Nucl. Phys. 67, 542 (1965).
[61]J. S. Forster, K. Bearpark, J. L. Hutton, and J. F. Sharpey-Schafer, Nucl. Phys. A150, 30 (1970).
[62]H. A. Enge, E. J. Irvin, and D. H. Weaner, Phys. Rev. 115, 949 (1959).
[63]M. E. Rickey, E. Kashy, and D. Knudsen, Bull. Am. Phys. Soc. 10, 550 (1965).
[64]E. Kashy and J. L. Snelgrove, Phys. Rev. 172, 1124 (1968).
[65]K. Yagi et al., Phys. Letters 10, 186 (1964).

PHYSICAL REVIEW C VOLUME 6, NUMBER 3 SEPTEMBER 1972

# Proton Inelastic Scattering from $^{48}$Ca[†]

C. R. Gruhn,* T. Y. T. Kuo,‡ C. J. Maggiore,§ and B. M. Preedom¶
*Department of Physics and Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48823*
(Received 29 December 1971)

Inelastic proton scattering from $^{48}$Ca has been measured at beam energies 25, 30, 35, and 40 MeV. Angular distributions from 13 to 97° for 22 inelastic states were obtained. Analyses with the collective distorted-wave Born approximation are presented. A direct comparison of the excitation of the $^{48}$Ca 3.830-MeV $2^+$ and 6.342-MeV $4^+$ states is made with the low-lying excited $2^+$ and $4^+$ states of $^{50}$Ti and $^{52}$Co.

## I. INTRODUCTION

Doubly magic nuclei, in general, have been studied in great detail both experimentally and theoretically. Perhaps the exception to this statement is $^{48}$Ca. From the experimental standpoint only a few of the low-lying states of $^{48}$Ca have well established spin and parity. From the theoretical point of view $^{48}$Ca is of interest because of the purity of its double-closed-shell structure. Jaffrin and Ripka[1] have tested the occupation numbers and find that the $1f_{7/2}$ shell and the inner neutron shells are at least 97% closed. It is because of the strong theoretical motivation and of our interest in developing the $(p,p')$ reaction as a probe in microscopic structure that we undertook the present $(p,p')$ experiment on $^{48}$Ca.

The level structure of $^{48}$Ca has also been investigated in other experiments such as $(\alpha, \alpha')$,[2,3] $(e, e')$,[4] $(t,p)$,[5] $(p,p')$,[6] and $(p,p'\gamma)$.[7] The $(\alpha, \alpha')$ and $(e, e')$ experiments probably should be repeated with the now available better resolutions. In principle, then, at least some of the ambiguities in the present assignments of the low-lying levels could be removed.

## II. DESCRIPTION OF EXPERIMENT

The experiment was carried out using the proton beam from the Michigan State University sector-focused cyclotron. Figure 1 shows the cyclotron and beam-handling system. The two horizontal bending magnets M3 and M4 are used to momentum analyze the beam and M5 deflects the beam into the goniometer.[8] More complete descriptions of the properties of the energy analysis system have been published elsewhere.[9,10] During this experiment the slits S1 and S3 were set at 15 mils for beam energy resolution of ±5 keV. S2 was set at 100 mils to yield a beam divergence of ±2 mrad. The Faraday cup is located in a shielded beam dump 12 ft beyond the goniometer.

The scattered protons were detected with two surface-barrier Ge(Li) detectors designed specifically for this experiment.[11] The two detectors were separated by 14.7° and were located outside the 16-in. scattering chamber. The detectors coupled to the scattering chamber vacuum via a sliding seal. A monitor counter at a fixed angle viewed the scattered beam through a 1-mil Kapton window.

The target was a commercially prepared self-supporting foil of $^{48}$Ca approximately 1.08 mg/cm$^2$ thick. The composition of the target as determined by the Isotopes Division of Oak Ridge National Laboratory is listed in Table I. The target was stored in vacuum when not in use and transferred to the scattering chamber in vacuum via a target-transfer system.[8]

Inelastic proton spectra were taken every 5° from 13 to 97°. The over-all energy resolution was 25–30 keV full width at half maximum. Each counter subtended an angle of about 0.5° in the scattering plane. The scattering angle was checked by comparing the positions of the H and $^{12}$C contaminant peaks relative to the $^{48}$Ca ground state and found to be accurate to within 0.1°. The energy of the incident protons determined by measuring the fields of the energy analyzing magnets with NMR probes was accurate to within 0.1%.[9,10]

The relative normalization for each run was obtained dividing by the total counts in the elastic peak observed by the monitor counter. The ratio of monitor counts to the integrated current for each run was found to be constant to within 5%. It is believed that the monitor counter yields the better relative normalization since the Faraday cup was subject to some charge leakage.

Since data were taken simultaneously at two angles during the experiment, it was necessary to determine the relative normalization between counters. This was determined by taking data over the same angular range with each counter and then normalizing one angular distribution to the other. This was done at each energy and found to be constant within 2%.

At scattering angles greater than 65° this experiment was able to resolve completely the elastic scattering of $^{40}$Ca from $^{48}$Ca. Accurate elastic scattering data for $^{40}$Ca already exist,[12,13] therefore it was decided to normalize the present $^{48}$Ca data using the ratio of isotopic abundances in the target. The absolute cross sections determined in this manner are believed to be accurate to 10%. The principal contributions to the uncertainty are the statistics of the $^{40}$Ca elastic peak and the absolute normalization of the $^{40}$Ca elastic scattering.

At the angles where a contaminant peak overlapped an inelastic state in $^{48}$Ca an estimate was made for the number of counts associated with the contaminant. In particular, peaks due to $^{12}$C, $^{16}$O, and $^{40}$Ca were subtracted from the forward-angle elastic scattering. To make these corrections, the cross sections of the elastic scattering from $^{12}$C and $^{16}$O were used, where kinematically separated, to determine the relative amounts of carbon and oxygen on the target at each energy. Using the known $^{12}$C and $^{16}$O cross sections,[14] the background subtraction could be made at smaller angles. The total amount of $^{12}$C and $^{16}$O on the target was demonstrated to be constant throughout the experiment. The same technique was used to

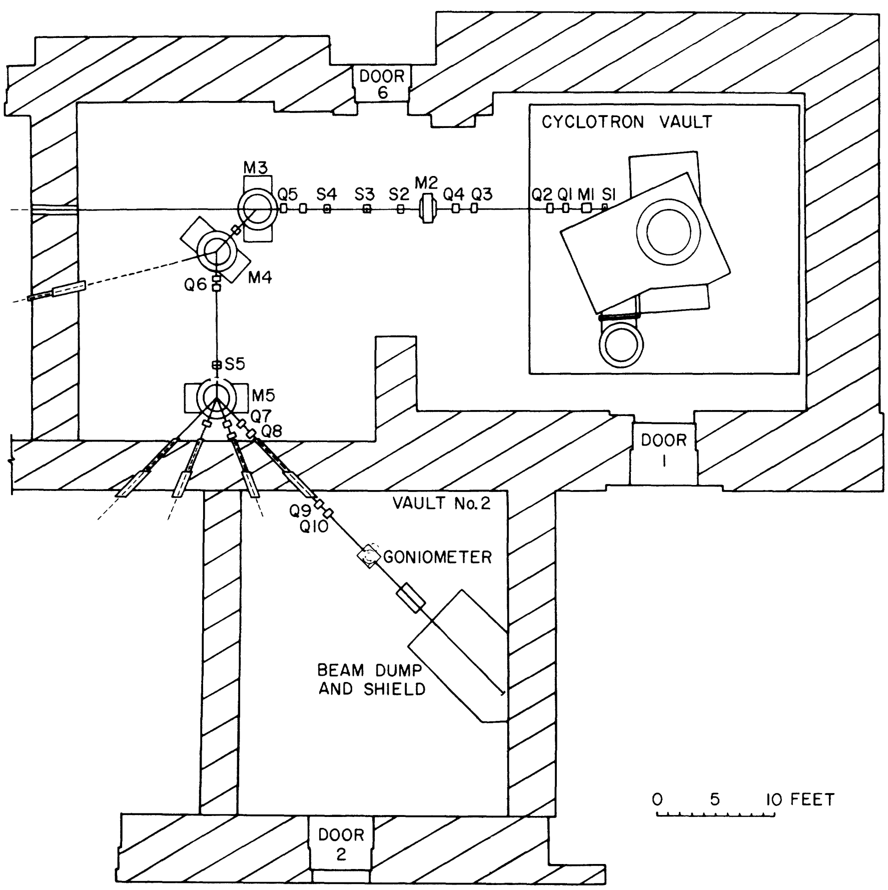


FIG. 1. The Michigan State University cyclotron and beam-handling system.

