

Spectroscopy of ^{25}Al by Means of the Reaction $^{24}\text{Mg}(p, p'_1\gamma)^{24}\text{Mg}^\dagger$

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The triple-correlation geometry suggested by Goldfarb and Seyler is employed in determining the spins of excited states in ^{25}Al by means of the reaction $^{24}\text{Mg}(p, p'_1)^{24}\text{Mg}^*$ (1.37 MeV, $J^\pi = 2^+$). Correlations were measured at 35 laboratory energies between $E_p = 3.4\text{--}4.9$ MeV. Excitation functions were measured at five angles for $^{24}\text{Mg}(p, p_0)$ and (p, p'_1) between $E_p = 2.5\text{--}5.9$ MeV in 20-keV steps. Nine previously unreported resonances are observed, corresponding to excitation energies in ^{25}Al ; (energy in MeV, tentative J^π in parentheses) 5.13, 5.58 ($\geq \frac{5}{2}$), 5.76 ($\frac{1}{2}^+$), 6.41 ($\frac{3}{2}^-$), 6.53 ($\geq \frac{5}{2}$), 6.78, 7.02, 7.42, and 7.58.

I. INTRODUCTION

The properties of the excited states of the mirror nuclei ^{25}Al and ^{25}Mg have been studied for many years and the parameters describing the states, particularly below 5-MeV excitation energy, have for the most part been determined. A review of the current experimental and theoretical understanding of these states is contained in a review article by Litherland.¹ The excited states of ^{25}Al below 5-MeV excitation have been described in terms of the Nilsson model viz., four rotational bands based on $K^\pi = \frac{5}{2}^+$, $\frac{1}{2}^+$, $\frac{1}{2}^+$, and $\frac{1}{2}^-$. Recent work² has provided evidence for a $K^\pi = \frac{3}{2}^+$ band which is most likely formed by the coupling of two nucleons (with total $\Omega = 0$) to the intrinsic state of the ^{23}Na ground-state rotational band. The present paper contains the results of experiments performed to measure the spins and parity of the unbound levels of ^{25}Al between 5.30- and 7.14-MeV excitation energy.

The states of ^{25}Al were studied using the reactions $^{24}\text{Mg}(p, p'_1\gamma)^{24}\text{Mg}^*$ and $^{24}\text{Mg}(p, p_0)^{24}\text{Mg}$. For the inelastic scattering studies, the protons leaving the residual nucleus ^{24}Mg in its first excited state at $E_x = 1.37$ MeV were detected in coincidence with γ rays from the decay of this state. These measurements were conducted in the geometry described by Goldfarb and Seyler.³ The measurements exploit only the azimuthal dependence of the γ -ray distribution. From these measurements one can extract information from the correlation coefficients in such a way as to identify, or place a lower limit on the spin of an isolated resonance in the compound nucleus. Information concerning the parity of the resonances studied was obtained by line-shape analysis of the elastic scattering excitation functions. In addition to the above measurements, the absolute excitation function for inelastic proton decay to the first excited

state of ^{24}Mg was measured. Nine previously unreported resonances were observed in the elastic and inelastic excitation functions. Not all of these resonances were strongly excited in both channels. Details of the attempts to assign spin and parity for these resonances are described in Sec. IV.

II. DESCRIPTION OF THE TRIPLE-CORRELATION PROCEDURE

The triple-correlation procedure outlined by Goldfarb and Seyler utilizes a particular geometry such that the complexity of the angular correlation is a function of the spin of the compound-nucleus state formed in the reaction. Instead of choosing the z axis coincident with the incident beam direction, the z axis is chosen to correspond to the momentum direction of the light outgoing reaction product in the c.m. system. In coincidence with this reaction product, the deexcitation γ ray is detected at the surface of a cone, the axis of which is coincident with the z axis. For the choice of a cone half-angle of 90° (and taking the resulting plane of the γ -ray detector as the x - y plane), the form of the correlation function becomes

$$W(\theta_\gamma = \frac{1}{2}\pi, \varphi_\gamma) = \sum_{\kappa} A_{\kappa} \cos \kappa \varphi_\gamma,$$

where only even values of κ are allowed. The properties of the vector coupling coefficients for this geometry limit the complexity (κ) of the correlation function for a reaction proceeding through an isolated resonance in the compound system to

$$\kappa_{\max} \leq \min[(2j_a), (2L)_{\max}, (2l_1)_{\max}, (2j_b - 1)_{\max}],$$

where j_a is the spin of the excited state of the residual nucleus, L is the γ -ray multipolarity, l_1 is the orbital angular momentum of the incident projectile, and j_b is the spin of the compound state. [For j_b equal to an integer, the complexity is

limited by $(2j_b)_{\text{max}}$.] In Table I are indicated the values of κ and the corresponding spin of the compound system. Note that for κ equal to 4, only a lower limit may be placed on the compound-state spin (j_b). For the reaction $^{24}\text{Mg}(p, p'_1)\gamma^{24}\text{Mg}$, inelastic protons to the first excited state of ^{24}Mg at 1.37 MeV ($j_a=2$) are detected in coincidence with the pure electric quadrupole decay ($L=2$) of this state. Both j_a and L then limit the correlation function to a maximum of three terms involving even values of κ , up to $\kappa=4$. Thus on resonance in the inelastic channel, the nonzero coefficient A_κ with the largest value of κ determines (or places a lower limit on) the spin of the state.

