Systematic study of the effect of short range correlations on the form factors and densities of s-p and s-d shell nuclei

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(Received 5 March 1999; published 23 July 1999)

Analytical expressions of the one- and two-body terms in the cluster expansion of the charge form factors and densities of the s-p and s-d shell nuclei with $N=Z$ are derived. They depend on the harmonic oscillator parameter b and the parameter β which originates from the Jastrow correlation function. These expressions are used for the systematic study of the effect of short range correlations on the form factors and densities and of the mass dependence of the parameters b and β. These parameters have been determined by fit to the experimental charge form factors. The inclusion of the correlations reproduces the experimental charge form factors at the high momentum transfers ($q \gtrsim 2$ fm^{-1}). It is found that while the parameter β is almost constant for the closed shell nuclei ^{4}He, ^{16}O, and ^{40}Ca, its values are larger (less correlated systems) for the open shell nuclei, indicating a shell effect in the closed shell nuclei. [S0556-2813(99)03408-1]

I. INTRODUCTION

The calculation of the charge form factors $F_{ch}(q)$ and density distributions $\rho_{ch}(r)$ of nuclei is a challenging and appealing problem [1]. A possibility to face this problem is by means of an independent-particle model. This approach, which is particularly attractive because of its simplicity, fails to reproduce the high momentum transfer data from electron scattering in nuclei [2–11]. For this reason a modification of the single particle (SP) potentials has to be suitably made.

In fact a short range repulsion in this potential seems advisable for light nuclei [12]. For example, with an harmonic oscillator (HO) potential having in addition an infinite soft core, the $F_{ch}(q)$ of ^{4}He can be well reproduced, but for the heavier nuclei, such as ^{12}C and ^{16}O, state-dependent potentials seem necessary and even then the fit is not so good for higher q values [12]. Another way is the introduction of a modified shell model with fractional occupation numbers for the various states [13–16].

An approach, which is rather similar, is the introduction of the short range correlations (SRC's) in the Slater determinant. Many attempts have been made in this direction, concerning mainly light closed shell nuclei in the framework of the Born approximation [4–11,17–19].

Czyz and Lesniak [2] first showed that the diffraction character of the ^{4}He form factor can be qualitatively explained by means of Jastrow-type [20] correlations. Khana [3], using Iwamoto-Yamada cluster expansion and retaining only one- and two-body cluster terms, showed that the inclusion of short range nucleon-nucleon correlations provides an adequate description for the known data on the elastic scattering of electrons by ^{40}Ca and makes predictions for the behavior of the cross section at large momentum transfers. Ciofi Degli Atti, using the ''single pair approximation'' [4] and the Iwamoto-Yamada cluster expansion [5] in s-p shell nuclei, showed that elastic electron scattering at high momentum transfers seems to give a strong indication of the presence of SRC's in nuclei. Bohigas and Stringari [8] and

Dal Ri *et al.* [9] evaluated the effect of SRC's on the one- and two-body densities by developing a low order approximation (LOA) in the framework of the Jastrow formalism. They showed that one-body quantities like the form factor provide an adequate test for the presence of SRC's in nuclei, which indicates that the independent-particle wave functions cannot reproduce simultaneously the form factor and the momentum distribution of a correlated system. Stoitsov *et al.* [11] generalized the model of Jastrow correlations, suggested by Bohigas and Stringari [8] within the LOA of Ref. [7], to heavier nuclei like ^{16}O, ^{36}Ar, and ^{40}Ca reproducing very well the experimental data.

In the above approaches, different types of expansions were used. The expansions were connected with the number of the simultaneously correlated nucleons and the order of the Jastrow correlation function $f(r_{ij})$, which were retained in the cluster expansion. Usually they are truncated up to the two-body terms which give significant contributions to various expectation values.

In a series of papers [17–19] an expression of the elastic charge form factor $F_{ch}(q)$, truncated at the two-body term, was derived using the factor cluster expansion of Clark and co-workers [21–23]. This expression, which is a sum of one-body and two-body terms, and depends on the HO parameter and the correlation parameter through a Jastrow-type correlation function, is used for the calculation of $F_{ch}(q)$ of ^{4}He, ^{16}O, and ^{40}Ca and in an approximate way for the open s-p and s-d shell nuclei.

The motivation of the present work is the systematic study of the effect of SRC's on the s-p and s-d shell nuclei by completely avoiding the approximation made in earlier work [17–19] for open shell nuclei, that is, the expansion of the two-body terms in powers of the correlation parameter, in which only the leading terms had been retained. General expressions for the $F_{ch}(q)$ and $\rho_{ch}(r)$ were found using the factor cluster expansion of Clark and co-workers and Jastrow correlation functions which introduce SRC's. These expressions are functionals of the SP wave functions and not of the

wave functions of the relative motion of two nucleons as was the case in many previous works [4,10,17]. Because of that, it is easy to extrapolate them to the case of open shell nuclei and use them either for analytical calculations with HO wave functions or for numerical calculations when more realistic SP wave functions are used. An advantage of the present method is that the mass dependence of the HO parameter b (with the presence of correlations) and the correlation parameter β can be studied. These parameters have been determined, for the various s-p and s-d shell nuclei by fit of the theoretical $F_{ch}(q)$ to the experimental ones. It is found that while the parameter β is almost constant for the closed shell nuclei ^{4}He, ^{16}O, and ^{40}Ca, it takes larger values (less correlated systems) in the open shell nuclei, indicating a shell effect for the closed shells. The present method has also been used with wave functions of Skyrme-type interactions. In this case there is only one free parameter, the correlation parameter β, and the two-body term is a small correction to the one-body term of the density.

The paper is organized as follows. In Sec. II, the general expressions of the correlated form factors and density distributions are derived using a Jastrow correlation function. In Sec. III the analytical expressions of the above quantities for the s-p and s-d shell nuclei, in the case of the HO orbitals, are given. Numerical results are reported and discussed in Sec. IV, while an outline of the present work is given in Sec. V.

II. CORRELATED DENSITY DISTRIBUTIONS AND FORM FACTORS

If we denote the model operator, which introduces SRC's, by $\mathcal{F}$, an eigenstate Φ of the model system corresponds to an eigenstate

$$\Psi = \mathcal{F}\Phi \qquad (1)$$

of the true system.

Several restrictions can be made on the model operator $\mathcal{F}$, as, for example, that it depend on (the spins, isospins, and) relative coordinates and momenta of the particles in the system, that it be a scalar with respect to rotations, etc. [24]. Further, it is required that $\mathcal{F}$ be translationally invariant and symmetrical in its argument $1 \cdots i \cdots A$ and possess the cluster property. That is, if any subset $i_1 \cdots i_p$ of the particles is removed far from the rest $i_{p+1} \cdots i_A$, $\mathcal{F}$ decomposes into a product of two factors, $\mathcal{F}(1 \cdots A) = \mathcal{F}(i_1 \cdots i_p)\mathcal{F}(i_{p+1} \cdots i_A)$ [23]. In the present work $\mathcal{F}$ is taken to be of the Jastrow type [20],

$$\mathcal{F} = \prod_{i<j}^A f(r_{ij}), \qquad (2)$$

where $f(r_{ij})$ is the state-independent correlation function of the form

$$f(r_{ij}) = 1 - \exp[-\beta(\mathbf{r}_i - \mathbf{r}_j)^2]. \qquad (3)$$

The correlation function $f(r_{ij})$ goes to 1 for large values of $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ and it goes to 0 for $r_{ij} \to 0$. It is obvious that

the effect of SRC's, introduced by the function $f(r_{ij})$, becomes large when the SRC parameter β becomes small and vice versa.

The charge form factor of a nucleus, in the Born approximation, can be written

$$F_{ch}(q) = f_p(q)f_{DF}(q)f_{c.m.}(q)F_p(q), \qquad (4)$$

where $f_p(q)$ and $f_{DF}(q)$ are the correction for the finite proton size and the Darwin-Foldy relativistic correction, respectively [25], $f_{c.m.}(q)$ is the Tassie-Barker [26] center-of-mass correction, and $F_p(q)$ the point form factor of the nucleus which is the expectation value of the one-body operator,

$$\mathbf{O}_q = \sum_{i=1}^A \mathbf{o}_q(i) = \sum_{i=1}^A \exp[i\mathbf{q}\mathbf{r}_i]. \qquad (5)$$

That is,

$$F_p(q) = \frac{\langle\Psi|\mathbf{O}_q|\Psi\rangle}{\langle\Psi|\Psi\rangle} = N\langle\Psi|\mathbf{O}_q|\Psi\rangle = N\langle\mathbf{O}_q\rangle, \qquad (6)$$

where $N = \langle\Psi(r_1, r_2, \ldots, r_A)|\Psi(r_1, r_2, \ldots, r_A)\rangle^{-1}$ is the normalization factor which is determined so that $F_{ch}(0) = F_p(0) = 1$ or $4\pi\int_0^\infty \rho(r)r^2 dr = 1$.

