Effects of Λ-Σ Coupling in $^4_\Lambda$H, $^4_\Lambda$He, and $^5_\Lambda$He†

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We consider the effect on the binding energies of $^4_\Lambda$He-$^4_\Lambda$H of inclusion of admixing the Σ particle, based on a Λ-N potential derived from $^5_\Lambda$He. This mixing explains the discrepancy in Coulomb energy of $^4_\Lambda$He-$^4_\Lambda$H and leads to a consistent set of potential parameters.

I. INTRODUCTION

It has long been recognized that in theoretical treatments of the hypernuclear problem the coupling between the Λ and Σ hyperons will have significant effects.[1-3] Many authors have speculated on the consequences of this coupling,[1-13] in particular on what role the virtual coupling term might play in explaining a number of puzzling difficulties in our understanding of the $^5_\Lambda$He isosinglet and the $^4_\Lambda$H-$^4_\Lambda$He isodoublet systems.[3,9-13] We wish to present here certain quantitative estimates of the Λ-Σ coupling effects and to show that these effects are, in fact, of the size and sign required to resolve several discrepancies in our models of these nuclei.

The experimental Λ-separation energy of $^5_\Lambda$He is 3.08 MeV, and those of $^4_\Lambda$H and $^4_\Lambda$He are 2.31 and 2.02 MeV, respectively.[14] A number of theoretical attempts have been made to determine these energies in terms of basic Λ-N potentials.[13,15-23] The conclusions resulting from these calculations may be summarized as follows:

(1) It is possible to determine a reasonable Λ-N potential well depth and range that will produce the correct Λ-separation energy in $^5_\Lambda$He. It is found that the presence of the Λ tends to compress the ^{4}He core only slightly.[13,16] This potential, however, is then insufficient to bind the Λ in the $^4_\Lambda$H-$^4_\Lambda$He hypernuclei. It proves necessary to increase the Λ-N well depth appreciably to achieve the empirical binding energies of these two hypernuclear systems. Since the potentials for the isosinglet and isodoublet cases represent different combinations of the Λ-N triplet and singlet interactions, one is lead to infer a strong spin dependence for the Λ-N potential. This spin dependence is much stronger than that required to understand the free Λ-N scattering data.

(2) The difference in the Λ-separation energies (ΔB_Λ) of the $^4_\Lambda$H-$^4_\Lambda$He isodoublet, which has generally been assumed to be purely a Coulomb effect, is not explainable as such. As in the case of $^5_\Lambda$He, the presence of the Λ tends to compress the nuclear core. Unfortunately, the corresponding ^{3}He core compression produces a shift in the Coulomb energy that is of the wrong sign and generally too small to account for the observed difference in the Λ-separation energies. This has led several authors to propose a charge-symmetry breaking term in the Λ-N interaction.[9,24-26]

Bodmer first recognized that the effects of the Λ-Σ coupling had been neglected in these calculations, and that this coupling would tend to alleviate the theoretical difficulties.[3] For reasons discussed in Sec. II, the Λ-Σ coupling term, which is present in the $^4_\Lambda$H-$^4_\Lambda$He doublet, is greatly suppressed in the $^5_\Lambda$He singlet leading to a corresponding reduction in the Λ-separation energy. Furthermore, the admixture of the $\Sigma(T=1)$ state into the $^4_\Lambda$H wave function gives a contribution of the correct sign to account for the ΔB_Λ energy difference.

In this paper we wish to show quantitatively that these speculations tend to be confirmed. Furthermore, we show that the $^4_\Lambda$H-$^4_\Lambda$He energy difference arises not only from the admixture of the charged Σ hyperons, but equally from the mass splitting in the Σ isotriplet state. In Sec. II we outline the methods used in the calculation. In Sec. III we present numerical results. Finally, in Sec. IV we use results from previous calculations to generate a spin-flip excited state in the $^4_\Lambda$He system, in agreement with that recently observed.

II. THEORETICAL ANALYSIS

Although a number of quite elaborate models for the $^5_\Lambda$He and $^4_\Lambda$H-$^4_\Lambda$He doublet have been developed and explored, for our purposes a simple extension of the original Dalitz-Downs model[16] is quite sufficient. In this model the $^5_\Lambda$He hypernucleus is assumed to consist of a Λ interacting with an inert ^{4}He core. The potential acting between the Λ and the core is then the basic Λ-N potential integrated over the ^{4}He mass density. The rationale behind this model lies in the fact that while the Λ-N potential is not very strong [i.e., not strong enough to form a bound (Λ-N) system], the ^{4}He core is quite compact and tightly bound. Such a model can also be used to describe the ${}^4_\Lambda$H-${}^4_\Lambda$He doublet. Here the justification is not as strong, since the ^{3}H or ^{3}He cores are more diffuse than ^{4}He. Nevertheless, the model can be expected to give relatively reasonable results and has done so in the past when the Σ coupling was omitted.