TABLE I. Isotopic analysis of the $^{48}$Ca target determined by the Isotopes Division of the Oak Ridge National Laboratory where the target was made.

| Ca Isotope | (at.%) | Precision |
|---|---|---|
| 40 | 3.58 | ±0.05 |
| 42 | 0.05 | ±0.01 |
| 43 | 0.01 | |
| 44 | 0.11 | ±0.02 |
| 46 | 0.01 | |
| 48 | 96.25 | ±0.05 |

subtract the excited states of $^{40}$Ca from the $^{48}$Ca excited states when they were not kinematically separated.

The excitation energies were measured at each angle and each beam energy. The data of Marinov and Erskine[6] were used as the calibration standard. Corrections were made for target thickness and relativistic kinematics were used throughout. The reference peaks used for calibration were the ground and 3833-, 4506-, 5368-, and 6338-keV states of $^{48}$Ca. A typical spectrum is shown in Fig. 2.

A linear calibration curve was found to give the best fit to the calibration points. In Table II we list the average excitation energy as measured for each beam energy along with the rms deviation. In the last column of Table II we show the average excitation energy derived from all measurements in this experiment.

In Table III we compare our average excitation energies with those determined in other experiments. In all cases we agree within the experimental errors reported.

## III. ANALYSIS

The theory and use of the distorted-wave born approximation (DWBA) to analyze inelastic proton scattering have been presented extensively elsewhere.[15-17] In particular, we shall use the procedures and terminology set forth in Preedom *et al.*[18] The DWBA calculations were made using a Fortran-IV version of the Oak Ridge computer code JULIE[19] implemented to run on the Michigan State University Cyclotron Laboratory XDS sigma-7 computer.

### Elastic Scattering

In order to obtain parameters for the distorted waves used in the DWBA calculations, the angular distributions of elastic scattering were analyzed for each energy using a standard optical potential

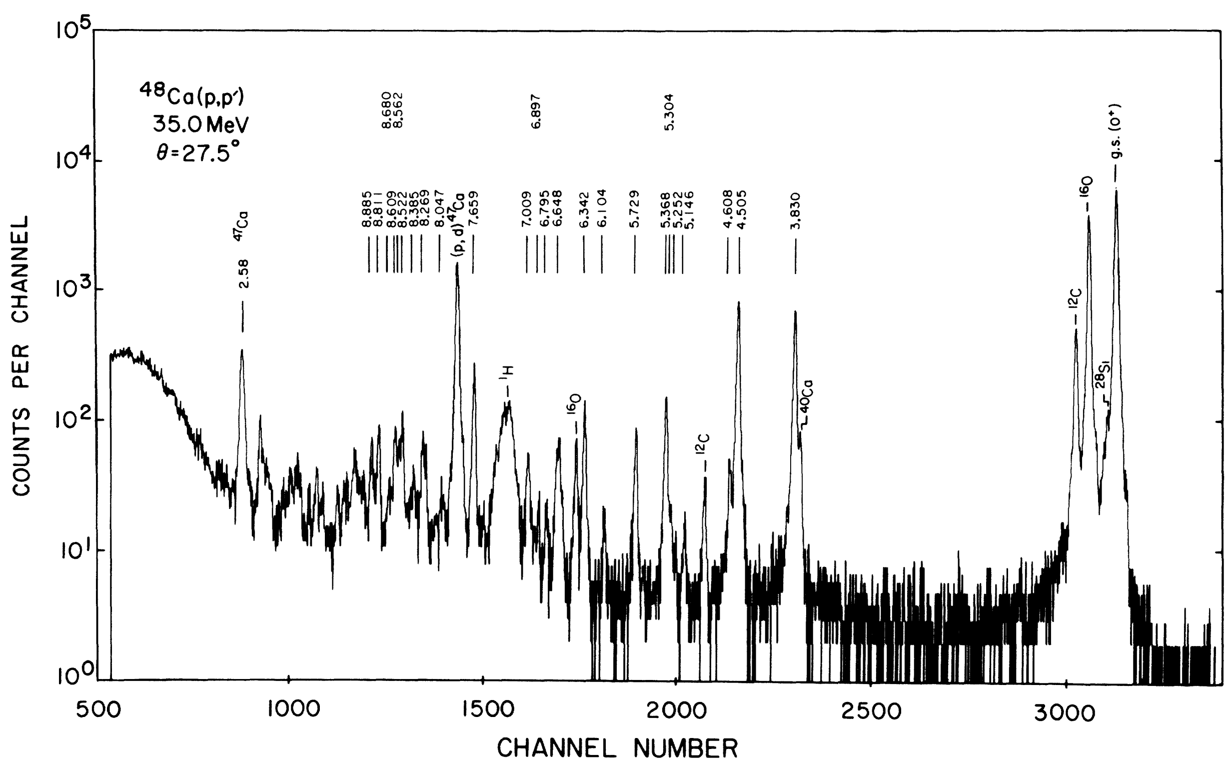


FIG. 2. A typical $^{48}$Ca($p$, $p'$) spectrum taken at 35 MeV.

(cf. Ref. 12). The geometrical parameters ($r_0$ and $a$) for the various terms in the optical potential along with the potential well depths are presented in Table IV. Note that the real and imaginary geometries were set equal. Fits to the data are shown in Fig. 3.

The elastic scattering from $^{48}$Ca was compared with the elastic scattering from $^{40}$Ca using the optical model. It was found that the rms radius of $^{48}$Ca is 0.15 fm larger than $^{40}$Ca, in agreement with the $A^{1/3}$ trend.[18] The fits obtained using the average geometry parameters of Fricke[12] with the real and imaginary geometries being different were comparable and did not affect the DWBA analysis.

### Collective DWBA

The form factors used for the collective-model calculation involved deformations of both the real and imaginary parts of the optical potential. Coulomb excitation was included for $L$ transfers of 2 and 3. Spin-flip contributions were not included. The entrance channel was described by the optical-model parameters listed in Table IV. The optical parameters for the exit channel were adjusted to account for the $Q$ value.

The deformation parameters, $\beta_L$, were obtained by calculating the ratio of the cross sections $\sigma$(exp) and $\sigma_L$(JULIE), each integrated over the angular range of this experiment. The deformation, $\delta_L$, is defined as $\beta_L R_0$ where $R_0$ is the real radius of the target nucleus. $R_0 = 1.20A^{1/3}$ F was used for all the calculations.

Figures 4 and 5 compare the collective-model calculations with states of known $L$ transfer[2] for $L = 2, 3, 4$, and 5 at 40 and 35 MeV. The structure of the angular distribution is more pronounced for the higher-energy data, and therefore for making $L$ assignments the 35- and 40-MeV data are more useful than the 25- and 30-MeV data[20] not shown here. The lower-energy data tend to be less structured. The shapes of the angular

TABLE II. Excitation energy measurements: $^{48}$Ca.