Of concern in the interpretation of the angular correlations is the accidental cancellation of one (or more) of the vector coupling coefficients which comprise the A_κ (in addition to the nuclear matrix elements). This possibility has previously⁴ been discussed and eliminated as a source of erroneous interpretation for the present case. The presence of a nonresonant background in the inelastic channel is also of concern. Below an incident proton bombarding energy of 4.2 MeV, there is good evidence⁵ that no direct-reaction mechanism is present although the $^{24}\text{Mg}(p, p'_1)\gamma^{24}\text{Mg}$ data show the presence of a slowly varying background that apparently begins nearby. Our procedure then has been to measure a series of particle γ -ray correlations on and off resonance (in the inelastic channel) assuming that the presence of such a background across the resonance will contribute incoherently to the A_κ .

III. EXPERIMENTAL PROCEDURE

The proton elastic and inelastic excitation functions were measured with a surface-barrier counter and an enriched (99%) self-supporting ^{24}Mg target that was approximately $60 \mu\text{g}/\text{cm}^2$ thick. The data are normalized to the absolute cross-section measurements of Mooring *et al.*⁶ The absolute uncertainty in each point in the present data is less than 10%; relative uncertainties involved in taking the present data are much

TABLE I. Correspondence between the maximum value of κ in the angular-correlation function, $W(\frac{1}{2}\pi, \varphi_\gamma)$ = $\sum_\kappa A_\kappa \cos\kappa\varphi_\gamma$, correlations were measured.

Maximum κ	J
0	$\frac{1}{2}$
2	$\frac{3}{2}$
4	$\geq \frac{5}{2}$

smaller. The data were taken in 20-keV steps for incident proton energies between 2.5 and 5.9 MeV at five angles corresponding to zeros in the Legendre polynomials. These are shown in Figs. 1 and 2 as smooth curves drawn through the data.

The 35 particle- γ -ray correlations were measured with the proton detector at a laboratory angle of 141.3° and with a similar target that was approximately twice as thick, approximately 10 keV at 4 MeV, as that used in the measurement of the excitation functions. A 7.5-cm \times 7.5-cm NaI crystal was used to detect the 1.37-MeV γ ray that is in coincidence with the inelastic proton that leaves ^{24}Mg in its first excited state.

The crystal was approximately 13 cm from the target which was inside a hemispherical stainless-steel chamber of 0.16-cm wall thickness. The

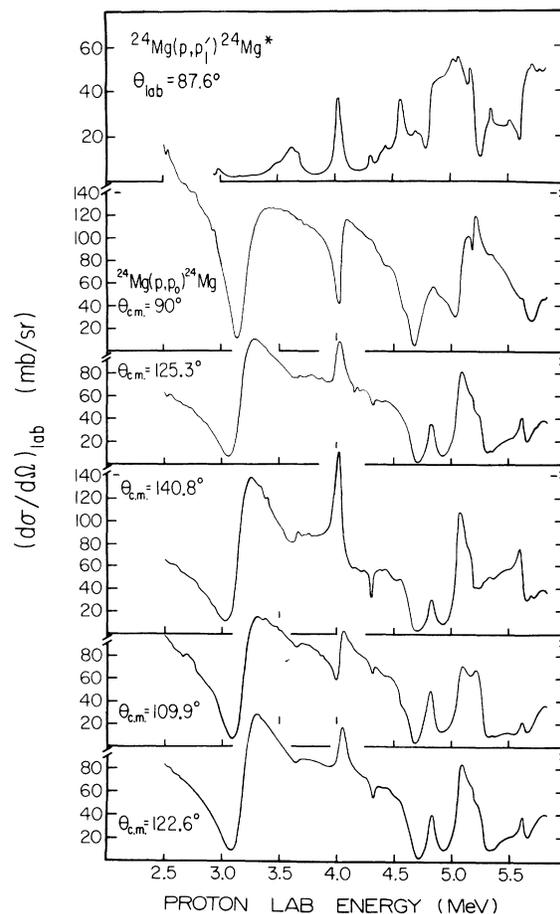


FIG. 1. Elastic scattering excitation functions for c.m. angles corresponding to Legendre-polynomial zeroes beginning with $l=0$ (90°) and increasing consecutively to $l=5$. At the top of the figure is shown the inelastic excitation function for the first excited state of ^{24}Mg at 1.37 MeV taken at $\theta_{\text{lab}}=87.6^\circ$. The data are normalized to the absolute cross-section measurements of Ref. 6. A smooth curve has been drawn through the data.

γ -ray detector was varied in angle about the x axis in the x - y plane (cone angle of 90°). The orientation of this plane is defined by the target-particle detector line (z axis). The $\varphi_\gamma = 0$ position of the γ -ray detector was determined by measuring an angular correlation for plus and minus values of φ_γ . Because of the required symmetry, the angular correction to φ_γ was determined by means of a nonlinear least-squares fit for the angular correlation taken with $\pm\varphi_\gamma$. Conventional fast-slow coincidence techniques incorporating pileup rejection in the fast channels were employed. Total and accidental coincidence spectra were recorded simultaneously and then subtracted. For each datum in an angular correlation, roughly 10^3 net coincidence counts under the 1.37-MeV photopeak were required so that the coefficients A_k obtained from a least-squares fit of $W(\frac{1}{2}\pi, \varphi_\gamma)$ to the

