# Magnetic Multipole Properties of $^{38}$Cl and $^{40}$K†

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The effects of weak particle-hole admixtures on magnetic multipole properties of $^{38}$Cl and $^{40}$K are investigated and the results are compared with experiment, with particular emphasis on $M1$ transitions between members of the lower quartet of states in each nucleus. In $^{40}$K, the admixtures tend to decrease the differences between theoretical and experimental values except for the magnetic moment. However, the same procedure leads to larger discrepancies with experiment in $^{38}$Cl. In addition, in the latter nucleus the abnormally large $M1$ transition strength from the second $3^-$ level to the first $4^-$ state cannot be understood if only weak-configuration mixing is assumed. On the other hand, one can account for the experimentally observed strongly inhibited $M3$ transition between the $5^-$ and $2^-$ states of $^{38}$Cl. Our prime interest is in exhibiting the effects of the separate particle-hole admixtures in order to isolate the source of the discrepancies.

## I. INTRODUCTION

About fifteen years ago, Goldstein and Talmi[1] and Pandya[2] pointed out that if the low-lying quartet of negative-parity states of $^{38}$Cl and $^{40}$K can be described by the configurations $(\pi d_{3/2} \times \nu f_{7/2})$ and $((\pi d_{3/2})^{-1} \times \nu f_{7/2})$, respectively, the excitation energies of the two nuclei are related by the particle-hole transformation. This relationship is fulfilled remarkably well experimentally. The experimental data on excited states have been extended to identify an expected higher multiplet[3] wherein a $2p_{3/2}$ neutron replaces the $1f_{7/2}$ neutron. Again the particle-hole transformation has been applied,[4] and although not obeyed nearly so well as in the lower multiplet, it appears to provide good correlation. However, it should be noted that other levels near this upper multiplet in both $^{38}$Cl and $^{40}$K are reported[5] to have an $l_n = 1$ stripping pattern in $(d, p)$ transfer, so the $2p$ strength seems to be quite fragmented. The energy levels attributed to the basic multiplets are shown in Fig. 1, where one sees that the multiplets are considerably closer together in $^{38}$Cl than they are in $^{40}$K. This means that levels of the same angular momentum are more likely to be mixed, and indeed the lower $3^-$ state in $^{38}$Cl is found in stripping experiments to exhibit a mixed $l_n = 1$ and 3 pattern corresponding to a 15% admixture of $2p_{3/2}$ neutron component. The energy relationships are not very sensitive to this degree of admixture.

A more stringent test of the multiplet description of these states is provided by the $\gamma$ transition probabilities. We shall concentrate on the many measured values[6,7] for $M1$ transitions in $^{40}$K and[4,8] in $^{38}$Cl. The magnetic moments and an isolated $M3$ transition in $^{38}$Cl are also considered. In this paper we summarize how well the values calculated from the basic multiplets compare with experiment and also the changes produced in these values by the use of effective moments. We then investigate the effect of weak-configuration mixing as proposed long ago by Arima, Horie, and Sano[9] and by Blin-Stoyle,[10] and show that while this helps to interpret the data in $^{40}$K, it leads to larger disagreements in $^{38}$Cl. Thus the conventional procedure is not adequate in $^{38}$Cl and an anomaly remains to be interpreted.

## II. $M1$ TRANSITIONS

The quantity selected for comparing calculated $M1$ matrix elements with magnitudes extracted from experiment is $b(I_1 \to I_2)$, which is related to the mean life for transition from the state $I_1$ to $I_2$ by

$$\tau E_\gamma{}^3 b^2(I_1 \to I_2) = 0.239\,, \tag{1}$$

where $\tau$ is in psec, $E_\gamma$ is in MeV, and $b$ is in $\mu_N$ and is related[11] to the reduced transition probability $B_{M1}$ by

$$b(I_1 \to I_2) = [\tfrac{4}{3}\pi B_{M1}(I_1 \to I_2)]^{1/2}\,. \tag{2}$$

The values of $b$ extracted from experiment are given in Table I, wherein the subscript $L$ or $H$ refers to a state ascribed to a multiplet having a $1f_{7/2}$ or $2p_{3/2}$ neutron, respectively. The values of $b$ listed under the heading of zero-order theory are calculated with the basic multiplet wave functions

$$\begin{aligned}\psi^{I_L} &= [(\pi d_{3/2})^{\pm 1} \times \nu f_{7/2}]^I\,,\\ \psi^{I_H} &= [(\pi d_{3/2})^{\pm 1} \times \nu p_{3/2}]^I\,,\end{aligned} \tag{3}$$

