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Observability of Nuclear Rearrangement and Choices of Single-Particle Potentials*

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Using a removal-time-dependent separation energy, the relationship of nucleon removal energies to observed spectra and single-particle eigenvalues is demonstrated. Three different choices of the nuclear single-particle potential are discussed and the observability of their eigenvalues is explained. It is shown that "observed" spectroscopic factors should vary with energy and that single-particle energies determined by measuring centroids will also vary with incident energy. Possible evidence for these removal-time-dependent effects in (d,t) and $(p,2p)$ experiments is presented and other experiments are suggested to verify these effects.

I. INTRODUCTION

The lingering problem of the relation of eigenvalues to observables where little progress was reported beyond Koopmans's 40-yr-old result[1] has recently been clarified for atoms (Meldner and Perez,[2] hereafter referred to as MP). In the present paper, we will extend and apply the principles of MP to atomic nuclei. The solution of this problem for the strongly interacting particles in the nucleus pertains to: (1) the theoretical foundations of the shell model,[3] (2) the validity of the Nilsson-Strutinsky[4] prescription, (3) the calculation of nuclear charge densities as measured in elastic electron scattering, (4) the proper interpretation of particle-removal spectra, and (5) the extraction of spectroscopic factors from experimental data.

In the general framework of Brueckner theory, we investigate three choices of nuclear single-particle potentials currently in use in the literature. These choices can be viewed as varying degrees of approximation, and so far no unconditional criterion for which is best seems to exist. In a separate paper,[5] we deal with the basic question of the optimal description of finite many-nucleon systems, i.e., what single-particle potential yields the best wave functions, particle and hole spectra, and other observables. Our conditional result is that the Brueckner-Goldman[6] choice (No. 3, cf. Sec. II) is best. However, it clearly requires complex and more involved calculations. The other choices or approximations could therefore be more practical in many cases. It is thus important to investigate the relationship to observables for the other choices, too.

The connection to the nuclear physics literature is made by showing how the removal-time-dependent effects are described by the three choices. In this way, we clarify major features inherent in the three choices of approximation.

We discuss how the removal-time-dependent effects described in MP can be measured in nuclei. We show that "observed" spectroscopic factors should vary with removal time and that single-particle energies determined by measuring the centroids of removal spectra also depend on the incident energy.

We begin with a review of those results of MP which are relevant to the present work. An observable *separation energy* $s_h(\tau)$ for the removal of the hth particle, which depends on the removal time τ, is defined as the weighted average of the energies of the states of the residual system minus the (initial) ground-state energy:

$$s_h(\tau) \equiv \sum_{\alpha_h} |c_{\alpha_h}(\tau)|^2 [E^{\alpha_h}(N-1) - E(N)]. \qquad (1)$$

The index α_h denotes states of the residual system with a hole in the hth level, beginning with 0_h which has no extra excitation besides the one hole. All other states in the sum integral are of the same spin and parity and have monopole excitations of the remaining particles. The α states can be divided into two groups: (1) "shake-up" states with discrete energies and monopole excitations of some of the remaining particles into bound states, and (2) "shake-off" states with particles excited into the continuum.

The weighting factors $c_{\alpha_h}(\tau)$ contain the removal-

time dependence. If the removal is adiabatic or infinitely slow, $\tau \to \infty$, only the configuration denoted by $\alpha = 0$, which has the lowest missing energy, is excited; i.e.,

$$|c_{\alpha_h}(\infty)|^2 = \delta_{\alpha,0} \tag{2}$$

and

$$s_h(\infty) = E^{0_h}(N-1) - E(N). \tag{3}$$

[Note: It is this adiabatic limit of our quantity $s_h(\tau)$ which is usually referred to as *the* separation energy.]

In the atomic case, one can closely approximate the adiabatic limit experimentally.[2] However, in the nuclear case, hole lifetimes for deeply bound nucleons are too short to allow a good approximation to adiabaticity. (See Sec. IV.)