The point density distribution has the form

$$\rho_p(r) = \frac{\langle\Psi|\mathbf{O}_r|\Psi\rangle}{\langle\Psi|\Psi\rangle} = N\langle\Psi|\mathbf{O}_r|\Psi\rangle = N\langle\mathbf{O}_r\rangle, \qquad (7)$$

where

$$\mathbf{O}_r = \sum_{i=1}^A \mathbf{o}_r(i) = \sum_{i=1}^A \delta(\mathbf{r} - \mathbf{r}_i). \qquad (8)$$

A. Expressions of the correlated density distributions and form factors

In order to evaluate the point density distribution $\rho_p(r)$, we consider, first, the generalized normalization integral

$$I(\alpha) = \langle\Psi|\exp[\alpha I(0)\mathbf{O}_r]|\Psi\rangle, \qquad (9)$$

corresponding to the operator $\mathbf{O}_r$, from which we have

$$\langle\mathbf{O}_r\rangle = \left[\frac{\partial \ln I(\alpha)}{\partial \alpha}\right]_{\alpha=0}. \qquad (10)$$

For the cluster analysis of Eq. (10), following the factor cluster expansion of Ristig, Ter Low, and Clark [21–23], we consider the sum-product integrals $I_i(\alpha)$, $I_{ij}(\alpha)$, $\ldots$ for a subsystems of the A-nucleon system and a factor cluster decomposition of these integrals. The expectation value of the density distribution operator is written in the form

$$\langle\mathbf{O}_r\rangle = \langle\mathbf{O}_r\rangle_1 + \langle\mathbf{O}_r\rangle_2 + \cdots + \langle\mathbf{O}_r\rangle_A, \qquad (11)$$

where

$$\langle \mathbf{O}_r \rangle_1 = \sum_{i=1}^{A} \left[\frac{\partial \ln I_i(\alpha)}{\partial \alpha} \right]_{\alpha=0} = \sum_{i=1}^{A} \langle i | \mathcal{F}_1^\dagger \mathbf{o}_r(1) \mathcal{F}_1 | i \rangle,$$

$$(12)$$

$$\langle \mathbf{O}_r \rangle_2 = \sum_{i<j}^{A} \frac{\partial}{\partial \alpha} [\ln I_{ij}(\alpha) - \ln I_i(\alpha) - \ln I_j(\alpha)]_{\alpha=0}$$

$$= \sum_{i<j}^{A} \langle ij | \mathcal{F}_{12}^\dagger [\mathbf{o}_r(1) + \mathbf{o}_r(2)] \mathcal{F}_{12} | ij \rangle_a$$

$$- \sum_{i<j}^{A} [\langle i | \mathbf{o}_r(1) | i \rangle + \langle j | \mathbf{o}_r(2) | j \rangle], \qquad (13)$$

and so on. $\mathcal{F}_1$ is chosen to be the identity operator.

The cluster expansion establishes the separation of one-body, two-body, ..., A-body correlation effects on the density. Three- and many-body terms will be neglected in the present analysis. Thus, in the two-body approximation, writing also the two sums of Eq. (13) in a different form, $\rho_p(r)$ is written as follows:

$$\rho_p(r) = N \langle \mathbf{O}_r \rangle \approx N[\langle \mathbf{O}_r \rangle_1 + \langle \mathbf{O}_r \rangle_{22} - \langle \mathbf{O}_r \rangle_{21}], \qquad (14)$$

where

$$\langle \mathbf{O}_r \rangle_1 = \sum_{i=1}^{A} \langle i | \mathbf{o}_r(1) | i \rangle, \qquad (15)$$

$$\langle \mathbf{O}_r \rangle_{22} = 2 \sum_{i<j}^{A} \langle ij | \mathcal{F}_{12}^\dagger \mathbf{o}_r(1) \mathcal{F}_{12} | ij \rangle_a, \qquad (16)$$

$$\langle \mathbf{O}_r \rangle_{21} = 2 \sum_{i<j}^{A} \langle ij | \mathbf{o}_r(1) | ij \rangle_a. \qquad (17)$$

If the two-body operator is taken to be the correlation function given by Eq. (3), then

$$\mathcal{F}_{12}^\dagger \mathcal{F}_{12} = 1 - 2g(r_1, r_2, \beta) + g(r_1, r_2, 2\beta), \qquad (18)$$

where

$$g(r_1, r_2, z) = \exp[-zr_1^2] \exp[-zr_2^2] \exp[2zr_1 r_2 \cos \omega_{12}],$$

$$z = \beta, 2\beta, \qquad (19)$$

and the term $\langle \mathbf{O}_r \rangle_{22}$ is written as

$$\langle \mathbf{O}_r \rangle_{22} = \langle \mathbf{O}_r \rangle_{21} - 2 O_{22}(r, \beta) + O_{22}(r, 2\beta), \qquad (20)$$

where

$$O_{22}(r, z) = 2 \sum_{i<j}^{A} \langle ij | \mathbf{o}_r(1) g(r_1, r_2, z) | ij \rangle_a. \qquad (21)$$

So $\rho_p(r)$ takes the form

$$\rho_p(r) \approx N[\langle \mathbf{O}_r \rangle_1 - 2 O_{22}(r, \beta) + O_{22}(r, 2\beta)]. \qquad (22)$$

The terms $\langle \mathbf{O}_r \rangle_1$ and $O_{22}(r, z)$ and the density $\rho_p(r)$ can be expressed also in the convenient form

$$\langle \mathbf{O}_r \rangle_1 = \rho_{SD}(\mathbf{r}), \qquad (23)$$

$$O_{22}(r, z) = \int g(r, r_2, z) [\rho_{SD}(\mathbf{r}) \rho_{SD}(\mathbf{r}_2) - \rho_{SD}^2(\mathbf{r}, \mathbf{r}_2)] d\mathbf{r}_2, \qquad (24)$$

$$\rho_p(r) \approx N \left[\rho_{SD}(\mathbf{r}) + \int [g(r, r_2, 2\beta) - 2g(r, r_2, \beta)] \right.$$

$$\left. \times [\rho_{SD}(\mathbf{r}) \rho_{SD}(\mathbf{r}_2) - \rho_{SD}^2(\mathbf{r}, \mathbf{r}_2)] d\mathbf{r}_2 \right], \qquad (25)$$

where $\rho_{SD}(\mathbf{r}_1, \mathbf{r}_2)$ is the uncorrelated density matrix associated with the Slater determinant,

$$\rho_{SD}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{i=1}^{A} \phi_i^*(\mathbf{r}_1) \phi_i(\mathbf{r}_2). \qquad (26)$$

The diagonal elements of this give the one-body density distribution

$$\rho_{SD}(\mathbf{r}_1) = \rho_{SD}(\mathbf{r}_1, \mathbf{r}_1). \qquad (27)$$

It should be noted that a similar expression for $\rho_p(r)$, given by Eq. (25), was derived by Gaudin *et al.* [7] in the framework of the LOA. In the LOA the Jastrow wave function Ψ of the nucleus was expanded in terms of the functions $\tilde{g} = f^2(r_{ij}) - 1$ and $h = f(r_{ij}) - 1$ and was truncated up to the second order of h and the first order of $\tilde{g}$. This expansion contains one- and two-body terms and a part of the three-body term which was chosen so that the normalization of the wave function was preserved. Expression (25) of the present work has only one- and two-body terms and the normalization of the wave function is preserved by the normalization factor N.

In the above expression of $\rho_p(r)$, the one-body contribution to the density is well known and given by the equation

$$\langle \mathbf{O}_r \rangle_1 = 4 \sum_{nl} \eta_{nl} (2l+1) \frac{1}{4\pi} \phi_{nl}^*(r) \phi_{nl}(r), \qquad (28)$$

where η_{nl} is the occupation probability of the state nl (0 or 1 in the case of closed shell nuclei) and $\phi_{nl}(r)$ is the radial part of the SP wave function.

