$\Lambda_c^+ \to p + K^- + \pi^+$ decay

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The proton energy spectrum and the angular distribution of the probability of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay for the polarized Λ_c^+ and the unpolarized proton are calculated in the quark model with chiral $U(3)\times U(3)$ symmetry incorporating heavy quark effective theory and chiral perturbation theory at the quark level. The application of the obtained result to the analysis of the polarization of the Λ_c^+ produced in the processes of photo and hadroproduction is discussed. We draw the similarity between the measurements of the polarization of the Λ_c^+ in the $\Lambda_c^+ \to p + K^- + \pi^+$ decay and the μ^- meson in the $\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$ decay. [S0556-2813(99)02807-1]

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I. INTRODUCTION

It is rather likely that in the reactions of photoproduction and hadroproduction the charmed baryon Λ_c^+ is produced polarized [1]. The analysis of the Λ_c^+ polarization by means of the investigation of the decay products should clarify the mechanism of the charmed baryon production at high energies.

The most favorable mode of the Λ_c^+ decays to be detected experimentally is $\Lambda_c^+ \to p + K^- + \pi^+$. The experimental probability of this mode equals $B(\Lambda_c^+ \to pK^-\pi^+)_{\rm exp} = (5.0 \pm 1.3)\%$ [2]. However, from the theoretical point of view this mode is the most difficult case due to the impossibility to factorize the baryonic and mesonic degrees of freedom for the computation of the matrix element of the transition [1].

The theoretical investigation of nonleptonic decays of charmed baryons without factorization of baryonic and mesonic degrees of freedom can be carried out in the quark model with chiral $U(3)\times U(3)$ symmetry incorporating heavy quark effective theory (HQET) [3,4] and chiral perturbation theory at the quark level $(\mathrm{CHPT})_q$ [5]. The quark model with chiral $U(3)\times U(3)$ symmetry is motivated by the low-energy effective QCD with a linearly rising interquark potential responsible for a quark confinement [6]. The application of this model to the description of the low-energy properties of charmed mesons: mass spectra [7], coupling constants [7–9], the form factors of the semileptonic decays [10], and the probabilities of the decays [7,11] gave the results agreeing good with experimental data.

Recently [12] the quark model with chiral $U(3)\times U(3)$ symmetry has been extended by the inclusion of the low-lying baryon octet and decuplet coupled with the three-quark currents. Due to the dynamics of strong low-energy interactions caused by a linearly rising interquark potential there has been shown [6] that (i) baryons are the three-quark states [13] and do not contain any bound diquark states, then (ii) the spinorial structure of the three-quark currents is defined as the products of the axial-vector diquark densities $\overline{q^c}_i(x)\gamma^\mu q_j(x)$ and a quark field $q_k(x)$ transforming under the $SU(3)_f\times SU(3)_c$ group as $(\underline{6}_f,\tilde{\underline{3}}_c)$ and $(\underline{3}_f,\underline{3}_c)$ multiplets, respectively, where i, j, and k are the color indices running through $i=1,2,3$ and $q=u$, d, or s quark field. This agrees with the structure of the three-quark currents used for the investigation of the properties of baryons within QCD sum rules approach [14]. The fixed structure of the three-quark currents allows us to describe all variety of low-energy interactions of baryon octet and decuplet in terms of the phenomenological coupling constant g_B describing the coupling of the baryon octet and decuplet with the three-quark currents [12]:

$$\mathcal{L}_{\rm int}(x) = \frac{1}{\sqrt{2}} g_B \bar{B}_{\underline{8}}(x)\eta_{\underline{8}}(x) + g_B \bar{B}_{\underline{10}}(x)\eta_{\underline{10}}(x) + \mathrm{H.c.}, \tag{1.1}$$

where $B_{\underline{8}}(x) = [\psi_p(x), \ldots]$ and $B_{\underline{10}}(x) = (\psi_{\Delta^{++}}, \ldots)$ are the fields of the baryon octet and decuplet, respectively, and $\eta_{\underline{8}}(x) = \{-\varepsilon^{ijk}[\overline{u_i^c}(x)\gamma^\mu u_j(x)]\gamma_\mu\gamma^5 d_k(x), \ldots\}$ and $\eta_{\underline{10}}(x) = \{\varepsilon^{ijk}[\overline{u_i^c}(x)\gamma^\mu u_j(x)]u_k(x), \ldots\}$ are the three-quark currents. The numerical value of g_B, calculated in terms of the coupling constant $g_{\pi NN} = 13.4$ of the πNN interaction, has been found equal $g_B = 1.34\times 10^{-4}$ MeV [12]. The coupling constants $g_{\pi N\Delta}$ and $g_{\gamma N\Delta}$ of the $\pi N\Delta$ and $\gamma N\Delta$ interactions relative to the coupling constant $g_{\pi NN}$ and the $\sigma_{\pi N}$ term of the low-energy πN scattering have been calculated in good agreement with the experimental data and other phenomenological approaches based on QCD [12].

In this paper we apply the quark model with chiral $U(3)\times U(3)$ symmetry to the calculation of the proton energy spectrum of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay of the polarized Λ_c^+ and the analysis of the Λ_c^+ polarization in the dependence of the energies and momenta of the decay

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products. For the analysis of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay at the quark level we assume that the Λ_c^+ is a three-quark state coupled with the three-quark current $\eta_{\Lambda_c^+}(x) = -\varepsilon^{ijk}[\overline{u_i^c}(x)\gamma^\mu d_j(x)]\gamma_\mu\gamma^5 c_k(x)$ defined as the product of the axial-vector light-diquark density $\overline{u_i^c}(x)\gamma^\mu d_j(x)$ and the c-quark field $c_k(x)$.

The paper is organized as follows. In Sec. II we discuss the effective low-energy Lagrangian describing nonleptonic decays of charmed hadrons and reduce the calculation of the amplitude of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay to the calculation of the matrix element of the low-energy transition $\Lambda_c^+ \to p + K^-$. In Sec. III we calculate the matrix element of the low-energy transition $\Lambda_c^+ \to p + K^-$. In Sec. IV we calculate the proton energy spectrum of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay for the polarized Λ_c^+ and the unpolarized proton. In Sec. V we calculate the probability of the $\Lambda_c^+ \to p + \bar{K}^0$ decay relative to the probability of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay. The theoretical result on the ratio of the probabilities agrees well with the experimental data. In the Conclusion we discuss the obtained results. In the Appendix we calculate the momentum integrals describing the the matrix elements of the low-energy transitions $\Lambda_c^+ \to p + K^-$ and $\Lambda_c^+ \to p + \bar{K}^0$.

II. EFFECTIVE LAGRANGIAN FOR WEAK NONLEPTONIC TRANSITIONS OF CHARMED HADRONS

The effective low-energy Lagrangian responsible for nonleptonic decays of charmed hadrons reads [15]

$$\mathcal{L}_{\rm eff}(x) = -\frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud}\{C_1(\mu)[\bar{s}(x)\gamma^\mu(1-\gamma^5)c(x)] \times [\bar{u}(x)\gamma_\mu(1-\gamma^5)d(x)] + C_2(\mu) \times [\bar{u}(x)\gamma^\mu(1-\gamma^5)c(x)][\bar{s}(x)\gamma_\mu(1-\gamma^5)d(x)]\}, \tag{2.1}$$

where $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi weak constant, V^*_{cs} and V_{ud} are the elements of the CKM-mixing matrix, $C_i(\mu)$ $(i=1,2)$ are the Wilson coefficients caused by the strong quark-gluon interactions at scales $p > \mu$ (short-distance contributions), where μ is a normalization scale. In the absence of quark-gluon interactions the coefficients $C_1(\mu)$ and $C_2(\mu)$ do not depend on μ and amount to $C_1 = 1$ and $C_2 = 0$. In $(\text{CHPT})_q$ we should identify μ with the scale of spontaneous breaking of chiral symmetry (SBχS) $\Lambda_\chi = 940$ MeV [5–11], i.e., $\mu = \Lambda_\chi = 940$ MeV. Therefore, below we would deal with $C_1(\Lambda_\chi)$ and $C_2(\Lambda_\chi)$. The contribution of strong low-energy interactions at scales $p \leqslant \mu = \Lambda_\chi$ (long-distance contributions) is described by $(\text{CHPT})_q$ in terms of constituent quark loop diagrams, where the momenta of virtual quarks are restricted from above by the SBχS scale Λ_χ [5–12].