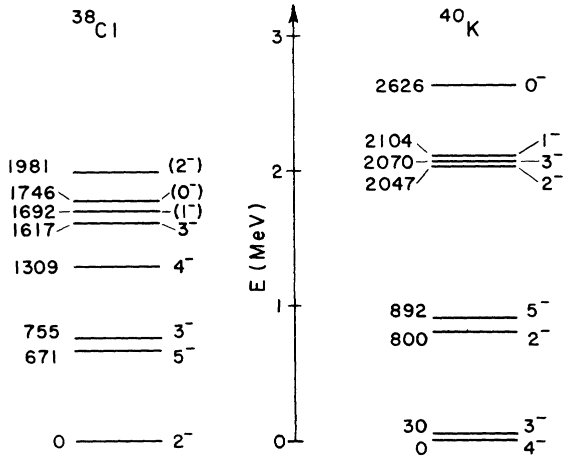


FIG. 1. Experimental identification of levels of $^{38}$Cl and $^{40}$K. The lower quartets are attributed to coupling a $1d_{3/2}$ proton or proton hole with a $1f_{7/2}$ neutron; for the upper quartets the neutron is $2p_{3/2}$.

where the bracket indicates vector coupling. The free-nucleon values $\mu_p = 2.79$ and $\mu_n = 1.91$ were used in the calculation. These wave functions give the same values for $^{40}$K and $^{38}$Cl, and forbid transitions between multiplets since this would be a $\nu p_{3/2} \to \nu f_{7/2}$ transition. The experimental results in $^{40}$K clearly exhibit the calculated feature of strong intramultiplet transitions and weaker intermultiplet transitions. The same is true in $^{38}$Cl with the exception of the strong intermultiplet transition $3_H \to 4$. Aside from this anomaly, the similarity of magnitudes between theory and experiment favors the expectation that minor changes in the basic configurations can lead to close agreement with the experimental values.

### III. EFFECTIVE MOMENTS IN THE LOWER MULTIPLET

Since the excitation energies of the lower multiplet fulfill the particle-hole relationship of the $(\pi d_{3/2} \times \nu f_{7/2})$ configuration so closely, attempts have been made to account for the $M1$ properties by using effective moments obtained from the neighboring odd-$A$ nuclei. With values $\mu_n = -1.595$ from $^{41}$Ca, $\mu_p = 2.35$ from $^{39}$K, or $\mu_p = 1.86$ from $^{37}$Cl, one obtains the values given in Table II. This procedure works very well for the magnetic moment of $^{40}$K, as noted by de-Shalit,[12] but increases the discrepancy with experiment for the $4 \to 5$ transition in $^{40}$K and for the $3 \to 4$ transition in $^{38}$Cl. It has been pointed out[6] that the ratios of $M1$ transition strengths between members of the multiplet $((\pi j_1)^{\pm 1} \times \nu j)$ are independent of the effective moments used since for $|I_1 - I_2| = 1$ one obtains

$$b(I_1 \to I_2) = [j(j+1)(2j+1)(2I_2+1)]^{1/2} \times W(j_1 j I_2 1; I_1 j)[g(\nu j) - g(\pi j_1)], \quad (4)$$

where the factor in braces contains the gyromagnetic factors of the odd neutron and odd proton. It is clear from comparing the experimental ratios of transition strengths with the calculated ratios that neither $^{40}$K or $^{38}$Cl can be satisfied by Eq. (4), regardless of what $g$ factors are used.

An $M3$ transition in $^{38}$Cl from the first excited state ($5^-$) to the ground state ($2^-$) has[5] a mean

TABLE I. Comparison between experimental and zero-order theoretical values of $b(I_1 \to I_2)$ for $M1$ transitions. This quantity is defined in Eqs. (1) and (2). The designation $I_L$ refers to states attributed to the configuration $[(\pi d_{3/2})^{\pm 1} \times \nu f_{7/2}]^{I_L}$ and $I_H$ refers to states formed by promoting the neutron from $f_{7/2}$ to $p_{3/2}$. The calculation assumes pure configurations and uses the values $\mu_p = 2.79\mu_N$ and $\mu_n = -1.91\mu_N$.