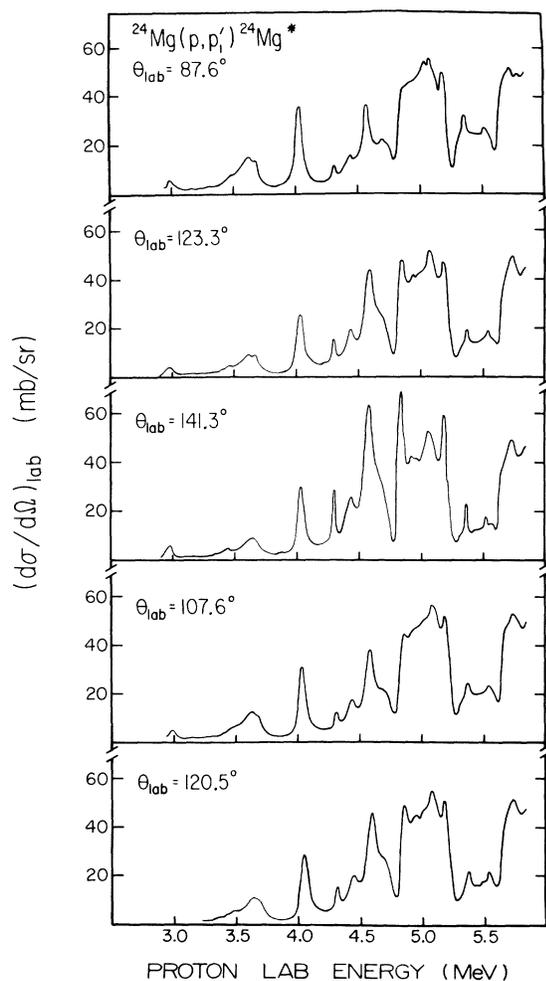


FIG. 2. Inelastic excitation functions to the first excited state of ^{24}Mg taken concurrently with the elastic scattering excitation functions. A smooth curve has been drawn through the data.

data would have uncertainties that were $\pm 20\%$ or less. Typical running time per point was about 3 h.

The yield curves and the correlation data were accumulated with the aid of an IBM 1800 computer interfaced with the experimental electronics.⁷ With the computer operating in a time-sharing mode, previously taken data could be analyzed while new data were being accumulated. This allowed background subtraction, correction for pileup losses, statistical uncertainty estimation, etc., to be performed on line.

In Fig. 3, arrows indicate the resonances on and about which a series of angular correlations were measured with the proton detector fixed at a laboratory angle of 141.3° . An uncertainty of ± 10 keV is assigned to the resonance energies due to beam-energy uncertainties, target-thickness uncertainties, and location of the resonance peak. A typical series of angular correlations for the geometry described is shown in Fig. 4. Shown are six particle- γ -ray correlations taken about the $E_p = 4.02$ MeV resonance. The curve is the result of the least-squares fit of the correlation function $W(\frac{1}{2}\pi, \varphi_\gamma)$ to the data. Since only the relative variation of the correlation coefficients A_k with energy is needed, no geometrical corrections have been made. The uncertainties determined for the coefficients shown in Figs. 6–11 are largely statistical with estimates of charge collection and angular uncertainties also included.

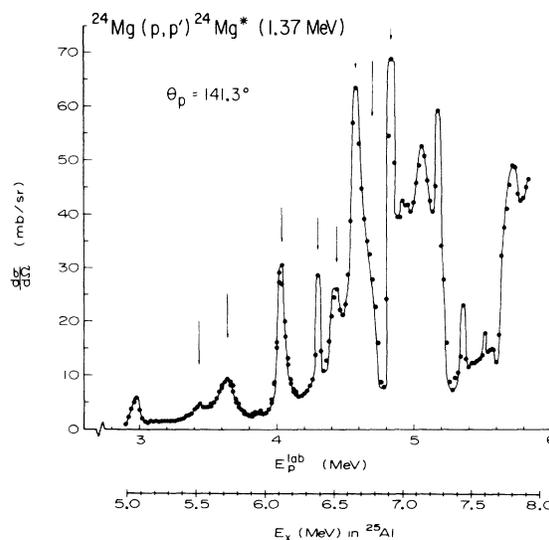


FIG. 3. Inelastic excitation functions to the first excited state of ^{24}Mg showing resonances (arrows) at which a series of particle- γ -ray angular correlations were measured. The proton laboratory angle of 141.3° is the same angle used in taking the angular correlations.