The structure of the term proportional to $C_2(\Lambda_\chi)$ can be reduced to the structure of the first one by means of the Fierz transformation [15]. The resultant coefficient of the first term would contain $C_2(\Lambda_\chi)$ in the form $C_2(\Lambda_\chi)/N$, where $N=3$ is the number of quark colors. Thus, the effective low-energy Lagrangian responsible for the $\Lambda_c^+ \to p + K^- + \pi^+$ decay can be taken in the form

$$\mathcal{L}_{\rm eff}(x) = -\frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud} \bar{C}_1(\Lambda_\chi)[\bar{s}(x)\gamma_\mu(1-\gamma^5)c(x)] \times [\bar{u}(x)\gamma^\mu(1-\gamma^5)d(x)], \tag{2.2}$$

where $\bar{C}_1(\Lambda_\chi) = C_1(\Lambda_\chi) + C_2(\Lambda_\chi)/3$. The amplitude of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay is then defined

$$\frac{\mathcal{M}[\Lambda_c^+(Q) \to p(q)K^-(q_-)\pi^+(q_+)]}{\sqrt{2E_{\Lambda_c^+}V 2E_p V 2E_{K^-}V 2E_{\pi^+}V}} = \langle p(q)K^-(q_-)\pi^+(q_+)|\mathcal{L}_{\rm eff}(0)|\Lambda_c^+(Q)\rangle, \tag{2.3}$$

where E_i $(i = \Lambda_c^+, p, K^-, \pi^+)$ are the energies of the Λ_c^+, the proton and the K^- and π^+ mesons, respectively, and V is the normalization volume.

In $(\text{CHPT})_q$ at the tree-meson approximation we can factorize the pionic degrees of freedom and represent the amplitude Eq. (2.3) as follows:

$$\frac{\mathcal{M}[\Lambda_c^+(Q) \to p(q)K^-(q_-)\pi^+(q_+)]}{\sqrt{2E_{\Lambda_c^+}V 2E_p V 2E_{K^-}V 2E_{\pi^+}V}} = \langle p(q)K^-(q_-)\pi^+(q_+)|\mathcal{L}_{\rm eff}(0)|\Lambda_c^+(Q)\rangle = -\frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud}\bar{C}_1(\Lambda_\chi)\langle p(q)K^-(q_-)||\bar{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle\langle\pi^+(q_+)|\bar{u}(0)\gamma^\mu(1-\gamma^5)d(0)|0\rangle, \tag{2.4}$$

where the matrix element $\langle\pi^+(q_+)|\bar{u}(0)\gamma^\mu(1-\gamma^5)d(0)|0\rangle$ can be expressed in terms of the leptonic constant of the π^+ meson $F_\pi = 92.4$ MeV [5]:

$$\sqrt{2E_{\pi^+}V}\langle\pi^+(q_+)|\bar{u}(0)\gamma^\mu(1-\gamma^5)d(0)|0\rangle = i\sqrt{2}F_\pi q_+^\mu. \tag{2.5}$$

Substituting Eqs. (2.5) in (2.4) we arrive at the amplitude of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay

$$\frac{\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_-)\pi^+(q_+)]}{\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V}} = -iG_F V_{cs}^* V_{ud} F_\pi \bar{C}_1(\Lambda_\chi) q_+^\mu \langle p(q)K^-(q_-)|\bar{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle, \quad (2.6)$$

where the matrix element $\langle p(q)K^-(q_-)|\bar{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle$ describes the transition $\Lambda_c^+\to p+K^-$ induced by the current $\bar{s}(0)\gamma^\mu(1-\gamma^5)c(0)$ and defined by strong low-energy interactions.

III. STRONG LOW-ENERGY TRANSITION $\Lambda_c^+\to p+K^-$

By applying the reduction technique we bring up the matrix element of the strong low-energy transition $\Lambda_c^+\to p+K^-$ to the form

$$\sqrt{2E_{\Lambda_c^+}V\,2E_pV\,2E_{K^-}V\,2E_{\pi^+}V}\;< p(q)K^-(q_-)|\bar{s}(0)\,\gamma_\mu(1-\gamma^5)\,c(0)|\Lambda_c^+(Q)>=$$
$$\lim_{Q^2\to M^2_{\Lambda_c^+},\,q^2\to M_p^2,\,q_-^2\to M_K^2} i\int d^4x_1 d^4x_2 d^4x_3\, e^{iq\cdot x_1}\, e^{iq_-\cdot x_2}\, e^{-iQ\cdot x_3}\,\bar{u}_p(q,\sigma')$$
$$\overrightarrow{\left(i\gamma^\nu\frac{\partial}{\partial x_1^\nu}-M_p\right)}(\Box_2+M_K^2)< 0|\mathrm{T}(\psi_p(x_1)\varphi_{K^-}(x_2)[\bar{s}(0)\gamma^\mu(1-\gamma^5)c(0)]\bar{\psi}_{\Lambda_c^+}(x_3))|0>$$
$$\overleftarrow{\left(-i\gamma^\alpha\frac{\partial}{\partial x_3^\alpha}-M_{\Lambda_c^+}\right)}u_{\Lambda_c^+}(Q,\sigma), \quad (3.1)$$

where $\psi_p(x_1)$, $\varphi_{K^-}(x_2)$, and $\bar{\psi}_{\Lambda_c^+}(x_3)$ are the operators of the proton, the K^- meson and the Λ_c^+ interpolating fields, $\bar{u}_p(q,\sigma')$ and $u_{\Lambda_c^+}(Q,\sigma)$ are the Dirac bispinors of the proton and the Λ_c^+, respectively.

Following Refs. [6–12] in order to describe the right-hand side (RHS) of Eq. (3.1) at the quark level we suggest to use the equations of motion

$$\overrightarrow{\left(i\gamma^\nu\frac{\partial}{\partial x_1^\nu}-M_p\right)}\psi_p(x_1)=\frac{g_{\rm B}}{\sqrt{2}}\eta_{\rm N}(x_1),$$
$$(\Box_2+M_K^2)\varphi_{K^-}(x_2)=\frac{g_{Kqq}}{\sqrt{2}}\bar{u}(x_2)i\gamma^5 s(x_2),$$
$$\bar{\psi}_{\Lambda_c^+}(x_3)\overleftarrow{\left(-i\gamma^\alpha\frac{\partial}{\partial x_3^\alpha}-M_{\Lambda_c^+}\right)}=\frac{g_{\rm C}}{\sqrt{2}}\bar{\eta}_{\Lambda_c^+}(x_3). \quad (3.2)$$

Here g_B and g_C are the phenomenological coupling constants of the proton and the Λ_c^+ coupled with three-quark currents $\eta_N(x_1)=-\varepsilon^{ijk}[\overline{u_i^c}(x_1)\gamma^\mu u_j(x_1)]\gamma_\mu\gamma^5 d_k(x_1)$ and $\bar{\eta}_{\Lambda_c^+}(x_3)=\varepsilon^{ijk}\bar{c}_i(x_3)\gamma_\mu\gamma^5[\bar{d}_j(x_3)\gamma^\mu u_k^c(x_3)]$ [12], respectively,

$$\mathcal{L}_{\rm int}^{(B)}(x)=\frac{g_B}{\sqrt{2}}\bar{\psi}_p(x)\,\eta_N(x)+\frac{g_C}{\sqrt{2}}\bar{\eta}_{\Lambda_c^+}(x)\,\psi_{\Lambda_c^+}(x)+\mathrm{H.c.} \quad (3.3)$$

Then, i, j, and k are color indices and $\bar{u}^c(x)=u(x)^TC$ and $C=-C^T=-C^\dagger=-C^{-1}$ is a matrix of a charge conjugation, T is a transposition. The three-quark current $\eta_{\Lambda_c^+}(x)=-\varepsilon^{ijk}[\overline{u_i^c}(x)\gamma^\mu d_j(x)]\gamma_\mu\gamma^5 c_k(x)$ coupled with the Λ_c^+ is constructed by analogy with $\eta_N(x)=-\varepsilon^{ijk}[\overline{u_i^c}(x)\gamma^\mu u_j(x_1)]\gamma_\mu\gamma^5 d_k(x)$ due to the similar spinorial properties of the Λ_c^+ and the proton. Then, $M_p=938$ MeV and $M_{\Lambda_c^+}=2285$ MeV are the masses of the proton and the Λ_c^+.