| $E_p$ = 25 MeV $E^*$ ± (keV) | $\Delta E$ | $E_p$ = 30 MeV $E^*$ ± (keV) | $\Delta E$ | $E_p$ = 35 MeV $E^*$ ± (keV) | $\Delta E$ | $E_p$ = 40 MeV $E^*$ ± (keV) | $\Delta E$ | All energies $E^*$ ± (keV) | $\Delta E$ |
|---|---|---|---|---|---|---|---|---|---|
| 3830 | 2 | 3830 | 1 | 3829 | 1 | 3830 | 2 | 3830 | 2 |
| 4505 | 1 | 4504 | 1 | 4504 | 1 | 4505 | 1 | 4505 | 1 |
| 4608 | 4 | 4609 | 5 | 4608 | 4 | 4605 | 4 | 4608 | 4 |
| 5145 | 2 | 5147 | 5 | 5147 | 2 | 5143 | 2 | 5146 | 5 |
| 5250 | 4 | 5255 | 2 | 5252 | 3 | 5239 | 5 | 5252 | 5 |
| 5301 | 3 | 5306 | 5 | 5304 | 2 | 5301 | 10 | 5304 | 6 |
| 5369 | 2 | 5368 | 2 | 5369 | 2 | 5368 | 4 | 5368 | 3 |
| 5729 | 3 | 5729 | 3 | 5730 | 2 | 5729 | 3 | 5729 | 3 |
| 6103 | 1 | 6105 | 3 | 6104 | 1 | 6103 | 2 | 6104 | 3 |
| 6342 | 2 | 6342 | 2 | 6341 | 3 | 6343 | 1 | 6342 | 2 |
| 6645 | 5 | 6646 | 3 | 6649 | 4 | 6652 | 5 | 6648 | 5 |
| 6798 | 6 | 6793 | 3 | 6796 | 4 | 6793 | 8 | 6795 | 6 |
| 6898 | 4 | 6899 | 9 | 6887 | 3 | 6899 | 8 | 6897 | 8 |
| 7021 | 5 | 7023 | 8 | 7012 | 5 | 7018 | 4 | 7019 | 7 |
| 7298 | 4 | 7301 | 4 | | | 7291 | 3 | 7298 | 5 |
| 7404 | 4 | 7402 | 5 | 7402 | 1 | 7401 | 4 | 7401 | 4 |
| 7468 | 5 | 7467 | 6 | 7469 | 3 | 7468 | 4 | 7468 | 5 |
| 7538 | 5 | 7535 | 10 | 7529 | 10 | 7541 | 5 | 7536 | 8 |
| 7660 | 5 | 7660 | 3 | 7658 | 1 | 7660 | 3 | 7659 | 3 |
| 7797 | 4 | 7805 | 4 | 7801 | 4 | 7802 | 4 | 7801 | 4 |
| 8054 | 2 | 8051 | 4 | 8042 | 6 | | | 8047 | 8 |
| 8270 | 6 | 8270 | 5 | 8269 | 5 | 8268 | 6 | 8269 | 6 |
| 8390 | 11 | 8387 | 9 | 8384 | 7 | 8382 | 7 | 8385 | 10 |
| 8523 | 5 | 8520 | 7 | 8523 | 2 | 8522 | 4 | 8522 | 5 |
| 8563 | 7 | 8560 | 8 | 8562 | 3 | 8561 | 6 | 8562 | 7 |
| 8611 | 8 | 8608 | 7 | 8609 | 3 | 8606 | 6 | 8608 | 6 |
| 8684 | 6 | 8678 | 8 | 8677 | 6 | 8678 | 2 | 8680 | 7 |
| 8807 | 6 | 8806 | 6 | 8806 | 2 | 8805 | 3 | 8806 | 5 |
| 8885 | 7 | 8884 | 6 | 8885 | 4 | 8884 | 6 | 8885 | 6 |

distributions are sufficiently different to enable one to make apparent $L$-transfer assignments based on shape alone. However, an ambiguity does exist for the case of unnatural-parity states. It is known empirically that $4^-$ and $2^-$ states in $^{40}$Ca look like $L=5$ and $L=3$ transfer states, respectively.[13] In addition the microscopic theory also predicts that $L=3$ and $L=1$ transfer giving rise to $4^-$ and $2^-$ states have angular distributions which are similar to $L=5$ and $L=3$ transfer.[13]

Figures 6–8 show the angular distributions obtained at 40 and 35 MeV grouped according to apparent $L$ transfer. The solid curve is the shape of the angular distribution observed in the $^{40}$Ca$(p,p')$ experiment for states of known $L=3$, 4, and 5.[13] The shape of the $(p,p')$ angular distribution is relatively independent of the target nucleus.

TABLE III. Comparison of excitation energy measurements. Error ±5 keV relative, ±10 keV absolute.

| Present experiment $E^*\pm$ (keV) | $\Delta E$ | $(p,p')$ [a] $E^*$ (keV) | $(p,p')$ [b] $E^*\pm$ (keV) | $\Delta E$ | $(p,p')$ [c] $E^*\pm$ (keV) | $\Delta E$ | $(t,p)$ [d] $E^*\pm$ (keV) | $\Delta E$ |
|---|---|---|---|---|---|---|---|---|
| 3830 | 2 | 3835 | 3833 | 4 | 3818 | 10 | 3827 | 10 |
| | | 4286 | 4284 | 6 | 4272 | 10 | 4281 | 10 |
| 4505 | 1 | 4512 | 4506 | 4 | 4498 | 10 | 4496 | 10 |
| 4608 | 4 | 4619 | 4613 | 4 | 4604 | 10 | | |
| 5146 | 5 | 5152 | 5146 | 5 | 5130 | 20 | | |
| 5252 | 5 | 5265 | | | 5266 | 10 | | |
| 5304 | 6 | | | | | | | |
| 5368 | 3 | 5376 | 5368 | 5 | 5370 | 20 | | |
| | | 5465 | | | | | 5459 | 10 |
| 5729 | 3 | 5737 | 5728 | 8 | 5724 | 10 | | |
| 6104 | 3 | 6108 | 6106 | 6 | 6096 | 10 | | |
| 6342 | 2 | 6351 | 6338 | 10 | 6340 | 24 | 6329 | 15 |
| 6648 | 5 | 6654 | | | 6610 | 20 | 6645 | 15 |
| 6795 | 6 | | | | 6790 | 20 | 6793 | 15 |
| 6897 | 8 | 6897 | | | | | | |
| 7019 | 7 | 7028 | | | | | | |
| 7298 | 5 | 7305 | | | | | | |
| 7401 | 4 | 7402 | | | | | | |
| 7468 | 5 | | | | | | | |
| 7536 | 8 | | | | | | | |
| 7659 | 3 | 7652 | | | | | 7650 | 20 |
| 7801 | 4 | | | | 7970 | 20 | | |
| 8047 | 8 | 8041 | | | | | 8018 | 20 |
| | | | | | | | 8237 | 20 |
| 8269 | 6 | 8276 | | | | | 8268 | 20 |
| 8385 | 10 | 8384 | | | | | 8473 | 20 |
| 8522 | 5 | 8527 | | | | | 8513 | 20 |
| 8562 | 7 | | | | | | 8538 | 20 |
| 8608 | 6 | 8603 | | | | | 8604 | 20 |
| 8680 | 7 | 8672 | | | | | 8697 | 20 |
| 8806 | 5 | 8811 | | | | | 8782 | 20 |
| 8885 | 6 | 8888 | | | | | | |

[a] Reference 7.
[b] Reference 6.
[c] Reference 23.
[d] Reference 5.

TABLE IV. Optical parameters: $r_R=r_I=1.20$ F, $r_c=1.25$ F, $a_R=a_I=0.68$ F.

| $E_p$ (MeV) | $V_0$ (MeV) | $W_0$ (MeV) | $W_0$ (MeV) | $\chi^2/N$ |
|---|---|---|---|---|
| 25 | 51.72 | 0.36 | 6.95 | 13 |
| 30 | 45.93 | 0.15 | 6.57 | 6.4 |
| 35 | 46.50 | 3.49 | 4.62 | 3.6 |
| 40 | 46.58 | 4.13 | 4.57 | 2.0 |

Table V lists the states with their apparent $L$ transfer. The $\delta_L$'s and the reduced transition probabilities were determined for the 40-MeV data. $G_{sp}$ is the reduced transition probability in Weisskopf single-particle units. The state at 6.642 MeV is a multiplet which could not be resolved in the present experiment, but the angular distribution could be fitted at the four energies by assuming a combination of 50% $L=2$ and 50% $L=4$. The calculations for the reduced transition probabilities are based on that decomposition of the multiplet.

The deformations observed in the present experiment are compared with results of previous experiments in Table VI. The deformations observed in the 12-MeV $(p,p'\gamma)$ experiment are always larger than observed in this experiment except for the 5.729-MeV state for which the results are comparable.

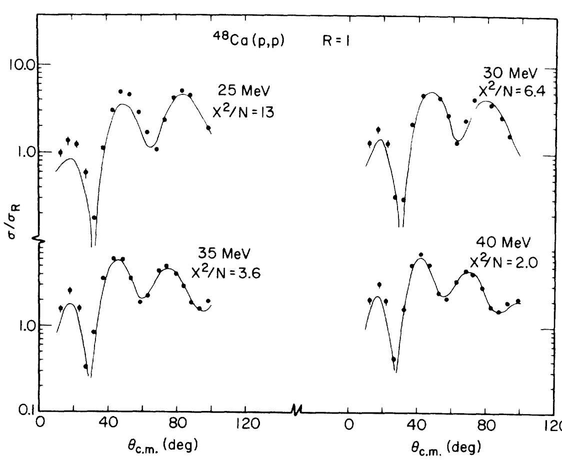


FIG. 3. Proton elastic scattering from $^{48}$Ca as a function of beam energy.

