=1$, $f_n=0$, for $n>1$), the standard results for the shell-model one-body mixed density matrices,

$$\rho_{SM}(\mathbf{r},\mathbf{r}')=\sum_\alpha \phi^*_\alpha(\mathbf{x})\phi_\alpha(\mathbf{x}')=4\rho_0(\mathbf{r},\mathbf{r}'), \tag{20}$$

$$\rho_{SM}(\mathbf{r}_i,\mathbf{r}_j)=\sum_\alpha \phi^*_\alpha(\mathbf{x}_i)\phi_\alpha(\mathbf{x}_j)=4\rho_0(\mathbf{r}_i,\mathbf{r}_j), \tag{21}$$

and the one-body diagonal matrix,

$$\rho_{SM}(\mathbf{r}_i)\equiv\rho_{SM}(\mathbf{r}_i,\mathbf{r}_i)=\sum_\alpha |\phi_\alpha(\mathbf{x}_i)|^2=4\rho_0(\mathbf{r}_i), \tag{22}$$

are obtained, where

$$\rho_0(\mathbf{r}_i,\mathbf{r}_j)=\sum_a \varphi^*_a(\mathbf{r}_i)\varphi_a(\mathbf{r}_j) \tag{23}$$

and

$$\rho_0(\mathbf{r}_i)=\sum_a |\varphi_a(\mathbf{r}_i)|^2 \tag{24}$$

are the spin- and isospin-independent matrices. In the above equations, the notation $\alpha\equiv\{a,\sigma,\tau\}$, $a\equiv\{n,l,m\}$, and $\mathbf{x}\equiv\{\mathbf{r},\mathbf{s},\mathbf{t}\}$ has been used, which means that the single particle orbitals have the representation $\phi_\alpha(\mathbf{x})=\varphi_a(\mathbf{r})\chi_\sigma^{1/2}\xi_\tau^{1/2}$.

We have developed a linked cluster expansion in the quantity $\eta(\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_i',\mathbf{r}_j')=1+f^*(\mathbf{r}_i,\mathbf{r}_j)f(\mathbf{r}_i',\mathbf{r}_j')$ which includes, at each order in $\eta(\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_i',\mathbf{r}_j')$, the Glauber operator to all orders, and the result at first order reads as follows:

$$\rho_D(\mathbf{r}_1,\mathbf{r}_1')\simeq\left\langle \Psi_o\left|\prod G^\dagger(\mathbf{r}_1,\mathbf{r}_i)\hat{O}(\mathbf{r}_1,\mathbf{r}_1')\prod G(\mathbf{r}_1',\mathbf{r}_j')\right|\Psi_o'\right\rangle+\left\langle \Psi_o\left|\prod G^\dagger(\mathbf{r}_1,\mathbf{r}_i)\sum \eta(\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_i',\mathbf{r}_j')\hat{O}(\mathbf{r}_1,\mathbf{r}_1')\right.\right.$$
$$\left.\left.\times\prod G(\mathbf{r}_1',\mathbf{r}_j')\right|\Psi_o'\right\rangle-\left\langle \Psi_o\left|\sum \eta(\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_i,\mathbf{r}_j)\right|\Psi_o\right\rangle\left\langle \Psi_o\left|\prod G^\dagger(\mathbf{r}_1,\mathbf{r}_i)\hat{O}(\mathbf{r}_1,\mathbf{r}_1')G(\mathbf{r}_1',\mathbf{r}_i')\right|\Psi_o'\right\rangle. \tag{25}$$

Placing Eq. (14) in the above equation, one obtains

$$\rho_D(\mathbf{r}_1,\mathbf{r}_1')=\tilde{A}+\tilde{B}_1+\tilde{B}_2^L+\tilde{B}_2^U-\tilde{C}^U-\tilde{C}^L, \tag{26}$$

where

$$\tilde{A}=\rho_{SM}(\mathbf{r}_1,\mathbf{r}_1')\times\Phi(\mathbf{r}_1,\mathbf{r}_1')^{(A-1)}, \tag{27}$$