The interaction of the K^- meson with quarks is described by the coupling constant $g_{Kqq} = \sqrt{2}m/F_\pi$

$$\mathcal{L}_{\rm int}^{(M)}(x) = \frac{g_{Kqq}}{\sqrt{2}}\bar{s}(x)i\gamma^5 u(x)\varphi_{K^-}(x) + \cdots + \text{H.c.}, \tag{3.4}$$

where $m = 330$ MeV is the constituent quark mass calculated in the chiral limit [5].

Substituting Eq. (3.2) in Eq. (3.1) we obtain

$$\begin{aligned}
&\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V2E_{\pi^+}V}\langle p(q)K^-(q_-)\\
&\quad\times|\overline{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle\\
&\quad= g_B g_C \frac{i}{2}\frac{m}{F_\pi}\int d^4x_1 d^4x_2 d^4x_3 e^{iqx_1}e^{iq_-x_2}e^{-iQx_3}\\
&\quad\times\bar{u}_p(q,\sigma')\langle 0|T\{\eta_N(x_1)[\bar{u}(x_2)i\gamma^5 s(x_2)]\\
&\quad\times[\bar{s}(0)\gamma^\mu(1-\gamma^5)c(0)]\bar{\eta}_{\Lambda_c^+}(x_3)\}|0\rangle u_{\Lambda_c^+}(Q,\sigma),
\end{aligned} \tag{3.5}$$

where the external particles are kept on-mass shell, i.e., $Q^2 = M^2_{\Lambda_c^+}$, $q^2 = M_p^2$, and $q_-^2 = M_K^2$.

By applying the formulas of quark conversion [5] (Ivanov) we can determine the vacuum expectation value in Eq. (3.3) in terms of the constituent quark diagrams. In the momentum representation we get [5–12]

$$\begin{aligned}
&\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V2E_{\pi^+}V}\langle p(q)K^-(q_-)\\
&\quad\times|\overline{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle\\
&\quad= -ig_B g_C\frac{3m}{F_\pi}\left[\frac{1}{16\pi^2}\right]^2\int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\bar{u}_p(q,\sigma')\\
&\quad\times\gamma_\alpha\gamma^5\frac{1}{m-\hat{k}_1}\gamma^\beta\frac{1}{m+\hat{k}_2}\gamma^\alpha\frac{1}{m-\hat{q}+\hat{k}_1+\hat{k}_2}\\
&\quad\times\gamma^5\frac{1}{m-\hat{q}-\hat{q}_-+\hat{k}_1+\hat{k}_2}\gamma_\mu(1-\gamma^5)\\
&\quad\times\frac{1}{M_c-\hat{Q}+\hat{k}_1+\hat{k}_2}\gamma_\beta\gamma^5 u_{\Lambda_c^+}(Q,\sigma),
\end{aligned} \tag{3.6}$$

where $M_c = 1860$ MeV is the mass of the constituent c quark [7–11]. In the HQET the RHS of Eq. (2.4) is given by [3,4,7–11]

$$\begin{aligned}
&\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V2E_{\pi^+}V}\langle p(q)K^-(q_-)\\
&\quad\times|\overline{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle\\
&\quad= -ig_B g_C\frac{3m}{F_\pi}\left[\frac{1}{16\pi^2}\right]^2\\
&\quad\times\int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\bar{u}_p(q,\sigma')\gamma_\alpha\gamma^5\frac{1}{m-\hat{k}_1}\\
&\quad\times\gamma^\beta\frac{1}{m+\hat{k}_2}\gamma^\alpha\frac{1}{m-\hat{q}+\hat{k}_1+\hat{k}_2}\\
&\quad\times\gamma^5\frac{1}{m-\hat{q}-\hat{q}_-+\hat{k}_1+\hat{k}_2}\gamma_\mu(1-\gamma^5)\\
&\quad\times\left(\frac{1+\hat{v}}{2}\right)\frac{1}{[(k_1+k_2)v+i0]}\gamma_\beta\gamma^5 u_{\Lambda_c^+}(Q,\sigma),
\end{aligned} \tag{3.7}$$

where v^μ is the four velocity of the Λ_c^+ [3,4,7–11] normalized by $v^\mu v_\mu = 1$.

For the computation of the momentum integral we assume [12] that the proton is a very heavy state and its four momentum is much larger than other momenta in the integrand of Eq. (3.7). Keeping the leading terms in the large M_p expansion we reduce the RHS of Eq. (3.7) to the form

$$\begin{aligned}
&\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V2E_{\pi^+}V}\langle p(q)K^-(q_-)\\
&\quad\times|\overline{s}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle\\
&\quad= i\frac{g_B g_C}{M_p^2}\frac{3m}{F_\pi}\left[\frac{1}{16\pi^2}\right]^2\int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\bar{u}_p\\
&\quad\times(q,\sigma')\gamma_\alpha\gamma^5\frac{1}{m-\hat{k}_1}\gamma^\beta\frac{1}{m+\hat{k}_2}\gamma^\alpha\gamma_\mu(1-\gamma^5)\\
&\quad\times\left(\frac{1+\hat{v}}{2}\right)\frac{1}{[(k_1+k_2)v+i0]}\gamma_\beta\gamma^5 u_{\Lambda_c^+}(Q,\sigma).
\end{aligned} \tag{3.8}$$

The replacement of the constituent quark Green function

$$\frac{1}{m-\hat{q}+\hat{k}_1+\hat{k}_2}\to-\frac{\hat{p}_1}{M_p^2} \tag{3.9}$$

agrees with the heavy baryon [16,17] and HQET [3,4] approaches. Indeed, in accordance with Refs. [16,17] and HQET [3,4] we obtain

$$\frac{1}{m-\hat{q}+\hat{k}_1+\hat{k}_2} = \frac{m+\hat{q}-\hat{k}_1-\hat{k}_2}{m^2-(q-k_1-k_2)^2-i0} = -\frac{m+\hat{q}-\hat{k}_1-\hat{k}_2}{M_p^2-2(k_1+k_2)q-m^2+(k_1+k_2)^2+i0}$$
$$= -\frac{1}{M_p}\frac{\dfrac{m-\hat{k}_1-\hat{k}_2}{M_p}+\hat{v}}{1+2(k_1+k_2)v/M_p-m^2/M_p^2+(k_1+k_2)^2/M_p^2+i0}, \quad (3.10)$$

where we have set $q^\mu = M_p v^\mu$ [3,4,16,17]. In the case of $m = M_p$ and in the limit $M_p \to \infty$ we arrive at the well-known expression for the Green function of a heavy baryon (or a heavy quark in HQET [3,4]) [16,17]

$$\frac{1}{m-\hat{q}+\hat{k}_1+\hat{k}_2} = \frac{1}{M_p+\hat{k}_1+\hat{k}_2-M_p\hat{v}} \to \left(\frac{1+\hat{v}}{2}\right)\frac{1}{(k_1+k_2)v+i0}. \quad (3.11)$$

In our case $m \ll M_p$, therefore, in the limit $M_p \to \infty$ [16,17] we arrive at Eq. (3.9).

