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Nuclear sizes and temperatures from two-body correlations

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The inter-relationship arising from the use of two-body correlation functions for determining both nuclear source size and freeze-out temperature is discussed. Features which influence the interpretation of each are explored. Estimates of the differences between *true* and *apparent* volumes are made. Differences between volume measurements employing different correlation pairs are discussed and related to different ''temperatures'' examined in nuclear reactions. [S0556-2813(99)01807-5]

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At the present time experimental measurements of two-body nuclear correlations are being studied to obtain information about two extremely important features of excited nuclear systems. In some studies, these measurements are performed to learn about the the space-time structure of excited nuclear systems. For example, evidence for source size is sought by using techniques suggested by Koonin [1] and Pratt [2,3] for this purpose. The underlying physics behind these measurements is the two-body final-state interaction between the correlated particles. At the same time, two-body correlations are also being employed to learn about temperatures [4–10] at freeze-out in hot nuclear systems. For this purpose, the relative yields of excited nuclear states are used, assuming Boltzmann-like populations, in an attempt to extract temperature. It was suggested that unbound states should be included among the species of fragments emitted from excited nuclear systems [11]. The greatest sensitivity for temperature measurement comes from states widely separated in energy. Thus there has been particularly great interest in unbound states (resonances). This, of course, requires the measurements of two-body correlations. The underlying physics here is that each of the unbound states is populated according to thermal equilibrium at a freeze-out temperature.

This work is concerned with the inter-relationship between these two attempts to obtain nuclear information. It is also concerned with the problems of interpretation involved with the use of a single class of measurements, i.e., two body correlations, for two disparate purposes. For example, in measuring particle-particle correlations to determine the size of emitting sources, one must be concerned about the influence coming from the decay of resonant states created during the reactions. These states may arise through either statistical or nonequilibrium processes. This problem is encountered not only in nuclear physics but also in high energy physics. On the other hand, if one seeks to determine the temperature using relative yields measured by correlation functions, one must be concerned with the influence of final state interactions between originally uncorrelated particles.

An extremely useful step toward handling this problem was provided by the formal work of Boal [12,13] and his group several years ago. The work presented here grows out of considerations examined by those earlier references.

To obtain information for source size (and time separations) from two-body correlations, one uses a procedure suggested by Koonin and refined by Pratt. The procedure, in essence, involves integrating the relevant two-body wave function over the relative distribution of particles provided by the source. One thus introduces the influence of the final state interaction which creates the correlation between the interacting particles. Koonin showed how, by using the size of the correlation signal, one can unfold the effect of the final-state interaction to learn about the distribution of pairs of particles existing before the interaction. This permits one to learn about the size of the source. If particles are emitted sequentially, however, one of the pair travels some distance before the partner is emitted. It was realized that the apparent source size, determined by the correlation function, could be quite large, even if the actual size were not. What, however, is the effect of particles emitted in a resonant state? Clearly, if these resonances are quite numerous they will enhance the correlation signal and provide an apparent size much smaller than the actual source.

The following question may be asked: For what number of decaying resonances is the apparent source size equal to the actual size? Another question is: What corrections to the estimate of source size must be made in light of the number of decaying resonances? We look into these questions in what follows.

To simplify the discussion we assume that the source is of uniform density and that it is large. By this we mean that the two integrations, one over the center-of-mass, and the other over the relative coordinates, can be decoupled when performed over the location of the correlated particles in the Koonin procedure. We further assume, for simplicity, that the effect of the time delay in the emission of the the particles is unimportant. Under these conditions Boal's group established a simple formal relationship between the Koonin procedure (source integration over the two-body wave function for interacting pairs) and the fundamental virial relationship for the modification of phase-space which is required in going from the case of a noninteraction system of particles to one with two-body forces.

They showed formally that the Koonin correlation function R can be expressed in the form

$$R(q)=(2\pi^2\hbar^3)/(q^2V)\times(1/\pi)(d\delta/dq)(2J+1)/[(2s_1+1)\times(2s_2+1)]. \tag{1}$$

This is exactly the expression which one would obtain by taking the ratio between the interaction-induced change in the phase-space density and the noninteracting phase-space density. If more than one resonance is important Eq. (1) is extended to include a sum over the channels containing these resonances. In the following discussion, we build on Eq. (1) to explore how emitted resonances can influence the apparent source size obtained with the Koonin procedure.