$$\tilde{B}_1=4\Phi(\mathbf{r}_1,\mathbf{r}_1')^{(A-2)}$$
$$\times\int d\mathbf{r}_2\{[4H^{\rm dir}(r_{12},r_{1'2})\rho_0(\mathbf{r}_1,\mathbf{r}_1')\rho_0(\mathbf{r}_2)$$
$$-H^{\rm ex}(r_{12},r_{1'2})\rho_0(\mathbf{r}_1,\mathbf{r}_2)\rho_0(\mathbf{r}_2,\mathbf{r}_1')]$$
$$\times G^\dagger(\mathbf{r}_1,\mathbf{r}_2)G(\mathbf{r}_1',\mathbf{r}_2)\}, \tag{28}$$

$$\tilde{B}_2^L = -4\Phi(\mathbf{r}_1,\mathbf{r}_1')^{(A-3)}\sum_{a\neq b}\varphi_a^*(\mathbf{r}_1)\varphi_b(\mathbf{r}_1') \times\int d\mathbf{r}_2 d\mathbf{r}_3\{[4H^{\rm dir}(r_{23})\varphi_b^*(\mathbf{r}_2)\varphi_a(\mathbf{r}_2)\rho(\mathbf{r}_3) - H^{\rm ex}(r_{23})\varphi_b^*(\mathbf{r}_2)\varphi_a(\mathbf{r}_3)\rho(\mathbf{r}_3,\mathbf{r}_2)]G^\dagger(\mathbf{r}_1,\mathbf{r}_2)G^\dagger(\mathbf{r}_1,\mathbf{r}_3) \times G(\mathbf{r}_1',\mathbf{r}_2)G(\mathbf{r}_1',\mathbf{r}_3)\}, \quad (29)$$

$$\tilde{B}_2^U = 4\Phi(\mathbf{r}_1,\mathbf{r}_1')^{(A-3)}\sum_{a\neq b}\varphi_a^*(\mathbf{r}_1)\varphi_a(\mathbf{r}_1') \times\int d\mathbf{r}_2 d\mathbf{r}_3\{[4H^{\rm dir}(r_{23})|\varphi_b(\mathbf{r}_2)|^2\rho(\mathbf{r}_3) - H^{\rm ex}(r_{23})\varphi_b^*(\mathbf{r}_2)\varphi_b(\mathbf{r}_3)\rho_0(\mathbf{r}_3,\mathbf{r}_2)]G^\dagger(\mathbf{r}_1,\mathbf{r}_2) \times G^\dagger(\mathbf{r}_1,\mathbf{r}_3)G(\mathbf{r}_1',\mathbf{r}_2)G(\mathbf{r}_1',\mathbf{r}_3)\}, \quad (30)$$

$$\tilde{C}^L = 4\Phi(\mathbf{r}_1,\mathbf{r}_1')^{(A-1)}\sum_{a}\varphi_a^*(\mathbf{r}_1)\varphi_a(\mathbf{r}_1') \times\int d\mathbf{r}_2 d\mathbf{r}_3\{[4H^{\rm dir}(r_{23})|\varphi_a(\mathbf{r}_2)|^2\rho_0(\mathbf{r}_3) - H^{\rm ex}(r_{23})\varphi_a^*(\mathbf{r}_2)\varphi_a(\mathbf{r}_3)\rho_0(\mathbf{r}_3,\mathbf{r}_2)]\}, \quad (31)$$

$$\tilde{C}^U = 4\Phi(\mathbf{r}_1,\mathbf{r}_1')^{(A-1)}\sum_{a\neq b}\varphi_a^*(\mathbf{r}_1)\varphi_a(\mathbf{r}_1') \times\int d\mathbf{r}_2 d\mathbf{r}_3\{[4H^{\rm dir}(r_{23})|\varphi_b(\mathbf{r}_2)|^2\rho_0(\mathbf{r}_3) - H^{\rm ex}(r_{23})\varphi_b^*(\mathbf{r}_2)\varphi_b(\mathbf{r}_3)\rho_0(\mathbf{r}_3,\mathbf{r}_2)]\}, \quad (32)$$

where $\rho_0(\mathbf{r}_i,\mathbf{r}_j)$ and $\rho_0(\mathbf{r}_i)$ are defined by Eqs. (23) and (24), respectively, $H^{\rm dir(ex)}(r_{12},r_{1'2})$ and $H^{\rm dir(ex)}(r_{23})$, where dir (ex) stands for *direct (exchange)*, respectively, depend upon the form of the correlation operator in Eq. (18) and will be defined in Sec. IV, and

$$[\Phi(\mathbf{r}_1,\mathbf{r}_1')]^n \equiv \left[\int \rho_o(\mathbf{r}_j)G^\dagger(\mathbf{r}_1,\mathbf{r}_j)G(\mathbf{r}_1',\mathbf{r}_j)d\mathbf{r}_j\right]^n, \quad (33)$$

with $n=(A-3),(A-2),(A-1),A$. In the above equations the sum over a and b runs over shell-model occupied states below the Fermi sea.