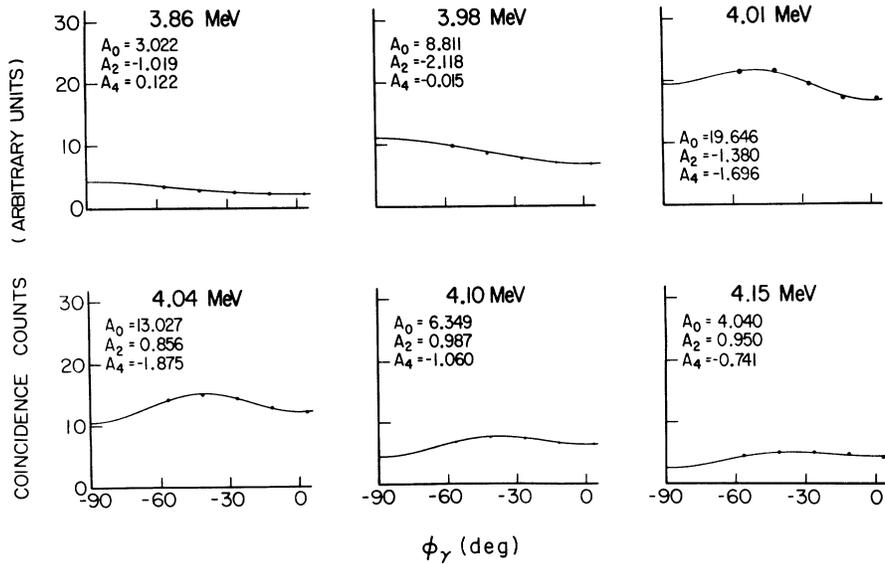


FIG. 4. A series of six particle- γ -ray angular correlations taken on and about the $E_p = 4.02$ MeV resonance. The ordinates are in arbitrary units and each is normalized to accumulated charge. The curve is the result of least-squares fits to the data of $W(\theta_\gamma = \frac{1}{2}\pi, \phi_\gamma)$; the resulting coefficients are shown for each fit. The uncertainties in the data are smaller than the size of the plotted data points.

TABLE II. Resonant structure observed in this experiment showing correspondence to Endt and van der Leun (see Ref. 10). No correspondence was made in column 4 (elastic channel) for the last four resonances observed in the inelastic channel. Column 6 indicates the spin J based solely on the triple correlation, while columns 7, 8, and 9 are the results of best fits to the elastic scattering data. The last column is the assignment based on both, or either of, columns 6 and 7 (see text).

E_p (lab) (MeV)	$^{25}\text{Al}^*$ (MeV)	Endt and van der Leun	Where observed (p, p_0)	(p, p'_1)	Assignment based on ($p, p'_1\gamma$)	(p, p_0)	Γ (keV)	Γ_p (keV)	J^π
2.98	5.13			...					
3.14	5.28			$\frac{1}{2}^+$	184.9	181	$\frac{1}{2}^+$
3.45	5.58			...	$\geq \frac{5}{2}$				$\geq \frac{5}{2}$
3.64	5.76		$\frac{1}{2}$	$\frac{1}{2}^+$	27.6	0.66	$\frac{1}{2}^+$
3.67	5.79					
4.02	6.13	$\geq \frac{5}{2}$	($\frac{3}{2}^+, \frac{5}{2}^+$)	56	26	$\frac{5}{2}^+$
4.31	6.41		$\frac{3}{2}$	$\frac{3}{2}^-$	47.1	8.8	$\frac{3}{2}^-$
4.44	6.53		$\geq \frac{5}{2}$				$\geq \frac{5}{2}$
4.59	6.67	$\geq \frac{5}{2}$				$\geq \frac{5}{2}$
4.70	6.78						
4.85	6.92	$\frac{3}{2}$				$\frac{3}{2}$
4.95	7.02						
5.05	7.12		($\frac{1}{2}^+$)			($\frac{1}{2}^+$)
5.18	7.24					
5.37	7.42						
5.54	7.58						
5.73	7.77					

IV. RESULTS

A. General Remarks

The elastic scattering yield curves were analyzed according to the formalism of Blatt and Biedenharn.⁸ The fits obtained from a simultaneous five-angle search using the code EORCS⁹ in the "single-level approximation" are shown in Fig. 5. Since the data were taken in 20-keV steps, resonances having a natural width of about 20 keV or less are not necessarily observed. Therefore such contributions to the elastic cross section are not included in the fitting analysis. This omission is not serious, since our primary purpose in the yield curve measurements was the location of resonant structure in the inelastic channel that were suitable for the Goldfarb-Seyler triple-correlation

procedure. In general a detailed analysis of the elastic scattering was not necessary to provide parity information.

Fits could not be obtained for laboratory energies greater than $E_p = 4.4$ MeV. To extend the analysis, a "two-level approximation" of the scattering matrix was tried in order to include the effects of interfering resonances of identical spin and parity. An obvious example of such interference is the resonances at $E_p = 4.70$ and 5.05 MeV which, from the general character of their line shape, are taken as $J^\pi = \frac{1}{2}^+$ (see Fig. 5). The result of using the "two-level approximation" worsened the fits for $E_p < 4.4$ MeV while only qualitatively reproducing the shape of the elastic scattering data up to $E_p = 5.1$ MeV. In view of this, further analysis was not pursued. The widths ex-