In a sudden removal, $\tau \to 0$, all the states are excited to their maximal strength – the value given by spectroscopic factors, and we have

$$|c_{\alpha_h}(0)|^2 = |\langle N-1, \alpha_h | a_h | N\rangle|^2 \tag{4}$$

and

$$s_h(0) = \sum_{\alpha_h} |\langle N-1, \alpha_h | a_h | N\rangle|^2 E^{\alpha_h}(N-1) - E(N). \tag{5}$$

MP showed that the separation energy or centroid in the sudden limit,

$$s_h(0) = -\epsilon_h. \tag{6}$$

This is the complete statement of Koopmans's theorem[1] conveniently quoted as $s_h(\infty) = -\epsilon_h + \text{corrections}$. This relation holds in the single-particle approximation where $|N\rangle$ is given by a single determinant and where

$$\epsilon_h = t_{hh} + \sum_{\substack{j \text{ occ} \\ \text{orbitals}}} (v_{hjhj} - v_{hjjh}), \tag{7}$$

which corresponds to the Hartree-Fock (HF) choice of the single-particle potential. Therefore it can be directly applied to atoms. In Sec. II, we show the relationship of single-particle eigenvalues to observed spectra for the three proposed choices of the *nuclear* single-particle potentials.

The removal-speed-dependent rearrangement energy is defined by the equation

$$\Delta_h(\tau) = s_h(\tau) - s_h(\infty). \tag{8}$$

This rearrangement energy moves from zero in the adiabatic limit to the difference between the energy of the threshold peak, $s_h(\infty)$, and the single-particle eigenvalue, ϵ_h, in the sudden limit; i.e., the maximum of $\Delta_h(\tau)$ is $\Delta_h(0) = |\epsilon_h| - s_h(\infty)$.

It is clear that the quantity $s_h(\tau)$, defined above, is in principle observable. For the case of hard x-ray photoionization of a K electron in neon, MP showed that it is in fact observable. However, for the removal of a nucleon from the nucleus, the existence of small far-off contributions from the continuum[7] may make a quantitative determination of the centroid almost impossible. In fact, in the atomic case, the continuum or shake-off states were the major part of the centroid shift in the sudden limit. We certainly expect a similar effect for nuclei.

In Sec. II, we relate the results of MP to nuclear single-particle pictures and describe the relationship of the eigenvalues of three choices of the potential to observables. In Sec. III, a model is presented to illustrate the magnitude of the nuclear rearrangement energy. This allows us to make some interesting observations about the Nilsson-Strutinsky[4] shell-model phenomenology.

Removal-speed-dependent effects can be observed by noting that the full spectroscopic factors are displayed only in the sudden limit. We expect an increase in the ratio of the strength of the shake-up and shake-off states to that of the lowest missing energy peak as the removal speed is increased. In low-resolution experiments, we expect the removal-speed dependence to cause shifts in the measured removal energies. In Sec. IV, we quote experimental evidence which is consistent with our predictions, but which is not absolutely conclusive. We therefore suggest experiments to verify these effects. Section V contains our conclusions.

II. SINGLE-PARTICLE POTENTIALS

The relationship of the single-particle eigenvalues to the observable quantities $s_h(\infty)$ and $s_h(0)$ clearly depends on the choice of the single-particle potential. There have been three proposals for the nuclear single-particle potential that make use of the Brueckner reaction matrix[8] which provides an effective two-body interaction without singularities and for which we assume that an expansion in terms of the number of independent hole lines converges.[9]

The choice which is simplest and most closely analogous to the HF choice in atoms is

$$\epsilon_h^{(1)} = t_h + \sum_{\substack{j \text{ occ} \\ \text{orbitals}}} [K_{hjhj} - K_{hjjh}]. \tag{9}$$

This is the original choice of Brueckner[8] and was advocated and used by Eden and collaborators.[10] By a derivation perfectly analogous to that of MP for Eq. (6), we can show that $\epsilon_h^{(1)}$ is given by a centroid which is weighted by a step-function-occupa-

tion probability, i.e., a "one-sided centroid,"

$$\epsilon_h^{(1)} = \sum_{\alpha_h} |\langle A-1, \alpha_h | a_h | A\rangle|^2 E^{\alpha_h}(A-1) - E(A) . \tag{10}$$

It is straightforward to show[11] that this single-particle spectrum gives large differences of eigenvalues and separation thresholds $s(\infty)$, i.e., "rearrangement" energies. The spacing of eigenvalues $\epsilon^{(1)}$ is normally 2 to 3 times larger than that of separation energies, $s_h(\infty)$, and implies an effective nucleon mass ratio of only $\frac{1}{2}$ or even less.[11,12] This clashes with the conventional notion of the shell model in which it is essentially assumed that eigenvalues equal real separation thresholds.