Equation (25) holds for any value of A. We will now consider the usual Glauber condition of large A: i.e., we consider $n=(A-3)\simeq(A-2)\simeq(A-1)\simeq A$. In such a case the various terms of Eq. (25) can be properly rearranged to finally obtain the following compact result:

$$\rho_D(\mathbf{r}_1,\mathbf{r}_1') \simeq \rho_{\rm SM}(\mathbf{r}_1,\mathbf{r}_1') + \rho_{\rm ISC}^H(\mathbf{r}_1,\mathbf{r}_1') + \rho_{\rm ISC}^S(\mathbf{r}_1,\mathbf{r}_1') + \rho_{\rm FSI}^{\rm SM}(\mathbf{r}_1,\mathbf{r}_1') + \rho_{\rm FSI}^H(\mathbf{r}_1,\mathbf{r}_1') + \rho_{\rm FSI}^S(\mathbf{r}_1,\mathbf{r}_1'). \quad (34)$$

The physical meaning of the various terms in Eq. (34) will be discussed later on; their explicit form is as follows:

$$\rho_{\rm ISC}^H(\mathbf{r}_1,\mathbf{r}_1') = 4\int d\mathbf{r}_2\{[4H^{\rm dir}(r_{12},r_{1'2})\rho_0(\mathbf{r}_1,\mathbf{r}_1')\rho_0(\mathbf{r}_2) - H^{\rm ex}(r_{12},r_{1'2})\rho_0(\mathbf{r}_1,\mathbf{r}_2)\rho_0(\mathbf{r}_2,\mathbf{r}_1')]\}, \quad (35)$$

$$\rho_{\rm ISC}^S(\mathbf{r}_1,\mathbf{r}_1') = -4\int d\mathbf{r}_2 d\mathbf{r}_3\{[4H^{\rm dir}(r_{23})\rho_0(\mathbf{r}_2,\mathbf{r}_1')\rho_0(\mathbf{r}_3) - H^{\rm ex}(r_{23})\rho_0(\mathbf{r}_2,\mathbf{r}_3)\rho_0(\mathbf{r}_3,\mathbf{r}_1')]\rho_0(\mathbf{r}_1,\mathbf{r}_2)\}, \quad (36)$$

$$\rho_{\rm FSI}^{\rm SM}(\mathbf{r}_1,\mathbf{r}_1') = \{\rho_{\rm SM}(\mathbf{r}_1,\mathbf{r}_1') + \rho_{\rm ISC}^H(\mathbf{r}_1,\mathbf{r}_1') + \rho_{\rm ISC}^S(\mathbf{r}_1,\mathbf{r}_1')\} \times\{\Phi(\mathbf{r}_1,\mathbf{r}_1')^A - 1\}, \quad (37)$$

$$\rho_{\rm FSI}^H(\mathbf{r}_1,\mathbf{r}_1') = \Phi(\mathbf{r}_1,\mathbf{r}_1')^A \times 4\int d\mathbf{r}_2\{[4H^{\rm dir}(r_{12},r_{1'2})\rho_0(\mathbf{r}_1,\mathbf{r}_1')\rho_0(\mathbf{r}_2) - H^{\rm ex}(r_{12},r_{1'2})\rho_0(\mathbf{r}_1,\mathbf{r}_2)\rho_0(\mathbf{r}_2,\mathbf{r}_1')] \times\Gamma(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_2)\}, \quad (38)$$

$$\rho_{\rm FSI}^S(\mathbf{r}_1,\mathbf{r}_1') = \rho_{\rm FSI}^{S,L}(\mathbf{r}_1,\mathbf{r}_1') + \rho_{\rm FSI}^{S,U}(\mathbf{r}_1,\mathbf{r}_1'), \quad (39)$$