| Transition | $^{38}$Cl experiment Argonne (Ref. 8) | Brookhaven (Ref. 4) | Zero-order theory | $^{40}$K experiment Argonne (Refs. 6 and 8) | Frankfurt (Ref. 7) |
|---|---|---|---|---|---|
| $4 \to 5$ | $1.13 \pm 0.30$ | $0.83 \pm 0.24$ | 0.71 | $0.52 \pm 0.06$ | $0.52 \pm 0.07$ |
| $3_L \to 4$ | $0.77 \pm 0.20$ | $0.57 \pm 0.16$ | 0.92 | $1.06 \pm 0.01$ [a] | |
| $2_L \to 3_L$ | $1.62 \pm 0.15$ | $1.21 \pm 0.19$ | 0.94 | $0.97 \pm 0.11$ | $1.02 \pm 0.30$ |
| $3_H \to 4$ | 1.69 | 1.34 | 0 | 0.15 | 0.11 |
| $3_H \to 3_L$ | 0.25 | 0.20 | 0 | 0.19 | 0.14 |
| $2_L \to 3_H$ | 0.11 | 0.09 | 0 | 0.18 | 0.14 |
| $2_H \to 3_L$ | 0.36 | 0.27 | 0 | 0.14 | 0.01 |
| $2_H \to 2_L$ | 0.20 | 0.15 | 0 | 0.31 | 0.27 |
| $1 \to 2_L$ | 0.19 | 0.15 | 0 | 0.24 | 0.17 |
| $2_H \to 3_H$ | 2.45 | 1.86 | 1.39 | | |
| $1 \to 2_H$ | 2.78 | 2.12 | 1.92 | | |
| $0 \to 1$ | | | 2.63 | 1.88 | 1.22 |

[a] Reference 5.

lifetime of 1.07 sec. This transition is inhibited by 2 orders of magnitude from the Weisskopf estimate of $1.2\times10^{-2}$ sec. The large inhibition is inherent in a $(\pi d_{3/2}\times\nu f_{7/2})$ description of these states. As in the $M1$ case, we define a quantity $b_3(I_1\rightarrow I_2)$ in units of $\mu_N\,\mathrm{fm}^2$, where $(b_3)^2=\frac{4}{7}\pi B_{M3}$ and the lifetime is given by

$$\tau E_\gamma{}^7 b_3{}^2(I_1\rightarrow I_2)=0.285\,, \tag{5}$$

where $\tau$ is in sec and $E$ is in MeV. The $M3$ operator is taken from the general $M\lambda$ single-particle expression

$$M^\lambda=(e\hbar/2mc)(4\pi\lambda)^{1/2}r^{\lambda-1}\times\{\mu_{t_3}[Y^{\lambda-1}\times\sigma]^\lambda+2g_{lt_3}(\lambda+1)^{-1}[Y^{\lambda-1}\times l]^\lambda\},$$

wherein $t_3$ refers to neutron or proton and the square bracket implies vector coupling of the spherical harmonic with $\sigma$ or $l$ to a resultant of $\lambda$. The transition rate depends on radial matrix elements of $r^2$ in $1f$ and $1d$ states, which we evaluate with harmonic-oscillator functions using $\hbar\omega=(41/A^{1/3})$ MeV. This leads to

$$b_3(5\rightarrow2)=0.5155[3\mu_n+7(\mu_p-2)]\,, \tag{6}$$

which has nearly complete cancellation between the neutron and proton contributions for free-nucleon values of $\mu_p$ and $\mu_n$. However, as one sees in Table II, the value obtained with empirical effective moments is about 1.5 times the experimental value, so this transition is very sensitive to configuration admixture.

## IV. CONFIGURATION ADMIXTURE

There are large $M1$ matrix elements connecting a single-particle state to its spin-orbit partner, so that weak admixtures of such configurations to the basic multiplets can have a large effect. In the present case, the relevant orbitals are the $1d_{5/2}$ proton-hole state and the $1f_{5/2}$ and $2p_{1/2}$ neutron-particle states. Recently Becker and Warburton[13] have pointed out that the inclusion of such configurations effects in $^{40}$K tends to remove the discrepancies with experiment for the $M1$ transitions within the lower multiplet. In their calculation, the admixtures are taken from a paper by Perez,[14] who used a central nuclear interaction with a Yukawa range dependence. Very small admixture coefficients produced large effects.

We have extended the calculation to include such weak admixtures in the wave functions of both the $[(\pi d_{3/2})^{\pm1}\times\nu f_{7/2}]$ and $[(\pi d_{3/2})^{\pm1}\times\nu p_{3/2}]$ multiplets by means of perturbation theory. We also diagonalize the interaction between the $I=2^-$ and $I=3^-$ states of the basic multiplets, which produces energy shifts in addition to changes in the transition probabilities. The interaction is taken to be of spin-dependent surface-$\delta$[15] form

$$V=-4\pi V_0\delta(\Omega_{12})[0.9+0.1\vec{\sigma}_1\cdot\vec{\sigma}_2]\,, \tag{7}$$

with the strength $V_0$ determined by the energy spread of the lower multiplet. For both nuclei one finds a strength of 0.9 MeV for the product $V_0$ times the radial wave functions evaluated at the surface.