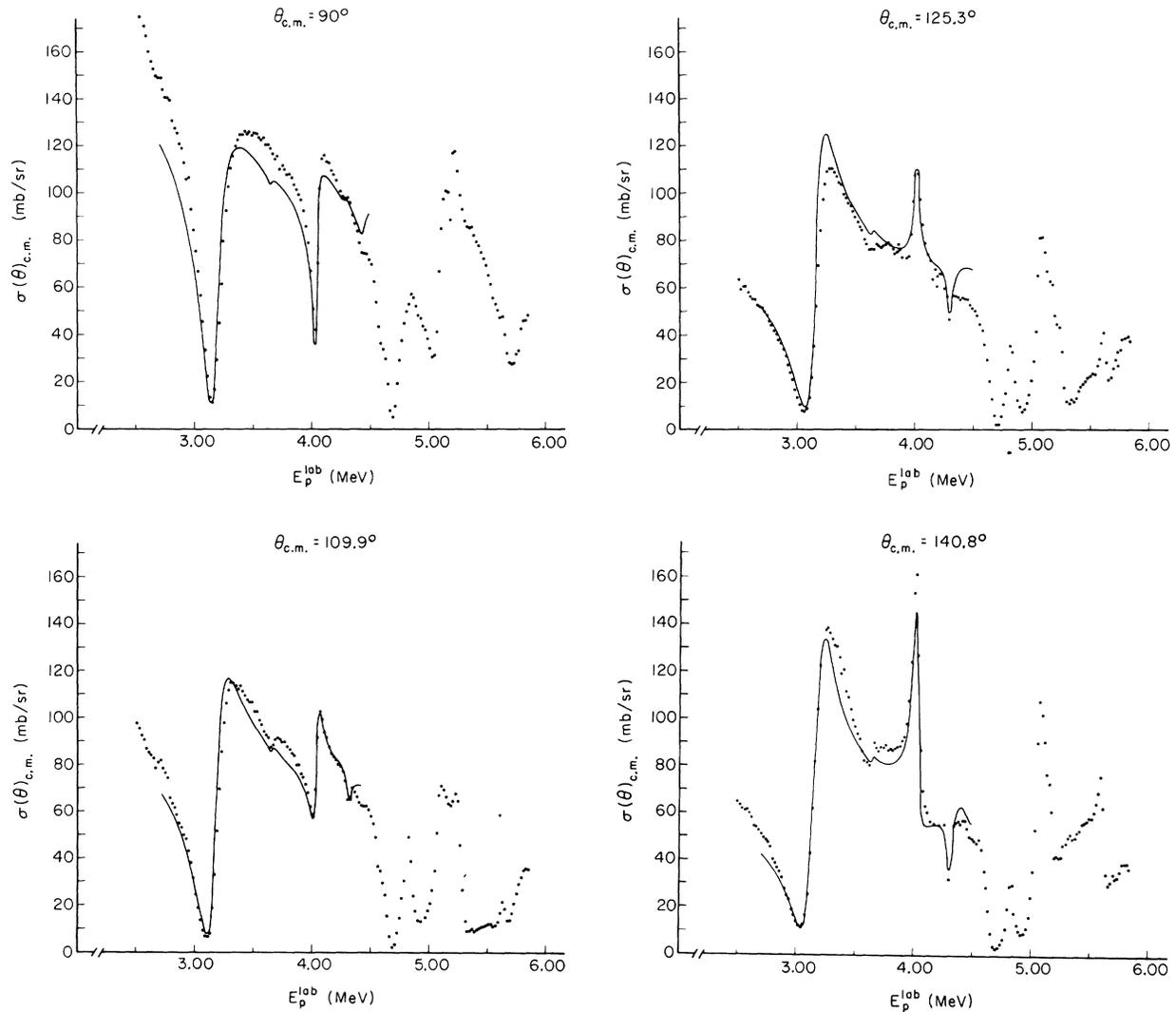


FIG. 5. Final fits obtained from a four-angle search of the elastic scattering data employing the "single-level approximation." Resonant structure observed in the inelastic channel at $E_p = 3.45$ and 3.76 MeV are not included in the fits.

tracted using only the one-level approximation are listed in Table II. The uncertainties in the widths are estimated as $\pm 20\%$.

The resonance at a bombarding energy of 3.14 MeV which corresponds to 5.28-MeV excitation in ^{25}Al could not be measured by means of the present angular-correlation procedure because of the lack of structure in the inelastic channel. However, in the elastic channel the fitting analysis is unambiguous in a $J^\pi = \frac{1}{2}^+$ assignment having a total width of 185 keV.

In the inelastic channel, previously unreported resonances are observed for $E_p = 2.98, 3.45, 3.64, 4.31, 4.44,$ and 4.70 MeV which correspond to excitation energies in ^{25}Al of 5.13, 5.58, 5.76, 6.41, 6.53, and 6.78 MeV, respectively, and of which all but the first are covered by the angular-correlation data. In addition, new resonant structure is observed at $E_p = 4.95, 5.37,$ and 5.54 MeV corresponding to excitation energies of 7.02, 7.42, and 7.58 MeV, respectively. The level reported⁴ at 6.328-MeV excitation was not observed, because of its small width ($\Gamma < 0.3$ keV).

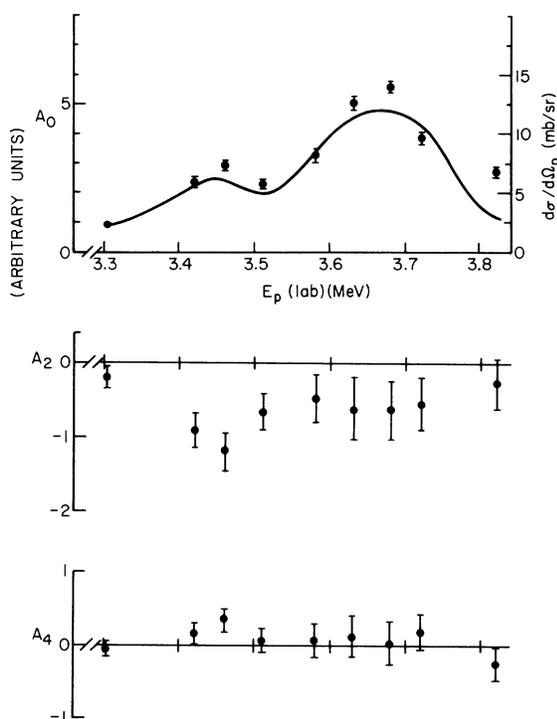


FIG. 6. The angular-correlation coefficients A_k in the region of the 3.45- and 3.64-MeV resonances in ^{25}Al . The solid curve is a superposition of the $^{24}\text{Mg}(p, p_1)$ yield. The ordinate for the yield curve is to the right. The coefficients A_2 and A_4 are plotted on the same relative scale. The statistical uncertainty of the data points is indicated if it is larger than the size of the data points.

