(unpublished). It is included in the ABACUS-M code as distributed to several laboratories [E. H. Auerbach and M. M. Sternheim, Brookhaven National Laboratory Report No. BNL-12696, July 1968 (unpublished)]. More recently it has been discussed by C. Wilkin [CERN Report No. CERN 71-14, 1971 (unpublished), p. 289], H. K. Lee and H. McManus [Nucl. Phys. A167, 257 (1971)], and G. Fäldt [Phys. Rev. C 5, 400 (1972)].

[15]R. M. Edelstein, W. F. Baker, and J. Rainwater, Phys. Rev. 122, 252 (1961).

[16]L. S. Kisslinger, M. M. Sternheim, R. L. Burman, and J. H. Koch (to be published).

[17]C. Wilkin, Ref. 4.

[18]H. K. Lee and H. McManus, Ref. 14.

[19]J. Beiner and P. Huguenin, as quoted by C. Wilkin, Ref. 14, p. 291.

[20]M. P. Locher, O. Steinmann, and N. Straumann, Nucl. Phys. B27, 590 (1971).

[21]J. P. Dedonder, Institut de Physique Nucléaire, Orsay, Report No. IPNO/TH 71-30 (to be published).

PHYSICAL REVIEW C VOLUME 6, NUMBER 3 SEPTEMBER 1972

Reactions (*p*, *pd*) and (*p*, 2*p*) on Helium-3 at 590 MeV

P. Kitching, G. A. Moss, and W. C. Olsen*
Nuclear Research Centre, University of Alberta, Edmonton 7, Canada

and

W. J. Roberts
N.A.S.A. Lewis Research Center, Cleveland, Ohio 44135

and

J. C. Alder,† W. Dollhopf, W. J. Kossler, and C. F. Perdrisat
College of William and Mary, Williamsburg, Virginia 26506

and

D. R. Lehman‡
National Bureau of Standards, Washington, D. C. 20234

and

J. R. Priest
Miami University, Oxford, Ohio 45056
(Received 31 May 1972)

The reaction $^3He(p,pd)$ has been studied for recoil momenta of the spectator proton up to 230 MeV/c. The cross section of $^3He(p,2p)$ was measured up to 100 MeV/c. The results are interpreted in terms of the plane-wave impulse approximation and the form factors for $^3He \to dp$ and $^3He \to p(pn)$ are compared with a calculation of the ^3He wave function in which the two-nucleon interaction is described by a separable potential which reproduces the low-energy properties of the two-nucleon system.

A high-pressure gaseous helium-3 target containing $(1.67 \pm 0.025) \times 10^{21}$ atoms/cm^3, 30-cm long and 8-cm inside diameter, was bombarded with 590-MeV protons in the external beam of the National Aeronautics and Space Administration synchrocyclotron at the Space Radiation Effects Laboratory.[1] A magnetic spectrometer-range telescope arrangement, including wire spark chambers, was used to detect charged-particle events. The momentum and mass of the particle on the magnet side, and the energy of the particle on the range side were used, in combination with the scattering angles, to determine the momentum of the recoiling nucleus event by event. The layout of the apparatus is shown inFig. 1. (For more details see Alder *et al.*[2])

The data for $^3He(p,pd)$ were obtained with the proton telescope at $\bar{\theta}_1 = 58°$ and the magnet telescope at $\bar{\theta}_2 = 43°$ to the beam for recoil momenta smaller than 100 MeV/c, and at $\bar{\theta}_1 = 68°$ and $\bar{\theta}_2 = 48°$, respectively, for recoil momenta larger than 100 MeV/c. For each event, the recoil momentum, $\vec{q}(q_{\parallel}, q_{\perp}, q_{\text{out}})$ of the residual proton was calculated from the measured angles θ_1 and θ_2, and the kinetic energy E_1 and the momentum p_2. Here $q_{\parallel}$, $q_{\perp}$, and q_{out} are the components of the recoil momentum