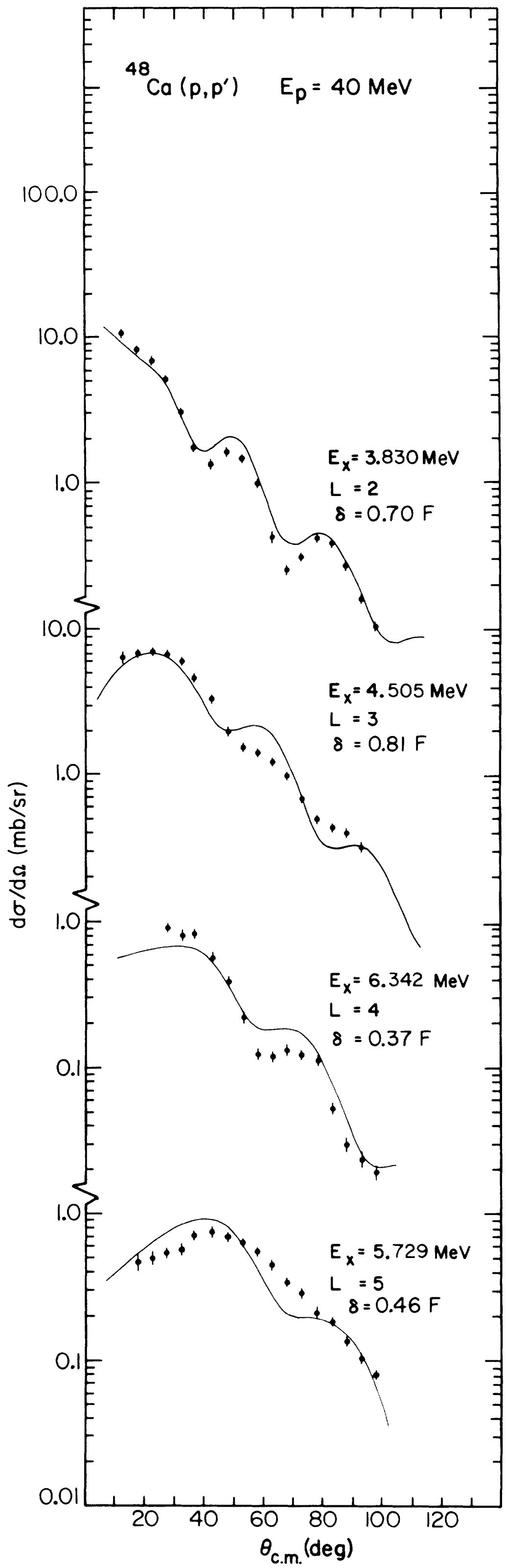


FIG. 4. Typical $L = 2$, 3, 4, and 5 transfers in the reaction $^{48}$Ca$(p, p')$ at 40 MeV.

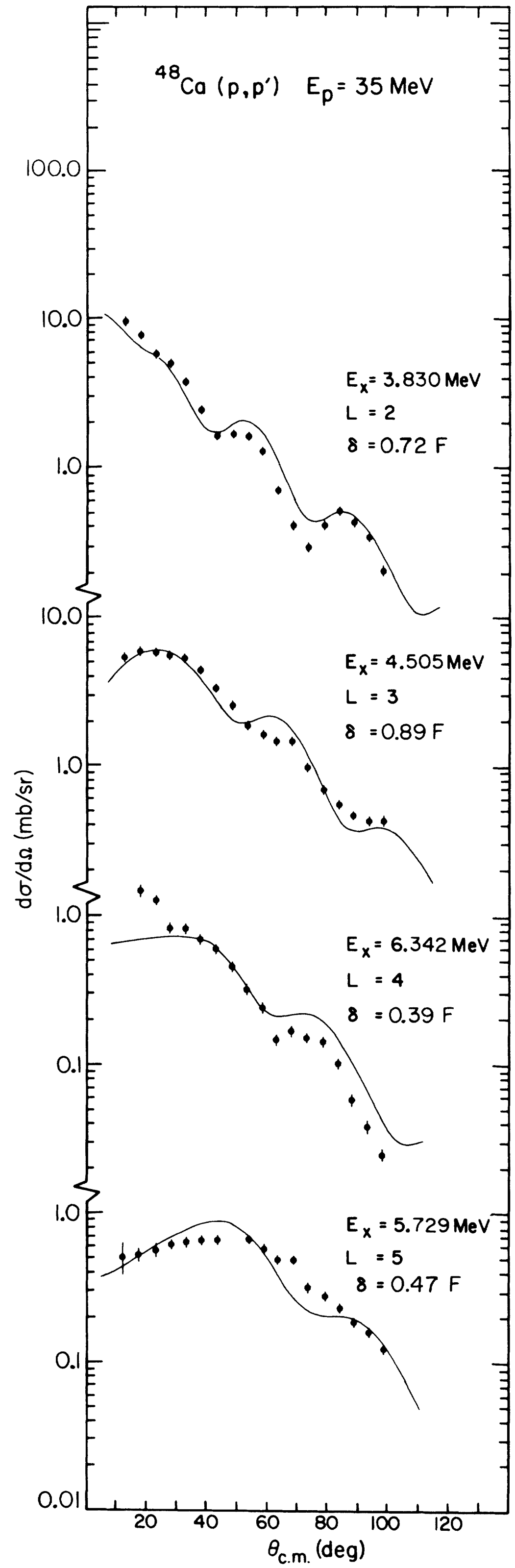


FIG. 5. Typical $L = 2$, 3, 4, and 5 transfers in the reaction $^{48}$Ca$(p, p')$ at 35 MeV.

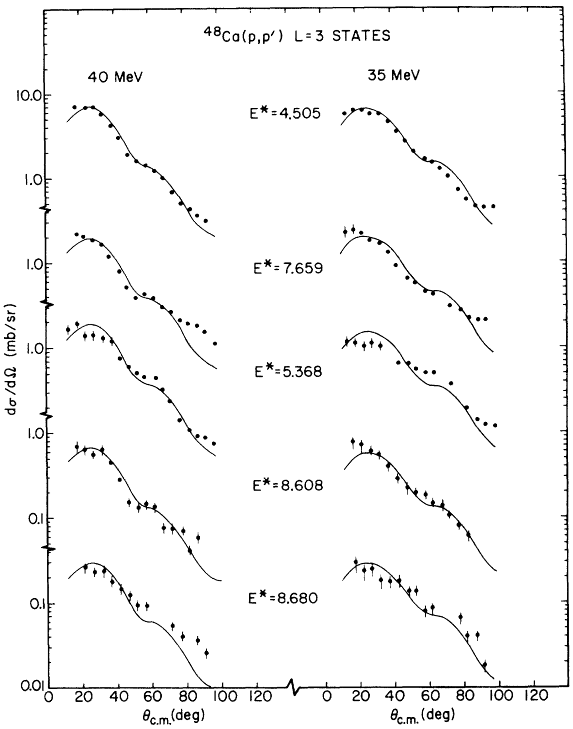


FIG. 6. A comparison of $L=3$ transfers in the reaction $^{48}$Ca($p$,$p'$) at 35 and 40 MeV.

TABLE V. Reduced transition probabilities in single-particle Weisskopf units.

| $E^*$ (keV) | $L$ | $S_L$ | $G_{sp}$ |
|---|---|---|---|
| 3830 | 2 | 0.70 | 4.7 |
| 4505 | 3 | 0.81 | 7.9 |
| 4604 | 4 | 0.22 | 0.9 |
| 5146 | 5 | 0.22 | 1.4 |
| 5252 | 5 | 0.11 | 0.4 |
| 5368 | 3 | 0.46 | 2.6 |
| 5729 | 5 | 0.46 | 6.2 |
| 6104 | 5 | 0.15 | 0.6 |
| 6342 | 4 | 0.37 | 2.5 |
| 6648 | 4 | 0.25 | 1.1 |
| 6897 | 5 | 0.15 | 0.6 |
| 7401 | 3 | 0.15 | 0.3 |
| 7659 | 3 | 0.49 | 2.9 |
| 7801 | 4 | 0.22 | 0.9 |
| 8269 | 4 | 0.22 | 0.9 |
| 8385 | 5 | 0.25 | 1.8 |
| 8522 | 3 | 0.26 | 0.8 |
| 8562 | 5 | 0.27 | 2.1 |
| 8608 | 3 | 0.27 | 0.9 |
| 8680 | 3 | 0.07 | 0.1 |
| 8806 | 5 | 0.41 | 4.9 |
| 8885 | 5 | 0.30 | 2.6 |

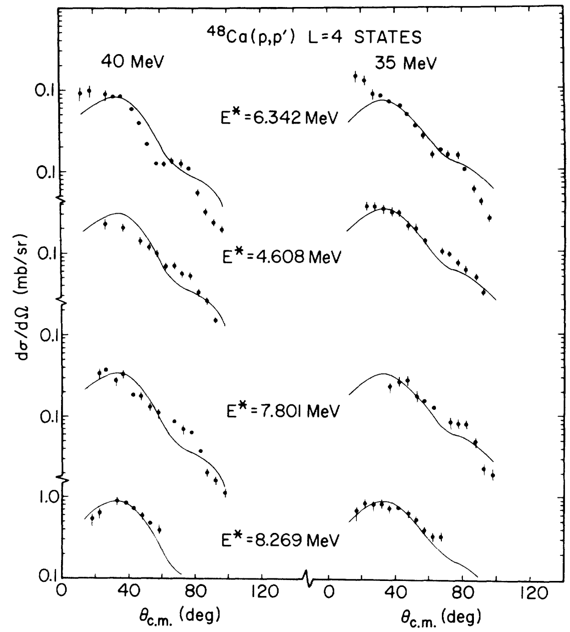


FIG. 7. A comparison of $L=4$ transfers in the reaction $^{48}$Ca($p$,$p'$) at 35 and 40 MeV.