B. 3.45-MeV Resonance

The $A_0, A_2,$ and A_4 correlation coefficients shown in Fig. 6 all resonate across the 3.45-MeV state. Hence the spin of this state is assigned as $\geq \frac{5}{2}$. The parity of the state could not be determined, since there is no observable structure corresponding to this resonance in the elastic channel.

C. 3.64-MeV Resonance

There is a broad resonance centered at 3.64 MeV which occurs at all angles in both the inelastic and elastic channels. At detector angles of 87.6 and 123.3° there appears to be evidence in the inelastic channel that this resonance may be two unresolved states centered at $E_p = 3.64$ MeV and at $E_p = 3.67$ MeV. Only the resonance at 3.64 MeV appears appreciably in the elastic channel.

The energy variation of the correlation coefficients measured across the resonance structure is shown in Fig. 6. Only the A_0 coefficient resonates, whereas the A_2 and A_4 coefficients remain essentially constant across the resonance. The en-

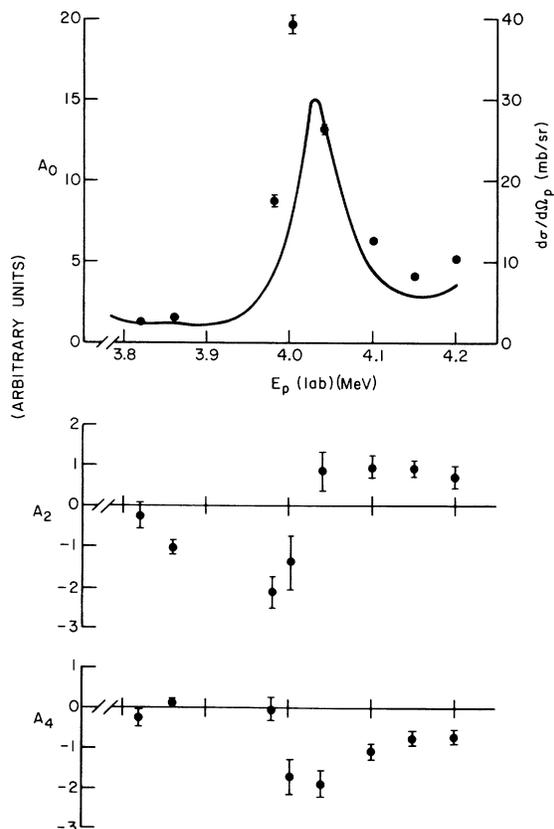


FIG. 7. The angular-correlation coefficients in the region of the 4.02-MeV resonance in ^{25}Al . See caption for Fig. 6.

ergy variation of the coefficients indicate that the spin of the state is $\frac{1}{2}$. The results of the simultaneous five-angle fits to the 3.64-MeV resonance in the elastic yield curves indicate $J^\pi = \frac{1}{2}^+$, total width $\Gamma = 27.6$ keV, and partial width $\Gamma_p = 0.66$ keV. Since the 3.67-MeV state is not resolved in the inelastic channel and does not appear in the elastic channel, no assignment is made.

In the compilation of nuclear energy levels edited by Endt and van der Leun,¹⁰ there is a state in ^{25}Al reported in the $^{24}\text{Mg} + p$ elastic channel at $E_p = 3.67$ MeV, and having $J^\pi = \frac{3}{2}^+$ or $\frac{5}{2}^+$ and $\Gamma < 4$ keV. In the present work a state as narrow as 4 keV could easily have been missed.

D. 4.02-MeV Resonance

The result of a five-angle search on the elastic scattering data was inconclusive for this D -wave resonance. For the $J^\pi = \frac{3}{2}^+$ fit, the ratio of the partial width Γ_p to the total width Γ was 0.6, while for the $J^\pi = \frac{5}{2}^+$ fit, the ratio $\Gamma_p/\Gamma = 0.46$ was obtained. For a given set of resonance parameters ($E_{\text{res}}, \Gamma, \Gamma_p$), the D -wave line shape for a $\frac{5}{2}^+$ resonance is identical in shape but slightly stronger than that for a $\frac{3}{2}^+$ resonance, so that for the 4.02-MeV resonance the strength of the line shape as determined by Γ_p/Γ obtained from our fits to the