The computation of the momentum integrals in Eq. (3.8) is carried out in the Appendix. Thus, the matrix element $\langle p(q)K^-(q_-)|\overline{s(0)}\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle$ is defined

$$\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V2E_{\pi^+}V}$$
$$\times\langle p(q)K^-(q_-)|\overline{s(0)}\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle$$
$$= ig_{\pi NN}\frac{2}{5}\frac{g_C}{g_B}\frac{\Lambda_\chi}{m^2}\bar{u}_p(q,\sigma')\gamma_\alpha\gamma^5\hat{v}\gamma^\beta\hat{v}\gamma^\alpha\gamma_\mu(1-\gamma^5)$$
$$\times\left(\frac{1+\hat{v}}{2}\right)\gamma_\beta\gamma^5 u_{\Lambda_c^+}(Q,\sigma), \quad (3.12)$$

where $g_{\pi NN}$ is the coupling constant of the πNN interaction expressed in terms of the parameters of the model through the relation $g_{\pi NN} = g_B^2(2m/3F_\pi)(\langle\bar{q}q\rangle^2/M_p^2)$ [12]. After some algebra the matrix element Eq. (3.12) amounts to

$$\sqrt{2E_{\Lambda_c^+}V2E_pV2E_{K^-}V2E_{\pi^+}V}$$
$$\times\langle p(q)K^-(q_-)|\overline{s(0)}\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle$$
$$= ig_{\pi NN}\frac{4}{5}\frac{g_C}{g_B}\frac{\Lambda_\chi}{m^2}\bar{u}_p(q,\sigma')$$
$$\times[2v_\mu(1-\gamma^5)+\gamma_\mu(1+\gamma^5)]u_{\Lambda_c^+}(Q,\sigma)$$
$$= ig_{\pi NN}\frac{4}{5}\frac{g_C}{g_B}\frac{\Lambda_\chi}{m^2}\bar{u}_p(q,\sigma')(1-\gamma^5)$$
$$\times(2v_\mu+\gamma_\mu)u_{\Lambda_c^+}(Q,\sigma), \quad (3.13)$$

where we used the Dirac equation of motion $\hat{v}u_{\Lambda_c^+}(Q,\sigma) = u_{\Lambda_c^+}(Q,\sigma)$ for the free Λ_c^+.

IV. PROTON ENERGY SPECTRUM OF THE $\Lambda_c^+ \to p+K^- + \pi^+$ DECAY

The amplitude of the $\Lambda_c^+ \to p+K^-+\pi^+$ decay is given by

$$\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_-)\pi^+(q_+)]$$
$$= G_F V_{cs}^* V_{ud}\bar{C}_1(\Lambda_\chi)$$
$$\times\frac{4}{5}\frac{g_{\pi NN}}{M_{\Lambda_c^+}}\left[\frac{g_C}{g_B}\frac{F_\pi\Lambda_\chi}{m^2}\right]\bar{u}_p(q,\sigma')(1-\gamma^5)$$
$$\times(2Qq_+ + M_{\Lambda_c^+}\hat{q}_+)u_{\Lambda_c^+}(Q,\sigma). \quad (4.1)$$

The partial width of the $\Lambda_c^+ \to p+K^-+\pi^+$ decay determined in the rest frame of the Λ_c^+ reads

$$d\Gamma(\Lambda_c^+ \to pK^-\pi^+)$$
$$= \frac{1}{2M_{\Lambda_c^+}}\overline{|\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_-)\pi^+(q_+)]|^2}$$
$$\times(2\pi)^4\delta^{(4)}(Q-q-q_--q_+)$$
$$\times\frac{d^3q}{(2\pi)^3 2E_p}\frac{d^3q_-}{(2\pi)^3 2E_{K^-}}\frac{d^3q_+}{(2\pi)^3 2E_{\pi^+}}. \quad (4.2)$$

We define the quantity

$$\overline{|\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_-)\pi^+(q_+)]|^2}$$

for the polarized Λ_c^+ and the unpolarized proton

$$\overline{|\mathcal{M}[\Lambda_c^+(Q)\to p(q)K^-(q_-)\pi^+(q_+)]|^2}$$
$$= |G_F V_{cs}^* V_{ud}\bar{C}_1(\Lambda_\chi)|^2\left[\frac{4}{5}\frac{g_{\pi NN}}{M_{\Lambda_c^+}}\frac{g_C}{g_B}\frac{F_\pi\Lambda_\chi}{m^2}\right]^2$$
$$\times\frac{1}{2}\mathrm{tr}\{(\hat{Q}+M_{\Lambda_c^+})(1+\gamma^5\hat{\omega}_{\Lambda_c^+})$$
$$\times(2Qq_+ + M_{\Lambda_c^+}\hat{q}_+)(1+\gamma^5)(\hat{q}+M_p)$$
$$\times(1-\gamma^5)(2Qq_+ + M_{\Lambda_c^+}\hat{q}_+)\}, \quad (4.3)$$

where $\omega^{\mu}_{\Lambda_c^+}$ is the spacelike unit four vector ($\omega^2_{\Lambda_c^+}=-1$) orthogonal to the four momentum of the Λ_c^+ ($\omega_{\Lambda_c^+}Q=0$) and related to the direction of its spin. In the rest frame of the Λ_c^+ we have $\omega^{\mu}_{\Lambda_c^+}=(0,\vec{\omega}_{\Lambda_c^+})$ such as $\vec{\omega}^2_{\Lambda_c^+}=1$.

Neglecting the terms proportional to M_π^2 and M_K^2 the result of the calculation of the trace reads

$$\frac{1}{2}\mathrm{tr}\{\cdots\}=16(Qq_+)^2(Qq)+24M^2_{\Lambda_c^+}(Qq_+)(qq_+)-32M_{\Lambda_c^+}(Qq_+)^2(\omega_{\Lambda_c^+}q)+16M_{\Lambda_c^+}(Qq_+)(Qq)\times(\omega_{\Lambda_c^+}q_+)+8M^3_{\Lambda_c^+}(qq_+)(\omega_{\Lambda_c^+}q_+). \tag{4.4}$$

For the derivation of the proton energy spectrum it is convenient to use the formula

$$\int q_+^\alpha q_+^\beta \delta^{(4)}(Q-q-q_--q_+)\frac{d^3q_-}{2E_{K^-}}\frac{d^3q_+}{2E_{\pi^+}}=\frac{1}{12}[-(Q-q)^2g^{\alpha\beta}+4(Q-q)^\alpha(Q-q)^\beta]\times\int \delta^{(4)}(Q-q-q_--q_+)\frac{d^3q_-}{2E_{K^-}}\frac{d^3q_+}{2E_{\pi^+}}=\frac{\pi}{24}\times[-(Q-q)^2g^{\alpha\beta}+4(Q-q)^\alpha(Q-q)^\beta], \tag{4.5}$$

which is valid when the contributions proportional to M_π^2 and M_K^2 are neglected. Using Eq. (4.5) we get

$$\int \frac{1}{2}\mathrm{tr}\{\cdots\}\delta^{(4)}(Q-q-q_--q_+)\frac{d^3q_-}{2E_{K^-}}\frac{d^3q_+}{2E_{\pi^+}}=\frac{\pi}{3}\{[15M^4_{\Lambda_c^+}(Qq)-18M^2_{\Lambda_c^+}(Qq)^2+8(Qq)^3+7M^2_{\Lambda_c^+}M_p^2(Qq)-12M^4_{\Lambda_c^+}M_p^2]-(\omega_{\Lambda_c^+}q)M_{\Lambda_c^+}[13M^4_{\Lambda_c^+}-14M^2_{\Lambda_c^+}(Qq)+8(Qq)^3-7M^2_{\Lambda_c^+}M_p^2]\}=\frac{5\pi}{2}M^6_{\Lambda_c^+}x\left[\left(1-\frac{3}{5}x+\frac{2}{15}x^2+\frac{7}{60}\xi^2-\frac{2}{5}\frac{\xi^2}{x}\right)+(\vec{\omega}_{\Lambda_c^+}\vec{n}_p)\frac{13}{15}\sqrt{1-\frac{\xi^2}{x^2}}\times\left(1-\frac{7}{13}x+\frac{2}{13}x^2-\frac{7}{52}\xi^2\right)\right], \tag{4.6}$$

where the final expression is taken in the rest frame of the Λ_c^+, $x=2E_p/M_{\Lambda_c^+}$ is the scaled proton energy, $\xi=2M_p/M_{\Lambda_c^+}$, and $\vec{n}_p=\vec{q}/|\vec{q}|$. The scaled proton energy x ranges the region $\xi\leqslant x\leqslant 1+\xi^2/4$.