An expression for the two-body term is usually found by making a transformation to the relative and the center-of-mass coordinates of the two interacting nucleons [4,10,17]. This is because the Jastrow function $f(r_{ij})$ depends on the relative coordinates of the two nucleons. Here, the expression for the two-body term, that is, of the term $O_{22}(r, z)$, will be found by expanding the factor $\exp[2zr_1 r_2 \cos \omega_{12}]$ in spherical harmonics [27,28]. That is,

$$\exp[2zr_1 r_2 \cos \omega_{12}]$$

$$= 2\pi \sum_{km_k} U_k(2zr_1 r_2) Y_{km_k}^*(\Omega_1) Y_{km_k}(\Omega_2), \qquad (29)$$

where

$$U_k(2zr_1r_2) = \int_{-1}^{1} \exp(2zr_1r_2 \cos \omega_{12})$$

$$\times P_k(\cos \omega_{12}) d(\cos \omega_{12})$$

$$= 2i_k(2zr_1r_2), \qquad (30)$$

and $i_k(x)$ is the modified spherical Bessel function.

Using the algebra of spherical harmonics, the term $O_{22}(r,z)$ takes the form

$$O_{22}(r,z) = 4 \sum_{n_il_i,n_jl_j} \eta_{n_il_i} \eta_{n_jl_j} (2l_i+1)(2l_j+1)$$

$$\times \left[4A_{n_il_in_jl_j}^{n_il_in_jl_jk}(r,z) \right.$$

$$\left. - \sum_{k=0}^{l_i+l_j} \langle l_i0l_j0|k0\rangle^2 A_{n_il_in_jl_j}^{n_jl_jn_il_ik}(r,z) \right],$$

$$z = \beta, 2\beta, \qquad (31)$$

where

$$A_{n_1l_1n_2l_2}^{n_3l_3n_4l_4k}(r,z) = \frac{1}{4\pi} \phi_{n_1l_1}^*(r) \phi_{n_3l_3}(r) \exp[-zr^2]$$

$$\times \int_0^\infty \phi_{n_2l_2}^*(r_2) \phi_{n_4l_4}(r_2) \exp[-zr_2^2]$$

$$\times i_k(2zrr_2) r_2^2 dr_2. \qquad (32)$$

Thus the expression of the term $O_{22}(r,z)$ depends on the SP wave functions and so it is suitable to be used either for analytical calculations with the HO potential or for numerical calculations with more realistic SP potentials. Expressions (28) and (31) were derived for the closed shell nuclei with $N=Z$, where η_{nl} is 0 or 1. For the open shell nuclei (with $N=Z$) we use the same expressions, where now $0 \leq \eta_{nl} \leq 1$. In this way the mass dependence of the correlation parameter β and the HO parameter b can be studied.

Finally, using the known values of the Clebsch-Gordan coefficients and provided that $\phi_{nl}^*(r)=\phi_{nl}(r)$, i.e., $A_{n_1l_1n_2l_2}^{n_3l_3n_4l_4k}(r,z) = A_{n_3l_3n_4l_4}^{n_1l_1n_2l_2k}(r,z)$, Eq. (31), for the case of s-p and s-d shell nuclei, takes the form

$$O_{22}(r,z) = 4 \left[3A_{0000}^{00000}(r,z) \eta_{1s}^2 + [33A_{0101}^{01010}(r,z) - 6A_{0101}^{01012}(r,z)] \eta_{1p}^2 + 3A_{1010}^{10100}(r,z) \eta_{2s}^2 + \left(95A_{0202}^{02020}(r,z) - \frac{50}{7}A_{0202}^{02022}(r,z) \right. \right.$$

$$\left. - \frac{90}{7}A_{0202}^{02024}(r,z) \right) \eta_{1d}^2 + [12A_{0001}^{00010}(r,z) + 12A_{0100}^{01000}(r,z) - 6A_{0001}^{01001}(r,z)] \eta_{1s}\eta_{1p} + [20A_{0002}^{00020}(r,z) + 20A_{0200}^{02000}(r,z)$$

$$- 10A_{0002}^{02002}(r,z)] \eta_{1s}\eta_{1d} + [60A_{0102}^{01020}(r,z) + 60A_{0201}^{02010}(r,z) - 12A_{0102}^{02011}(r,z) - 18A_{0102}^{02013}(r,z)] \eta_{1p}\eta_{1d}$$

$$+ [4A_{0010}^{00100}(r,z) + 4A_{1000}^{10000}(r,z) - 2A_{0010}^{10000}(r,z)] \eta_{1s}\eta_{2s} + [12A_{0110}^{01100}(r,z) + 12A_{1001}^{10010}(r,z) - 6A_{0110}^{10011}(r,z)] \eta_{1p}\eta_{2s}$$

$$+ [20A_{0210}^{02100}(r,z) + 20A_{1002}^{10020}(r,z) - 10A_{0210}^{10022}(r,z)] \eta_{1d}\eta_{2s} \right]. \qquad (33)$$

The point form factor $F_p(q)$ can be derived in two equivalent ways. The first one is to follow the same cluster expansion as in the case of the density distribution and the second one is to take the Fourier transform of the density distribution $\rho_p(r)$,

$$F_p(q) = \int \exp[i\mathbf{q}\mathbf{r}]\rho_p(r)d\mathbf{r}. \qquad (34)$$

In both cases, the form factor takes the following form:

$$F_p(q) \approx N[\langle \mathbf{O}_q \rangle_1 - 2\tilde{O}_{22}(q,\beta) + \tilde{O}_{22}(q,2\beta)]. \qquad (35)$$

In the above expression, the one-body term is given by the equation

$$\langle \mathbf{O}_q \rangle_1 = 4 \sum_{nl} \eta_{nl}(2l+1) \int_0^\infty \phi_{nl}^*(r) \phi_{nl}(r) j_0(qr) r^2 dr, \qquad (36)$$

while the two-body term $\tilde{O}_{22}(q,z)$ is given by the right hand side of Eqs. (31) and (33) by replacing the matrix elements $A_{n_1l_1n_2l_2}^{n_3l_3n_4l_4k}(r,z)$ by $\tilde{A}_{n_1l_1n_2l_2}^{n_3l_3n_4l_4k}(q,z)$ given by the equation

$$\tilde{A}_{n_1l_1n_2l_2}^{n_3l_3n_4l_4k}(q,z) = \int_0^\infty \phi_{n_1l_1}^*(r_1) \phi_{n_3l_3}(r_1)$$

$$\times \exp[-zr_1^2] j_0(qr_1) r_1^2 dr_1$$

$$\times \int_0^\infty \phi_{n_2l_2}^*(r_2) \phi_{n_4l_4}(r_2)$$

$$\times \exp[-zr_2^2] i_k(2zr_1r_2) r_2^2 dr_2, \qquad (37)$$

where $j_0(x)$ and $i_k(x)$ are the spherical Bessel and the modified spherical Bessel functions, respectively.

B. *l-s* coupling scheme

In the cases of mean field potentials with *l-s* coupling, the wave function of the nlj state has the form

$$\psi_{nlj}(r_1)=\phi_{nlj}(r_1)\sum_{m_l,m_s}\langle lm_lsm_s|jm_j\rangle$$

$$\times Y_l^{m_l}(\Omega)\chi_s^{m_s}(1)\chi_t^{m_t}(1), \qquad (38)$$

where $\phi_{nlj}(r_1)$ is the radial part of the wave function of the state nlj, and $\chi_s^{m_s}(1)$ and $\chi_t^{m_t}(1)$ are the spin and the isospin wave functions, respectively.