The calculated excitation energies are very close to observation for the lower multiplets in both $^{40}$K and $^{38}$Cl if one includes the amounts by which the energies of the $I=2^-$ and $3^-$ states are shifted by the mixing with the corresponding states of the upper multiplet. With the experimentally observed mixing of 15% in intensity for the $I=3^-$ states of $^{38}$Cl, 5% for the much more widely separated $I=2^-$ states, and an arbitrary 10% mixing for the $^{40}$K states, the root-mean-square deviations between calculated and experimental excitation energies are 10–15 keV for the lower multiplets. On the other hand, the calculated excitation energies for the upper multiplets do not bear much resemblance

TABLE II. Magnetic properties of the presumed $[(\pi d_{3/2})^{\pm1}\times\nu f_{7/2}]$ multiplets in $^{38}$Cl and $^{40}$K. The quantities $b\,(I_1\rightarrow I_2)$ and $b_3(5\rightarrow2)$ are defined in Eqs. (2) and (5), respectively. The theoretical estimates are based on the indicated values of the magnetic moments $\mu_p$ and $\mu_n$ with $\mu_p=3-\frac{5}{2}g(\pi_{3/2})$ and $\mu_n=\frac{7}{2}g(\nu_{7/2})$.

| | $^{38}$Cl Experiment (Ref. 8) | Theoretical estimates $\mu_p$ ($^{37}$Cl), $\mu_n$ ($^{41}$Ca) [a] | Free-nucleon | $\mu_p$ ($^{39}$K), $\mu_n$ ($^{41}$Ca) [b] | $^{40}$K Experiment (Ref. 6) |
|---|---|---|---|---|---|
| $b\,(4\rightarrow5)$ | $1.13\pm0.30$ | 1.03 | 0.71 | 0.81 | $0.52\pm0.06$ |
| $b\,(3\rightarrow4)$ | $0.77\pm0.20$ | 1.33 | 0.92 | 1.05 | $1.06\pm0.01$ [c] |
| $b\,(2\rightarrow3)$ | $1.62\pm0.15$ | 1.37 | 0.94 | 1.07 | $0.97\pm0.11$ |
| $\mu$(g.s.) | | −1.82 | −1.72, −1.68 | −1.25 | −1.298 [c] |
| $b_3(5\xrightarrow{M3}2)$ | 2.08 [c] | 2.98 | 0.10 | | |

[a] Effective moments $\mu_p=1.86\mu_N$ from $^{37}$Cl, $\mu_n=-1.595\mu_N$ from $^{41}$Ca.
[b] Effective moments $\mu_p=2.348\mu_N$ from $^{39}$K, $\mu_n=-1.595\mu_N$ from $^{41}$Ca.
[c] Reference 5.

TABLE III. Effects of particle-hole admixture in $^{40}$K. The tabulated quantity $b(I_1 \to I_2)$ is defined in Eq. (2).

| Transition $(I_1 \to I_2)$ | $^{40}$K Experiment [a] | Zero-order contribution | Theoretical values of $b(I_1 \to I_2)$ for $^{40}$K: $\nu f_{5/2}$ or $\nu p_{1/2}$ contribution [b] | $\pi d_{5/2}^{-1}$ contribution [c] | Total unmixed [d] | Total with mixing [e] |
|---|---|---|---|---|---|---|
| $4 \to 5$ | $0.52 \pm 0.06$ | 0.71 | −0.08 | −0.31 | +0.32 | |
| $3_L \to 4$ | $1.06 \pm 0.01$ [f] | 0.92 | +0.10 | +0.27 | +1.29 | +1.02 |
| $2_L \to 3_L$ | $0.97 \pm 0.11$ | 0.94 | +0.10 | −0.45 | +0.59 | +0.80 |
| $3_H \to 4$ | 0.15 | 0 | −0.19 | −0.47 | −0.65 | −1.02 |
| $3_H \to 3_L$ | 0.19 | 0 | −0.07 | −0.38 | −0.45 | −0.51 |
| $2_L \to 3_H$ | 0.18 | 0 | +0.02 | +0.29 | +0.31 | +0.35 |
| $2_H \to 3_L$ | 0.14 | 0 | +0.10 | +0.17 | +0.27 | +0.31 |
| $2_H \to 2_L$ | 0.31 | 0 | +0.04 | +0.35 | +0.39 | +0.61 |
| $1 \to 2_L$ | 0.24 | 0 | +0.08 | −1.18 | −1.10 | −0.29 |
| $0 \to 1$ | 1.88 | 2.63 | −0.33* | +0.10 | +2.40 | |