A partial squeezing of the single-particle levels was proposed by Brandow[13] and Baranger,[14] and calculated by Becker and others.[15] A set of diagrams whose leading term is given by Fig. 1(a) is added to the potential of Eq. (9). The series can be summed and expressed in terms of partial-occupation probabilities. The eigenvalues are then given by the two-sided centroid:

$$\epsilon_h^{(2)} = \sum_{\alpha_h} |\langle A-1, \alpha_h | a_h | A\rangle|^2 [E^{\alpha_h}(A-1) - E(A)] - \sum_{\beta_h} |\langle A+1, \beta_h | a_h^\dagger | A\rangle|^2 [E^{\beta_h}(A+1) - E(A)] . \tag{11}$$

Baranger[14] discussed how this eigenvalue is determined by the short-time behavior of the system so that it is related to the results of a fast reaction.[16] This choice of the potential is considered appealing because more diagrams are included than in $\epsilon_h^{(1)}$ and because of its symmetrical appearance. However, this centroid is even more difficult to determine experimentally than the "one-sided centroid" of Eq. (10) because it requires both a complete particle-addition and particle-removal spectrum.

A proposal made in the development of the

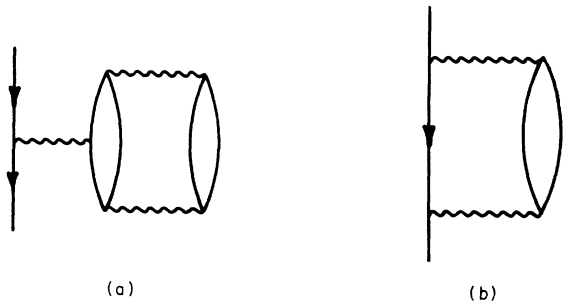

FIG. 1. Lowest order (in K) Brueckner-Goldman rearrangement diagrams: (a) resulting from δe, and (b) from δQ, as discussed in the text.

Brueckner theory by Brueckner and Goldman[6] (BG) is to have eigenvalues equal to the energy required to remove a particle, i.e.,

$$\epsilon_h^{(3)} \equiv \frac{\delta E}{\delta n_h} , \tag{12}$$

where E is the total energy of the many-body system, and n_h is the occupation number of the level h. This is the definition of the single-particle energies used in Landau's[17] theory of a normal system of fermions. It results in nearly independent quasiparticles with a long lifetime from which the transport properties of the Fermi liquid can be predicted.[18] At the Fermi energy, $\epsilon_h^{(3)}$ is the mean energy per particle, and thus fulfills the Hugenholtz-VanHove theorem.[19] Recent papers by Jones and Mohling[20] and Becker and Jones[20] claim that this choice cancels most diagrams in the perturbation expansion for the ground-state energy. It is also closer to the idea of a local shell-model potential, and gives an "effective mass ratio" of unity.[12]

For *nuclear matter*, BG showed that $\epsilon_h^{(3)}$ defined by Eq. (12) differs from $\epsilon_h^{(1)}$ due to the changes in the Pauli operator, δQ, as well as changes in the single-particle spectrum, δe. The δe corrections are those contained in Brandow's[13] partial-occupation probabilities, and consequently in $\epsilon_h^{(2)}$. The leading term in the series for the δQ corrections is shown in Fig. 1(b).

The original BG[6] estimate yielded almost equal energies (about 6 MeV) for both diagrams 1(a) and 1(b) near the Fermi energy, and considerably more for 1(b) than for 1(a) at the deepest bound states (27 vs 9 MeV). Similar results were reported more recently.[21,22] The estimates for 1(b) are rather safe, since this diagram depends mostly on the unambiguous long-range part of the interaction. Becker[23] recently calculated 19.5 MeV for the energy difference of the deepest bound states in No. 2 and No. 3 in Ca. This number times $(0.8)^{-2}$ is to be compared with the above 27 MeV estimated for diagram 1(b).