with

$$\rho_{\rm FSI}^{S,L}(\mathbf{r}_1,\mathbf{r}_1') = -\Phi(\mathbf{r}_1,\mathbf{r}_1')^A \times 4\int d\mathbf{r}_2 d\mathbf{r}_3\{[4H^{\rm dir}(r_{23})\rho_0(\mathbf{r}_2,\mathbf{r}_1')\rho_0(\mathbf{r}_3) - H^{\rm ex}(r_{23})\rho_0(\mathbf{r}_2,\mathbf{r}_3)\rho_0(\mathbf{r}_3,\mathbf{r}_1)]\rho_0(\mathbf{r}_2,\mathbf{r}_1') \times\Gamma(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_2,\mathbf{r}_3)\}, \quad (40)$$

$$\rho_{\rm FSI}^{S,U}(\mathbf{r}_1,\mathbf{r}_1') = \Phi(\mathbf{r}_1,\mathbf{r}_1')^A 4\rho_0(\mathbf{r}_1,\mathbf{r}_1')\int d\mathbf{r}_2 d\mathbf{r}_3 \times\{[4H^{\rm dir}(r_{23})\rho_0(\mathbf{r}_2)\rho_0(\mathbf{r}_3) - H^{\rm ex}(r_{23})\rho_0(\mathbf{r}_2,\mathbf{r}_3)^2]\Gamma(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_2,\mathbf{r}_3)\}, \quad (41)$$

where $\Gamma(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_j)$ and $\Gamma(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_2,\mathbf{r}_3)$ denote the product of the Glauber factors G appearing in Eqs. (28), (29), and (30) minus 1 [see Eq. (45) below], and the superscripts S, H, L, and U stand for *spectator*, *hole*, *linked*, and *unlinked*, respectively.

Let us now discuss the meaning of the various terms appearing in Eq. (34). The first term represents the trivial shell-model contribution whereas $\rho_{\rm ISC}^{H(S)}$ represents the contribution from initial-state correlations. If only these three contributions are considered, the correlated momentum distribution calculated in several papers [16–18] are obtained, i.e.,

$$n(\mathbf{k}) = (2\pi)^{-3}\int e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}\rho_1(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}', \quad (42)$$

where

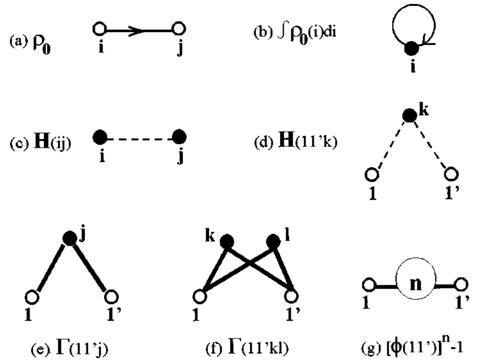

FIG. 1. The various diagrams corresponding to the terms in Eq. (45).

$$\rho_1(\mathbf{r}_1,\mathbf{r}_1')\equiv\rho_{\rm SM}(\mathbf{r}_1,\mathbf{r}_1')+\rho_{\rm ISC}^{H}(\mathbf{r}_1,\mathbf{r}_1')+\rho_{\rm ISC}^{S}(\mathbf{r}_1,\mathbf{r}_1'). \tag{43}$$

As will be clear later on using a diagrammatic representation, $\rho_{\rm ISC}^{H}(\mathbf{r}_1,\mathbf{r}_1')$ represents the contribution when particle ''1'' is correlated with a second particle, whereas $\rho_{\rm ISC}^{S}(\mathbf{r}_1,\mathbf{r}_1')$ represents the contribution from the correlation in a spectator pair composed of particles ''2'' and ''3.'' The last three terms of Eq. (34) represent the contribution from ISC's *and* FSI's; namely, $\rho_{\rm ISC}^{SM}$ represents the contribution when ISC's are present but a struck proton interacts in the final state with uncorrelated nucleons, whereas $\rho_{\rm ISC}^{H(S)}$ represents the contributions when initial state correlations are present but the struck nucleon interacts either with a partner, correlated nucleon ($\rho_{\rm ISC}^{H}$), or with a nucleon which is correlated with a third one ($\rho_{\rm ISC}^{S}$). By taking the Forier transform of Eq. (34) the distorted momentum distribution is obtained:

$$n_D(\mathbf{k}_m)=(2\pi)^{-3}\int e^{i\mathbf{k}_m(\mathbf{r}-\mathbf{r}')}\rho_D(\mathbf{r},\mathbf{r}')d\mathbf{r}d\mathbf{r}'. \tag{44}$$

