

# Dominance of $J^P=0^+$ ground states in even-even nuclei from random two-body interactions

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Recent calculations using random two-body interactions showed a preponderance of  $J^P=0^+$  ground states, despite the fact that there is no strong pairing character in the force. We carry out an analysis of a system of identical particles occupying orbits with  $j=1/2, 3/2,$  and  $5/2$  and discuss some general features of the spectra derived from random two-body interactions. We show that for random two-body interactions that are not time-reversal invariant the dominance of  $0^+$  states in this case is more pronounced, indicating that time-reversal invariance cannot be the origin of the  $0^+$  dominance. [S0556-2813(99)50608-0]

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In a recent paper [1], Johnson *et al.* discussed the low-energy spectra of many-body even-even nuclear systems arising from random two-body interactions. Surprisingly, their results showed a preponderance of  $J^P=0^+$  ground states for these nuclei, despite the fact that there is no obvious pairing character in the assumed random forces. This is contrary to the traditional assumption whereby the favoring of  $0^+$  ground states is thought to be a reflection of nuclear pairing arising from the short-range nuclear force.

What is it that produces this preponderance of  $0^+$  ground states in even-even many-body systems? In this work, we carry out a detailed analysis of a system of  $N=2, 4,$  and  $6$  identical particles occupying orbits with  $j=1/2, 3/2,$  and  $5/2$ . We address one of the oft-stated suggestions, that it may arise because of the time-reversal invariance of the random Hamiltonian. Since time-reversed states play an important role in the formation of correlated  $0^+$  (Cooper) pairs which in turn can give rise to favored collective many-body states, it is conceivable that time-reversal invariance may contain a built-in preference for  $J^P=0^+$  many-body ground states.

To see whether this is indeed the case, we have carried out an analysis very similar to that of [1], but now relaxing the assumption of time-reversal invariance in the random two-body interaction. This can be done by introducing Gaussian unitary ensembles (GUE's) rather than Gaussian orthogonal ensembles (GOE's) to randomly generate the two-body matrix elements.

More specifically, to investigate the effect of time-reversal symmetry breaking we consider a two-body Hamiltonian matrix of the form [2,3]

$$H_{\alpha\alpha'} = \frac{S_{\alpha\alpha'} + i\epsilon A_{\alpha\alpha'}}{\sqrt{1 + \epsilon^2}}, \quad (1)$$

where  $S$  and  $A$  are real symmetric and antisymmetric random matrices, respectively, and  $\alpha$  and  $\alpha'$  label the two-body states  $|(j_1 j_2)JT\rangle$  with angular momentum  $J$  and isospin  $T$ .

The matrix elements  $S_{\alpha\alpha'}$  and  $A_{\alpha\alpha'}$  are chosen independently using a Gaussian distribution of random numbers with zero mean and variances

$$\begin{aligned} \langle S_{\alpha\alpha'}^2 \rangle &= v_{JT}^2 (1 + \delta_{\alpha\alpha'}), \\ \langle A_{\alpha\alpha'}^2 \rangle &= v_{JT}^2 (1 - \delta_{\alpha\alpha'}). \end{aligned} \quad (2)$$

Here  $\langle \rangle$  denote ensemble averages. For  $\epsilon=0$  and  $1$  they correspond to GOE and GUE, respectively. The Hamiltonian is time-reversal invariant if the two-body matrix elements are real, i.e.,  $\epsilon=0$ . The breaking of time-reversal symmetry can be studied by taking  $0 \leq \epsilon \leq 1$ . For a given value of  $J$  and  $T$ , the above ensemble for two-body interactions gives a semicircle level density. The normalization was chosen such that the radius of this semicircle distribution does not depend on  $\epsilon$  [3].