For *finite* nuclei, there are additional terms due to changes, $\delta\Psi$, in the basis wave functions. The resulting finite size effects have been called orbital rearrangement.[22,24] Such distinctions are problematic: If an orbit changes in a strongly interacting finite system, the interference of all three shifts, $\delta\Psi$, δQ, δe, is important.

For the No. 3 choice of the single-particle potential, the eigenvalue is no longer given by a centroid, but is rather a complex number with an imaginary part which is zero at the Fermi surface and which increases with distance from the Fermi level[25]:

$$\epsilon_h^{(3)} = s_h(\infty) . \tag{13}$$

All three proposals for the single-particle potential coincide in the weak coupling limit, i.e., if terms in the energy of higher order than the first can be neglected (as well as orbital rearrangement in No. 3). If the hole-line expansion converges to the degree estimated by Day,[26] all three give the same result for $s_h(\infty)$ in Eq. (3) up to negligible corrections. However, *finite system wave functions are different for the three choices of the potential* and, consequently, the density distributions are not identical.

Note that the single-particle wave functions of No. 3 have a tail which falls off with a decay constant corresponding to the real hole energy, and thus typically have "longer" tails than the wave functions corresponding to choices Nos. 1 and 2.

As mentioned above, the rearrangement diagrams 1(a) and 1(b) each give a repulsive contribution of about 6 MeV at the Fermi level. Hence, the rms radius of the most weakly bound particle increases in choice No. 2 relative to No. 1, and No. 3 relative to No. 2 by approximately the same amount. Brandow[27] recently estimated 0.4 fm for the radial shift of Fermi level neutrons in No. 2 relative to No. 3.

III. ILLUSTRATIVE MODEL

As a practical example, we derive the potential of choice No. 3, $v^{(3)}$, from a semiempirical nonlocal nuclear $v^{(1)}$ which was determined from a fit to mass defects, radii, etc. throughout the Periodic Table[28]; viz.,

$$v^{(1)}(\vec{r},\vec{r}') = v\frac{\exp(-|\vec{r}-\vec{r}'|)/a}{|\vec{r}-\vec{r}'|/a}\frac{\rho}{2}\left[1-\left(\frac{\rho}{2\rho_1}\right)^{2/3}\right], \tag{14}$$

where v, a, and ρ_1 are parameters (cf. Ref. 28 and Meldner and Shakin[29]). The Fourier transform $v^{(1)}(k)$ is plotted in Fig. 2. Given this $v^{(1)}$, we can calculate the total binding energy

$$E = \sum(t+\tfrac{1}{2}v^{(1)}) = \sum\frac{\epsilon^{(1)}+t}{2}, \tag{15}$$

and thus the difference between the total energy of the ground state of A particles and the energy of the one-hole system, i.e., the separation threshold. In this section, we neglect imaginary parts so that $v^{(3)}$ has this real threshold energy as eigenvalues, $\epsilon^{(3)}$. Therefore, $v^{(3)}$ can be calculated from $v^{(1)}$. Since the density dependence is explicitly given in this model, we can estimate the "rearrangement potential,"

$$\Delta \equiv v^{(3)} - v^{(1)} = \epsilon^{(3)} - \epsilon^{(1)}, \tag{16}$$

from the density derivative of $v^{(1)}$ at constant volume. Such an approximation yields for Δ

$$v^{(3)}(\vec{r},\vec{r}') - v^{(1)}(\vec{r},\vec{r}') \approx -\tfrac{1}{2}v^{(1)}(\vec{r},\vec{r}') + \frac{2v}{\pi}\delta(\vec{r}-\vec{r}')\left[1-\frac{5}{3}\left(\frac{\rho}{2\rho_1}\right)^{2/3}\right] \times\left\{a\left(\frac{3\pi^2\rho}{2}\right)^{1/3} - \tan^{-1}\left[a\left(\frac{3\pi^2\rho}{2}\right)^{1/3}\right]\right\}, \tag{17}$$