[a] Argonne values, Refs. 6 and 8.
[b] Perturbation-theory effects from neutron excitation to $f_{5/2}$ (no asterisk) or to $p_{1/2}$ (asterisk).
[c] Perturbation-theory effects from creating a $d_{5/2}$ proton hole.
[d] Theoretical values calculated with $\mu_n = -1.91\mu_N$, $\mu_p = 2.79\mu_N$. The effects of particle-hole admixture have been included.
[e] Values resulting from mixing the two $I=2^-$ state and the two $I=3^-$ states. The intensity mixtures are (90% $I_L$, 10% $I_H$) and (90% $I_H$, 10% $I_L$) with the relative phases of the components determined by the surface $\delta$ interaction.
[f] Reference 5.

TABLE IV. Effects of particle-hole admixture in $^{38}$Cl. The tabulated quantity $b(I_1 \to I_2)$ is defined in Eq. (2). The last row shows the effect on the $M3$ transition for which the particle-hole admixures are dominant.

| Transition $(I_1 \to I_2)$ | $^{38}$Cl Experiment [a] | Zero-order contribution | Theoretical values of $b(I_1 \to I_2)$ for $^{38}$Cl: $\nu f_{5/2}$ or $\nu p_{1/2}$ contribution [b] | $\pi d_{5/2}^{-1}$ contribution [c] | Total unmixed [d] | Total with mixing [e] |
|---|---|---|---|---|---|---|
| $4 \to 5$ | $1.13 \pm 0.30$ | 0.71 | +0.11 | −0.10 | +0.72 | |
| $3_L \to 4$ | $0.77 \pm 0.20$ | 0.92 | −0.02 | +0.41 | +1.31 | +1.37 |
| $2_L \to 3_L$ | $1.62 \pm 0.15$ | 0.94 | −0.13 | −0.34 | +0.47 | +0.48 |
| $3_H \to 4$ | 1.69 | 0 | +0.05 | −0.48 | −0.43 | +0.11 |
| $3_H \to 3_L$ | 0.25 | 0 | +0.02 | −0.21 | −0.19 | −0.11 |
| $2_L \to 3_H$ | 0.11 | 0 | −0.01 | +0.23 | +0.22 | +0.07 |
| $2_H \to 3_L$ | 0.36 | 0 | −0.13 | +0.08 | −0.05 | −0.53 |
| $2_H \to 2_L$ | 0.20 | 0 | −0.05 | +0.43 | +0.38 | +0.11 |
| $1 \to 2_L$ | 0.19 | 0 | −0.29 | −1.25 | −1.54 | −1.87 |
| $2_H \to 3_H$ | 2.45 | 1.39 | +0.19* | −0.08 | +1.50 | +1.42 |
| $1 \to 2_H$ | 2.78 | 1.92 | −0.35* | +0.07 | +1.64 | +1.17 |
| $5_L \to 2_L$ | 2.08 [f] | 0.10 | −0.54 | −5.12 | −5.57 | +1.63 |

[a] Argonne values, Refs. 6 and 8.
[b] Perturbation-theory effects from neutron excitation to $f_{5/2}$ (no asterisk) or to $p_{1/2}$ (asterisk).
[c] Perturbation-theory effects from creating a $d_{5/2}$ proton hole.
[d] Theoretical values calculated with $\mu_n = -1.91\mu_N$, $\mu_p = 2.79\mu_N$. The effects of particle-hole admixture have been included.
[e] Values resulting from a 5% intensity mixture for $I=2$ and 15% for $I=3$. The latter mixture is indicated by stripping experiments.
[f] Reference 5.

to the experimental values shown in Fig. 1. For $^{40}$K the calculated level order is $I=2^-$, $1^-$, $3^-$, and $0^-$ with an energy spread of 1 MeV, while for $^{38}$Cl it is $I=0^-$, $1^-$, $3^-$, and $2^-$ with a spread of 1600 keV. These are large discrepancies with observation, and our treatment is not adequate for the upper multiplets. However, in view of the experimental fragmentation of the $l=1(d,p)$ strength, this is not too surprising.