Following the same procedure as in Sec. II A, the same expression (22) for $\rho_p(r)$ has been found. In this case the terms $\langle \mathbf{O}_r\rangle_1$ and $O_{22}(r,z)$ have the form

$$\langle\mathbf{O}_r\rangle_1=2\sum_{nlj}\eta_{nlj}(2j+1)\frac{1}{4\pi}\phi_{nlj}^*(r)\phi_{nlj}(r) \qquad (39)$$

and

$$O_{22}(r,z)=2\sum_{n_il_ij_i,n_jl_jj_j}\eta_{n_il_ij_i}\eta_{n_jl_jj_j}(2j_i+1)(2j_j+1)$$

$$\times\left[2A_{n_il_ij_in_jl_jj_j}^{n_il_ij_in_jl_jj_jk}(r,z)\right.$$

$$\left.-\frac{1}{2}\sum_{k=0}^{l_i+l_j}\langle l_i0l_j0|k0\rangle^2A_{n_il_ij_in_jl_jj_j}^{n_jl_jj_jn_il_ij_ik}(r,z)\right],$$

$$z=\beta,2\beta, \qquad (40)$$

where the matrix elements $A_{n_1l_1j_1n_2l_2j_2}^{n_3l_3j_3n_4l_4j_4k}(r,z)$ are given again by Eq. (32) replacing the wave functions $\phi_{nl}(r)$ by the wave functions $\phi_{nlj}(r)$. Equation (40) can also be written in a form similar to that of Eq. (33) with the difference that there are now more terms. This expression goes to Eq. (33) if there is no *l-s* coupling.

In the evaluation of the density matrix $\rho_{SD}(\mathbf{r}_1,\mathbf{r}_2)$, which is necessary for the derivation of the expression (40) of the term $O_{22}(r,z)$, we made the following approximation: In the sum over the spin coordinates, only terms of pairs of particles having the same third spin component are taken into account. The contribution of the terms which contain pairs with opposite third spin component is small and is neglected [29,30]. In this scheme the form factor is calculated numerically by Fourier transform of $\rho_p(r)$ employing Eq. (34).

III. ANALYTICAL EXPRESSIONS

In the case of the HO wave functions, with radial part,

$$\phi_{nl}(r)=N_{nl}b^{-3/2}\left(\frac{r}{b}\right)^lL_n^{l+1/2}\left(\frac{r^2}{b^2}\right)\exp\left[-\frac{r^2}{2b^2}\right], \qquad (41)$$

$$N_{nl}=\left(\frac{2n!}{\Gamma\left(n+l+\frac{3}{2}\right)}\right)^{1/2},$$

where the associated Laguerre polynomial is defined as in Ref. [31], analytical expressions of the one-body term and of the matrix elements $A_{n_1l_1n_2l_2}^{n_3l_3n_4l_4k}(r,z)$ and $\tilde{A}_{n_1l_1n_2l_2}^{n_3l_3n_4l_4k}(q,z)$, defined by Eqs. (32) and (37), can be found. From these expressions, the analytical expressions of the terms $O_{22}(r,z)$ and $\tilde{O}_{22}(q,z)$, defined by Eq. (33), can also be found.

The expression of the one-body term of the density and form factor has the form

$$\langle\mathbf{O}_x\rangle_1=Ce^{-\xi^2}\sum_{k=0}^2C_{2k}\xi^{2k}, \qquad x=r,q, \qquad (42)$$

where for the $\langle\mathbf{O}_r\rangle_1$ the variable ξ and the coefficients C and C_{2k} are

$$\xi=\frac{r}{b}, \qquad C=\frac{2}{\pi^{3/2}b^3},$$

$$C_0=2\eta_{1s}+3\eta_{2s},$$

$$C_2=4(\eta_{1p}-\eta_{2s}), \qquad (43)$$

$$C_4=\frac{4}{3}(2\eta_{1d}+\eta_{2s}),$$

while for the $\langle\mathbf{O}_q\rangle_1$ the corresponding quantities are

$$\xi=\frac{1}{2}bq, \qquad C=2,$$

$$C_0=2(\eta_{1s}+\eta_{2s}+3\eta_{1p}+5\eta_{1d}),$$

$$C_2=-\frac{4}{3}(3\eta_{1p}+10\eta_{1d}+2\eta_{2s}), \qquad (44)$$

$$C_4=\frac{4}{3}(2\eta_{1d}+\eta_{2s}).$$

The analytical expression of the matrix element $A_{n_1l_1n_2l_2}^{n_3l_3n_4l_4k}(r,z)$, which is given by Eq. (32), has the form

$$A_{n_1 l_1 n_2 l_2}^{n_3 l_3 n_4 l_4 k}(r,z) = \left(\prod_{i=1}^{4} N_{n_i l_i} \right) \frac{y^k}{16\sqrt{\pi}b^3} \xi^{l_1+l_3+k} L_{n_1}^{l_1+1/2}(\xi^2) L_{n_3}^{l_3+1/2}(\xi^2) \exp\left[-\frac{1+2y}{1+y}\xi^2 \right] \sum_{w=0}^{n_2} \sum_{s=0}^{n_4} \frac{(-1)^{w+s}}{w!s!} \binom{n_2+l_2+\frac{1}{2}}{n_2-w}$$

$$\times \binom{n_4+l_4+\frac{1}{2}}{n_4-s} \frac{[\frac{1}{2}(l_2+l_4-k)+w+s]!}{(1+y)^{(l_2+l_4+k+3)/2+w+s}} L_{(l_2+l_4-k)/2+w+s}^{k+1/2}\left(\frac{-y^2}{1+y}\xi^2 \right), \tag{45}$$

where $\xi = r/b$ and $y = zb^2$ ($z = \beta, 2\beta$). It is mentioned that the Clebsch-Gordan coefficients $\langle l_i 0 l_j 0 | k 0 \rangle$, which appear in Eq. (31) (where k runs from 0 to l_i+l_j), are different from zero only when l_i+l_j+k is an even integer and therefore l_2+l_4-k is an even integer. Because of that, the lower index of the Laguerre polynomial of Eq. (45) is an integer.

The above expression of $A_{n_1 l_1 n_2 l_2}^{n_3 l_3 n_4 l_4 k}(r,z)$ is of the form

$$f(\xi^2)\exp\left[-\frac{1+2y}{1+y}\xi^2 \right],$$

where $f(\xi^2)$ is a polynomial of ξ^2. The substitution of the expression of $A_{n_1 l_1 n_2 l_2}^{n_3 l_3 n_4 l_4 k}(r,z)$ to the expression of $O_{22}(r,z)$, which is given by Eq. (33), leads to the analytical expression of the two-body term of the density. This expression, for the case of s-p and s-d shell nuclei, is again of the form

$$f(\xi^2)\exp\left[-\frac{1+2y}{1+y}\xi^2 \right],$$

where now $f(\xi^2)$ is a polynomial of the form

$$f(\xi^2) = \sum_{k=0}^{4} C_{2k}(\eta_{n_k l_k}, y)\xi^{2k}. \tag{46}$$

The corresponding analytical expression of the matrix element $\tilde{A}_{n_1 l_1 n_2 l_2}^{n_3 l_3 n_4 l_4 k}(q,z)$, which is given by Eq. (37), has the form

$$\tilde{A}_{n_1 l_1 n_2 l_2}^{n_3 l_3 n_4 l_4 k}(q,z) = \frac{\pi}{16}\left(\prod_{i=1}^{4} N_{n_i l_i} \right) \exp\left[-\frac{1+y}{1+2y}\xi^2 \right] \sum_{p=0}^{n_1} \sum_{v=0}^{n_3} \sum_{w=0}^{n_2} \sum_{s=0}^{n_4} \sum_{t=0}^{(l_2+l_4-k)/2+w+s} \frac{(-1)^{p+v+w+s+t}}{p!v!w!s!t!} \binom{n_1+l_1+\frac{1}{2}}{n_1-p}$$

$$\times \binom{n_3+l_3+\frac{1}{2}}{n_3-v} \binom{n_2+l_2+\frac{1}{2}}{n_2-w} \binom{n_4+l_4+\frac{1}{2}}{n_4-s}$$

$$\times \frac{\Gamma(\frac{1}{2}(l_2+l_4+k+3)+w+s)[\frac{1}{2}(l_1+l_3+k)+p+v+t]![\frac{1}{2}(k-l_2-l_4)-w-s]_t}{\Gamma(t+k+\frac{3}{2})}$$

$$\times y^{2t+k} \frac{(1+y)^{(l_1+l_3-l_2-l_4)/2+p+v-w-s}}{(1+2y)^{(l_1+l_3+k+3)/2+p+v+t}} L_{(l_1+l_3+k)/2+p+v+t}^{1/2}\left(\frac{1+y}{1+2y}\xi^2 \right), \tag{47}$$

where $\xi = qb/2$ and $y = zb^2$ ($z = \beta, 2\beta$). The lower index of the Laguerre polynomial of Eq. (47) is an integer because l_1+l_3+k is an even integer.