Let us consider the general case of a correlation between particles labeled 1 and 2. We assume that these particles are also the end products of a resonance labeled 3, where energy conservation provides

$$P_3^2/(2M_3)=P_1^2/(2M_1)+P_2^2/(2M_2)-Q. \tag{2}$$

Here $Q=(M_3-M_1-M_2)c^2$, is positive for an unstable resonance. Let us assume that the number of each species (1,2,3) produced in the reaction, is given by N_j, and that S_j provides the respective spin. The number N_3 includes resonances that are formed as the result of a final state interaction between 1 and 2, as well as those emitted in the reaction. To simplify the argument, we assume that all three types of species move isotropically in the source frame, and that each appears to have a thermal spectrum proportional to $\exp(-E_j/T)$ before Coulomb acceleration. Let us next consider the differential cross section for particles of type *j*. This takes the form

$$dN_j/dP_j^3=N_j/(2\pi M_jT)^{3/2}\times\exp(-P_j^2/(2M_jT)), \tag{3}$$

where the result is normalized to the total number of *j* particles, N_j, and *T* is the spectral temperature, T_{spect}. It follows, that the differential cross section for uncorrelated particles of type 1 and 2 with center of mass moment $\vec{K}$ and relative momentum $\vec{q}$ is provided by

$$dn_1n_2/dK^3dq^3=N_1/(2\pi M_1T)^{3/2}\times N_2/(2\pi M_2T)^{3/2}\times\exp(-[P_1^2/(2M_1)+P_2^2/(2M_2)]/T), \tag{4}$$

where the individual momenta, $\vec{P}_1$ and $\vec{P}_2$, can be expressed in terms of the center of mass and relative momenta $\vec{K}$ and $\vec{q}$.

We next describe a similar differential cross section for the particles arising from the decay of particles of type 3, which has momentum $\vec{K}$ before decay. Following decay, the relative momentum of the two fragments (1 and 2) is given by $\vec{q}$. That differential cross section has the following form:

$$dn_3/dK^3dq^3=N_3/(2\pi M_3T)^{3/2}\times\exp(-K^2/(2M_3T))f(\vec{q}), \tag{5}$$

where $f(\vec{q})$ is normalized to unity when integrated over the three-dimensional space of momentum $\vec{q}$. It reflects the shape of the momentum distribution of the decaying pair. We take $f(\vec{q})=[1/(4\pi q^2)](1/\pi)(d\delta/dq)$. Since the integral across the resonance results in a phase increase of π this function is properly normalized. The dependence on relative momentum is essentially the same as that provided by the final-state interaction.

The correlation function *R* can be obtained from the ratio of the differential cross section of Eq. (5) to the cross section of Eq. (4).

$$\begin{aligned}R&=dn_3/dK^3dq^3)/(dn_1n_2/dK^3dq^3)\\&=N_3/(2\pi M_3T)^{3/2}\exp(Q/T)/[N_1/(2\pi M_1T)^{3/2}\\&\quad\times N_2/(2\pi M_2T)^{3/2}]f(\vec{q}),\end{aligned} \tag{6}$$

where we have used the conservation of energy, Eq. (2), relating the initial and final energies. In this form R is written in terms of the yields N_j.

It is also convenient to consider the particles in a volume *V*, and to introduce the density of each species, by ρ_j, such that $N_j=V\rho_j$. In addition, it is convenient to multiply and divide each yield by the respective degeneracy factors $(2J_j+1)$ and to introduce the expression for the ''thermal wavelengths'' $\lambda_{T_j}=(2\pi\hbar)/(2\pi M_jT)^{1/2}$. We then obtain for the *R* of Eq. (6)

$$\begin{aligned}R&=(2J_3+1)/[(2J_1+1)(2J_2+1)]/(4\pi q^2V)(1/\pi)(d\delta/dq)\\&\quad\times(2\pi\hbar)^3\times G,\end{aligned} \tag{7}$$

where

$$\begin{aligned}G&=[\rho_3\lambda_{T_3}^3/(2J_3+1)][\exp(Q/T)]/\{[(\rho_1\lambda_{T_1}^3/(2J_1+1)]\\&\quad\times[\rho_2\lambda_{T_2}^3/(2J_2+1)]\}.\end{aligned} \tag{8}$$

Notice, that, apart from the factor *G*, Eq. (7) is the same as Eq. (1) which links the shape of the correlation function to the apparent volume of the source obtained with the Koonin prescription.

To explore the factor *G*, let us consider a system in chemical and thermal equilibrium for which the chemical potentials for particles of species 1, 2, and 3, are related by $\mu_1/T+\mu_2/T=\mu_3/T+Q/T$. In a quantum nuclear system which is highly clustered, i.e., where the partial density of any given species is quite low, the chemical potential can be well approximated by that of classical gas with, $\mu/T=\log(\rho\lambda_T^3/(2J+1))$. It is thus clear from Eq. (8) that when the yields of the three species N_j are exactly those predicted by chemical equilibrium for temperature T_{spect} in the true volume V, then the value of G is exactly 1.

In this case the correlation function is that given by Boal's group, and by his analysis, in the limit of large source size, it is the same as the one provided by the Koonin prescription. Thus, when the yields are those predicted by chemical and thermal equilibrium, the apparent volume obtained from the Koonin analysis is the actual volume.

On the other hand, if the yields, volume, and spectral temperatures do not correspond to chemical and thermal equilibrium the link to actual volume is modified. For G larger than 1, the correlation is stronger than expected and the apparent size from the correlation analysis is smaller than the actual size. The converse is true for G smaller than one, where the actual size is smaller than the apparent size. The apparent volume given by Eq. (7) is approximately

$$V_{\mathrm{apparent}} = V/G. \tag{9}$$

Several features can cause G to vary from its value at chemical equilibrium. Secondary decay can boost the yields of particles of type 1, and 2, and hence reduce the G from unity. Other mechanisms (nonequilibrium, for example) can boost the yield of the resonances above equilibrium and force G above one. The spectral temperature can also be modified by nonstatistical features.