The proton energy spectrum of the $\Lambda_c^+\to p+K^-+\pi^+$ decay in the rest frame of the Λ_c^+ is determined:

$$d\Gamma(\Lambda_c^+\to pK^-\pi^+)=|G_FV^*_{cs}V_{ud}\overline{C}_1(\Lambda_\chi)|^2\left[g_{\pi NN}\frac{4}{5}\frac{g_C}{g_B}\frac{F_\pi\Lambda_\chi}{m^2}\right]^2\times\left[\frac{5M^5_{\Lambda_c^+}}{512\pi^3}\right]\left(1-\frac{3}{5}x+\frac{2}{15}x^2+\frac{7}{60}\xi^2-\frac{2}{5}\frac{\xi^2}{x}\right)\times[1+\alpha(x)(\vec{\omega}_{\Lambda_c^+}\vec{n}_p)]x\sqrt{x^2-\xi^2}dx\frac{d\Omega_{\vec{n}_p}}{4\pi}, \tag{4.7}$$

where $d\Omega_{\vec{n}_p}$ is the solid angle of the unit vector $\vec{n}_p=\vec{q}/|\vec{q}|$ and $\alpha(x)$, the parameter of the asymmetry related to the polarization of the Λ_c^+, is given by

$$\alpha(x)=\frac{13}{15}\sqrt{1-\frac{\xi^2}{x^2}}\times\frac{1-(7/13)x+(2/13)x^2-(7/52)\xi^2}{1-(3/5)x+(2/15)x^2+(7/60)\xi^2-(2/5)(\xi^2/x)}. \tag{4.8}$$

In order to apply Eq. (4.7) to the analysis of the polarization of the Λ_c^+ in the processes of photoproduction and hadroproduction we have to remove an uncertainty related to the arbitrary coupling constant g_C^2. For this aim we suggest to normalize the proton energy spectrum to the partial width of the mode $\Lambda_c^+\to p+K^-+\pi^+$ and replace the coupling constant g_C^2 by the experimental value of the probability. Integrating Eq. (4.7) over all variables we obtain the partial width of the mode

$$\Gamma(\Lambda_c^+\to pK^-\pi^+)=|G_FV^*_{cs}V_{ud}\overline{C}_1(\Lambda_\chi)|^2\left[g_{\pi NN}\frac{4}{5}\frac{g_C}{g_B}\frac{F_\pi\Lambda_\chi}{m^2}\right]^2\times\left[\frac{5M^5_{\Lambda_c^+}}{512\pi^3}\right]f(\xi). \tag{4.9}$$

The function $f(\xi)$ is determined by the integral

$$f(\xi)=\int_\xi^{1+\xi^2/4}\left(1-\frac{3}{5}x+\frac{2}{15}x^2+\frac{7}{60}\xi^2-\frac{2}{5}\frac{\xi^2}{x}\right)x\sqrt{x^2-\xi^2}dx=0.065. \tag{4.10}$$

The numerical value has been obtained at $M_{\Lambda_c^+}=2285$ MeV and $M_p=938$ MeV.

The proton energy spectrum of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay is then given by

$$\frac{dB(\Lambda_c^+ \to pK^-\pi^+)}{B(\Lambda_c^+ \to pK^-\pi^+)} = 15.40\left(1 - \frac{3}{5}x + \frac{2}{15}x^2 + \frac{7}{60}\xi^2 - \frac{2}{5}\frac{\xi^2}{x}\right) \times [1 + \alpha(x)(\vec{\omega}_{\Lambda_c^+}\vec{n}_p)]x\sqrt{x^2-\xi^2}dx\,\frac{d\Omega_{\vec{n}_p}}{4\pi}. \tag{4.11}$$

By using the experimental value of the probability $B(\Lambda_c^+ \to pK^-\pi^+)_{\rm exp}$=(0.050±0.013) [2] we obtain

$$dB(\Lambda_c^+ \to pK^-\pi^+) = (0.77 \pm 0.20)\left(1 - \frac{3}{5}x + \frac{2}{15}x^2 + \frac{7}{60}\xi^2 - \frac{2}{5}\frac{\xi^2}{x}\right) \times [1 + \alpha(x)(\vec{\omega}_{\Lambda_c^+}\vec{n}_p)]x\sqrt{x^2-\xi^2}dx\,\frac{d\Omega_{\vec{n}_p}}{4\pi}. \tag{4.12}$$

Integrating over the energy of the proton we derive the angular distribution of the probability of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay

$$\frac{dB(\Lambda_c^+ \to pK^-\pi^+)}{d\Omega_{\vec{n}_p}} = \frac{0.050 \pm 0.013}{4\pi}[1 + 0.77(\vec{\omega}_{\Lambda_c^+}\vec{n}_p)]. \tag{4.13}$$

Thus, the proton energy spectrum Eq. (4.12) and the angular distribution of the probability Eq. (4.13) of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay do not contain arbitrary parameters and, therefore, can be applied to the analysis of the polarization of the Λ_c^+ in the processes of photoproduction and hadroproduction.

The formulas (4.12) and (4.13) define the polarization of the Λ_c^+ relative to the momentum of the proton. If the spin of the Λ_c^+ is parallel to the momentum of the proton, the right-handed (R) polarization, the scalar product $\vec{\omega}_{\Lambda_c^+}\vec{n}_p$ amounts to $\vec{\omega}_{\Lambda_c^+}\vec{n}_p = \cos\vartheta$. The angular distribution of the probability reads

$$\frac{dB(\Lambda_c^+ \to pK^-\pi^+)_{(R)}}{d\Omega_{\vec{n}_p}} = \frac{0.050 \pm 0.013}{4\pi}(1 + 0.77\cos\theta). \tag{4.14}$$

In turn, for the left-handed (L) polarization of the Λ_c^+, the spin of the Λ_c^+ is antiparallel to the momentum of the proton, the scalar product reads $(\vec{\omega}_{\Lambda_c^+}\vec{n}_p) = -\cos\vartheta$ and the angular distribution becomes equal

$$\frac{dB(\Lambda_c^+ \to pK^-\pi^+)_{(L)}}{d\Omega_{\vec{n}_p}} = \frac{0.050 \pm 0.013}{4\pi}(1 - 0.77\cos\theta). \tag{4.15}$$

For the right- and left-handed polarizations of the Λ_c^+ the proton energy spectrum is given by

$$dB(\Lambda_c^+ \to pK^-\pi^+)_{(R,L)} = (0.77 \pm 0.20)\left(1 - \frac{3}{5}x + \frac{2}{15}x^2 + \frac{7}{60}\xi^2 - \frac{2}{5}\frac{\xi^2}{x}\right) \times [1 \pm \alpha(x)\cos\vartheta]x\sqrt{x^2-\xi^2}dx\,\frac{d\Omega_{\vec{n}_p}}{4\pi}. \tag{4.16}$$

The formulas (4.12) and (4.13) resemble the electron energy spectrum and the angular distribution of the probability of the β decay of the μ^- meson, i.e., $\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$. Therefore, the procedure of the investigation of the polarization of the Λ_c^+ in the $\Lambda_c^+ \to p + K^-\pi^+$ decay is in complete analogy to the procedure of the measurement of the polarization of the μ^- meson in the $\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$ decay [2].