i.e., a local and a *repulsive* nonlocal potential with the same nonlocality as $v^{(1)}$. Finite nucleus calculations[30] show that this is, in fact, a good approximation to get eigenvalues $\epsilon^{(3)}$ close to separation thresholds. Brueckner,[3] and Johnson and Teller[11] have shown a long time ago that the empirical nuclear $v^{(1)}$ has a strong nonlocality, corresponding roughly to an effective mass ratio[12] of $\frac{1}{2}$. Equation (16) shows that half the nonlocality of $v^{(1)}$ is removed in $v^{(3)}$. This is a general result for models where $v^{(1)}$ depends explicitly on momentum and density. Combined with the purely local part, this yields effectively a rather weak momentum dependence for the single-particle potential, $v^{(3)}$, as shown in Fig. 2.

To the extent that the local potential in Eq. (17) can be neglected, the *total binding energy* becomes the sum of the separation energies, i.e., simply the *sum of the eigenvalues* in $v^{(3)}$. The latter being widely used in connection with the Nilsson-Strutinsky prescription.[4] We find it holds here in a rough approximation only for somewhat exotic parameters. Realistic parameter sets in this model

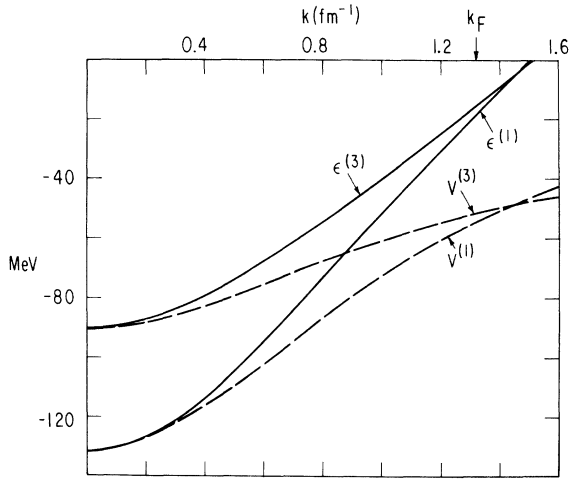

FIG. 2. Lower and upper solid lines are $\epsilon^{(1)}$, $\epsilon^{(3)}$ and dashed lines are $v^{(1)}$, $v^{(3)}$, respectively, as defined by Eqs. (15) through (17). The parameters are very close to those of Refs. 20 and 26, namely, nonlocality $a = 0.8$ fm, potential strength $v = 370$ MeV, and saturation parameter $\rho_1 = 0.36$ fm^{-3}. This corresponds to $k_F = 1.32$ fm^{-1}, and a compressibility $\kappa = 196$ MeV. For details, see also Ref. 27.

yield local potentials that have approximately the same magnitude as the nonlocal one (i.e., $\frac{1}{2}v^{(1)}$) near the Fermi level. Note in this connection that the sum of "realistic" Nilsson eigenvalues is equal to at least *three times* the observed nuclear binding energy. It is interesting to note that the parameters determined from the fit to finite nuclei in Ref. 28 give $\epsilon^{(1)}$ and $\epsilon^{(3)}$ curves (cf. Fig. 2) which lie within a few percent of the original Brueckner-theory calculations (cf. Fig. 2 of Brueckner, Gammel, and Kubis[31]).

IV. DETECTION OF REMOVAL-SPEED-DEPENDENT EFFECTS

For all three choices of the nuclear single-particle potential, we have shown how the eigenvalues are related to observable spectra. For choice No. 3, measurement of the position and width of the leading peak (the one with the lowest missing energy) in the removal spectrum gives the eigenvalue. Whereas, the eigenvalues of choices No. 1 and No. 2 require the measurement of the energies and strengths of all shake-up and shake-off contributions.

The existence of high momentum components in the nucleus will give rise to shake-off contributions to the centroid which are very far from the lowest energy peak. So, even if the strength is small, a significant shift in the centroid energy may occur.[7] Since these would be extremely difficult to observe, a quantitative determination of the centroids may be impossible.