Equation (43) clearly illustrates the number conserving property of the expansion; as a a matter of fact, it can be readily checked that when $\mathbf{r}_1=\mathbf{r}_1'$, the integrals over $\mathbf{r}_1$ of $\rho_{\rm ISC}^{H}(\mathbf{r}_1,\mathbf{r}_1)$ and $\rho_{\rm ISC}^{S}(\mathbf{r}_1,\mathbf{r}_1)$ are identical and of opposite sign, so that the number of particles is conserved; such a property holds to all orders of the expansion.

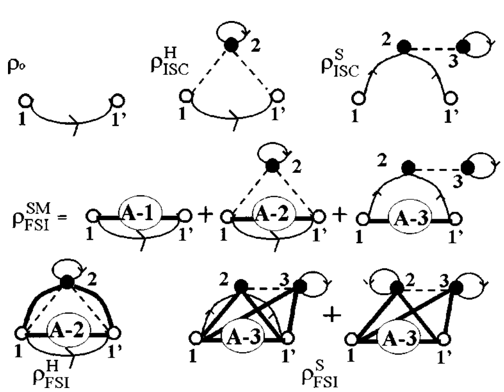

FIG. 2. The various diagrams corresponding to the terms in Eq. (34) (only the direct contributions are shown).

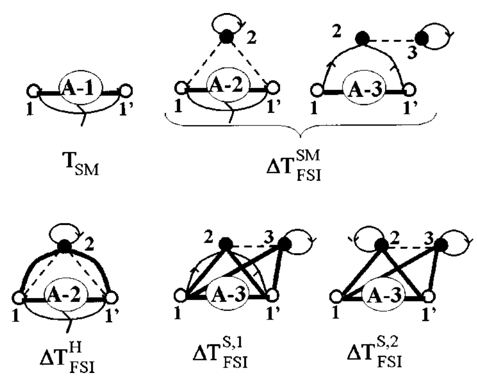

FIG. 3. The various diagrams corresponding to the terms of the nuclear transparency, Eq. (48) (only the direct contributions are shown).

A transparent diagrammatic representation of Eq. (34) can be given representing the generalization of the one given in [15–17] for the (undistorted) momentum distributions. The basic elements appearing in Eq. (34) are the following ones:

$$\text{(a)}\quad \rho_{\rm SM}(\mathbf{r}_i,\mathbf{r}_j)\equiv 4\rho_{\rm o}(\mathbf{r}_i,\mathbf{r}_j),$$

$$\text{(b)}\quad \int\rho_{\rm SM}(\mathbf{r}_i)d\mathbf{r}_i=4\int\rho_{\rm o}(\mathbf{r}_i)d\mathbf{r}_i,$$

$$\text{(c)}\quad H^{\rm dir(ex)}(r_{ij}),$$

$$\text{(d)}\quad H^{\rm dir(ex)}(r_{1k}r_{1'k}),$$

$$\text{(e)}\quad \Gamma(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_j)\equiv G^{\dagger}(\mathbf{r}_1,\mathbf{r}_j)G(\mathbf{r}_1',\mathbf{r}_j)-1,$$

$$\text{(f)}\quad \Gamma(\mathbf{r}_1,\mathbf{r}_1',\mathbf{r}_k,\mathbf{r}_l)\equiv G^{\dagger}(\mathbf{r}_1,\mathbf{r}_k)G(\mathbf{r}_1',\mathbf{r}_k)\times G^{\dagger}(\mathbf{r}_1,\mathbf{r}_l)G(\mathbf{r}_1',\mathbf{r}_l)-1,$$

$$\text{(g)}\quad [\Phi(\mathbf{r}_1,\mathbf{r}_1')]^n\equiv\left[\int\rho_{\rm o}(\mathbf{r}_j)G^{\dagger}(\mathbf{r}_1,\mathbf{r}_j)G(\mathbf{r}_1',\mathbf{r}_j)d\mathbf{r}_j\right]^n. \tag{45}$$

The diagrammatic representation of the various quantities defined in Eq. (45) is presented in Fig. 1, whereas the diagrammatic representation of Eq. (34) is given in Fig. 2, where only the direct terms are shown. The diagrams corresponding to the exchange terms can be readily drawn.