V. PROBABILITY OF THE $\Lambda_C^+ \to P + \overline{K}^0$ DECAY

Most modes of the Λ_c^+ decays are measured relative to the mode $\Lambda_c^+ \to p + K^- + \pi^+$ [2]. For the theoretical description of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay at the quark level we have introduced the phenomenological low-energy interaction of the Λ_c^+ with the three-quark current $\eta_{\Lambda_c^+}(x)$ containing an arbitrary phenomenological coupling constant g_C, Eq. (3.3). The spinorial structure of the three-quark current $\eta_{\Lambda_c^+}(x) = -\varepsilon^{ijk}[\overline{u_i^c}(x)\gamma^\mu d_j(x)]\gamma_\mu\gamma^5 c_k(x)$ defined as the product of the axial-vector light diquark density $[\overline{u_i^c}(x)\gamma^\mu d_j(x)]$ transforming under the $\mathrm{SU(3)}_f \times \mathrm{SU(3)}_c$ group as $(6_f, \overline{3}_c)$ and the c-quark field $c_k(x)$ is caused by the dynamics of the quark confinement given by a linearly rising interquark potential [6,11]. In order to verify the validity of the approach applied to the computation of the proton energy spectrum and the angular distribution of the probability of the mode $\Lambda_c^+ \to p + K^- + \pi^+$ it is not sufficient to be restricted by the consideration only this mode. For the confirmation of the result obtained for the mode $\Lambda_c^+ \to p + K^- + \pi^+$ one needs the computation of the probabilities of other modes relative to the probability of the main mode $\Lambda_c^+ \to p + K^- + \pi^+$. In the ratio the coupling constant g_C cancels itself and the theoretical result turns out to be dependent on the Wilson coefficients, determined by the short-distance quark-gluon interactions, and the long-distance dynamics, described by the quark model with chiral $\mathrm{U(3)} \times \mathrm{U(3)}$ symmetry motivated by QCD with a linearly rising confinement potential. The agreement between the experimental data and the theoretical predictions for the ratios should testify both the self-consistency of the approach and the consistency of it with a short-distance QCD. Below we obtain the evidence of the self-consistency and the consistency of the approach by example of the calculation of the decay mode $\Lambda_c^+ \to p + \overline{K}^0$. We have chosen this mode due to the following reasons. First, it contains the proton in the final state, and, second, the computation of the matrix element of this mode is completely different to the computation of the matrix element of the mode $\Lambda_c^+ \to p + K^- + \pi^+$. Indeed, the mode $\Lambda_c^+ \to p + \overline{K}^0$ unlike the mode $\Lambda_c^+ \to p + K^- + \pi^+$ admits the factorization of the baryonic and mesonic degrees of freedom.

The effective low-energy Lagrangian responsible for the decay $\Lambda_c^+ \to p + \overline{K}^0$ can be obtained from the effective Lagrangian (2.1) at $\mu = \Lambda_\chi$:

$$\mathcal{L}_{\rm eff}(x) = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \overline{C}_2(\Lambda_\chi)[\overline{u}(x)\gamma_\mu(1-\gamma^5)c(x)] \times [\overline{s}(x)\gamma^\mu(1-\gamma^5)d(x)], \tag{5.1}$$

where $\overline{C}_2(\Lambda_\chi) = C_2(\Lambda_\chi) + C_1(\Lambda_\chi)/3$. The amplitude of the $\Lambda_c^+ \to p + \overline{K}^0$ decay can be defined in analogy with Eq. (2.6):

$$\frac{\mathcal{M}[\Lambda_c^+(Q) \to p(q)\overline{K}^0(q_0)]}{\sqrt{2E_{\Lambda_c^+}V2E_pV}} = -iG_F V_{cs}^* V_{ud} \overline{C}_2(\Lambda_\chi) F_K q_0^\mu \times \langle p(q)|\overline{u}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle, \tag{5.2}$$

where $F_K = 113$ MeV [2] is the leptonic constant of the K mesons.

To the computation of the matrix element $\langle p(q)|\overline{u}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle$ we apply the reduction technique. By using the equations of motion (2.2) we arrive at the expression

$$\begin{aligned}\sqrt{2E_{\Lambda_c^+}V2E_pV}&\langle p(q)|\overline{u}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle \\ &= g_B g_C \frac{1}{2}\int d^4x_1 d^4x_2 e^{iqx_1} e^{-iQx_2} \overline{u}_p(q,\sigma') \\ &\times \langle 0|T\{\eta_N(x_1)[\overline{u}(0)\gamma^\mu(1-\gamma^5)c(0)] \\ &\times \overline{\eta}_{\Lambda_c^+}(x_2)\}|0\rangle u_{\Lambda_c^+}(Q,\sigma),\end{aligned} \tag{5.3}$$

where the Λ_c^+ and the proton are kept on-mass shell, i.e., $Q^2 = M^2_{\Lambda_c^+}$ and $q^2 = M_p^2$. In terms of the constituent quark diagrams represented by the momentum integrals the RHS of Eq. (5.3) defined in HQET reads

$$\begin{aligned}\sqrt{2E_{\Lambda_c^+}V2E_pV}&\langle p(q)|\overline{u}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle \\ &= -3g_B g_C\left[\frac{1}{16\pi^2}\right]^2 \int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\overline{u}_p(q,\sigma') \\ &\times \gamma_\alpha\gamma^5\frac{1}{m-\hat{k}_1}\gamma^\beta\frac{1}{m+\hat{k}_2}\gamma^\alpha\frac{1}{m-\hat{q}+\hat{k}_1+\hat{k}_2}\gamma_\mu \\ &\times (1-\gamma^5)\left(\frac{1+\hat{v}}{2}\right)\frac{1}{[(k_1+k_2)v+i0]} \\ &\times \gamma_\beta\gamma^5 u_{\Lambda_c^+}(Q,\sigma).\end{aligned} \tag{5.4}$$

Keeping the leading terms in the large M_p expansion we reduce the RHS of Eq. (5.4) to the form

$$\begin{aligned}\sqrt{2E_{\Lambda_c^+}V2E_pV}&\langle p(q)|\overline{u}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle \\ &= 3\frac{g_B g_C}{M_p^2}\left[\frac{1}{16\pi^2}\right]^2 \int\frac{d^4k_1}{\pi^2 i}\int\frac{d^4k_2}{\pi^2 i}\overline{u}_p(q,\sigma') \\ &\times \gamma_\alpha\gamma^5\frac{1}{m-\hat{k}_1}\gamma^\beta\frac{1}{m+\hat{k}_2}\gamma^\alpha\hat{q}\gamma_\mu(1-\gamma^5) \\ &\times \left(\frac{1+\hat{v}}{2}\right)\frac{1}{[(k_1+k_2)v+i0]}\gamma_\beta\gamma^5 u_{\Lambda_c^+}(Q,\sigma).\end{aligned} \tag{5.5}$$

The integrals over k_1 and k_2 have been calculated in the Appendix. Using Eq. (A7) and making some algebraic transformations with the Dirac matrices we get

$$\begin{aligned}\sqrt{2E_{\Lambda_c^+}V2E_pV}&\langle p(q)|\overline{u}(0)\gamma_\mu(1-\gamma^5)c(0)|\Lambda_c^+(Q)\rangle \\ &= -g_{\pi NN}\left[\frac{4}{5}\frac{g_C}{g_B}\frac{F_\pi\Lambda_\chi}{m^3}\right]\overline{u}_p(q,\sigma')(2\hat{v}\gamma_\mu\hat{q}+\hat{v}\hat{q}\gamma_\mu) \\ &\times (1-\gamma^5)u_{\Lambda_c^+}(Q,\sigma).\end{aligned} \tag{5.6}$$

The amplitude of the $\Lambda_c^+ \to p + \overline{K}^0$ decay is defined

$$\begin{aligned}\mathcal{M}[\Lambda_c^+(Q)&\to p(q)K^0(q_0)] \\ &= iG_F V_{cs}^* V_{ud}\overline{C}_2(\Lambda_\chi)\frac{F_K}{m} \\ &\times \left[g_{\pi NN}\frac{4}{5}\frac{g_C}{g_B}\frac{F_\pi\Lambda_\chi}{m^2}\right] M^2_{\Lambda_c^+}\overline{u}_p(q,\sigma') \\ &\times (A+B\gamma^5)u_{\Lambda_c^+}(Q,\sigma),\end{aligned} \tag{5.7}$$

where the constants A and B are given by

$$A = 1 + \frac{M_p}{M_{\Lambda_c^+}} - 2\frac{M_p^2}{M_{\Lambda_c^+}^2} - \frac{M_{\bar{K}^0}^2}{M_{\Lambda_c^+}^2} = 1.03,$$

$$B = 1 - \frac{M_p}{M_{\Lambda_c^+}} - 2\frac{M_p^2}{M_{\Lambda_c^+}^2} - \frac{M_{\bar{K}^0}^2}{M_{\Lambda_c^+}^2} = 0.21. \qquad (5.8)$$