Within our model the ${}^5_\Lambda$He wave function is simply $|{}^5_\Lambda\text{He}\rangle = \Phi_\Lambda(\vec{r}) \times |{}^4\text{He}\rangle$, where $|{}^4\text{He}\rangle$ represents the free ^{4}He eigenstate and $\Phi_\Lambda(\vec{r})$ describes the relative motion of the Λ with respect to the ^{4}He center of mass. Dalitz and Downs[16] were slightly more general in that they allowed for a radial compression in the $|{}^4\text{He}\rangle$ function. The resulting compression was small (~5%) and for our purposes can be neglected. The $|{}^5_\Lambda\text{He}\rangle$ state described necessarily has isospin $T=0$. And since the lowest-energy state of the system is presumed to have $l=0$ in $\Phi_\Lambda(r)$, the spin and parity is $J^\pi = \frac{1}{2}^+$.

In a similar fashion, the model wave function for the ${}^4_\Lambda$He (or ${}^4_\Lambda$H) system is

$$|{}^4_\Lambda\text{He}\rangle = \Phi_\Lambda(\vec{r}) \times |{}^3\text{He}\rangle ,$$

where we have $T=\frac{1}{2}$, $J^\pi = \frac{1}{2}^+$. [Note that $\Phi_\Lambda(r)$ does *not* represent the same wave function in $|{}^5_\Lambda\text{He}\rangle$ and $|{}^4_\Lambda\text{He}\rangle$.]

If we then write $\Phi_\Lambda(r) = [\phi_\Lambda(r)/\sqrt{4\pi} r]\chi$, where χ is a spin-$\frac{1}{2}$ spinor, the equation satisfied by $\phi_\Lambda(r)$ for each of the hypernuclear systems is

$$-\frac{\hbar^2}{2\mu}\frac{d^2r}{dr^2}\phi_\Lambda(r) + V_\Lambda(r)\phi_\Lambda(r) = E\phi_\Lambda(r) , \qquad (1)$$

where μ is the reduced mass appropriate to the problem in question[13]: $\mu = (4m_p m_\Lambda/4m_p + m_\Lambda)$ for ${}^5_\Lambda$He and $\mu = (3m_p m_\Lambda/3m_p + m_\Lambda)$ for ${}^4_\Lambda$He, with m_p and m_Λ representing the N and Λ masses respectively. Here $V_\Lambda(r)$ is the potential describing the interaction between the Λ and the core nucleus and is given by

$$V_\Lambda(r) = \int d^3r' \rho(\vec{r}\,') V_{\Lambda N}(\vec{r} - \vec{r}\,') ,$$

where $V_{\Lambda N}(\vec{r} - \vec{r}\,')$ is the basic Λ-N potential and $\rho(\vec{r}\,')$ is the mass density of the core nucleus normalized according to $\int d^3r \rho(\vec{r}) = A$ ($A=3$ for ${}^4_\Lambda$He and $A=4$ for ${}^5_\Lambda$He).

Any admixture of the Σ isotriplet state into the wave functions will be small, because this represents a virtual excitation of at least 80 MeV (the Λ-Σ mass difference). By virtue of the Σ's isospin this admixture enters into the ${}^5_\Lambda$He and ${}^4_\Lambda$H-${}^4_\Lambda$He hypersystems in qualitatively different ways. In the case of the ${}^4_\Lambda$H-${}^4_\Lambda$He doublet, the core state (^{3}H or ^{3}He) has isospin $T=\frac{1}{2}$ as does the hypernucleus. Thus the $(T=1)$ Σ can couple directly with the unexcited core maintaining the $T=\frac{1}{2}$ hypernuclear state. In the case of ${}^5_\Lambda$He, however, the core ^{4}He has in its ground state $T=0$ as does the hypernucleus. Hence, the $(T=1)$ Σ can couple only to $T=1$ excited states of the core, ^{4}He. Since the threshold for excitation of the $T=1$ ^{4}He states is at least 20 MeV above the ground state, the effective Λ-Σ mass difference is increased and the admixture of the Σ is presumably decreased. For this reason it had been thought that the Σ coupling could be neglected entirely in the ^{5}He system but could play an important role in the ${}^4_\Lambda$H-${}^4_\Lambda$He systems. While this conclusion appears to be correct, the above reasoning is incomplete. One must note that the *observed* $T=1$ resonant states in ^{4}He all have odd parity. Hence, to preserve the quantum numbers of the primary component of ${}^5_\Lambda$He ($T=0$, $J^\pi = 0^+$) the relative Σ-^{4}He state must have odd l ($l=1, 3, \ldots$). Such a Σ wave function is strongly excluded from the region of the origin by the centrifugal barrier. On the other hand the large excitation energy concentrates the Σ within a very small region about the origin. The net effect of these competing mechanisms is to reduce the Σ probability considerably. It is this combination of odd l and large excitation energy, rather than just the high ^{4}He excitation threshold, that provides the physical basis for the Σ suppression in ${}^5_\Lambda$He.