As a practical matter, for resonances near threshold the factor Q/T in G is generally very small, and exp(Q/T) can be set to one. In that case

$$G \approx [N_3/(N_2 N_1)] \times V T^{3/2} \times C, \tag{10}$$

where C is a constant depending only on the masses and spins of the species.

To judge the reliability of the volume obtained by the Koonin procedure, it is sufficient to use the apparent value of volume for V in Eq. (10), along with the measured yields N_j and the spectral temperature to obtain the estimate, G_{est}. In the ideal case G_{est} will give 1.0, and the true and apparent volumes will match. If G_{est} is greater than 1.0, then the true value of V is greater than the apparent one. By using the apparent volume in Eq. (10), we obtain a lower estimate for the true volume, $V > G_{\mathrm{est}} \times V_{\mathrm{apparent}}$. Conversely if G_{est} is less than 1.0, then $V < G_{\mathrm{est}} \times V_{\mathrm{apparent}}$.

A great deal of effort has gone into the measurement of isotope yields, and resonance yields for the purpose of determining nuclear temperatures. It is enlightening to examine what the trends found in those investigations suggest for the determination of source sizes. Let us consider two different two-particle correlation measurements aimed at determining source size: for one, consider correlation measurements involving protons and α-particles near the the unbound resonance associated with the ground state of ^{5}Li; for the second, consider the correlation of d and ^{3}He at the resonance which lies 16.66 MeV above the ground state in ^{5}Li. In each case the application of the Koonin prescription is expected to provide an apparent volume, approximately equal to the true source volume divided by the respective correction factor, G, for each correlation pair. As shown in Eqs. (8) and (9), the value of the effective volume in each case depends on the yields of each resonance and the yields of the respective products in its decay. It also depends on simple mass and spin factors, as well as the spectral temperature, T_{spect}. Let us assume that the latter is the same for all particles involved. Furthermore, let us suppose that the apparent temperature found from taking the ratio of yields for the two ^{5}Li resonances is found to be $T_{^5\mathrm{Li}}$. Also, suppose that the Albergo [14] determination of of temperature from the ratio of yield, (p/d) to (^{3}He/^{4}He) gives $T_{pd\mathrm{He}}$. The latter is determined by the double ratio of measured yields and ground state spin, binding energies, and mass factors [15],

$$T_{pd\mathrm{He}} = B/\ln(aR), \tag{11}$$

where R is the yield ratio, B involves binding energies, $B = \mathrm{Be}(^4\mathrm{He}) - \mathrm{Be}(d) - \mathrm{Be}(^3\mathrm{He}) = 18.37$, and a is found from masses and spins to be 5.51. From Eqs. (8) and (9), we find that the ratio of the apparent volumes, $V_{p\text{-}\alpha}/V_{d\text{-}^3\mathrm{He}}$ can be expressed in terms of the three different ‘‘temperatures’’

$$(V_{p\text{-}\alpha}/V_{d\text{-}^3\mathrm{He}}) = \exp(-1.71/T_{\mathrm{spect}})\exp(18.37/T_{pd\mathrm{He}}) \times \exp(-16.66/T_{^5\mathrm{Li}}). \tag{12}$$

From this it is clear that if all the ‘‘temperatures’’ are the same, one would expect the ratio of the apparent volumes to be 1.0. This is expected for simple equilibrium. Experience suggests that measurement of these three temperatures can differ from one another by large amounts. As can be seen, the influence of the spectral temperature is small if it is found to be much greater than 1.7 MeV. In all but the lowest energy experiments the spectral temperature is about ten times this value. The other general observation is that the Albergo temperatures for yields involving He isotopes ratios is generally larger than $T_{^5\mathrm{Li}}$. The latter is generally found to be about 5 MeV, while the former may be a factor of two larger [16]. If the general trends are followed we expect that apparent volume suggested by p-α correlations should be smaller than that suggested by d-^{3}He correlations. The ratio may decline to values of approximately 0.25, when $T_{pd\mathrm{He}}$ equals 10 MeV. This translates into determinations of apparent source radii that may differ by about 60%.

Finally the individual estimates of G_{est} as described above for the two types of particle pairs, would yield different lower (upper) limits on the true volume. The consistency of these two limits could be checked.

In summary, for full chemical and thermal equilibrium true estimates of the source size and true temperature determinations are both possible from two-body correlations. Existing temperature determinations suggest, however, that observed yields may differ from these conditions. We have suggested ways of setting limits on true source size through the measurement of both spectral shapes and yields. Using trends seen in previous temperature measurements, we have predicted the relative size of apparent source volumes obtained from different pairs of particles. Finally, we suggest that it is useful to combine the determination of source size, through correlations measurements, with careful measurements of the relative yields of the resonances as well as the stable particles.

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