IV. NUCLEAR TRANSPARENCY FOR ^{16}O AND ^{40}Ca

In this section the results of the calculation of the nuclear transparency of ^{16}O and ^{40}Ca obtained using Eq. (34) will be presented. The results for the momentum distributions will be given in a separate paper [21].

A. Nuclear wave function

The nuclear wave function, Eq. (18), was constructed with Ψ_0 built up from harmonic oscillator orbitals and the correlation operators corresponding to the $V6$ Reid soft core (RSC) interaction, i.e., $O_1(ij)=1$, $O_2(ij)=\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$, $O_3(ij)$

TABLE I. The nuclear transparency, Eq. (48), for ^{16}O.

	T_{SM}	ΔT^{SM}_{FSI}	ΔT^{H}_{FSI}	$\Delta T^{S,1}_{FSI}$	$\Delta T^{S,2}_{FSI}$	T
Central	0.51	0.020	0.032	−0.013	0.022	0.57
Realistic	0.51	0.003	0.009	0.001	−0.001	0.52

TABLE II. The nuclear transparency, Eq. (48), for ^{40}Ca.

	T_{SM}	ΔT^{SM}_{FSI}	ΔT^{H}_{FSI}	$\Delta T^{S,1}_{FSI}$	$\Delta T^{S,2}_{FSI}$	T
Central	0.41	0.020	0.028	−0.011	0.023	0.47
Realistic	0.41	0.002	0.008	−0.001	0.001	0.42

$=\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$, $O_4(ij)=\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$, $O_5(ij)=S_{ij}$, $O_6(ij)=S_{ij}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$, where $S_{ij}=3[(\boldsymbol{\sigma}_i\cdot\mathbf{r}_{ij})(\boldsymbol{\sigma}_j\cdot\mathbf{r}_{ij})]/(r_{ij})^2-\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$.

The harmonic oscillator length parameter and the form of the correlation functions $f_n(r_{ij})$ have been obtained by minimizing the expectation value of the Hamiltonian calculated at the second order in the cluster expansion. The results will be presented elsewhere [19]. Having fixed the form of the various f_n's the quantities $H^{\rm dir(ex)}$ can be readily obtained. In the case of the simple Jastrow wave function one has

$$H^{\rm dir}(r_{ij})=H^{\rm ex}(r_{ij})=f_1(r_{ij})^2-1,$$

$$H^{\rm dir}(r_{ij},r_{i'j})=H^{\rm ex}(r_{ij},r_{i'j})=f_1(r_{ij})f_1(r_{i'j})-1, \quad (46)$$

but when the spin, isospin, and tensor dependences of the correlation functions are considered, a complex structure of $H^{\rm dir(ex)}$ is generated. The expressions of $H^{\rm dir(ex)}$ for the general case of the $V6$ RSC interaction are rather involved and will be reported elsewhere [19]; here, below, the results corresponding to the case of the dominant correlation functions of the $V6$ RSC interaction, i.e., f_1, f_4, and f_6, are shown:

$$H^{\rm dir}(r_{ij})=f_1(r_{ij})^2-1+27g(r_{ij})^2,$$

$$H^{\rm ex}(r_{ij})=f_1(r_{ij})^2-1-27g(r_{ij})^2+18f_1(r_{ij})g(r_{ij}),$$

$$H^{\rm dir}(r_{ij},r_{i'j})=f_1(r_{ij})f_1(r_{i'j})-1+27g(r_{ij})g(r_{i'j}),$$

$$H^{\rm ex}(r_{ij},r_{i'j})=f_1(r_{ij})f_1(r_{i'j})-1-27g(r_{ij})g(r_{i'j})+9f_1(r_{ij})g(r_{i'j})+9f_1(r_{i'j})g(r_{ij}), \quad (47)$$

where we have used $f_4=f_6\equiv g$.