The variances  $v_{JT}^2$  depend on the particular ensemble chosen in the calculations. For the two-body random ensemble (TBRE) of [4] the variances are independent of the angular momentum  $J$  and isospin  $T$

$$\text{TBRE: } v_{JT}^2 = \bar{v}^2, \quad (3)$$

whereas for the random quasiparticle ensemble (RQE) of [1], which is obtained by requiring that the ensemble be invariant under particle-hole conjugation, the higher values of  $J$  and  $T$  are suppressed with respect to the lower ones

$$\text{RQE: } v_{JT}^2 = \frac{\bar{v}^2}{(2J+1)(2T+1)}. \quad (4)$$

Other choices for the ensembles of two-body matrix elements are discussed in [5]. In this paper we study the ensembles RQE and TBRE.

As a model space we take that of  $N$  identical particles in the  $sd$  shell, which consists of single-particle orbitals with  $j=1/2, 3/2,$  and  $5/2$ . The case of  $N=6$  particles is one of the examples considered in [1,5] and referred to as correspond-

TABLE I. Percentage of ground states with angular momentum  $J$  for RQE and TBRE for  $N=2, 4$ , and 6 identical particles in the  $sd$  shell (the nuclei  $^{18,20,22}\text{O}$ ). The results are obtained for 1000 runs with  $\epsilon=0.0$ .

$N$	$J$	$d_J$	Basis	RQE	TBRE
2	0	3	21.4%	64.0%	15.9%
	1	2	14.3%	5.3%	4.9%
	2	5	35.7%	29.3%	68.3%
	3	2	14.3%	0.9%	6.1%
	4	2	14.3%	0.5%	4.8%
4	0	9	11.1%	66.6%	55.9%
	1	12	14.8%	4.7%	4.9%
	2	21	25.9%	20.5%	22.7%
	3	15	18.5%	0.9%	1.4%
	4	15	18.5%	6.6%	12.3%
	5	6	7.4%	0.5%	1.5%
	6	3	3.7%	0.2%	1.3%
6	0	14	9.9%	74.2%	67.7%
	1	19	13.4%	0.8%	1.3%
	2	33	23.2%	12.7%	15.0%
	3	29	20.4%	5.6%	7.1%
	4	26	18.3%	5.7%	6.8%
	5	12	8.5%	0.4%	0.4%
	6	8	5.6%	0.6%	1.7%
	7	1	0.7%	0.0%	0.0%

ing to the nucleus  $^{22}\text{O}$  which has six active neutrons in the  $sd$  shell. For identical particles the isospin is the same for all states, and hence does not play a role.

Before turning to the issue of time-reversal invariance and its effect on the fraction of  $0^+$  ground states, we first consider some general issues regarding GOE ensembles. In these calculations we take  $\epsilon=0$ . In Table I we show the percentage of the total number of runs for which the ground state has a given angular momentum  $J$ . For  $N=2$  particles and a given value of  $J$  the ensemble is a GOE, whose level distribution is given by a semicircle with radius  $R = \sqrt{4d_J v_J^2}$  (here  $d_J$  is the

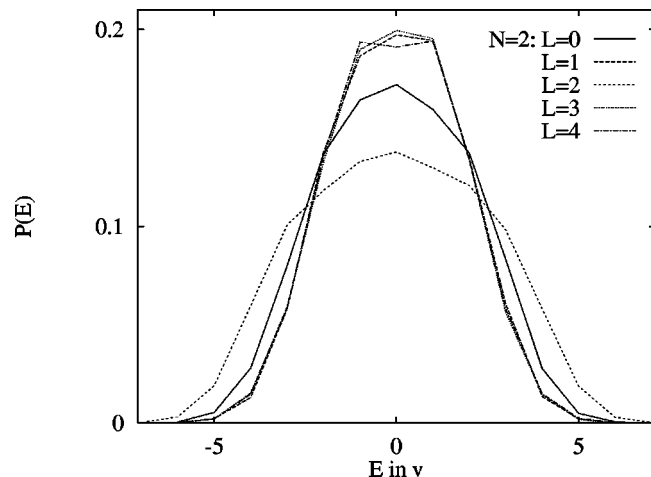


FIG. 1. Level distributions for  $N=2$  particles ( $^{18}\text{O}$ ).