For the reasons discussed above it appears valid to use the Dalitz-Downs model[16] to describe ${}^5_\Lambda$He, where the relative Λ-^{4}He motion is assumed to be in an s state and the radial dependence of the orbital is governed by Eq. (1).

The ${}^4_\Lambda$H-${}^4_\Lambda$He states, on the other hand, do permit mixing of the Σ component without exciting the core nucleus. One may write the states as

$$|{}^4_\Lambda\text{H}\rangle = \Phi_\Lambda(\vec{r})|{}^3\text{H}\rangle + \frac{1}{\sqrt{3}}\Phi_{\Sigma^0}(\vec{r})|{}^3\text{H}\rangle - \sqrt{\tfrac{2}{3}}\Phi_{\Sigma^-}(\vec{r})|{}^3\text{He}\rangle ,$$
$$|{}^4_\Lambda\text{He}\rangle = \Phi_\Lambda(\vec{r})|{}^3\text{He}\rangle - \frac{1}{\sqrt{3}}\Phi_{\Sigma^0}(\vec{r})|{}^3\text{He}\rangle + \sqrt{\tfrac{2}{3}}\Phi_{\Sigma^+}(\vec{r})|{}^3\text{H}\rangle \qquad (2)$$

The function $\Phi_{\Sigma^0, \Sigma^\pm}$ includes the radial dependence of the relative Σ core orbital and the spin-isospin properties of the Σ hyperons. The wave functions are subject to the normalization condition

$$\langle\Phi_\Lambda|\Phi_\Lambda\rangle + \tfrac{1}{3}\langle\Phi_{\Sigma^0}|\Phi_{\Sigma^0}\rangle + \tfrac{2}{3}\langle\Phi_{\Sigma^\pm}|\Phi_{\Sigma^\pm}\rangle = 1 .$$

We again assume isotropy (i.e., only relative $l=0$) in the spatial coordinate $\vec{r}$, the hyperon-nucleus relative coordinate,

$$\Phi_\Lambda = \frac{\phi_\Lambda(r)}{\sqrt{4\pi} r}\chi_\Lambda ,$$

where χ_Λ describes the Λ-nucleus spin properties. Similarly we can write

$$\Phi_{\Sigma^0(\Sigma^\pm)} = \frac{\phi_{\Sigma^0(\Sigma^{+-})}}{\sqrt{4\pi} r}\chi_{\Sigma^0(\Sigma^\pm)} ,$$

where, in this case, $\chi_{\Sigma^0(\Sigma^\pm)}$ includes also the Σ isospin projection. If we further assume that the Σ-N potentials are charge-independent, so that

$$\phi_{\Sigma^0}(r) = \phi_{\Sigma^\pm}(r),$$

then one obtains the following equations describing the ϕ_Λ and ϕ_Σ functions [for ($^4_\Lambda$He-$^4_\Lambda$H)]:

$$-\frac{\hbar^2}{2\mu_\Lambda}\frac{d^2}{dr^2}\phi_\Lambda(r) + V_\Lambda(r)\phi_\Lambda(r) + V_{\Lambda\Sigma}(r)\phi_\Sigma(r) = E\phi_\Lambda(r),$$
$$-\frac{\hbar^2}{2\mu_\Sigma}\frac{d^2}{dr^2}\phi_\Sigma(r) + \Delta m_\Sigma \phi_\Sigma(r) + V_{\Lambda\Sigma}(r)\phi_\Lambda(r) + V_\Sigma(r)\phi_\Sigma(r) = E\phi_\Sigma(r). \qquad (3)$$