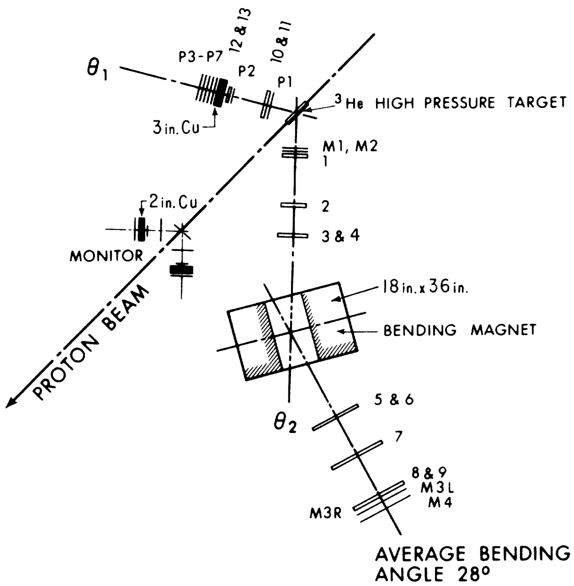

FIG. 1. Experimental arrangement used to investigate both reactions. Numbers 1 to 13 are wire spark planes; M1–M4 and P1–P7 are scintillation counters.

along the beam, perpendicular in the plane, and perpendicular out of the plane of the reaction.

A Monte Carlo simulation of the experiment showed that, at the first set of angles of the magnet and range telescopes, the transmission of the system was constant for values of the transverse recoil component, $q_\perp$, between −100 and +100 MeV/c (a positive transverse component occurs when the projected deuteron momentum is larger than the projected proton momentum). That part of the data was, therefore, analyzed in terms of the transverse recoil, $q_\perp$, summing over all values of the longitudinal and out-of-plane projection, $q_\parallel$ and q_{out}, accepted in the experiment. In the second position of the detectors, the transmission was a function of both the longitudinal and transverse components of $\vec{q}$. Here, a summation over the transverse and out-of-plane recoil projections, $q_\perp$ and q_{out}, was performed because the largest component was the longitudinal one.

If we call $T(E_{\text{miss}}, \vec{q})$ the transmission of the system for events with missing energy around $E_{\text{miss}} = E_0 - (E_1 + E_2 + E_{\text{recoil}})$ and with recoil momentum around $\vec{q} = \vec{p}_0 - (\vec{p}_1 + \vec{p}_2)$, where E_0, E_1, E_2 and $\vec{p}_0$, $\vec{p}_1$, and $\vec{p}_2$ are the kinetic energies and momenta of the incident proton and scattered proton and deuteron, respectively, then the cross section is given in both cases as

$$\frac{d^5\sigma(E_{\text{miss}}, \vec{q})}{d\Omega_1 d\Omega_2 dE_1} = \frac{N(E_{\text{miss}}, \vec{q})}{T(E_{\text{miss}}, \vec{q})} \frac{1}{\Delta\Omega_1 \Delta\Omega_2 \Delta E_1} \frac{1}{nI\epsilon}. \qquad (1)$$

$N(E_{\text{miss}}, \vec{q})$ is the number of experimental events within defined energy and recoil-momentum increments around E_{miss} and $\vec{q}$, respectively, $\Delta\Omega_1$ and $\Delta\Omega_2$ are the geometrical solid angles of the defining counters, and ΔE_1 is the total energy acceptance of the range telescope. The number of ^{3}He atoms within the effective length of target considered is n, and I is the number of incident protons; ϵ is the efficiency of the range telescope. The effective length of the target was defined by applying cuts in the reconstruction of the interaction point.

The cross-section data for ^{3}He(p, $2p$) were obtained from Eq. (1) with particle 2 now being a proton. Only one set of angles $\bar{\theta}_1 = \bar{\theta}_2 = 43°$ was studied for this reaction, and again a Monte Carlo calculation of the transmission of the spectrometer indicated a constant transmission as a function of $q_\perp$ between −90 and +90 MeV/c. With the two telescopes set at 43° the system selected reactions with nonvanishing average longitudinal recoil component, $\bar{q}_\parallel \sim 41$ MeV/c. Furthermore, because the over-all resolution in energy of

TABLE I. Cross section and momentum density as a function of recoil momentum (q).