B. Nuclear transparency for ^{16}O and ^{40}Ca

The nuclear transparency has been calculated by Eq. (16). Note that since the linked cluster expansion we are using is a number conserving one, the terms ρ^H_{ISC} and ρ^S_{ISC} give equal and opposite contributions to the integral in Eq. (16), so that ΔT gets contribution only from the terms ρ^{SM}_{FSI}, ρ^{H}_{FSI}, and ρ^{S}_{FSI}; therefore, the nuclear transparency can be represented in the following form:

$$T=1+\Delta T^{SM}_{FSI}+\Delta T^{H}_{FSI}+\Delta T^{S,1}_{FSI}+\Delta T^{S,2}_{FSI}, \quad (48)$$

where the spectator contribution has been split in two parts which, as will be seen later on, cancel to a large extent. Let us reiterate that ΔT^{SM}_{FSI} includes Glauber FSI's to all orders between the hit nucleon and uncorrelated nucleons. The diagrammatic representation of Eq. (48) is given in Fig. 3. Calculations have been performed by parametrizing the Glauber profile in the usual way [8]:

$$\Gamma(b)=\frac{\sigma_{\rm tot}(1-i\alpha)}{4\pi b_{\rm o}^2}e^{-b^2/(2b_{\rm o}^2)}, \quad (49)$$

with $\sigma_{\rm tot}$=43 mb, $\alpha=-0.33$, and $b_{\rm o}$=0.6 fm. Two different types of nuclear wave functions have been used, viz., the wave function, Eq. (18), corresponding to the Reid $V6$ interaction [20], with single particle and correlation parameters determined from the minimization of the nuclear Hamiltonian [17], and the phenomenological Jastrow wave function with central correlations, frequently used in the calculations of the transparency (see, e.g., [10]). The results of the calculations, which are presented in Tables I and II, deserve the following comments.

(1) Within the phenomenological central correlation approach, the effects of correlations on the nuclear transparency are sizable (about 12%).

(2) The contribution of the spectator term is almost zero, originating from two terms of opposite sign, and the effect of FSI's within correlated nucleons is almost entirely due to the hole contribution.

(3) Noncentral correlations affect very sharply the nuclear transparency, in that the overall effect of correlations reduces to about 2%, with the hole contribution remaining the dominant one and the spectator contribution canceling out.

It is important to stress that similar conclusions have been reached in [19], where the nuclear transparency in the process ^{4}He$(e,e'p)X$ has been obtained by an exact (to all order of correlations and Glauber multiple scattering) calculation performed using a realistic four-body wave function corresponding to the same interaction used in this paper.

Thus we have found a small effect of realistic correlations on the transparency, in apparent agreement with the results of, e.g., Ref. [8]; there, however, such a result is a consequence of a cancellation between hole and spectator contributions, whereas in our approach it is due to an overall decrease of the transparency generated by noncentral correlations, which lead to an almost vanishing contribution of the spectator effect, with the only surviving contributions being ΔT^{SM}_{FSI} and ΔT^{H}_{FSI}.[1] The reasons for the apparent overall agreement between our results and the ones of Ref. [8], are, at the moment, difficult to understand, also in view of the fact that the two approaches are formally different, with the latter one being based upon the Foldy-Walecka expansion [22], which requires the orthonormality condition

[1]Note that in the central Jastrow correlation approach, both for complex nuclei (cf. Table I) and for ^{4}He (cf. [11] and [19], where the Jastrow calculation has been carried out to all orders both in the correlations and the multiple scattering operators), correlations increase the transparency by more than 10%.

$\int d\mathbf{r}_1 \rho(\mathbf{r}_1) C(\mathbf{r}_1,\mathbf{r}_2)=0$, which, however, is not usually implemented in actual calculations.

V. SUMMARY AND CONCLUSIONS

Our work can be summarized as follows.

(1) A linked cluster expansion has been developed which includes both initial state correlations and final state interactions. The expansion holds for the most general form of the correlation function, which includes both central and noncentral correlations, and is such that at each order in the correlations, Glauber multiple scattering is included at all orders.