Nevertheless, for $M1$ transitions within the lower multiplet, we expect that our perturbation treatment should be adequate to include the effects of neutron excitation to the $1f_{5/2}$ and $2p_{1/2}$ orbitals and proton excitation from the $1d_{5/2}$ orbital. The procedure should also give a rough idea of the intermultiplet transition probabilities. In the perturbation expression

$$\alpha_{I_k} = -\langle\psi^{I_k}|V|\psi^{I_0}\rangle/(E_{I_k}-E_{I_0})$$

for the coefficients of admixture to the states of the basic multiplets, we set all energy denominators equal to 5 MeV. Subsequently we mix the $I=2^-$ and $I=3^-$ states of the basic multiplets with the previously indicated magnitudes, the relative signs of the mixing being determined by the surface $\delta$ interaction. As we show, our procedure can provide reasonable agreement with experiment for $^{40}$K, but it fails for $^{38}$Cl. Our objective is to show the source of the poor result in $^{38}$Cl by exhibiting how the separate ingredients contribute to the calculated values.

### A. Transitions in $^{40}$K

The effect of configuration admixture on the $^{40}$K matrix elements $b(I_1 \to I_2)$ is given in Table III. In column 4 we list the contributions due to excitation of a neutron to the $1f_{5/2}$ or $2p_{1/2}$ orbital. Since the $\delta$ interaction gives no $2p_{1/2}$ admixture to the lower multiplet, this admixture can only make a first-order contribution to transitions within the upper multiplet. In column 5 we list the contributions from excitation of a $1d_{5/2}$ proton hole, and in column 6 we list the sum of the contributions. Finally, in column 7 we list the $b$ values resulting from a 10% (intensity) mixing between upper and lower states for $I=2^-$ and $3^-$, with the relative sign determined by the surface $\delta$ interaction. Although the 10% admixture is an arbitrary figure, it is approximately what one would get if the zero-order states were separated by 2 MeV and is roughly the magnitude needed to preserve the particle-hole relationship between the lower multiplets in $^{38}$Cl and $^{40}$K. The following features are evident from Table III:

(1) For transitions within the lower multiplet we obtain practically the same total contribution (column 6) as Becker and Warburton.[13] This supports their statement that their result should be insensitive to the particular interaction.

(2) The neutron contribution (column 4) is always in the direction to improve agreement with experiment for intramultiplet transitions. The $d_{5/2}$ proton-hole contribution is large and generally outweighs the neutron contribution.

(3) The effect of mixing the multiplets (column 7) is destructive interference in $b(3_L \to 4)$ and constructive interference in $b(2_L \to 3_L)$. This interference tends to make the agreement with experiment better than it is for the values in column 6.

(4) The matrix elements for intermultiplet transitions are all larger than are observed experimentally. This is not a serious difficulty since we suspect that the upper multiplet is far from pure $[(\pi d_{3/2})^{-1} \times \nu p_{3/2}]$. Therefore the calculated values should be multiplied by the probability that the observed state is of this configuration. Also the surface-$\delta$ matrix elements pertinent to the intermultiplet transitions are of the type $\langle 1d\,1f|V|1d\,2p\rangle$, which are generally larger for the surface $\delta$ interaction than for other forms since this potential ignores the $2p$ radial node.

Therefore, it seems likely that with some modification – such as reduction of the $(\pi d_{5/2})^{-1}$ contributions – one can account for the $M1$ transitions in $^{40}$K with weak configuration mixing. It should be pointed out, however, that the calculated zero-order value of the magnetic moment of the $I=4^-$ ground state is $-1.68\mu_N$, the $\nu f_{5/2}$ contribution is $-0.04\mu_N$, and the $\pi d_{5/2}$ contribution is $-0.26\mu_N$. Thus configuration admixtures tend to increase the discrepancy with the experimental value of $-1.30\mu_N$, a feature originally noted by de-Shalit.[12]

### B. Transitions in $^{38}$Cl

Table IV displays the effects of configuration mixing on $M1$ transitions in $^{38}$Cl in the same form as in Table III. The number of states arising from creating a $\pi d_{5/2}$ hole is greater than for the $^{40}$K case as one sees from the wave functions

$$\psi^I = [(\pi d_{5/2})^{-1} \times (\nu j) \times (\pi d_{3/2})^2 J_0]^I . \tag{8}$$

The $d_{3/2}$ protons can have angular momentum $J_0 = 0$ or 2, whereas in the $^{40}$K case we had $(\pi d_{3/2})^4$ which has only $J_0 = 0$. We have calculated the total $d_{5/2}$ contribution by assuming a state-independent energy denominator of 5 MeV for all the $d_{5/2}$ admixture coefficients. This leads to an overestimate of the $J_0 = 2$ contribution, but the qualitative effect would be the same if we kept only the $J_0 = 0$ contribution, as we show later. The important features seen in Table IV are the following:

(1) The net result of adding weak configuration contributions to the zero-order values for the lower multiplet is to increase the discrepancy with experiment.
(2) The neutron-excitation contributions (column 4) have signs opposite to those in the analogous $^{40}$K transitions in Table III. This is because the admixture is determined by the attractive particle-particle interaction in $^{38}$Cl but by the repulsive particle-hole interaction in $^{40}$K. These contributions tend to reduce the discrepancy with experiment for $b(4\to 5)$ and $b(3_L\to 4)$.
(3) The $d_{5/2}$ proton contribution (column 5) has the same sign for analogous transitions in $^{40}$K and $^{38}$Cl. These terms strongly increase the disparity between calculated and observed $b$ values within the lower multiplet.
(4) The effect of mixing the multiplets (column 7) is constructive interference in $b(3_L\to 4)$, which further increases the discrepancy with observation. The associated destructive interference in the intermultiplet transition $b(3_H\to 4)$ means that this calculated value is more than an order of magnitude smaller than the experimental transition probability. The intermultiplet transition $b(1\to 2_L)$ is an order of magnitude stronger than is observed.

The major source of discrepancy is the $\pi d_{5/2}$ contribution, wherein both the $J_0=0$ and $J_0=2$ terms are lumped together with a common energy denominator despite the fact that $(\pi d_{3/2})^2{}_{J_0}$ with $J_0=2$ should lie about 2 MeV above the state with $J_0=0$. However, even if only the $J_0=0$ contribution is retained, one can show that its contribution to column 5 of Table IV is exactly half the value in column 5 of Table III for the analogous transition in $^{40}$K. This is due to the fact that there are half as many $d_{3/2}$ protons in $^{38}$Cl which can make the $M1$ transition to the $d_{5/2}$ proton state as there are in $^{40}$K. From Tables III and IV, we see that the sign of the $J_0=0$ part of the $d_{5/2}$ contribution (column 5) is the same as the sign of the total $J_0=0$ plus $J_0=2$ contribution, so that the discrepancy remains even if we increase the energy denominator in the $J_0=2$ terms.

The value of $b_3(5\to 2)$ for the $M3$ transition is also strongly affected by configuration admixtures whose contributions are much bigger than the zero-order value. The numerical results are similar to the experimental value, but they are sensitive both to the admixture strength and to the degree of $(\nu p_{3/2}\text{-}\nu f_{7/2})$ mixing in the $I=2^-$ state.

## V. CONCLUSIONS

Both in $^{38}$Cl and in $^{40}$K, the states of the lower quartet are seen with $l=3$ stripping patterns. If they are assumed to be the states of the $[(\pi d_{3/2})^{\pm 1}\times \nu f_{7/2}]$ configuration, the energy differences closely fulfill the particle-hole relationship. This assignment is further supported by the similarity between the observed $M1$ properties and the pure-multiplet predictions. The use of effective moments removes most of the remaining differences but leaves one prominent exception in each nucleus. Further, this assignment accounts for the strong inhibition of $b_3(5\to 2)$ for the $M3$ transition in $^{38}$Cl. On the other hand, the primary evidence for ascribing the upper quartet in these nuclei to the $[(\pi d_{3/2})^{\pm 1}\times \nu p_{3/2}]$ configuration is that the energy differences fulfill the particle-hole relationship quite well. Experimentally, 11 of the 12 intermultiplet transitions are relatively weak, as one would expect from the zero-order picture. The one glaring exception is the transition $3_H\to 4$ in $^{38}$Cl.

For $^{40}$K we find that the contributions from weak admixtures of configurations involving $\nu f_{5/2}$, $\nu p_{1/2}$, and $\pi d_{5/2}$ orbitals have phases relative to the zero-order $M1$ matrix elements such that the contributions tend to improve agreement with experiment. Although our assumed two-body force is different from that used by Becker and Warburton,[13] our results are quite similar to theirs. Thus their expectation that the general trend is relatively independent of the form of interaction is borne out. With some adjustment of the strength of admixture, mainly a weaker $\pi d_{5/2}$ contribution, one could obtain close agreement with experiment. An exception to this conclusion is the magnetic moment of the $^{40}$K ground state, for which these admixtures increase the discrepancy with experiment.