The above expression of $\tilde{A}_{n_1 l_1 n_2 l_2}^{n_3 l_3 n_4 l_4 k}(q,z)$ is of the form

$$f(\xi^2)\exp\left[-\frac{1+y}{1+2y}\xi^2 \right],$$

where $f(\xi^2)$ is a polynomial of ξ^2. The substitution of the

expression of $\tilde{A}_{n_1 l_1 n_2 l_2}^{n_3 l_3 n_4 l_4 k}(q,z)$ to the expression of $\tilde{O}_{22}(q,z)$, given by Eq. (33), leads to the analytical expression of the two-body term of the form factor. This expression, for the case of s-p and s-d shell nuclei, is again of the form

$$\tilde{f}(\xi^2)\exp\left[-\frac{1+y}{1+2y}\xi^2 \right],$$

where now $\tilde{f}(\xi^2)$ is a polynomial similar to that of Eq. (46).

TABLE I. The values of the parameters b and β, of the χ^2, and of the rms charge radii $\langle r_{ch}^2 \rangle^{1/2}$: contribution of the one-body density (column HO), contribution of SRC (column SRC) and of the total rms charge radii (column Total), for various s-p and s-d shell nuclei, determined by fit to the experimental $F_{ch}(q)$. Case 1 refers to the HO form factor, case 2 when SRC are included. Case 2* is the same as case 2 but with the occupation probability of the state $2s$ taken to be a free parameter. Cases 3 and 4 refer to the SK1 potential with and without SRC's, respectively. The experimental rms charge radii are from Ref. [32].

Case	Nucleus	b [fm]	β [fm^{-2}]	χ^2	$\langle r_{ch}^2 \rangle^{1/2}$ [fm]			
					HO	SRC	Total	Expt.
1	^{4}He	1.4320	–	31	1.7651	–	1.7651	1.676(8)
2	^{4}He	1.1732	2.3126	3.5	1.5353	0.5277	1.6234	
1	^{12}C	1.6251	–	177	2.4901	–	2.4901	2.471(6)
2	^{12}C	1.5923	3.7051	110	2.4463	0.2566	2.4597	
1	^{16}O	1.7610	–	199	2.7377	–	2.7377	2.730(25)
2	^{16}O	1.6507	2.4747	120	2.5853	0.7070	2.6802	
3	^{16}O	–		148	2.6518	–	2.6518	
4	^{16}O	–	3.0201	146	2.6518	0.1503	2.6561	
1	^{24}Mg	1.8495	–	188	3.1170	–	3.1170	3.075(15)
2	^{24}Mg	1.8270	6.6112	161	3.0823	0.3009	3.0969	
2*	^{24}Mg	1.8315	7.1226	155	3.0893	0.2841	3.1023	
1	^{28}Si	1.8941	–	148	3.2570	–	3.2570	3.086(18)
2	^{28}Si	1.8738	8.2245	114	3.2249	0.2438	3.2341	
2*	^{28}Si	1.8743	8.8104	112	3.2260	0.2315	3.2343	
1	^{32}S	2.0016	–	320	3.4830	–	3.4830	3.248(11)
2	^{32}S	1.9810	9.1356	270	3.4497	0.2114	3.4561	
2*	^{32}S	1.9056	15.579	194	3.3284	0.1445	3.3315	
1	^{36}Ar	1.8800	–	–	3.3270	–	3.3270	3.327(15)
2	^{36}Ar	1.8007	2.2937	–	3.1970	0.9470	3.3343	
1	^{40}Ca	1.9453	–	229	3.4668	–	3.4668	3.479(3)
2	^{40}Ca	1.8660	2.1127	160	3.3353	1.1115	3.5156	
3	^{40}Ca	–		181	3.4097	–	3.4097	
4	^{40}Ca	–	2.1729	178	3.4097	0.2349	3.4178	

From the analytical expression of the form factor, the analytical expression of the mean square radius of a nucleus, which is the coefficient of $-q^2/6$, can be found. This expression is of the form

$$\langle r^2 \rangle = N[\langle r^2 \rangle_1 - 2\langle r^2(\beta b^2) \rangle_{22} + \langle r^2(2\beta b^2) \rangle_{22}], \quad (48)$$

where

$$\langle r^2 \rangle_1 = [6(\eta_{1s} + \eta_{2s}) + 30\eta_{1p} + 70\eta_{1d}]b^2 \quad (49)$$

and

$$\langle r^2(y) \rangle_{22} = b^2(1+2y)^{-13/2} \sum_{k=0}^{5} a_k y^k, \quad y = \beta b^2, 2\beta b^2. \quad (50)$$

The coefficients a_k depend on the occupation probabilities of the various states. If we expand, the right hand side of Eq. (50), in powers of $1/y$ ($y = \beta b^2 > 1$), and keep powers of βb^2 up to $(\beta b^2)^{-3/2}$, an approximate expression of the contribution of the two-body term to the radius takes the simple form

$$\langle r^2(y) \rangle_{22} = Cb^2(\beta b^2)^{-3/2}, \quad (51)$$

where the coefficient C depends on the occupation probabilities of the various states.

The analytical expressions of the form factor and the density, which were found previously, will be used in Sec. IV for the fit of the theoretical charge form factors to the experimental ones and for the calculations of the charge density distributions for various $N = Z$ (s-p and s-d shell) nuclei.
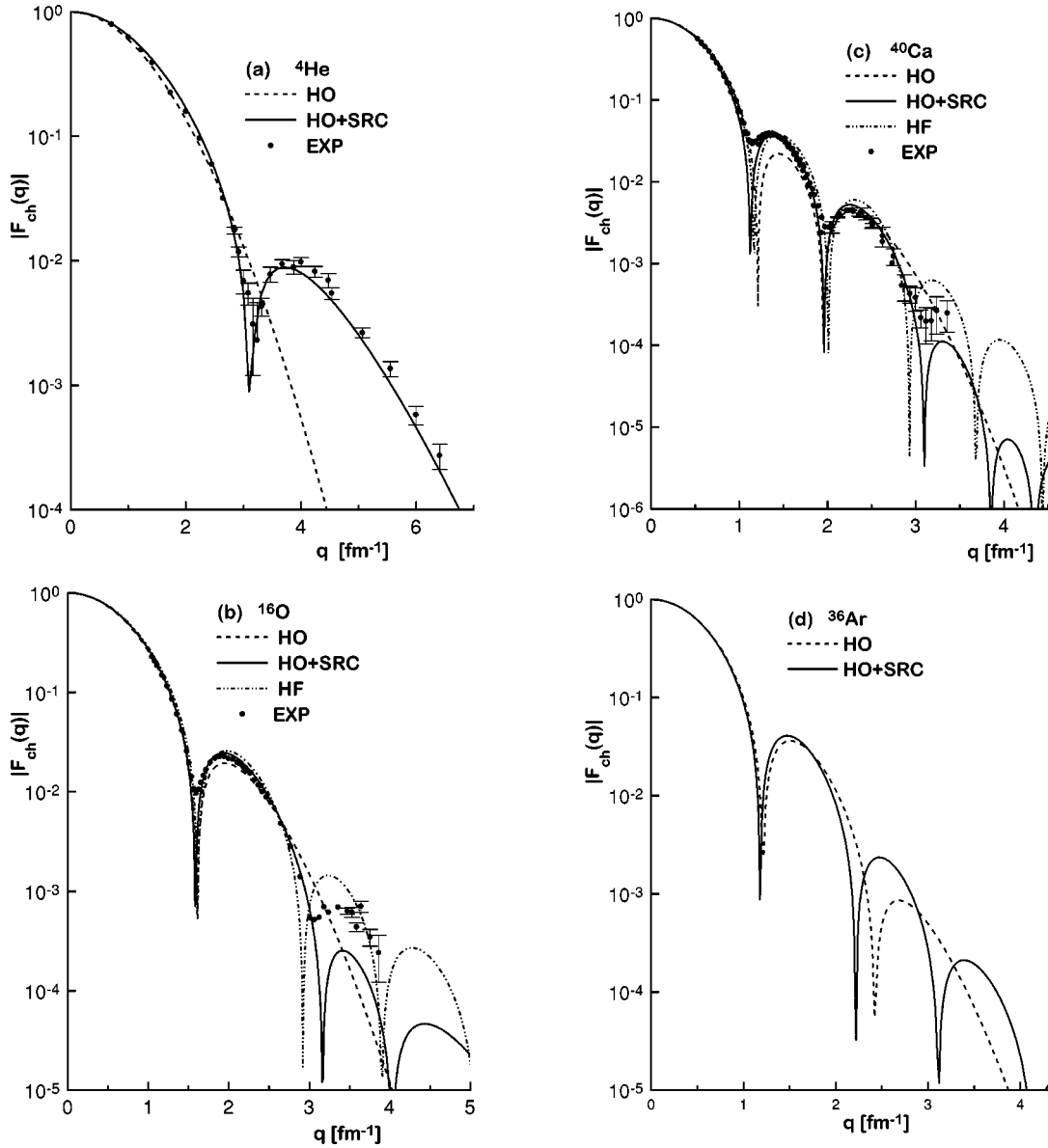
FIG. 1. The charge form factor of the nuclei ^{4}He (a), ^{16}O (b), ^{40}Ca (c), and ^{36}Ar (d) for various cases. The experimental points for ^{4}He, ^{16}O, and ^{40}Ca are from Refs. [33], [34], and [35], respectively.