The reduced transition probabilities in single-particle units are compared with the previous experimental results in Table VII. In comparison with the electron scattering results, the present ($p$,$p'$) and ($\alpha$, $\alpha'$) experiments generally yield higher values of the reduced transition probability. This indicates some differences with respect to the ($e$, $e'$) reaction.

The values of $J^\pi$ deduced for levels of $^{48}$Ca from the present data are listed in Table VIII and compared with previous spin and parity assignments. The present results are in agreement with previous experiments except for the state at 5.146 MeV which appears to have $L=5$ transfer, but this state may be an unnatural-parity state which may have a different apparent $L$ transfer in inelastic proton scattering than that observed in inelastic $\alpha$ scattering.

### Microscopic DWBA

A microscopic model using realistic nucleon-nucleon forces has been used to calculate the angular distributions of the low-lying $2^+$ and $4^+$ states in $^{50}$Ti and $^{52}$Cr.[18] These calculations indicate that 90% of the strength in the excitation of these states was contained in the $^{48}$Ca core. For this reason it was thought to be appropriate to compare the angular distributions for these states with those of $^{48}$Ca itself. Figures 9 and 10 show that the experimental angular distributions are virtually the same, both in shape and magnitude, giving experimental evidence that the valence nucleons appear to contribute little to the strength of excitation. The results of the microscopic calculations for $^{50}$Ti and $^{52}$Cr are also shown in the figures. The calculation assumes excitations within the $(1f_{7/2})^2$ configuration and is seen to provide a reasonable fit to the $^{48}$Ca data.

The calculations were carried out using the macroscopic vibrational description of the core and fixing the core parameters from the bound-state matrix elements of Kuo and Brown.[18,21] Since the microscopic model with realistic forces contains no free parameters, the comparison of the calculations based on this model with experiment provides a direct test of the theory.

It is also interesting to note that in $^{52}$Cr the full strength of the $2^+$ and $4^+$ states are required to give agreement with both shape and strength of the $^{48}$Ca core data.

The comparison with the shape of a known $L=4$ state in $^{40}$Ca shown in Fig. 7 indicates that overlap is not as good as in the case of the comparison with $^{50}$Ti and $^{52}$Cr. But this may be due to the different configurations giving rise to the $4^+$ states in $^{40}$Ca and $^{48}$Ca.

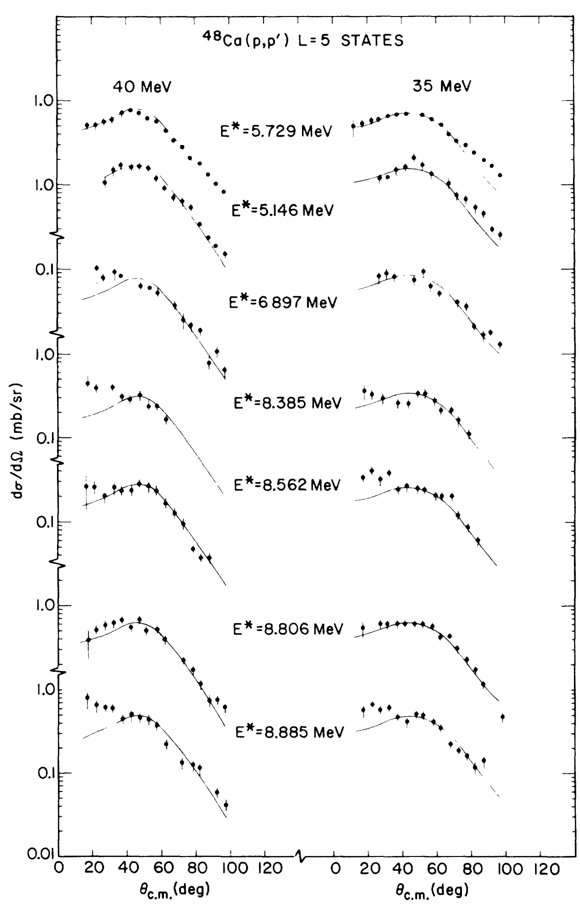


FIG. 8. A comparison of $L=5$ transfers in the reaction $^{48}$Ca$(p,p')$ at 35 and 40 MeV.

## IV. EXCITED STATES OF $^{48}$Ca

### Theory

#### *1. Odd-Parity States*

Jaffrin and Ripka have made a calculation for the odd-parity levels in $^{48}$Ca.[1] The calculation is based on particle-hole excitations with an interaction adjusted to fit the first $3^-$ state. Correlations in the ground state were accounted for by use of the random-phase approximation. The configuration space was restricted to the $2s$-$1d$, $2p$-$1f$ major shells and to the $1g_{9/2}$ subshell. Figure 11 shows the comparison between theory and the present experiment.

The theory predicts two groupings of odd-parity states. The lower group consists of two $3^-$ states, one $5^-$ state, two $4^-$ unnatural-parity states, and a $2^-$ unnatural-parity state. In the same region of excitation energy (4 to 6.5 MeV) we observe two $3^-$ states and four states with $L=5$ angular momentum transfer. The $L=5$ transfer implies that the states are negative parity. Perhaps the weak low-lying $L=5$ states are associated with the predicted $4^-$ unnatural-parity states.

The same process may be occurring in the higher grouping of odd-parity states between 8 and 10 MeV. The theory predicts two $5^-$, two $3^-$, four $2^-$, and two $4^-$ states. We observe three $L=3$ and four $L=5$ transfers in this region. There is close agreement between theory and experiment with the onset of these odd-parity states at higher excitation energy.

#### *2. Even-Parity States*

The low-lying even-parity states of the calcium isotopes ($^{42}$Ca through $^{50}$Ca) have been calculated by McGrory, Wildenthal, and Halbert within the framework of the conventional shell model.[22] Their calculations, using a modified Kuo-Brown interaction, predict three $J=2$ states, four $J=4$ states, and two $J=0$ states below 7 MeV excitation. The results of their calculation are shown in Fig. 12 compared with the present experimental results. The theory agrees with the present experimental results.

### Experiment

#### *1. Odd-Parity States*

*4.505 MeV.* The first $3^-$ state in $^{48}$Ca is well resolved and strongly excited in the present experiment. It is observed to have an $L=3$ transfer. In

TABLE VI. Comparison of nuclear deformations, $\delta_L$. The number in brackets indicates the $L$ value used when it differs from that found in the present experiment.

| | | $(p,p')$ | | $(\alpha, \alpha')$ |
|---|---|---|---|---|
| $E^*$ (MeV) | $L$ | Present experiment 40 MeV | Ref. 7 12 MeV | Ref. 2 30.5 MeV |
| 3.830 | 2 | 0.70 | 1.0 | 0.71 |
| 4.505 | 3 | 0.81 | 1.15 | 0.76 |
| 4.608 | 4 | 0.24 | 0.44[2] | |
| 5.146 | 5 | 0.22 | 0.87 | |
| 5.368 | 3 | 0.46 | 0.66[4] | 0.36 |
| 5.729 | 5 | 0.46 | 0.44[3] | 0.32[4] |
| 6.342 | 4 | 0.37 | | 0.41 |
| 6.648 | 2 +4 | 0.35 | | 0.36[4] |
| 7.659 | 3 | 0.49 | | 0.54 |

TABLE VII. Comparison of experimental reduced transition probabilities. $G_{sp}$ (single-particle Weisskopf units).