However, it is possible to detect the removal-speed-dependent effects by observing the strengths of a few shake-up peaks for a nearly adiabatic and then for a sudden removal.

The removal time cannot be made arbitrarily long, since the lifetime of the hole is finite. So, an adiabatic removal means a removal time which is long compared with single-nucleon-orbit periods, but not longer than the lifetime of the hole. These conditions cannot be realized to a good approximation when removing the deepest bound nucleons. The $(p, 2p)$ experiments[32] show that the width of the $1s$ proton-hole state is of order 15 MeV with a real separation energy of 45 MeV for $A \approx 40$. For less strongly bound nucleons, we can approach adiabaticity by choosing incident energies near the Coulomb barrier. This is above the single-hole-state width but below the average kinetic energy of the nucleons, 25 MeV, and allows experiments with conveniently large cross sections.

For sudden removals, the speed must be large as compared to nucleon velocities. This is complicated by the fact that short-range correlations give rise to very high momentum components.

The correlations are described by the defect wave function which has approximately 15% overlap with the full nucleon wave function. (For a list of references, see Ref. 22.) As an illustrative example, we have Fourier transformed the defect wave functions of a recent Brueckner-Hartree-Fock calculation for finite nuclei.[33] The corresponding momentum distributions in Fig. 3 show that roughly half the momenta in the defect wave functions are above 2.0 fm^{-1}, corresponding to 100-MeV kinetic energy. Therefore, the outgoing particle should have at least 100-MeV kinetic energy in order to achieve a reasonable approximation to the sudden limit.

We have seen that, in the adiabatic limit, the threshold peak has all the strength [see Eq. (3)]. As the removal speed increases, shake-up and shake-off peaks appear and increase in strength until the removal is sudden and their full strength (given by the spectroscopic factors) is reached. From our estimates of the high-momentum components, we expect a variation in the ratio of the area under the shake-up peaks to the area of the threshold peak of roughly 15%. This variation in the strengths should be no less than 10%, according to an analysis[34] of the strength depleted from the threshold peak to the shake-off states.

The present literature contains some evidence

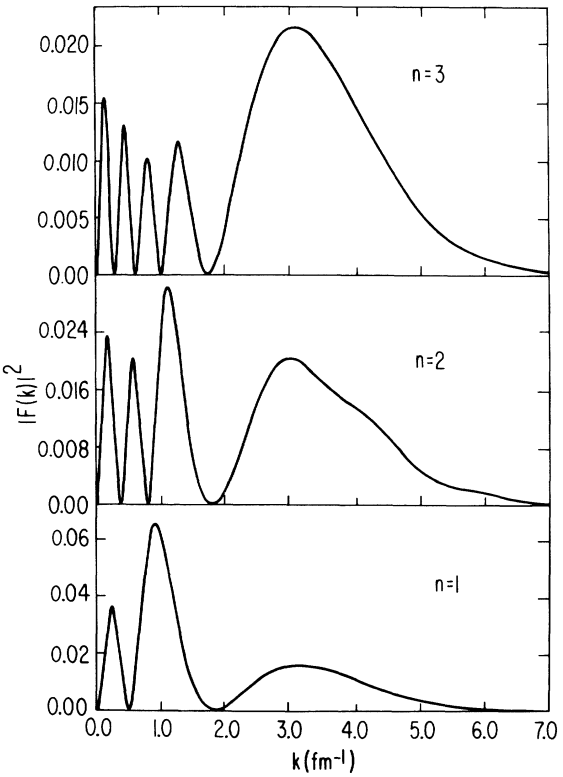

FIG. 3. Fourier transforms of s-state defect wave functions for finite nuclei.

for these effects. A plot of spectroscopic factors determined from neutron removal at energies between 15 and 50 MeV[35] seems to indicate the removal-speed dependence expected from our considerations above. Unfortunately, uncertainties in the distorted-wave Born-approximation analysis for spectroscopic factors make definite conclusions difficult.