Reaction	Angles (deg)	Recoil momentum (MeV/c) $\bar{q}_\perp$	$\bar{q}_\parallel$	q	Cross section $\frac{d\sigma}{d\Omega_1 d\Omega_2 dE_1}$ (mb/sr^2 MeV)	Momentum density $n\rho(q)$ (MeV/c)$^{-3}$ $\times 10^{-7}$
^{3}He(p, pd)p	43–58	0	0	33	0.0425 ± 0.0020	2.2 ± 0.1
		30	0	45	0.0362 ± 0.0016	1.87 ± 0.08
		60	0	69	0.0226 ± 0.0010	1.15 ± 0.05
		90	0	96	0.0109 ± 0.0010	0.55 ± 0.05
	48–68	0	125	134	0.0029 ± 0.0007	0.142 ± 0.034
		0	175	185	0.0008 ± 0.0002	0.041 ± 0.010
		0	225	233	0.0005 ± 0.0001	0.022 ± 0.005
^{3}He(p, $2p$)	43–43	0	41	46	1.76 ± 0.08	2.56 ± 0.11
		30	41	55	1.55 ± 0.06	2.26 ± 0.08
		60	41	76	0.97 ± 0.05	1.41 ± 0.07

(E_1+E_2) was 16 MeV [full width at half maximum (FWHM)], the data may contain events in which the proton-neutron pair is not a deuteron, the separation energy of the deuteron being 2.2 MeV, but has relative kinetic energy up to about 16 MeV.

The cross-section data can be interpreted in terms of the plane-wave impulse approximation, leading to the well-known relation

$$\frac{d^5\sigma(\vec{q}_2)}{d\Omega_1 d\Omega_2 dE_1} = k(d\sigma/d\Omega)^{\text{c.m.}}_{12\to 12} n_2\rho(\vec{q}_2)\,, \qquad (2)$$

where $\vec{q}_2$ is the internal-motion momentum of the struck particle 2 that is obtained from the recoil momentum $\vec{q}$ by $\vec{q}_2 = -\vec{q}$. The kinematic factor k is given in the literature (cf. for example, Jain *et al.*[3]), $(d\sigma/d\Omega)^{\text{c.m.}}_{12\to 12}$ is the free-scattering cross section for $1+2\to 1+2$, and $n_2\rho(\vec{q}_2)$ is the effective number of particles of type 2 participating in the reaction multiplied by the momentum density distribution of that particle with the condition $\int \rho(\vec{q}_2)d^3\vec{q}_2 = 1$. Using the above formula and the 90° c.m. values $(d\sigma/d\Omega)^{\text{c.m.}}_{pd} = 0.050 \pm 0.002$ mb/sr and $(d\sigma/d\Omega)^{\text{c.m.}}_{pp} = 2.58 \pm 0.03$ mb/sr,[4] one obtains the values $n\rho(q)$ shown in Table I and Figs. 2(a) and 2(b). The recoil momentum $q = |\vec{q}|$ in the table is calculated as

$$q = (q_\perp{}^2 + \langle q_\parallel{}^2\rangle + \langle q_{\text{out}}{}^2\rangle)^{1/2}$$

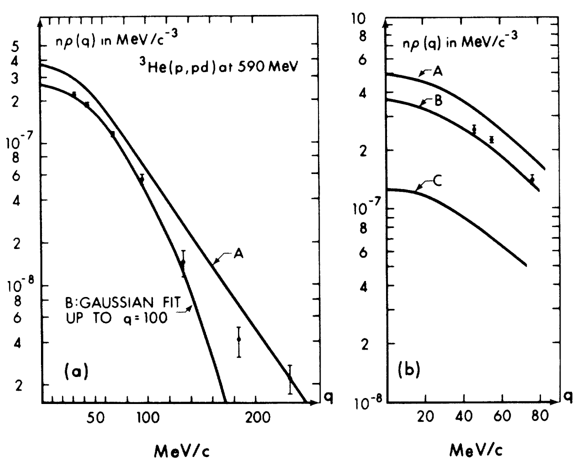

FIG. 2. (a) The momentum distribution $n_d\rho(q)$ for a deuteron in ^{3}He obtained assuming the validity of the impulse approximation. Curve A is calculated from a $^3\text{He}\to d+p$ vertex function that gives satisfactory fits to the quasielastic electron-scattering data of Johansson (Ref. 5) and Hughes *et al.* (Ref. 7). Curve B is the best fit with Gaussian shape to the data points below 100 MeV/c. (b) The momentum distribution for a proton on ^{3}He. Curve A is calculated for the total contribution of the deuteron, $^3S_1$ and $^1S_0$ continuum (np) states as explained in the text. Curves B and C show the bound- and continuum-state contributions separately.