| $E^*$ (keV) | $J^\pi$ | Present $(p,p')$ 40 MeV | Ref. 7 $(p,p')$ 12 MeV | Ref. 2 $(\alpha, \alpha')$ 31 MeV | Ref. 4 $(e, e')$ 20–60 MeV |
|---|---|---|---|---|---|
| 3830 | $2^+$ | $4.7 \pm 0.6$ | 7.70 | $5.4 \pm 0.8$ | $1.7 \pm 0.2$ |
| 4505 | $3^-$ | $7.9 \pm 1.3$ | 10.25 | $8.0 \pm 1.2$ | $6.8 \pm 1.0$ |
| 4608 | $(4^+)$ | $0.9 \pm 0.2$ | 1.60 | | |
| 5146 | $(4, 5)^-$ | $1.4 \pm 0.3$ | 6.66 | | |
| 5368 | $3^-$ | $2.6 \pm 0.4$ | 3.90 | $1.4 \pm 0.2$ | ~0.2 |
| 5729 | $5^-$ | $6.2 \pm 0.9$ | 1.65 | $3.9 \pm 0.6$ | |
| 6342 | $4^+$ | $2.5 \pm 0.4$ | | $3.3 \pm 0.8$ | |
| 6648 | $4^+$ | $1.1 \pm 0.3$ | | $2.7 \pm 0.7$ | |
| 7659 | $3^-$ | $2.9 \pm 0.4$ | | $4.1 \pm 0.6$ | ~1.5 |

TABLE VIII. Comparison of spin and parity assignments.

| Ref. 7 $E^*$(keV) | Excitation energy Present $E^*$(keV) | $\pm\Delta E$(keV) | $(p,p')$ Present 25–40 MeV $L$ | $(p,p'\gamma)$ Ref. 7 12 MeV $J^\pi$ | $(\alpha, \alpha')$ Ref. 2 31 MeV $J^\pi$ | $(\alpha, \alpha')$ Ref. 3 42 MeV $J^\pi$ | $(e, e')$ Ref. 4 60 MeV $J^\pi$ | $(t,p)$ Ref. 5 12 MeV $J^\pi$ | Probable $J^\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| 3835 | 3830 | 2 | 2 | $2^+$ | $2^+$ | $2^+$ | $2^+$ | $2^+$ | $2^+$ |
| 4286 | | | | $0^+$ | | $(4^+, 5^-)$ | | $0^+$ | $0^+$ |
| 4512 | 4505 | 1 | 3 | $3^-$ | $3^-$ | $3^-$ | $3^-$ | ... | $3^-$ |
| 4619 | 4608 | 4 | (4) | $2^+$ | | | | | (4) |
| 5152 | 5146 | 5 | 5 | 5 | $(4^+)$ | $3^-$ | | | $(4, 5)^-$ |
| 5265 | 5252 | 5 | | $(4^-)$ | | | | | $(4^-)$ |
| | 5304 | 6 | | | | | | | |
| 5376 | 5368 | 3 | 3 | $(4^+)$ | $3^-$ | $3^-$ | 3 | | $3^-$ |
| 5465 | ... | ... | ... | $0^+$ | ... | ... | ... | $0^+$ | $0^+$ |
| 5737 | 5729 | 3 | 5 | $(3^-)$ | $5^-$ | $2^+$ | | | $5^-$ |
| 6108 | 6104 | 3 | | | | $(2^+)$ | | | |
| 6351 | 6342 | 2 | 4 | | $4^+$ | $1^-$ | | $2^+$ | $(4^+)$ |
| 6654 | 6648 | 5 | (2), (4) | | $4^+$ | | | | $(2^+), (4^+)$ |
| | 6795 | 6 | | | | | | $2^+$ | $2^+$ |
| 6897 | 6897 | 8 | 5 | | | | | | |
| 7028 | 7019 | 7 | | | | $(3^-)$ | | | |
| 7305 | 7298 | 5 | | | | | | | |
| 7402 | 7401 | 4 | (3) | | | | | | |
| | 7468 | 5 | | | | | | | |
| | 7536 | 8 | | | | | | | |
| 7652 | 7659 | 3 | 3 | | $3^-$ | $3^-$ | $3^-$ | ... | $3^-$ |
| | 7801 | 8 | | | | | | | |
| 8041 | 8047 | 8 | | | | | | $(4^+)$ | $4^+$ |
| 8276 | 8269 | 6 | (4) | | | | | | |
| 8384 | 8385 | 10 | (5) | | | | | | |
| 8527 | 8522 | 5 | (3) | | | | | | |
| | 8562 | 7 | (5) | | | | | | |
| 8603 | 8608 | 6 | (3) | | | | | | |
| 8672 | 8680 | 7 | (3), (4) | | | | | | |
| 8811 | 8806 | 5 | (5) | | | | | | |
| 8888 | 8885 | 6 | (5), ( ) | | | | | | |

the lower energy $(p, p')$[7] and 30.5- and 42-MeV $(\alpha, \alpha')$[2,3] studies this state is also excited with an unambiguous $L = 3$ transfer. These results are further confirmed by the $(e, e')$ $E3$ excitation of this state.[4]

The reduced-transition probabilities observed for these reactions are all more or less in agreement with our present result of $G_{sp} = 7.2$. Jaffrin and Ripka[1] predict a strength for this state which is in agreement with this result ($G_{sp} = 7.8$).

*5.146 MeV.* The weakly excited state at 5.146 MeV is assigned an $L = 5$ transfer in the present experiment. Even though the state is only weakly excited, the data are free of contaminants and the state is well resolved. It is observed to be excited with a strength ($G_{sp} = 1.4$)(40 MeV) in this experiment in contrast with the strength ($G_{sp} = 6.66$) observed at 12 MeV in $(p, p')$. The reaction mechanism is presumably more purely direct at the higher energies. This is expected to be most noticeable for unnatural-parity states.

This state was not observed in the $(\alpha, \alpha')$ experiment[2] or in the $(e, e')$ experiment.[4] However, it was observed by Peterson[3] to be excited in $(\alpha, \alpha')$ with an $L = 3$ transfer. Because of the weakness of the state at the higher energies, and because the calculations of Jaffrin and Ripka predict only one low-lying $5^-$ state (observed at 5.729 MeV), whereas there are two low-lying $4^-$ states calculated, we believe this $(4, 5)^-$ state is probably $4^-$.

*5.252 MeV.* The angular distributions of the weakly excited 5.252-MeV state are consistent with the $(4^-)$ assignment of Tellez.[7]

*5.368 MeV.* The 5.368-MeV state is a strongly excited $L = 3$ state in agreement with all the previous $L$ assignments except one[7] which makes a tentative $L = 4$ assignment.

This state in $(e, e')$[4] is observed to be excited with approximately 10 times less strength (see Table VI) than in $(p, p')$ and $(\alpha, \alpha')$[2] again indicating a difference in the $(e, e')$ reaction. The calculations of Jaffrin and Ripka[1] predict a strength ($G_{sp} = 0.44$) for this state which is more in agreement with the $(e, e')$ results ($G_{sp} = 0.2$).

*5.729 MeV.* The state at 5.729 MeV is excited with an $L = 5$ transfer in agreement with the 31-MeV $\alpha$ experiment.[2] Peterson[3] in $(\alpha, \alpha')$ observed the state as a $2^+$ and Tellez[7] in $(p, p')$ tentatively assign $L = 3$, but the present evidence for $L = 5$ seems strong. The deformation of 0.46 F is again larger than that observed in $\alpha$ reactions (0.32 F).[2] Tellez[7] obtained a deformation of 0.44 F in agreement with the present value. The state is not observed in $(t, p)$[5] or $(e, e')$[4] experiments, which is also consistent with the $J^\pi = 5^-$ assignment. Jaffrin and Ripka[1] calculate one strong low-lying $5^-$ state based primarily on a $(d_{3/2}^{-1} f_{7/2})$ proton particle-hole excitation and this is possibly that state.

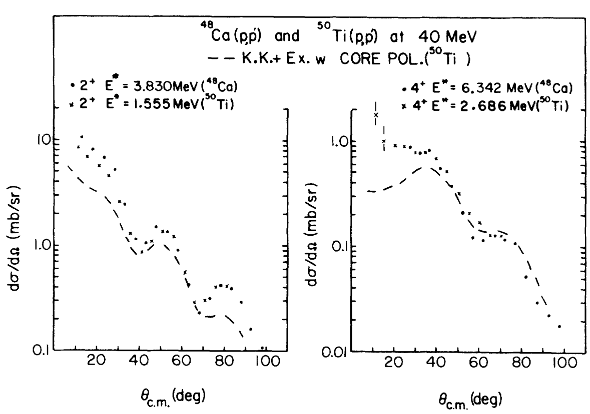


FIG. 9. A comparison of the differential cross sections of the $^{48}$Ca 3.830-MeV $2^+$ and 6.342-MeV $4^+$ states with the $^{50}$Ti 1.555-MeV $2^+$ and 2.686-MeV $4^+$ states. The dashed curves are the results of a microscopic calculation assuming a Kallio-Kolltveit force plus exchange with core polarization (K.K. + Ex. w CORE POL.).