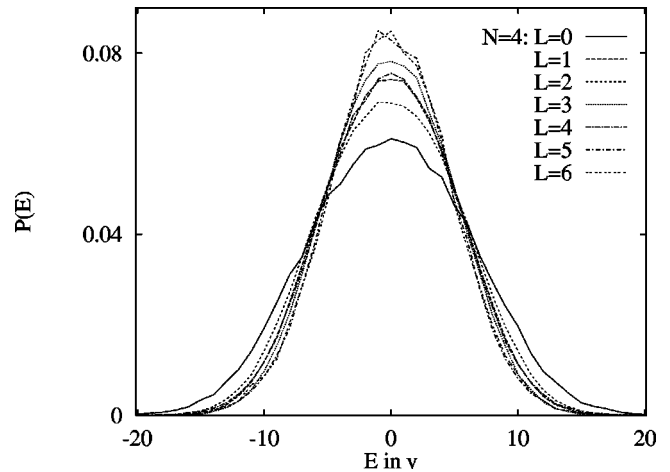


FIG. 2. Level distributions for  $N=4$  particles ( $^{20}\text{O}$ ).

dimension). The differences between TBRE and RQE arise from the  $J$  dependence of the variances, see Eqs. (3) and (4). Whereas the results for TBRE depend solely on the dimension of the matrices, for RQE there is an additional suppression of the higher values of the angular momentum by the  $J$  dependence of the variances. For  $N=2$  particles the RQE gives already a  $J^P=0^+$  ground state in 64.0% of the cases, compared to 15.9% for TBRE and 21.4% of  $0^+$  states in the model space.

For  $N>2$  particles the ensemble is the so-called embedded GOE, in which the  $N$ -body matrix elements are expressed in terms of the two-body matrix elements by the usual reduction formulas. Subsequently the two-body matrix elements are chosen randomly using either RQE or TBRE. For  $N=4$  particles there is a dramatic increase in the percentage of  $J^P=0^+$  ground states for TBRE to 55.9%, whereas the percentage of  $0^+$  states in the model space is now only 11.1%. For RQE there is only a slight increase. The same holds for  $N=6$  particles. For the latter case we confirm the results already obtained by Johnson [1,5].

The results of Table I can be better understood by examining the energy distributions for each value of the angular

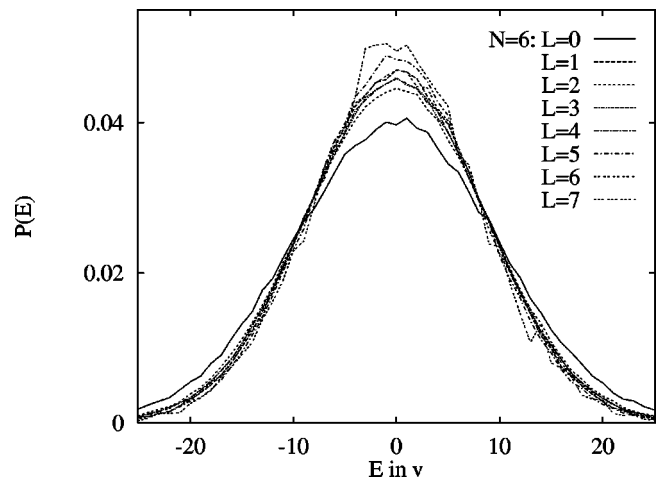


FIG. 3. Level distributions for  $N=6$  particles ( $^{22}\text{O}$ ).

TABLE II. Widths of level distributions for TBRE. The results are obtained for 10 000 runs.