for the first set of angles and

$$q = (q_\parallel{}^2 + \langle q_\perp{}^2\rangle + \langle q_{\text{out}}{}^2\rangle)^{1/2}$$

for the second, where $\langle q_x{}^2\rangle$ is the average square of a component x over which the recoil has been integrated. This procedure amounts to a first-order resolution correction if the shape of the cross section is Gaussian; in this experiment, the cross-section data are well fitted by Gaussians. In the first angular situation, each data point is an average for $q_\perp$ and $-q_\perp$, and accordingly, an average value of k, the kinematic factor, has been used in calculating $n\rho(q)$. The errors quoted in Table I are statistical only. For the second angular position, the errors have been set arbitrarily at 25% to take into account a possible uncertainty in shape of the $q_\parallel$ dependence of the transmission.

It must be emphasized that the resolution of the experiment did not allow a separation of the d and (np) final states for the $(p, 2p)$ reaction; therefore, the values of $n_p\rho(q)$ in the table and in Fig. 2(b), deduced from the $(p, 2p)$ cross-section data include an unknown contribution from (np) final states. The systematic errors arising from uncertainties in counter efficiencies, beam characteristics, target pressure, solid angles, range-counter efficiency, and reconstruction probability are estimated at ±15% for (p, pd) and ±10% for $(p, 2p)$.

The impulse approximation predicts that the function $n\rho(q)$ obtained from (p, pd) and $(p, 2p)$ on ^{3}He should be the same provided the residual nucleus in the second reaction is a bound deuteron. Ignoring for the time being the contribution from disintegrated deuterons and assuming Gaussian shapes for the momentum distributions, one can obtain the two parameters A and Γ in $n\rho(q) = Ae^{-q^2/\Gamma^2}$ by least χ^2 fitting to the data. The following results are obtained from the data points with $q < 100$ MeV/c:

$$(p, pd):\ n_d = 0.68 \pm 0.09\,,\quad \Gamma_d = 78 \pm 3 \text{ MeV}/c\,;$$

$$(p, 2p):\ n_p = 1.00 \pm 0.13\,,\quad \Gamma_p = 79.7 \pm 3 \text{ MeV}/c\,.$$

The (p, pd) fit was limited to 100 MeV/c to facilitate a comparison with the $(p, 2p)$ fit.

In the above values of n, the errors include the estimated systematic contribution, and take into account a cancellation of the uncertainties in the free-elastic cross section which were measured by the same method as the quasielastic ones. The widths of the distribution obtained from both reactions are equal within uncertainties, and both are well fitted by Gaussians for recoil momenta smaller than 100 MeV/c. The larger value of n_p probably reflects the contribution of (np) final states in the $(p, 2p)$ reaction.

It is interesting to compare the above results with those of a quasielastic electron-scattering experiment performed by Johansson.[5] In that experiment, the fivefold differential cross section for the reactions ^{3}He$(e, e'p)$ and ^{3}H$(e, e'p)$ was determined with 550-MeV electrons. The interpretation of this data has been the object of a number of studies. In particular, Lehman[6] has calculated the cross sections to compare with this data and with that of Hughes[7] for the quasielastic reactions ^{3}He(e, e') and ^{3}H(e, e'). Good agreement was obtained, except with the ^{3}He$(e, e'p)$ coincidence results. In this calculation, the ^{3}He-$d(p)$ and ^{3}He-$p(d)$ vertices are obtained from wave functions that are exact solutions of the three-nucleon Schröedinger equation with a separable potential for the two-nucleon interaction. The separable nonlocal potential of Yamaguchi[8] was used with Tabakin's[9] parameters which give reasonably good fits to the low-energy properties of the two-nucleon system. With these wave functions the momentum distributions measured in the present experiment, $n\rho(q)$, are calculated in the pole-dominance approximation which is equivalent to the plane-wave impulse approximation. The (p, pd) momentum distribution is obtained from the deuteron-exchange diagram, while the $(p, 2p)$ distribution is calculated as the sum of proton-exchange diagrams which have as the spectator pair the deuteron, the $(np)^3S_1$ continuum, or the $(np)^1S_0$ continuum states. Relative (np) momenta up to 87 MeV/c were used. This corresponds to one half of the experimental resolution on the sum (E_1+E_2). The result is relatively insensitive to the cutoff.