IV. RESULTS AND DISCUSSION

The calculations of the charge form factors for various s-p and s-d shell nuclei, with $N=Z$, have been carried out on the basis of Eqs. (4) and (35) and the analytical expressions of the one- and two-body terms which were given in Sec. III. Two cases have been examined, named case 1 and case 2, which correspond to the analytical calculations with HO wave functions without and with SRC's, respectively. In case 1 there is one free parameter, the HO parameter b, while in case 2 there are two free parameters, the parameter b and the correlation parameter β. The parameters, in both cases, have been determined, for each nucleus separately, by a least squares fit to the experimental $F_{ch}(q)$.

The best fit values of the parameters as well as of the values of χ^2,

$$\chi^2 = \frac{1}{K} \sum_{i=1}^{K} \{[F_i(\text{theory}) - F_i(\text{expt})]/\Delta F_i(\text{expt})\}^2,$$

(52)

are displayed in Table I. In the same table the calculated root mean square (rms) charge radii $\langle r_{ch}^2 \rangle^{1/2}$ and the contribution of the SRC's to them,

$$\langle r^2 \rangle_2 = \langle r_{ch}^2 \rangle - \langle r_{ch}^2 \rangle_1 / A,$$

(53)

are displayed and compared with the corresponding experimental rms radii. It is noted that $\langle r^2 \rangle_2$ is independent from the center-of-mass correction and finite proton size.

The experimental and the theoretical $F_{ch}(q)$, for the various cases, for the closed shell nuclei ^{4}He, ^{16}O, and ^{40}Ca, are shown in Fig. 1 while for the open shell nuclei ^{12}C, ^{24}Mg,
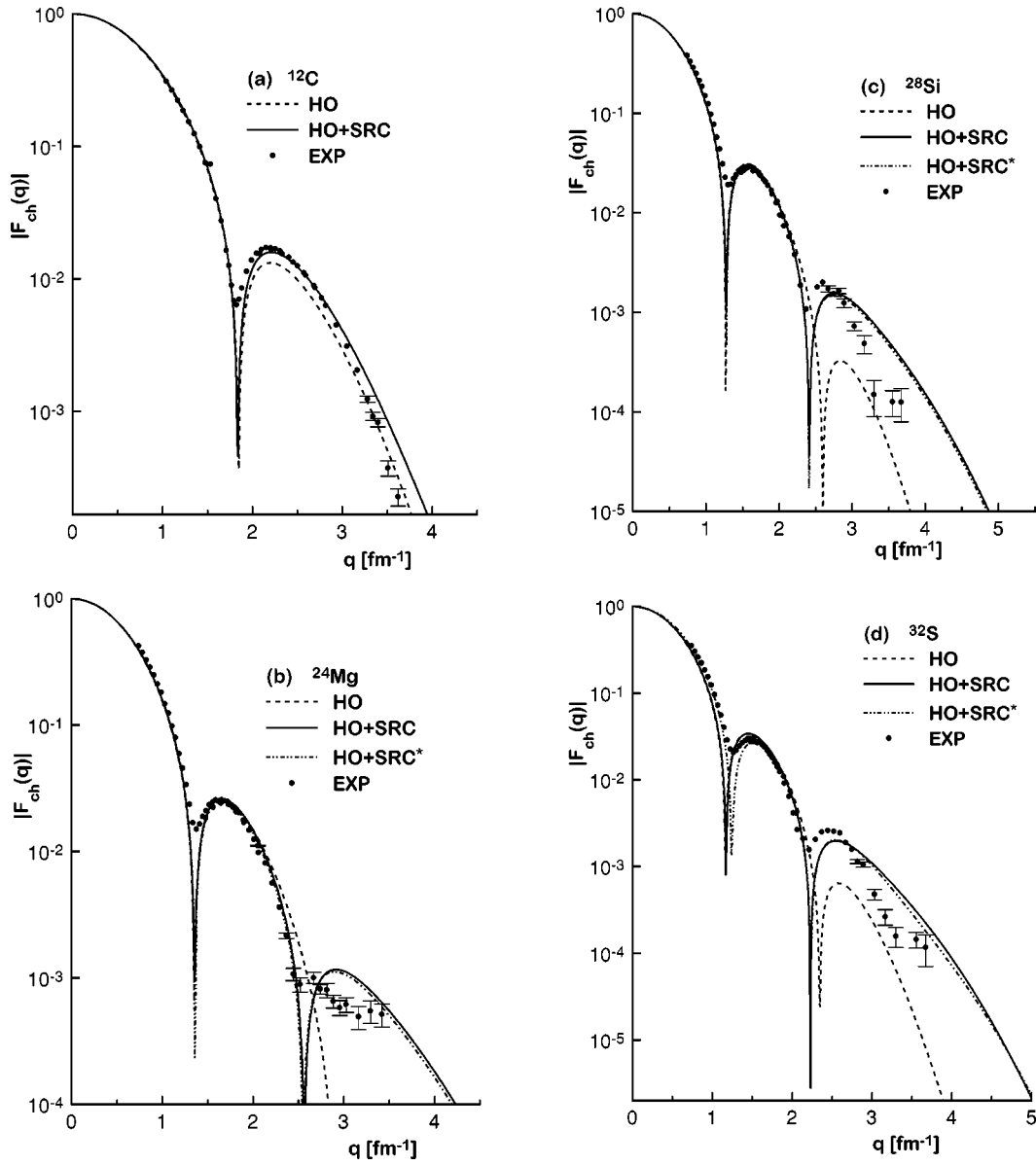
FIG. 2. The charge form factor of the nuclei ^{12}C (a), ^{24}Mg (b), ^{28}Si (c), and ^{32}S (d) for various cases. The case HO+SRC* corresponds to the case when the occupation probability η_{2s} is treated as free parameter. The experimental points of ^{12}C are from Ref. [34] and for the other nuclei from Ref. [36].
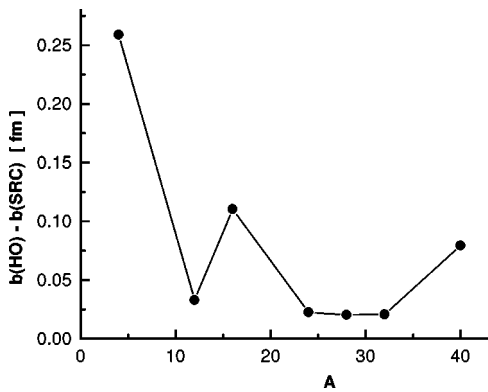

FIG. 3. The difference $\Delta b = b(\mathrm{HO}) - b(\mathrm{SRC})$ versus the mass number A. $b(\mathrm{HO})$ and $b(\mathrm{SRC})$ are the HO parameters in cases 1 (HO without SRC) and 2 (HO with SRC), respectively.
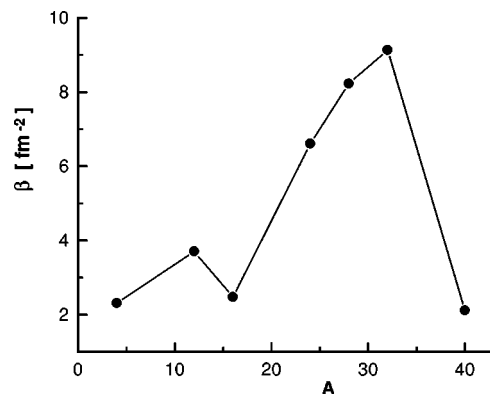