(2) The expansion has been applied to the calculation of the nuclear transparency in the processes $^{16}\mathrm{O}(e,e'p)X$ and $^{40}\mathrm{Ca}(e,e'p)X$. The results show that whereas central Jastrow correlations increase the transparency by about 12%, realistic central and noncentral correlations increase it by only 2%.

(3) A comparison of our results with the ones obtained for the nuclear transparency in the process $^{4}\mathrm{He}(e,e'p)X$ calculated by an exact treatment of realistic correlations and Glauber multiple scattering [19,21] shows similar results, indicating that the effects of correlations on triple- and higher-order Glauber multiple scattering contributions is negligible. A thorough investigation of the convergence of the distorted linked cluster expansion will be presented elsewhere [19], together with the results of the calculations for the distorted momentum distributions.

To sum up, the general conclusion can be drawn that a realistic calculation of the nuclear transparency in semi-inclusive processes $A(e,e'p)X$, for both light and heavy nuclei, can be performed, thus appreciably improving the pioneering estimates based on simple phenomenological nuclear wave functions embodying only central repulsive correlations.

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[1] S. Boffi, C. Giusti, and F. D. Pacati, Phys. Rep. **226**, 1 (1993).
[2] S. J. Brodsky, in *Proceedings of the 13th International Symposium on Multiparticle Dynamics*, edited by E. W. Kittel, W. Metzger, and A. Stergiou (World Scientific, Singapore, 1982), p. 964.
[3] A. Mueller, in *Proceedings of the 17th Rencontre de Moriond*, edited by J. Tranh Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1982), p. 13.
[4] L. L. Frankfurt, G. A. Miller, and M. Strikman, Annu. Rev. Nucl. Part. Sci. **45**, 501 (1994); N. N. Nikolayev, Surv. High Energy Phys. **7**, 1 (1994).
[5] S. C. Pieper, R. B. Wiringa, and V. R. Pandharipande, Phys. Rev. C **46**, 1741 (1992).
[6] O. Benhar *et al.*, Phys. Rev. Lett. **69**, 881 (1992); Phys. Lett. B **359**, 8 (1995).
[7] T. S. H. Lee and G. A. Miller, Phys. Rev. C **45**, 1863 (1992).
[8] N. N. Nikolayev *et al.*, Phys. Lett. B **317**, 281 (1993).
[9] N. N. Nikolaev, J. Speth, and B. G. Zakharov, J. Exp. Theor. Phys. **82**, 1046 (1996).
[10] A. Bianconi *et al.*, Phys. Lett. B **338**, 123 (1994).
[11] A. Bianconi *et al.*, Nucl. Phys. **A608**, 437 (1996).
[12] A. S. Rinat and B. K. Jennings, Nucl. Phys. **A568**, 873 (1994); A. S. Rinat and M. F. Taragin, Phys. Rev. C **52**, 28 (1995).
[13] L. L. Frankfurt, E. J. Moniz, M. M. Sargsyan, and M. I. Strikman, Phys. Rev. C **51**, 3435 (1995).
[14] A. Kohama, K. Yazaki, and R. Seki, Nucl. Phys. **A551**, 687 (1993); R. Seki *et al.*, Phys. Lett. B **383**, 133 (1996).
[15] M. Gaudin, J. Gillespie, and G. Ripka, Nucl. Phys. **A176**, 237 (1971).
[16] O. Bohigas and S. Stringari, Phys. Lett. **95B**, 9 (1980); M. F. Flynn *et al.*, Nucl. Phys. **A427**, 253 (1984).
[17] O. Benhar, C. Ciofi degli Atti, S. Liuti, and G. Salme, Phys. Lett. **28B**, 885 (1986).
[18] F. Arias de Saavedra, G. Co', and M. M. Renis, Phys. Rev. C **55**, 673 (1997).
[19] C. Ciofi degli Atti, H. Morita, and D. Treleani, nucl-th/9901086.
[20] I. E. Lagaris and V. R. Pandharipande, Nucl. Phys. **A359**, 331 (1981); O. Benhar *et al.*, Phys. Lett. B **359**, 8 (1995).
[21] H. Morita, C. Ciofi degli Atti, and D. Treleani (unpublished).
[22] L. L. Foldy and J. D. Walecka, Ann. Phys. (N.Y.) **54**, 447 (1969).