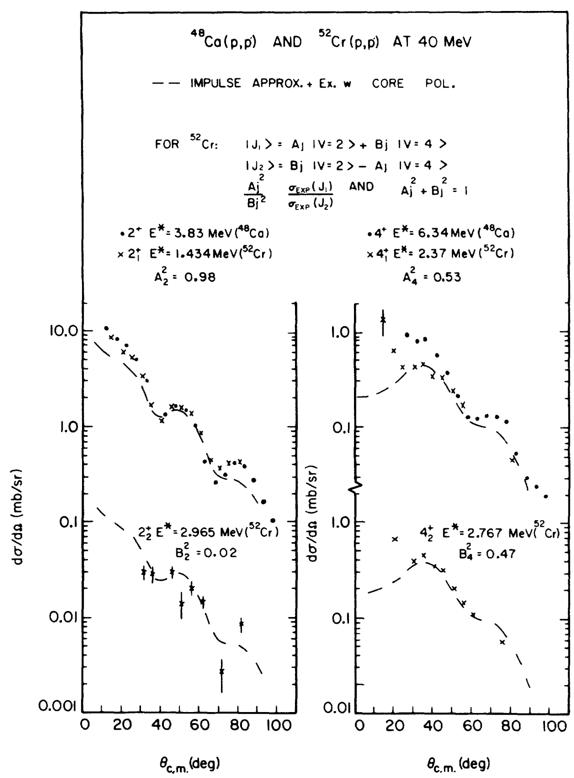


FIG. 10. A comparison of the differential cross sections of the $^{48}$Ca 3.830-MeV $2^+$ and 6.342-MeV $4^+$ states with the $^{52}$Cr 1.434-MeV and 2.965-MeV $2^+$ and 2.370-MeV and 2.767-MeV $4^+$ states.

*6.104 MeV.* The state at 6.104 MeV is believed to correspond to the 6.11-MeV state observed in the $(\alpha, \alpha')$[2,3] experiments. The state is only weakly excited in $(p, p')$ and the statistics on the angular distribution are not good enough to make a positive $L$ assignment, but the data appear to be most consistent with $L=5$.

*7.298, 7.401, 7.468, 7.536 MeV.* Lippincott[2] in $(\alpha, \alpha')$ observed a weak state at 7.53 MeV at a few large angles. Tellez[7] in $(p, p')$ observed a quartet of states in this region. Of the four states observed, only the strongest state of 7.401 had angular distributions at each energy consistent with a single $L$ value. On the basis of the present data the state would be tentatively assigned $L=3$.

*7.659 MeV.* The strong $L=3$ state at 7.659 MeV agrees with the $L$ assignment obtained previously in the $(\alpha, \alpha')$[2,3] experiments. The state is only weakly observed with the $(t, p)$[5] reaction. The deformation of 0.49 F is comparable to that observed in the 31-MeV $\alpha$ experiment[2] of 0.54 F. It is the only state observed to be excited with comparable strength in $(e, e')$.[4] The calculations of Jaffrin and Ripka[1] account for the strength and placement of this state reasonably well.

*8.385 MeV.* The state at 8.385 MeV is only weakly excited and may be a doublet as indicated by a broadened peak shape. The state appears to have a strong $L=5$ component.

*8.522, 8.562, 8.608 MeV.* The next three states are each separated from one another by about 40 keV and are just resolved in the present experiment. The 8.522- and 8.608-MeV states have $L=3$ angular distributions. The middle state at 8.562 MeV has an apparent $L=5$ angular distribution. The $(t, p)$ reaction[5] also excites three states in this region at 8.513, 8.538, and 8.604 MeV which are excited with 52, 68, and 31% of the ground-state strength, respectively. Inelastic electron scattering studies record a broad bump in this energy region.

*8.680 MeV.* The 8.680-MeV state is another state whose peak shape indicates that it is not a single state. A reasonable fit can be obtained

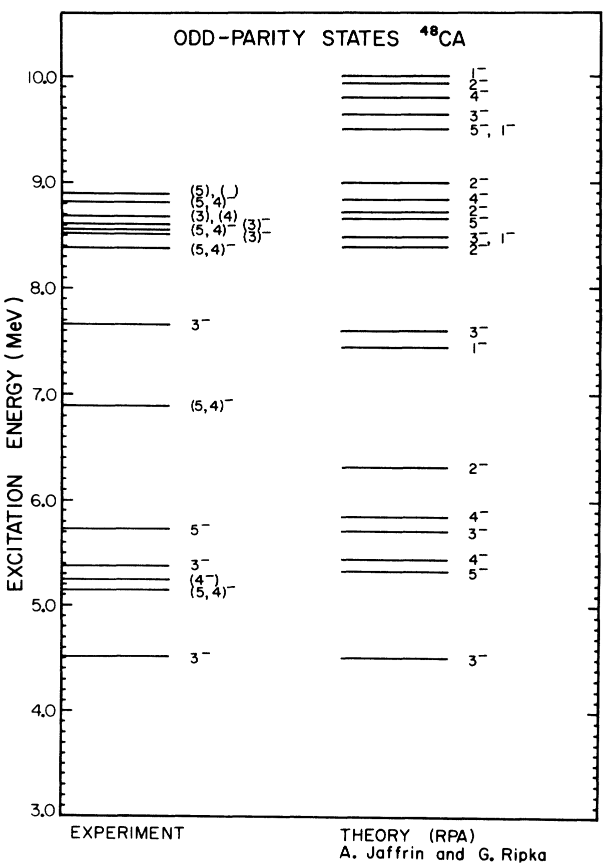


FIG. 11. A comparison of the calculated odd-parity states of Jaffrin and Ripka and the odd-parity states observed in this experiment.

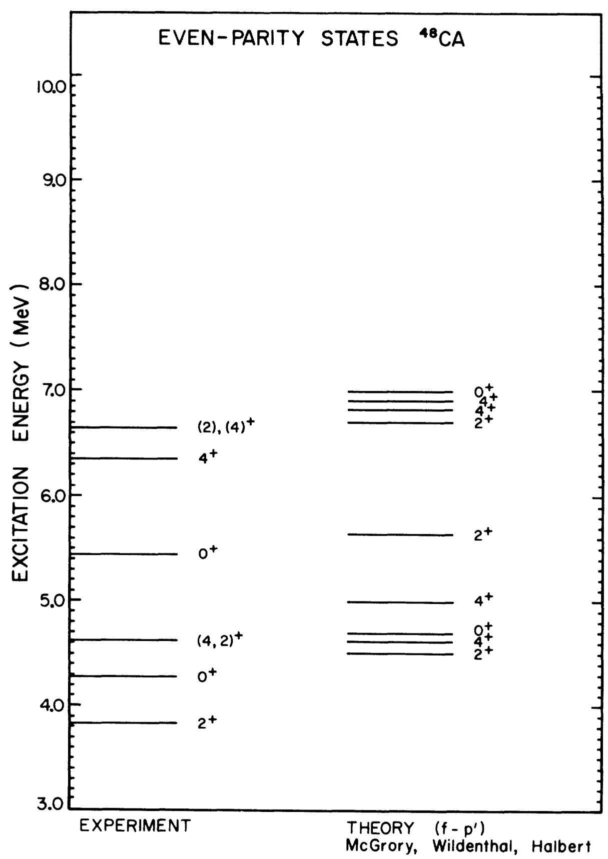


FIG. 12. A comparison of the calculated even-parity states of McGrory, Wildenthal, and Halbert and the even-parity states observed in this experiment.

using 67% $L=3$ and 33% $L=4$, but the evidence is not conclusive for these assignments.

*8.806 MeV.* The state at 8.806 MeV is a strongly excited state with an apparent $L=5$ angular distribution. The state is well resolved with no close contaminants and the angular distribution agrees almost point for point with the $L=5$ angular distribution at 5.729 MeV.

*8.885 MeV.* The last strong state observed in this experiment is a very close-lying doublet at 8.885 MeV. The evidence that this state is a doublet is the fact that the state consistently has a resolution about 5 to 10 keV broader than the 8.806-MeV state just below it. Also the shape of the angular distribution, while strongly $L=5$ in character, does rise at the forward angles. Inelastic electron scattering excites a state close to 9 MeV excitation which may correspond to this state; but the $(e, e')$ data were not analyzed.