$J$	$N=6$	$N=4$	$N=2$
0	10.16	6.24	2.00
1	8.53	5.05	1.73
2	9.01	5.37	2.45
3	8.80	4.79	1.73
4	8.80	5.12	1.74
5	8.27	4.65	
6	8.61	4.69	
7	8.07		

momentum. Especially interesting are the results for TBRE which show a large change between  $N=2, 4$  and  $6$  particles. This is illustrated in Figs. 1–3 in which we present the corresponding level distributions. The width of the distributions can be obtained from

$$w_J = \sqrt{\frac{\langle \text{Tr}(H^2) \rangle - \langle \text{Tr}(H) \rangle^2}{d_J}}. \quad (5)$$

The values of the widths are given in Table II. For  $N=2$  particles the semicircular level distributions have a width  $w_J = \sqrt{(d_J+1)\bar{v}^2}$  which only depends on the dimension [2]. Therefore,  $J^P=2^+$  has the largest value of the width, followed by  $0^+$  and then  $1^+, 3^+, 4^+$ . For  $N=4$  and  $6$  the level distributions are Gaussian. Here the  $J^P=0^+$  has the largest width, followed by  $2^+$  and the other values of  $J^+$ . This is the result of a competition between the dimensions of the Hamiltonian matrices and the correlations in the many-body matrix elements arising from the two-body matrix elements. This is another manifestation of the dominance of  $0^+$  ground states that was discussed above.

Next we turn our attention to the main point of the present study: the breaking of time-reversal invariance. The results of our calculations for  $N=6$  particles are presented in Table III. For  $\epsilon=0$  the Hamiltonian is time-reversal invariant and we confirm the results of [1,5]. For  $0 < \epsilon \leq 1$  the time-reversal invariance is broken. We see that the dominance of  $0^+$  ground states increases with  $\epsilon$  for both RQE and TBRE. The increase is from 74.2 to 85.7% for RQE and from 67.7 to 76.8% for TBRE. On the basis of these results, we conclude that time-reversal invariance of the two-body interaction terms is not the origin of the preponderance of  $0^+$  ground states that emerged in the analyses of [1,5].

In this paper, we have investigated the recent observation that the spectra of many-body systems with random two-body interactions show a predominance of  $J^P=0^+$  ground states [1]. This result is quite robust, and does not depend sensitively on the choice of the random matrix ensemble [5].

TABLE III. Percentage of  $J^P=0^+$  ground states for RQE and TBRE for  $N=6$  identical particles in the  $sd$  shell (the nucleus  $^{22}\text{O}$ ). The results are obtained for 1000 runs.

$\epsilon$	RQE	TBRE
0.00	74.2%	67.7%
0.25	76.3%	69.3%
0.50	79.7%	71.7%
0.75	83.4%	74.0%
1.00	85.7%	76.8%

However, we have shown that for  $N=2$  particles there is a large difference between the random quasiparticle ensemble and the two-body random ensemble. This can be understood from the suppression of the high angular momenta in the variances of the two-body matrix elements of the RQE, compared to TBRE. For  $N=4$  and  $6$  particles the results for the two ensembles become comparable. Since the RQE implies an additional suppression of higher angular momenta, we suggest that further efforts should concentrate on TBRE Hamiltonians.

A study of the TBRE level distributions for  $N=2, 4,$  and  $6$  particles in the  $sd$  shell shows a rapid change from semicircular distributions whose radius depends only on the dimension (for  $N=2$ ) to Gaussian distributions (for  $N=4$  and  $6$ ). In the latter case, the widths are determined by a competition between the dimension and the many-body dynamics. The widths for all  $J \neq 0$  values become roughly comparable, whereas the  $J=0$  width is larger.

Most importantly, we have shown that the dominance of  $0^+$  ground states is not a consequence of the time-reversal invariance of the two-body force, since the breaking of this symmetry leads to a slight increase of the percentage of  $0^+$  ground states. It seems that this effect arises solely from the differences in the correlations for the  $n$ -body matrix elements for each angular momentum. We are presently studying the angular momentum coupling behavior of TBRE Hamiltonians in an analytically tractable model [6]. The understanding of this problem and more generally that of the spectral properties of two-body random ensembles could have significant consequences on random matrix analyses of complex many-body systems [2].

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