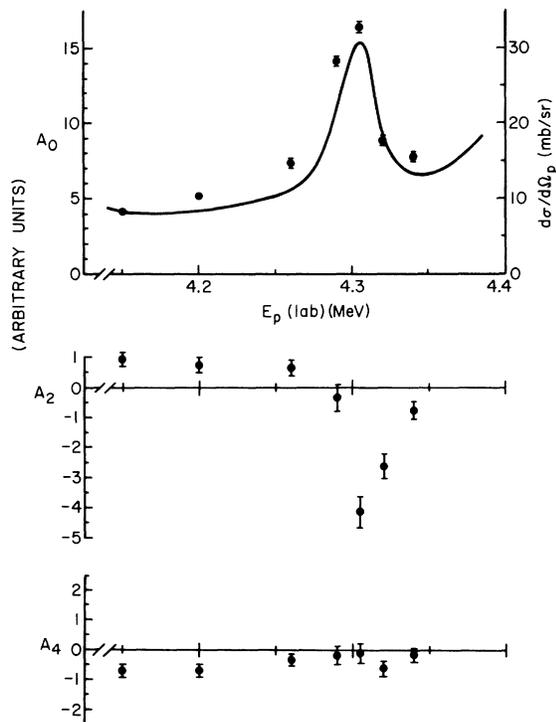


FIG. 8. The angular-correlation coefficients in the region of the 4.31-MeV resonance in ^{25}Al . See caption for Fig. 6.

data makes the distinction between $J^\pi = \frac{3}{2}^+$ or $\frac{5}{2}^+$ uncertain. However, the correlation coefficients $A_0, A_2,$ and A_4 , shown in Fig. 7 clearly show strong resonant structure which indicates a $J \geq \frac{5}{2}$; hence we assign $J^\pi = \frac{5}{2}^+$ for this level corresponding to $^{25}\text{Al}^* = 6.13$ MeV. Since previous^{5,11} line-shape analysis favored $J^\pi = \frac{3}{2}^+$ for this resonance, we remeasured this series of angular correlations both on and off resonance and obtained essentially identical resonant behavior for the coefficients as in the previous measurement. For both series of angular correlations the A_2 coefficient underwent a change of sign on resonance, the significance of which is not understood. The resonance in the inelastic channel appears to be well isolated. The recently reported² $J = \frac{7}{2}$ resonance at $E_p = 4.23$ MeV is not observed due to its width of less than 0.3 keV.

E. 4.31-MeV Resonance

The energy variation of the correlation coefficients across this state is shown in Fig. 8. The

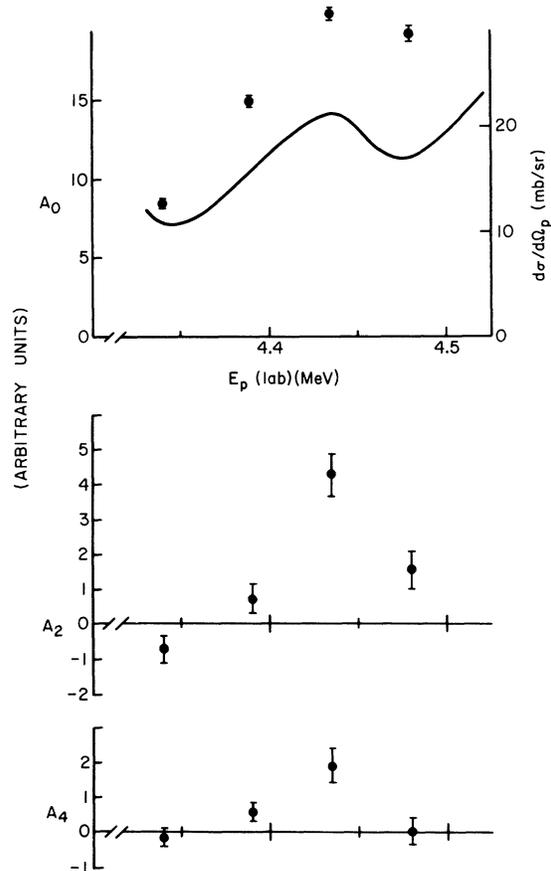


FIG. 9. The angular-correlation coefficients in the region of the 4.44-MeV resonance in ^{25}Al . See caption for Fig. 6.

A_0 and A_2 coefficients resonate strongly, while the A_4 coefficient is essentially constant over their energy range. Hence the spin of the state is assigned as $\frac{3}{2}$. The analysis of the elastic scattering data indicates a P -wave resonance with $J^\pi = \frac{3}{2}^-$. In this case the $J = \frac{1}{2}$ and $\frac{3}{2}$ line shapes are quite distinct.

F. 4.44-MeV Resonance

The results of the angular correlations shown in Fig. 9 indicate a spin of $J \geq \frac{5}{2}$ for this resonance since both the A_2 and A_4 coefficients are strongly resonating. For this and higher-lying resonances in the elastic channel, acceptable fits could not be obtained. The dominance of the strong 4.70-MeV resonance does not permit any useful information concerning the parity of the 4.44-MeV resonance to be obtained.

G. 4.59- and the 4.70-MeV Resonance

In the inelastic yield curves of Fig. 2, one can

observe a strong resonant structure between 4.4 and 4.8 MeV which consists of a dominant resonance at 4.59 MeV and perhaps one or more other resonances. At a laboratory angle of 87.6° the structure is most clearly evident. At this angle the structure appears to be composed of two resonances, one at 4.59 MeV and one at 4.70 MeV. In the elastic yield curves the resonance at 4.59 MeV, which is so dominant in the inelastic channel, is very weak; whereas the resonance at 4.70 MeV, which is weak in the inelastic channel, is very strong in the elastic channel.