When we apply the same procedure to $^{38}$Cl, the result is usually to increase the discrepancy with experiment. The phase of mixing of the $I=3^-$ states of the major multiplets is opposite to that which would tend to account for the large $b(3_H\to 4)$ matrix element. We have shown that the major source of difficulty is the phase of the contributions from the $\pi d_{5/2}$ admixture. While the magnitude may depend on the type of interaction, the phase of the contributions is not likely to change. We have in fact examined the predictions obtained by using an interaction containing an ordinary $\delta$ force plus a tensor force, which fits the energies of all eight states very well in $^{38}$Cl. The sign of the $\pi d_{5/2}$ contribution does not change. We conclude that the perturbation treatment is not adequate in $^{38}$Cl, especially for describing the upper multiplet.

Goode[16] has studied the transitions within the lower multiplets by use of a model space that allows for exciting at most two protons from the $d_{5/2}$ and $s_{1/2}$ single-particle states to the $d_{3/2}$ level. In his work, a modified form of the Kuo-Brown interaction is used and $M1$ transition strengths are given only for the lower multiplets. In $^{40}$K, for which only one proton can be excited from these orbitals, his results are quite similar to ours. This again confirms the expectation that the results are insensitive to the exact potential matrix elements used. In $^{38}$Cl, Goode's values are closer to experiment than our perturbation-theory results in the first three rows of Table IV. However, Goode's values of $b(2_L \rightarrow 3_L)$ and $b(3_L \rightarrow 4_L)$ also tend to have larger discrepancies with experiment than the values of the zero-order theory. Therefore the undesirable effects of the $d_{5/2}$ proton admixture seem to persist in this more extensive shell-model calculation.

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[1] S. Goldstein and I. Talmi, Phys. Rev. 102, 589 (1956).
[2] S. P. Pandya, Phys. Rev. 103, 956 (1956).
[3] R. M. Freeman and A. Gallmann, Nucl. Phys. A156, 305 (1970).
[4] G. A. P. Engelbertink and J. W. Olness, Phys. Rev. C 5, 431 (1972).
[5] P. M. Endt and C. van der Leun, Nucl. Phys. A105, 1 (1967).
[6] R. E. Segel, G. H. Wedberg, G. B. Beard, N. G. Puttaswamy, and N. Williams, Phys. Rev. Letters 25, 1352 (1970).
[7] R. Bass and R. Wechsung, Phys. Letters 32B, 602 (1970).
[8] G. H. Wedberg and R. E. Segel, to be published.
[9] A. Arima, H. Horie, and M. Sano, Progr. Theoret. Phys. (Kyoto) 17, 567 (1957).
[10] R. J. Blin-Stoyle, Proc. Phys. Soc. (London) A66, 1158 (1953).
[11] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. 28, 432 (1956).
[12] A. de-Shalit, Nucl. Phys. 22, 677 (1961).
[13] J. A. Becker and E. K. Warburton, Phys. Rev. Letters 26, 143 (1971).
[14] S. M. Perez, Nucl. Phys. A136, 599 (1960).
[15] I. M. Green and S. A. Moszkowski, Phys. Rev. 139, B790 (1965); R. Arvieu and S. A. Moszkowski, *ibid.* 145, 830 (1966).
[16] P. Goode, Bull. Am. Phys. Soc. 17, 34 (1972); and to be published.

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# Study of Even Calcium and Nickel Isotopes with Self-Consistent Approximations

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Employing the central Yukawa interaction, and renormalized matrix elements for the Hamada-Johnston and Tabakin potentials, self-consistent Hartree-Fock, Hartree-Fock-Bogoliubov (HFB), and spherical BCS calculations for even calcium and nickel isotopes are carried out. The existence of spherical $^{40}$Ca and $^{56}$Ni cores is assumed and the calcium and nickel isotopes are treated within the framework of single-closed-shell structure. It is found that for a given isotope and interaction the HFB and spherical BCS approximations always correspond to the same actual minimum energy solution. This feature is observed for all the isotopes and interactions considered here, though the quasiparticle energies and the occupation numbers are found to differ. For Ca isotopes the BCS results are also compared with the shell-model calculations of McGrory, Wildenthal, and Halbert, and are found to approximate the ground-state properties of all the even isotopes rather well.

## I. INTRODUCTION

The presence of approximate vibrational states in some nuclei with their characteristic enhanced transition rates from the $2_1^+$ to the ground state and the lack of transition from the $2_2^+$ to the ground state has led to their identification as single-closed-shell spherical nuclei. The spherical ground-state structure of these nuclei has been justified by the successful use of the spherical BCS approach to predict their level spectra. The most widely studied single-closed-shell nuclei with this approach are the nickel isotopes and to a lesser extent the tin isotopes. However, all the SCS nuclei do not have the above-mentioned vibrational characteristics. One such group of nuclei is, for ex-