Another way to observe removal-speed-dependent effects is by actually looking at shifts in partial centroids. In a low-resolution experiment, the main peak and some of the shake-up peaks will not be resolved so that effectively one measures a partial centroid. Possible evidence for such an effect can be seen in comparing $(p, 2p)$ data on ^{58}Ni for the removal of an $f_{7/2}$ proton with incident protons of energy 46 MeV[36] and 385 MeV.[32] The 46-MeV experiment yields outgoing protons of 15–20 MeV, and therefore should be near the adiabatic limit. The $f_{7/2}$ peak is centered at 8.77-MeV separation energy. At 385 MeV, the analogous peak starts at about 7 MeV, but is centered at approximately 12 MeV. It is suggested that at 385 MeV more strength is in the nearby shake-up peaks leading to a partial centroid with larger separation energy.

In order to verify the removal-time-dependent rearrangement effects, we propose that the relative heights of about three or more peaks with the same spin and parity be observed. Two classes of experiments could be performed to observe the shifts in the relative strengths of the peaks:
(1) A photonuclear, i.e., (γ, p) or (γ, n), experiment with monoenergetic γ rays at a series of energies (varying from slow to fast removals) to observe the energy spectrum of the outgoing nucleons, i.e., for energies near the Coulomb barrier of light nuclei to at least 100 MeV. This is the experiment most analogous to the photoionization experiments which were discussed in detail by MP. However, a drawback might be the small cross sections.
(2) A (p, d), $(p, 2p)$, or $(e, e'p)$ experiment where the energy of the incident particle is varied and the energy spectrum of the outgoing particle or particles observed.
The strong interaction experiments might be easier, but the analysis is not as clean as in the photonuclear or $(e, e'p)$ reactions.

V. CONCLUSIONS

We have defined a separation energy which depends on the removal time and have related it to observed binding energies for nucleon removal from the nucleus. Two limits, when the removal time is either very short, $\tau \to 0$, or very long, $\tau \to \infty$, have been considered.

In these two limits, it is possible to demonstrate the relationship of the observed spectra to various choices for the nuclear single-particle potential:
(1) For the choice in which no rearrangement graphs are included [see Eq. (9)], the eigenvalues are equal to a one-sided centroid of energies in the residual nucleus [see Eq. (10)]. This eigenvalue can be observed only by performing a *complete* spectral measurement in the sudden limit, $\tau \to 0$.
(2) The choice which includes partial occupation probabilities [diagrams of the type shown in Fig. 1(a)] has eigenvalues which are given by a two-sided centroid [see Eq. (11)]. Again, it can only be observed in the sudden limit and requires *complete* spectrum measurements for *both* removal and capture (pickup and stripping) experiments.
(3) The Brueckner-Goldman choice which adds all the rearrangement graphs is shown to have eigenvalues equal to removal thresholds. These can be observed for any value of the removal time which is shorter than the lifetime of the hole, as discussed in Sec. IV. As discussed in a separate paper,[5] we can also show that this choice is optimal under certain conditions. Phenomenological analysis with (local) Nilsson and Woods-Saxon potentials seem to lie very close to this choice.

In conclusion, we can state that although the real eigenvalues of choices No. 1 and No. 2 are simpler to calculate, the experimental determination of centroids is very difficult. Whereas choice No. 3 requires more elaborate and complex calculations, but is directly related to easily measured data.

In any case, it is possible to observe removal-time-dependent effects in nucleon-removal experiments. These effects would be displayed by an energy dependence of spectroscopic factors and indeed hints of such dependence have been observed. It is also possible to resolve these effects by investigating the shift of partial centroids in low-resolution experiments. We have cited evidence for such a shift from two $(p, 2p)$ experiments done with high and low incident proton energy.

But at this point evidence is certainly insufficient. We therefore propose experiments to observe the heights of two or more shake-up peaks relative to the threshold peak as the removal speed varies from the adiabatic to the sudden limit. By employing our present knowledge of the relative two-nucleon wave function, we have estimated the high-momentum components present in the nucleus. Thus, we are able to make predictions about the energy required to reach the sudden limit and how large a shift in the relative heights of the peaks is to be expected. On the other hand, analysis of such experiments would yield information about the correlations and high-momentum components present in the nucleus.

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