The numerical values are obtained for $M_{\Lambda_c^+} = 2285$ MeV, $M_p = 938$ MeV, and $M_{\bar{K}^0} = 498$ MeV [2]. The partial width of the $\Lambda_c^+ \to p + \bar{K}^0$ decay reads

$$\Gamma(\Lambda_c^+ \to p\bar{K}^0) = |G_F V_{cs}^* V_{ud} \bar{C}_2(\Lambda_\chi)|^2 \left[g_{\pi NN} \frac{4}{5} \frac{g_C}{g_B} \frac{F_\pi \Lambda_\chi}{m^2} \right]^2 \frac{F_K^2}{m^2} \frac{M_{\Lambda_c^+}^3}{32\pi} \times \mathrm{tr}\{(\hat{Q} + M_{\Lambda_c^+})(A - B\gamma^5)(\hat{q} + M_p)(A + B\gamma^5)\} \times \sqrt{\left[1 - \left(\frac{M_p - M_{\bar{K}^0}}{M_{\Lambda_c^+}}\right)^2\right]\left[1 - \left(\frac{M_p + M_{\bar{K}^0}}{M_{\Lambda_c^+}}\right)^2\right]}. \qquad (5.9)$$

When computing the trace over Dirac matrices we obtain the partial width in the form

$$\Gamma(\Lambda_c^+ \to p\bar{K}^0) = |G_F V_{cs}^* V_{ud} \bar{C}_2(\Lambda_\chi)|^2 \left[g_{\pi NN} \frac{4}{5} \frac{g_C}{g_B} \frac{F_\pi \Lambda_\chi}{m^2} \right]^2 \left[\frac{5M_{\Lambda_c^+}^5}{512\pi^3} \right] \times \frac{32\pi^2}{5} \frac{F_K^2}{m^2} \left\{ A^2 \left[\left(1 + \frac{M_p}{M_{\Lambda_c^+}}\right)^2 - \frac{M_{\bar{K}^0}^2}{M_{\Lambda_c^+}^2} \right] + B^2 \left[\left(1 - \frac{M_p}{M_{\Lambda_c^+}}\right)^2 - \frac{M_{\bar{K}^0}^2}{M_{\Lambda_c^+}^2} \right] \right\} \times \sqrt{\left[1 - \left(\frac{M_p - M_{\bar{K}^0}}{M_{\Lambda_c^+}}\right)^2\right]\left[1 - \left(\frac{M_p + M_{\bar{K}^0}}{M_{\Lambda_c^+}}\right)^2\right]} = 11.72 \times |G_F V_{cs}^* V_{ud} \bar{C}_2(\Lambda_\chi)|^2 \times \left[g_{\pi NN} \frac{4}{5} \frac{g_C}{g_B} \frac{F_\pi \Lambda_\chi}{m^2} \right]^2 \left[\frac{5M_{\Lambda_c^+}^5}{512\pi^3} \right]. \qquad (5.10)$$

Now we can define the partial width of the $\Lambda_c^+ \to p + \bar{K}^0$ decay with respect to the partial width of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay. By using Eqs. (4.9) and (4.10) we get

$$R_{\rm th} = \frac{\Gamma(\Lambda_c^+ \to p\bar{K}^0)}{\Gamma(\Lambda_c^+ \to pK^-\pi^+)} = 180.31 \times \frac{\bar{C}_2^2(\Lambda_\chi)}{\bar{C}_1^2(\Lambda_\chi)}. \qquad (5.11)$$

The numerical factor in front of the ratio of the squared Wilson coefficients is completely due the low-energy dynamics of our model induced by a linearly rising confinement potential. In order to verify the consistency of this dynamics with a short-distance QCD we should substitute in Eq. (5.11) the numerical values of the Wilson coefficients.

Following Buras *et al.* [15] we obtain $C_1(\Lambda_\chi) = 1.24$ and $C_2(\Lambda_\chi) = -0.47$.[1] These numerical values agree well with the numerical values of the Wilson coefficients calculated at the normalization scale $\mu \simeq 1.5$ GeV [15]: $C_1(\mu) \simeq 1.21$ and $C_2(\mu) \simeq -0.42$.

The ratio $R_{\rm th}$ calculated at $C_1(\Lambda_\chi) = 1.24$ and $C_2(\Lambda_\chi) = -0.47$ amounts to

$$R_{\rm th} = \frac{\Gamma(\Lambda_c^+ \to p\bar{K}^0)}{\Gamma(\Lambda_c^+ \to pK^-\pi^+)} = 0.50. \qquad (5.12)$$

The theoretical result agrees well with the experimental value averaged over all experimental data [2]: $R_{\rm exp} = (0.49 \pm 0.07)$. This agreement is nontrivial and testifies to not only the consistency of the model with short-distance QCD but the self-consistency of the quark model with chiral $U(3) \times U(3)$ symmetry incorporating HQET and $(\mathrm{CHPT})_q$. The former is due to the distinction between the computations of the matrix elements of the modes $\Lambda_c^+ \to p + \bar{K}^0$ and $\Lambda_c^+ \to p + K^- + \pi^+$. Indeed, the computation of the matrix element of the mode $\Lambda_c^+ \to p + \bar{K}^0$ admits the factorization of the baryonic and mesonic degrees of freedom, whereas for the computation of the matrix element of the mode $\Lambda_c^+ \to p + K^- + \pi^+$ such a factorization is not feasible.

Our theoretical result for ratio Eq. (5.12) also confirms the validity of our prediction for the proton energy spectrum and the angular distribution of the probability of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay given by Eqs. (4.12) and (4.13), respectively.

VI. CONCLUSION

The main result of the paper is in the prediction of the proton energy spectrum and the angular distribution of the probability of the mode $\Lambda_c^+ \to p + K^- + \pi^+$ of the Λ_c^+ decays. This mode is the most favorable for the measurement as it contains the proton and the charged mesons. However, from the theoretical point of view this mode is the most difficult for the computation due to the impossibility to factorize baryonic and mesonic degrees of freedom.

To the computation of the matrix element of the $\Lambda_c^+ \to p + K^- + \pi^+$ decay we have applied the quark model with chiral $U(3) \times U(3)$ symmetry incorporating HQET and

[1]The Wilson coefficients defining the effective Lagrangian describing weak hadronic transitions with $\Delta S = 1$ and $\Delta I = 1/2$ selection rules and taken at the renormalization point $\mu = \Lambda_\chi = 940$ MeV have been calculated previously in Ref. [19]. For this aim one had only to follow the explict expression of the Wislon coefficients as functions of μ obtained by Gilman and Wise [20] and Buras and Slominsky [21].

$(\mathrm{CHPT})_q$. This model is motivated by the effective low-energy QCD with a linearly rising confinement potential. Due to the dynamics of strong low-energy interactions caused by a linearly rising confinement potential the spinorial structure of the three-quark currents coupled to the baryons is fixed unambiguously. The effective low-energy interactions of the low-lying baryon octet and charmed baryons coupled to the three-quark currents can be described in terms of two phenomenological coupling constants g_B and g_C, respectively. These constants enter multiplicatively to the matrix elements of the strong low-energy transitions of baryons.