The results of the angular-correlation experiments for this resonance structure are shown in Fig. 10. All three of the coefficients initially peak at the 4.59-MeV resonance, leading to an assignment of $J \geq \frac{5}{2}$, and then continue to have finite values over the remainder of the resonant structure. The 4.70-MeV state is not resolved from the 4.59-MeV state at $\theta_p = 141.3^\circ$ and hence no assignment is made for the 4.70-MeV state, since the finite values of the A_2 and A_4 coefficients across this

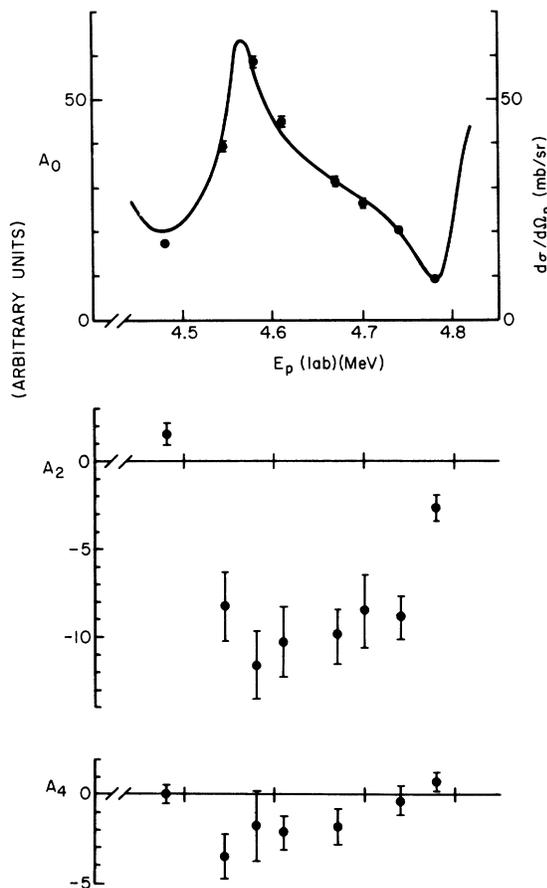


FIG. 10. The angular-correlation coefficients in the region of the 4.59-MeV resonance in ^{25}Al . See caption for Fig. 6.

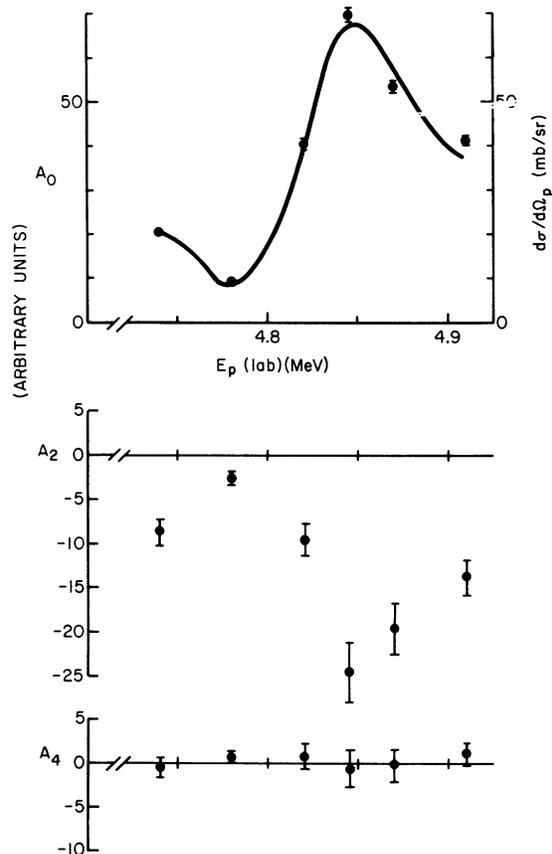


FIG. 11. The angular-correlation coefficients in the region of the 4.85-MeV resonance in ^{25}Al . See caption for Fig. 6.

resonance could be due to the tail of the 4.59-MeV resonance. Although the shape of the 4.70-MeV resonance in the elastic yield curves strongly suggest a $\frac{1}{2}$ spin, we were unable to fit the shape satisfactorily using either the "one-level" or "two-level" approximation.

H. 4.85-MeV Resonance

The correlation coefficients shown in Fig. 11 clearly indicate $J = \frac{3}{2}$ for this resonance. This is consistent with an earlier measurement⁴ employing the same correlation procedure and using a much thicker target. Remaining structure in the inelastic channel was not explored using the angular-correlation procedure. The nonresonant background and the obvious overlapping of a few states would not lead to a clear interpretation of the correlation coefficients. Table II indicates the remaining structure observed in the inelastic channel. No attempt was made to correlate this observed structure with the elastic channel, because of the apparent interference of resonant structure in this channel.

V. SUMMARY

In this experiment we were able to make unambiguous spin assignments for four of the nine resonances studied while lower limits were obtained on spins for three others. The two resonances for which spin assignments could not be made were weak states not completely resolved in energy from nearby states. For excitation energies above 7 MeV in ²⁵Al, most of the states are not resolved one from the other and it is questionable as to whether the Goldfarb-Seyler procedure would lead to unambiguous results for such complex structures. However, the triple-correlation geometry as outlined by Goldfarb and Seyler is a useful procedure for measuring the spins of isolated resonances in compound nuclei. The technique has also been employed for measuring the spins of isolated isobaric analog resonances.¹²

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