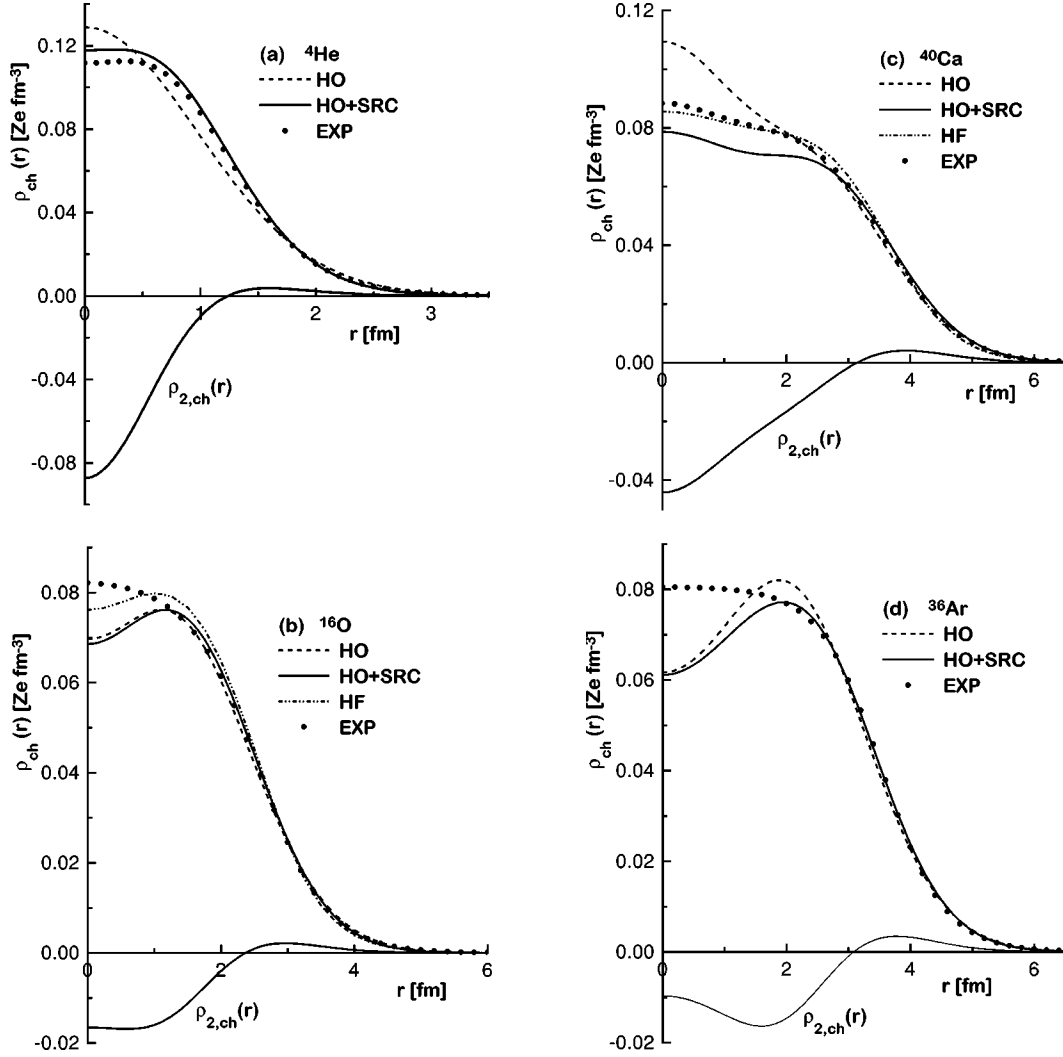FIG. 4. The correlation parameter β versus the mass number A in case 2 (HO+SRC).

FIG. 5. The charge density distribution $\rho_{ch}(r)$ and the contribution $\rho_{2,ch}(r)$ of SRC's to it of the nuclei ^{4}He (a), ^{16}O (b), ^{40}Ca (c), and ^{36}Ar (d) for various cases. The experimental points are from Ref. [32].

^{28}Si, and ^{32}S, are shown in Fig. 2.

From the values of χ^2, which have been found in cases 1 and 2 (see Table I) and also from Figs. 1 and 2, it can be seen that the inclusion of the correlations improves the fit of the form factor of all the nuclei we have examined. Almost all the diffraction minima which are known from the experimental data are reproduced in the correct place. There is a disagreement in the fit of the form factor of the open shell nuclei ^{24}Mg, ^{28}Si, and ^{32}S for $q \approx 3.5$ fm^{-1} where it seems that there is a third diffraction minimum in the experimental data, which cannot be reproduced in both cases.

It is seen from Table I that the parameter b has the same behavior as a function of the mass number A in the HO and the correlated model, while the following inequality holds:

$$b(\mathrm{HO}) > b(\mathrm{SRC}).$$

This is due to the fact that the introduction of SRC's tends to increase the relative distance of the nucleons, i.e., the size of the nucleus, while the parameter b, which is (on the average) proportional to the (experimentally fixed) radius of the

nucleus, should become smaller. Such a behavior should be expected in view also of relations (49) and (51).

It is also noted that the difference

$$\Delta b = b(\mathrm{HO}) - b(\mathrm{SRC})$$

is almost constant for the open shell nuclei and is larger for the closed shell nuclei ^{4}He, ^{16}O, and ^{40}Ca. This can also be seen from Fig. 3 where the values of Δb versus the mass number A have been plotted. The behavior of Δb as function of A indicates that the SRC's are stronger for the closed shell nuclei than in the open shell ones.

In Fig. 4 the values of the correlation parameter β versus the mass number A have been plotted. From this figure it is seen that the parameter β is almost constant for ^{4}He, ^{16}O, and ^{40}Ca and takes larger values (less correlated systems) in the open shell nuclei.

The behavior of the two parameters b and β indicates that there should be a shell effect in the case of closed shell nuclei. That is, there is a shell effect not only on the values
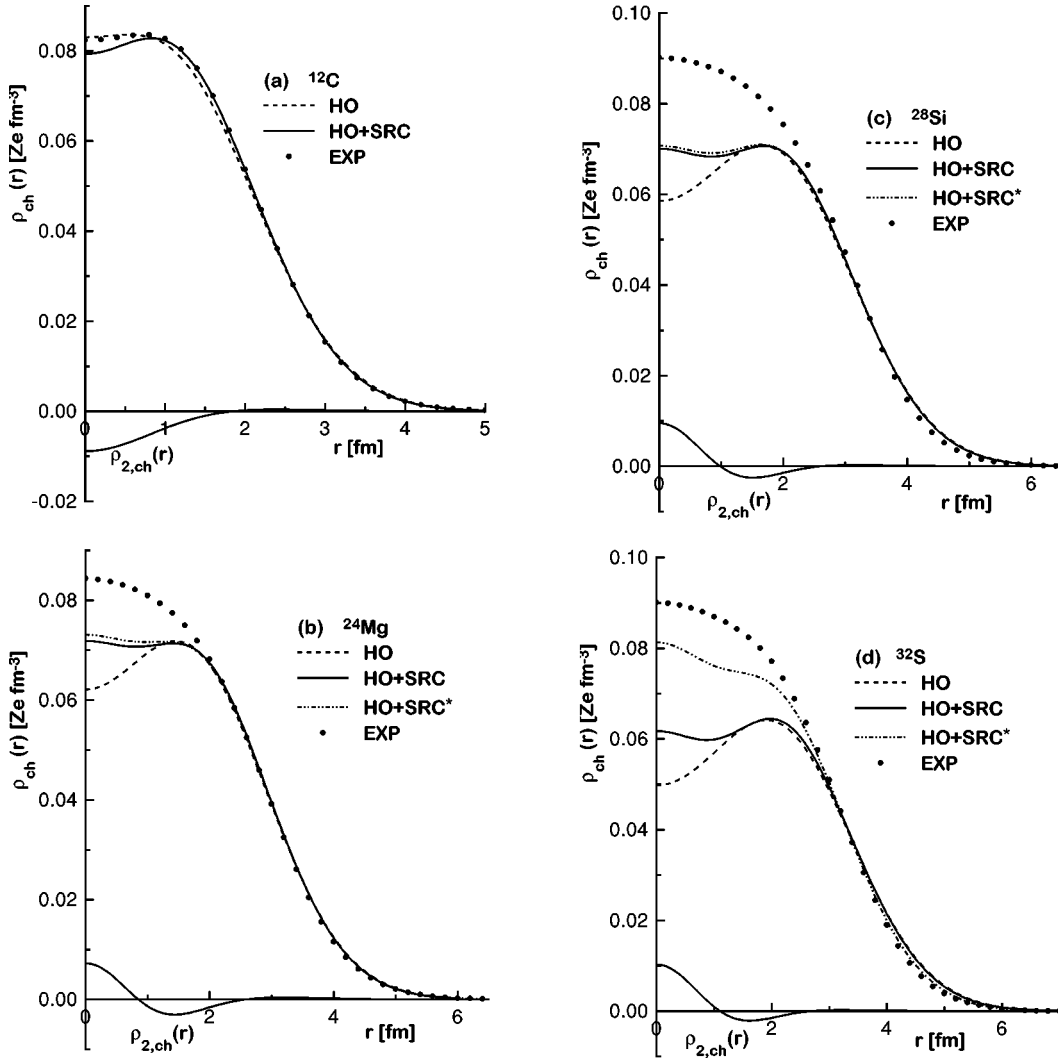
FIG. 6. The charge density distribution $\rho_{ch}(r)$ and the contribution $\rho_{2,ch}(r)$ of SRC to it of the nuclei ^{12}C (a), ^{24}Mg (b), ^{28}Si (c), and ^{32}S (d) for various cases. The case HO+SRC* corresponds to the case when the occupation probability η_{2s} is treated as a free parameter. The experimental points are from Ref. [32].