In the case of the nonleptonic decays of the Λ_c^+ the multiplicative character of the constants g_B and g_C allows us to replace the product of these constants by the experimental value of the probability of the mode $\Lambda_c^+ \to p + K^- + \pi^+$. This defines any mode of the Λ_c^+ decays relative to the mode $\Lambda_c^+ \to p + K^- + \pi^+$. As regards the proton energy spectrum and the angular distribution, the resultant expressions do not contain any arbitrary parameters and can be applied to the analysis of the polarization of the Λ_c^+ in the processes of photoproduction and hadroproduction. We have considered the simplest case, maybe most favorable from the experimental point of view, when the Λ_c^+ is polarized while the proton of the decay is unpolarized. This means that for the investigation of the polarization the Λ_c^+ one should follow only the geometry of the momenta of the protons of the decay but not their polarizations. In this case there is an obvious similarity between measurements of the polarization of the Λ_c^+ in the $\Lambda_c^+ \to p + K^- + \pi^+$ decay and the μ^- meson in the β decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$.

For the confirmation of the validity of our prediction for the proton energy spectrum and the angular distribution of the probability of the mode $\Lambda_c^+ \to p + K^- + \pi^+$, we have computed the probability of the mode $\Lambda_c^+ \to p + \bar{K}^0$ relative to the probability of the mode $\Lambda_c^+ \to p + K^- + \pi^+$. Our prediction for the ratio of the probabilities $R_{\mathrm{th}} = B(\Lambda_C^+ \to p\bar{K}^0)/B(\Lambda_c^+ \to pK^-\pi^+) = 0.50$ agrees well with the experimental value averaged over all experimental data $R_{\mathrm{exp}} = (0.49 \pm 0.07)$. This agreement is not trivial and confirms not only the self-consistency of our approach but the consistency of it with a short-distance QCD, since the computation of the matrix element of the mode $\Lambda_c^+ \to p + \bar{K}^0$ differs from the computation of the matrix element of the mode $\Lambda_c^+ \to p + K^- + \pi^+$. Indeed, if for the computation of the matrix element of the mode $\Lambda_c^+ \to p + \bar{K}^0$ one can factorize the baryonic and mesonic degrees of freedom, whereas in the case of the computation of the matrix element of the mode $\Lambda_c^+ \to p + K^- + \pi^+$ such a factorization is not feasible.

APPENDIX: COMPUTATION OF THE MOMENTUM INTEGRALS

We perform the integration over k_1 and k_2 of the momentum integral of Eq. (3.6). For this aim we consider the integral

$$\mathcal{J}(v) = \int \frac{d^4k_1}{\pi^2 i}\int \frac{d^4k_2}{\pi^2 i}\frac{1}{m - \hat{k}_1}\Gamma\frac{1}{m + \hat{k}_2}\frac{1}{[(k_1 + k_2)v + i0]}, \tag{A1}$$

where Γ is a Dirac matrix. The nontrivial contribution comes from the components of the k_1^μ and k_2^μ four vectors parallel to four vector v^μ. This gives

$$\mathcal{J}(v) = \int \frac{d^4k_1}{\pi^2 i}\int \frac{d^4k_2}{\pi^2 i}\frac{m + \hat{v}k_1 v}{[m^2 - k_1^2 - i0]} \times \Gamma\frac{m - \hat{v}k_2 v}{[m^2 - k_2^2 - i0]}\frac{1}{[(k_1 + k_2)v + i0]}. \tag{A2}$$

Now it is convenient to make the Wick rotation and pass to Euclidean momentum space [9]:

$$\mathcal{J}(v) = 4i\int_0^\infty \frac{dk_{E1}k_{E1}^3}{m^2 + k_{E1}^2}\int \frac{d\Omega_1}{2\pi^2}(m + i\hat{v}k_{E1}\cos\chi_1) \times \Gamma\int_0^\infty \frac{dk_{E2}k_{E2}^3}{m^2 + k_{E2}^2} \times \int \frac{d\Omega_2}{2\pi^2}\frac{m - i\hat{v}k_{E2}\cos\chi_2}{k_{E1}\cos\chi_1 + k_{E2}\cos\chi_2}, \tag{A3}$$

where $d\Omega_i = 4\pi\sin^2\chi_i d\chi_i$ $(i = 1,2)$ are the solid angles in Euclidean spaces of the momenta k_{E1}^μ and k_{E2}^μ, respectively, $k_{Ei} = \sqrt{k_{4i}^2 + \vec{k}_i^2}$ $(i = 1,2)$. Then we have used the relation $\sqrt{v_E^2} = -i$ [8].

In order to disconnect integrations over k_{E1} and k_{E2} we suggest to use the following integral representation:

$$\mathcal{J}(v) = 4\int_0^\infty dt\int_0^\infty \frac{dk_{E1}k_{E1}^3}{m^2 + k_{E1}^2}\int \frac{d\Omega_1}{2\pi^2}(m + i\hat{v}k_{E1}\cos\chi_1) \times e^{itk_{E1}\cos\chi_1}\Gamma\int_0^\infty \frac{dk_{E2}k_{E2}^3}{m^2 + k_{E2}^2}\int \frac{d\Omega_2}{2\pi^2} \times (m - i\hat{v}k_{E2}\cos\chi_2)e^{itk_{E2}\cos\chi_2}. \tag{A4}$$

Integrating out χ_1 and χ_2 we get

$$\mathcal{J}(v) = 16\int_0^\infty dt\int_0^\infty \frac{dk_{E1}k_{E1}^3}{m^2 + k_{E1}^2}\left\{m\left[\frac{J_1(k_{E1}t)}{k_{E1}t}\right] - i\hat{v}\frac{\partial}{\partial t}\left[\frac{J_1(k_{E1}t)}{k_{E1}t}\right]\right\}\Gamma\int_0^\infty \frac{dk_{E2}k_{E2}^3}{m^2 + k_{E2}^2}\left\{m\left[\frac{J_1(k_{E2}t)}{k_{E2}t}\right] + i\hat{v}\frac{\partial}{\partial t}\left[\frac{J_1(k_{E2}t)}{k_{E2}t}\right]\right\}, \tag{A5}$$

where $J_1(k_{Ei}t)$ $(i = 1,2)$ is the Bessel function. Now we can perform the integration over k_{Ei} $(i = 1,2)$:

$$\mathcal{J}(v) = 16m^2 \int_0^\infty dt \left\{ m\left[\frac{K_1(mt)}{t}\right] - i\hat{v}\frac{\partial}{\partial t}\left[\frac{K_1(mt)}{t}\right]\right\} \Gamma \left\{ m\left[\frac{K_1(mt)}{t}\right] + i\hat{v}\frac{\partial}{\partial t}\left[\frac{K_1(mt)}{t}\right]\right\}, \tag{A6}$$

where $K_1(mt)$ is the McDonald function. The integrals over t are divergent and should be regularized. We suggest to use the cutoff regularization restricting the region of the integration over t from below as $t \geqslant 1/\Lambda_\chi$. Keeping leading divergent contributions [5–12] it is convenient to represent the RHS of Eq. (A6) in the following form:

$$\mathcal{J}(v) = \frac{4}{5}\left[\frac{16\pi^2}{3}\right]^2 \frac{\Lambda_\chi}{m^2} \langle \bar{q}q \rangle^2 \hat{v} \Gamma \hat{v}, \tag{A7}$$

where $\langle \bar{q}q \rangle$ is the quark condensate defined in terms of the SBχS scale and the constituent quark mass as follows [5–12]

$$\langle \bar{q}q \rangle = -\frac{N}{16\pi^2} \int \frac{d^4k}{\pi^2 i} \mathrm{tr}\left\{\frac{1}{m - \hat{k}}\right\} = -\frac{Nm}{4\pi^2}\left[\Lambda_\chi^2 - m^2 \ln\left(1 + \frac{\Lambda_\chi^2}{m^2}\right)\right] = -(253 \ \mathrm{MeV})^3. \tag{A8}$$

The numerical value is calculated at $N=3$, $m=330$ MeV, and $\Lambda_\chi = 940$ MeV. As has been shown in Ref. [5] the quark condensate value $\langle \bar{q}q \rangle = -(253 \ \mathrm{MeV})^3$ describes with an accuracy better than 5% the mass spectrum of low-lying pseudoscalar mesons for the current quark masses $m_{0u} = 4$ MeV, $m_{0d} = 7$ MeV, and $m_{0s} = 135$ MeV quoted by QCD [18].

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