The resulting momentum distributions are shown in Figs. 2(a) and 2(b). For the $(p, 2p)$ result, the d and np final-state contributions are shown both separately and summed. The calculation has no free parameters. Both spectra have a shape consistent with the data. The (p, pd) experimental points appear to be lower by a factor of 1.40 ± 0.15 than the calculated values for $q < 100$ MeV/c. The corresponding number in the case of the $(p, 2p)$ reaction is 1.20 ± 0.18, where the error includes the estimated systematic errors.

Recently a ^{3}He$(p, 2p)$ experiment has been carried out by Frascaria *et al.*[10] with 155-MeV protons, with a resolution sufficiently good to separate the d from the (np) final states. The width at $1/e$ observed is about 58 MeV/c, to be compared with the value of 79 MeV/c found in the present experiment. As explained above, the present data are corrected in first order for finite resolution. Hence the difference in observed widths between the 590- and the 155-MeV results must have some other origin.

The value of $n\rho(q=0)$ from the 155-MeV experiment is in closer agreement with the one calculated according to Ref. 6 than with those at 590 MeV.[11] Another ^{3}He$(p, 2p)$ experiment at 35 MeV, reported by Slaus *et al.*[12] appears to be in fair agreement with the 590-MeV results as far as the width of the $n\rho(q)$ distribution is concerned, although the value of $n\rho(q=0)$ is about one half the 590-MeV value.

The authors wish to thank the Space Radiation Effects Laboratory Director, Dr. Robert T. Siegel, and his staff for their assistance during the course of this experiment.

---

*Work supported in part by the Atomic Energy Control Board of Canada.

†Work supported by the Swiss Institute for Nuclear Research (S.I.N.). Present address: Deutsches Electronen Synchrotron, Notkestieg 1, 2000 Hamburg-52.

‡National Academy of Science – National Research Council Postdoctoral Research Associate.

[1]The preliminary results of this experiment have been presented by J. C. Alder, W. J. Kossler, C. F. Perdrisat, W. K. Roberts, P. Kitching, G. A. Moss, W. C. Olsen, and J. R. Priest, Bull. Am. Phys. Soc. 16, 132 (1971).

[2]J. C. Alder, W. Dollhopf, W. J. Kossler, C. F. Perdrisat, W. K. Roberts, P. Kitching, G. A. Moss, W. C. Olsen, and J. R. Priest, Phys. Rev. C 6, 18 (1972).

[3]M. Jain, P. C. Roos, H. G. Pugh, and H. D. Holmgren, Nucl. Phys. A153, 49 (1970).

[4]W. K. Roberts, J. S. Vincent, E. T. Boschitz, K. Gutow, M. Blecher, P. Gugelot, C. F. Perdrisat, L. W. Swenson, and J. R. Priest, NASA Technical Note No. NASA TN D-5961, 1970 (unpublished).

[5]A. Johansson, Phys. Rev. 136, B1030 (1964).

[6]D. R. Lehman, Phys. Rev. C 3, 1827 (1971).

[7]E. B. Hughes, M. R. Yearian, and R. Hofstadter, Phys. Rev. 151, 841 (1966).

[8]Y. Yamaguchi, Phys. Rev. 95, 1628 (1954).

[9]F. Tabakin, Phys. Rev. 137, B75 (1965).

[10]R. Frascaria, V. Comparat, N. Murty, M. Morlet, A. Willis, and N. Willis, Nucl. Phys. A178, 307 (1971).

[11]D. R. Lehman, to be published.

[12]I. Šlaus, E. Epstein, F. Paić, J. R. Richardson, D. L. Shannon, J. W. Verba, H. H. Forster, C. C. Kim, D. Y. Park, and L. C. Welch, Phys. Rev. Letters 27, 751 (1971).