of the harmonic oscillator spacing $\hbar\omega$, as has been noted in Refs. [37,38], but also on the values of the correlation parameter β.

In the above analysis, the nuclei ^{24}Mg, ^{28}Si, and ^{32}S were treated as $1d$ shell nuclei. We have also considered case 2* in which the occupation probability η_{2s} of the nuclei ^{24}Mg, ^{28}Si, and ^{32}S is taken to be a free parameter besides the other two parameters b and β. We found that while the χ^2 values become better, comparing to those of case 2, the third diffraction minimum is not reproduced either and the behavior of the parameters b and β as functions of mass number A remains the same. The results in this case are shown in Table I and in Fig. 2. The values of the occupation probability η_{2s} of the above-mentioned three nuclei are 0.0355, 0.0245, and 0.2945, respectively, while the corresponding values of η_{1d}, which can be found from the values of η_{2s} through the relation

$$\eta_{1d}=[(Z-8)-2\eta_{2s}]/10,$$

are 0.3929, 0.5951, and 0.7411, respectively. The partial occupancy of the state $2s$ for the nucleus ^{32}S has the result of increasing the central part of the charge density significantly.

In Figs. 5 and 6 the charge densities $\rho_{ch}(r)$ (normalized to Z) of the above-mentioned nuclei for the various cases of Table I are shown and compared with the charge densities of Ref. [32]. In the same figures the contribution of the SRC's to $\rho_{ch}(r)$,

$$\rho_{2,ch}(r)=\rho_{ch}(r)-\rho_{1,ch}(r), \tag{54}$$

is shown. The introduction of short range correlations has the feature of reducing the central part of the densities of the closed shell nuclei, while $\rho_{2,ch}(r)$ is small and characterized by oscillations in the case of open shell nuclei.

From the determined mass dependence of the parameters b and β, the values of these parameters for other s-p or s-d shell nuclei can found. In Figs. 1(d) and 5(d) the $F_{ch}(q)$ and $\rho_{ch}(r)$ of the nucleus ^{36}Ar, treated as an $1d$ closed shell nucleus, are shown. As there are no experimental data for

$F_{ch}(q)$ for high q values, the value of the parameter β is taken to be the mean value of the corresponding values of ^{16}O and ^{40}Ca, that is, $\beta_{36} = 2.2937$ fm^{-2}, while the parameter b is determined assuming that

$$\Delta b_{36} = \Delta b_{40},$$

where

$$\Delta b_A = b_A(\text{HO}) - b_A(\text{SRC}).$$

Using the values of the parameters $b_{40}(\text{HO}) = 1.9453$ fm and $b_{40}(\text{SRC}) = 1.8660$ fm from Table I and choosing the parameter $b_{36}(\text{HO}) = 1.8800$ fm in order to reproduce the experimental rms charge radius of ^{36}Ar ($\langle r^2 \rangle_{\text{expt}}^{1/2} = 3.327 \pm 15$ fm [32]) the value $b_{36}(\text{SRC}) = 1.8007$ fm was found. These values of β_{36} and $b_{36}(\text{SRC})$ have been used for the calculations of the correlated $F_{ch}(q)$ and $\rho_{ch}(r)$ of ^{26}Ar, which are shown in Figs. 1(d) and 5(d), respectively. The calculated rms charge radius $\langle r^2 \rangle^{1/2} = 3.3343$ fm, which was found, is within the experimental error.

The effect of SRC's on the form factors has also been examined with various Skyrme-type wave functions. In this case, there is only one free parameter, the correlation parameter β. The potential parameters have been adjusted in Refs. [39,40] in order to reproduce various physical quantities such as rms radii, binding energies, etc. Thus, some effects of the SRC's we would like to study have already been averaged out. Because of that, we cannot study the effect of the SRC's on the parameters of the Skyrme interactions. This can be done if both the Skyrme parameters and the correlation parameter are readjusted by fit to various experimental data including the form factors and the fit is made for many nuclei at the same time. As this is out of the scope of the present work, we examined the form factors of ^{16}O and ^{40}Ca with various Skyrme interactions, namely, SK1–SK6 [39,40], without SRC's, named case 3 and with SRC's, named case 4.

In case 3 and for ^{16}O, only SK1 gives smaller value for χ^2 compared with that of case 1 (HO without SRC). The inclusion of SRC's (case 4) to the Skyrme-type wave functions gives better χ^2 only for the SK1, but still this value is about 10% larger than in case 2 (HO with SRC's) and 1% smaller than in case 3. For SK2–SK6 the correlated parameter β goes to very large values (very small correlations) without improving the quality of the fit. For ^{40}Ca and for case 3 all the Skyrme interactions we have examined give almost the same χ^2 value which is about 20% smaller than in case 1 and 10% larger than that of case 2, while the inclusion

of the correlations improves the quality of the fit, for all the Skyrme interactions, not more than 2%. See Table I and Fig. 1 for the results we have found with SK1.

From the results with Skyrme-type wave functions we could conclude that even if a mean field is more realistic than the HO one, the inclusion of the SRC's does not improve the fit, at least of the $F_{ch}(q)$, significantly. Significant improvement of the fit should be expected if the parameters of the mean field will be determined along with the correlation parameter. It should be noted also that a good fit to the experimental $F_{ch}(q)$ of ^{6}Li, ^{12}C, and ^{16}O has been found by Ciofi Degli Atti and Kabachnik [5], using Woods-Saxon wave functions, only if the correlation parameter and the depths and radii of the wells were free parameters.

V. SUMMARY

In the present work, general expressions for the correlated charge form factors and densities have been found using the factor cluster expansion of Clark and co-workers. These expressions can be used, either for analytical calculations, with HO orbitals or for numerical calculations of $F_{ch}(q)$ and $\rho_{ch}(r)$, with more realistic orbitals.

The systematic calculations, with HO orbitals, of these quantities, for a number of $N = Z$, s-p and s-d shell nuclei, show that there is a shell effect on the values of the HO parameter b and on the correlation parameter β. Regarding the parameter β it is almost constant for the closed shell nuclei while it takes larger values for the open shell nuclei. The mass dependence of these parameters indicates that the SRC's are stronger for the closed shell nuclei than for the open shell ones. This dependence can also be used to find reasonable values of these parameters for nuclei for which there are no experimental data of the $F_{ch}(q)$.

Numerical calculations with various Skyrme-type wave functions, taken from the literature [39,40], indicate that even if a mean field is more realistic than the HO one, the introduction of the SRC's does not improve the fit of the charge form factors significantly. The reason for this should be that the introduction of the correlations makes a change in the mean field in a way that there is a balance between the SRC's and the mean field, while for a given mean field there is not this flexibility. A way to overcome this difficulty should be to readjust the parameters of the mean field and the parameter of the correlations.

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