### 2. *Even-Parity States*

*3.830 MeV.* The first excited state of $^{48}$Ca is the strongly excited $2^+$ state at 3.830 MeV. The angular distributions observed in this experiment were all consistent with an $L=2$ transfer. This is in agreement with all the previous experiments on $^{48}$Ca.[2,3,5,7,23] Comparisons of these data with that of the first $2^+$ states in $^{50}$Ti and $^{52}$Cr indicate that the $^{48}$Ca core dominates in these excitations.

*4.281 MeV.* The first $0^+$ excited state in $^{48}$Ca is not observed in this experiment. Upper limits for the cross section are about 1.5% the strength of the first $2^+$ state. The $(t, p)$ experiment observes the state with a strength of 60% of ground state or 150% the strength of the first $2^+$ state. The fact that this state is so weak in $(p, p')$ compared to the $0^+$ state in $^{40}$Ca observed with the same $(p, p')$ reaction[13] has been used as evidence for the existence of a deformed admixture in the $^{40}$Ca ground state,[13] and conversely, for the greater purity of the shell closure in the ground state of $^{48}$Ca.

*4.608 MeV.* Analysis of the 4.608-MeV state is complicated by the fact that the strong $^{40}$Ca $5^-$ state at 4.49 MeV contaminates a large part of the angular distribution. The data seem to favor an $L=4$ transfer, but $L=3$ cannot be ruled out on the basis of these data alone. However, the 10-MeV proton experiment[23] observes an angular distribution which peaks beyond the $3^-$ state and is compatible with an $L=4$ DWBA calculation. The $\alpha$ experiments were not able to resolve the state from the stronger 4.505-MeV state, but the angular distribution obtained[2] is not inconsistent with an $L=4$ transfer. The state is not observed in the $(t, p)$ experiment.[5]

*5.304 MeV.* The angular distributions associated with the weakly excited state at 5.304 MeV appear to be consistent with an $L=2$ assignment. However, the data are too poor to make a tentative assignment.

*6.342 MeV.* The angular distribution for this state was found to be consistent with an $L=4$ transfer which is also in agreement with the $(\alpha, \alpha')$ data.[2] A comparison of the angular distribution of this state with the low-lying $4^+$ states in $^{50}$Ti and $^{52}$Cr indicates that it accounts for most of the core excitation in these nuclei. The $(t, p)$ experiment assigns $2^+$ for this state.

*6.468 MeV.* The group at 6.648 MeV is a multiplet of states as revealed by the broadened peak shape. This is in agreement with the 12-MeV $(p, p'\gamma)$ experiment which observes a triplet at 6.618, 6.654, and 6.687 MeV. The 31-MeV $\alpha$ data for this state[2] are consistent with a $4^+$ assignment beyond 40° but the first maximum appears to be washed out. This is what one would expect if the state were a combination of $L=2$ and $L=4$. The angular distribution observed in this experiment has the slope associated with a $2^+$ state, but the valleys are missing. This is also what one would expect if the state were a combination of $L=2$ and $L=4$. By combining the known $L=2$ and $L=4$ shapes in ratio of 1 to 1, a reasonable fit to the data can be obtained at each energy. For purposes of calculations, the strength of this 6.648-MeV state is assumed to be 50% $L=2$ and 50% $L=4$, and the total strength of the multiplet is the sum of the two strengths. The deformation observed in the $(\alpha, \alpha')$ experiment of 0.36 $F^2$ is comparable to the total observed strength of the multiplet of 0.35 F. The $(t, p)$ reaction[5] excites a single state at 6.645 MeV.

*8.269 MeV.* The state at 8.269 MeV has a broadened peak shape and probably corresponds to the 8.268, 8.237-MeV doublet observed in the $(t, p)$[5] work. The $(p, p')$ angular distribution is consistent with $L=4$ transfer. In the $(t, p)$ experiment, a strength of 50% of the ground-state strength is observed for the 8.268-MeV state. They tentatively identify the 8.268-MeV state as being $(4^+)$.

## V. CONCLUSIONS

Angular distributions for the reaction $^{48}$Ca-$(p, p')$ have been measured for 22 low-lying states. In most cases the $L$ transfer was obtained. Excitation energies were determined for 29 states and were found to agree with other measurements. The $^{50}$Ti and $^{52}$Cr low-lying $2^+$ and $4^+$ excitations are found to have nearly identical angular distributions to that of the corresponding core states in $^{48}$Ca. This gives further evidence concerning the role $^{48}$Ca plays as a core for these nuclei. Perhaps, the most striking aspect of the present experiment is the discovery of an unusual amount of strength is $L=5$ transfers to a group of states between 8.0 and 9.0 MeV. Further studies concerning these unusual transitions and studies directed to the removal of ambiguities in the spin and parity assignments are needed.

ACKNOWLEDGMENTS

We would like to express our gratitude to all of the staff at the Michigan State University Cyclotron Laboratory for making this work possible. In particular we wish to express thanks to our colleagues, Professor G. Crawley, Professor W. Kelly, Professor H. McManus, Professor H. Wildenthal, and Dr. M. Dworzecka for their many invaluable discussions and comments.

---

†Work supported in part by the National Science Foundation.

*On sabbatical leave to CERN, Geneva, Switzerland.

‡Present address: Department of Physics, Sloan-Kettering Institute, New York, New York.

§Submitted in partial requirement for Ph.D. degree. Present address: Department of Environmental Medicine, Mt. Sinai School of Medicine, New York, New York.

¶Present address: Physics Department, University of South Carolina, Columbia, South Carolina 29208.

[1]A. Jaffrin and G. Ripka, Nucl. Phys. A119, 529 (1968).

[2]E. P. Lippincott and A. M. Bernstein, Phys. Rev. 163, 1170 (1967).

[3]R. J. Peterson, Phys. Rev. 140, B1479 (1965).

[4]R. A. Eisenstein, D. W. Madsen, H. Theissen, L. S. Cardman, and C. K. Bockelman, Phys. Rev. 188, 1815 (1969).

[5]J. H. Bjerregaard, Ole Hansen, O. Nathan, R. Chapman, S. Hinds, and R. Middleton, Nucl. Phys. A103, 33 (1967).

[6]A. Marinov and J. R. Erskine, Phys. Rev. 147, 826 (1966).

[7]A. Tellez, R. Ballini, J. Delaunay, and J. P. Fouan, Nucl. Phys. A127, 438 (1969).

[8]K. M. Thompson, Ph.D. thesis, Michigan State University, 1969 (unpublished).

[9]G. H. Mackenzie, E. Kashy, M. M. Gordon, and H. G. Blosser, IEEE Trans. Nucl. Sci. NS-14, No. 3, 450 (1967).

[10]J. L. Snelgrove and E. Kashy, Nucl. Instr. Methods 52, 153 (1967).

[11]C. R. Gruhn, T. Kuo, C. Maggiore, B. Preedom, L. Samuelson, and J. Chander, IEEE Trans. Nucl. Sci. NS-15, No. 3, 337 (1968).

[12]M. P. Fricke, E. E. Gross, R. J. Morton, and A. Zucker, Phys. Rev. 156, 1207 (1967).

[13]C. R. Gruhn, T. Y. T. Kuo, C. J. Maggiore, H. McManus, F. Petrovich, and B. M. Preedom, preceding paper, Phys. Rev. C 6, 915 (1972).

[14]J. Cameron, Ph.D. thesis, University of California at Los Angeles, 1967 (unpublished).

[15]G. R. Satchler, Nucl. Phys. 55, 1 (1964).

[16]G. R. Satchler, Nucl. Phys. A100, 481 (1967).

[17]C. J. Maggiore, C. R. Gruhn, T. Y. T. Kuo, and B. M. Preedom, Phys. Letters 33B, 571 (1970).

[18]B. M. Preedom, C. R. Gruhn, T. Kuo, and C. J. Maggiore, Phys. Rev. C 2, 166 (1970).

[19]Oak Ridge National Laboratory Report No. ORNL-3240, 1962 (unpublished); and Oak Ridge National Laboratory Memorandum to the Users of the Code JULIE, 1966 (unpublished).

[20]C. J. Maggiore, Ph.D. thesis, Michigan State University, 1971 (unpublished).

[21]F. Petrovich, Ph.D. thesis, Michigan State University, 1970 (unpublished).

[22]J. B. McGrory, B. H. Wildenthal, and E. C. Halbert, Phys. Rev. C 2, 186 (1970).

[23]L. Lassen, T. Scholz, and H. J. Unsold, Phys. Letters 20, 516 (1966).