In these equations Δm_Σ represents the excitation energy of the virtual Σ state; i.e., the Λ-Σ mass difference, and $V_\Lambda(r)$ is again the Λ-nucleus potential given above. The $V_\Sigma(r)$ is the Σ-nucleus potential defined by

$$V_\Sigma(r) = \int d^3r' \rho(\vec{r}\,') V_{\Sigma N}(\vec{r} - \vec{r}\,'),$$

where $V_{\Sigma N}(\vec{r} - \vec{r}\,')$ is the Σ-N interaction. The term $V_{\Lambda\Sigma}(r)$ represents the Λ-Σ coupling:

$$V_{\Lambda\Sigma}(r) = \int d^3r' \rho(\vec{r}\,') V_{\Lambda\Sigma N}(\vec{r} - \vec{r}\,'),$$

where $V_{\Lambda\Sigma N}(\vec{r} - \vec{r}\,')$ represents the Λ-Σ transition coupling at $\vec{r}$ in the presence of a nucleon at $\vec{r}\,'$. In the next section we shall present results for the Λ-separation energy E for various depths and ranges of the hyperon-nucleon interactions $V_{\Lambda N}$, $V_{\Sigma N}$, and $V_{\Lambda\Sigma N}$.

We also have estimated the $\Delta_{B\Lambda}$ energy splitting of the $^4_\Lambda$H-$^4_\Lambda$He isodoublet. The Coulomb energy difference between $^4_\Lambda$He and $^4_\Lambda$H is

$$B_C = E_C(^4_\Lambda\text{He}) - E_C(^4_\Lambda\text{H}),$$

with E_C the Coulomb energy of the appropriate hypernucleus. The splitting in the Λ-separation energies, measured to be approximately 0.29 MeV, is then

$$\Delta B_C = B_C - E_C(^3\text{He}),$$

where $E_C(^3\text{He})$ is the Coulomb energy of ^{3}He, 0.764 MeV. These Coulomb energies are given by

$$E_C(^4_\Lambda\text{He}) = \frac{e^2}{2}\left\langle ^4_\Lambda\text{He}\left|\sum_{i\neq j}^{4}\frac{e_i e_j}{|\vec{r}_i - \vec{r}_j|}\right|^4_\Lambda\text{He}\right\rangle,$$

$$E_C(^4\text{H}) = \frac{e^2}{2}\left\langle ^4_\Lambda\text{H}\left|\sum_{i\neq j}^{4}\frac{e_i e_j}{|r_i - r_j|}\right|^4_\Lambda\text{H}\right\rangle,$$

$$E_C(^3\text{He}) = \frac{e^2}{2}\left\langle ^3\text{He}\left|\sum_{i\neq j}^{3}\frac{e_i e_j}{|r_i - r_j|}\right|^3\text{He}\right\rangle,$$

where e_i is the charge of the ith baryon ($e_i = +1$ for p and Σ^+, $e_i = 0$ for n, Λ, Σ^0, and $e_i = -1$ for Σ^-). It is seen that if we assume the simplified Dalitz-Downs[16] model without core distortion [Eq. (1)] then $E_C(^4\text{He}) = E_C(^3\text{He})$ and $E_C(^4_\Lambda\text{H}) = 0$, so that $\Delta B_C = 0$. All previous modifications which allow for core distortion result in $\Delta B_C < 0$. If, however, we use the Σ admixed state given by Eq. (2), we obtain

$$\Delta B_C = -\tfrac{4}{3}P_\Sigma E_C(^3\text{He}) + \tfrac{2}{3}e^2\int\frac{d^3r\,d^3r'}{4\pi r^2}\frac{\phi_\Sigma^2(r)}{|\vec{r} - \vec{r}\,'|} \times[\rho_C(^3\text{He}, r') + \rho_C(^3\text{H}, r')]. \qquad (4)$$

Here P_Σ is the probability of the admixed Σ,

$$P_\Sigma = \tfrac{1}{3}\langle\Phi_{\Sigma^0}|\Phi_{\Sigma^0}\rangle + \tfrac{2}{3}\langle\Phi_{\Sigma^\pm}|\Phi_{\Sigma^\pm}\rangle = \int dr\,\phi_\Sigma^2(r).$$

$\rho_C(^3\text{He}, r)$ and $\rho_C(^3\text{H}, r')$ are the charge densities of ^{3}He and ^{3}H, respectively, normalized according to $\int d^3r\,\rho_C(\vec{r}) = Z$.

III. NUMERICAL RESULTS

Many forms have been proposed for the basic Λ-N potential. Since one is attempting to compute only the binding energy of weakly bound systems, it should suffice to take a simple potential with variable depth (and perhaps range). We have chosen to assume a Gaussian form for the potential:

$$V_{\Lambda N}(\vec{r} - \vec{r}\,') = -V_1\exp[-(\vec{r} - \vec{r}\,')^2/b_1^2] + V_2\exp[-(\vec{r} - \vec{r}\,')^2/b_2^2].$$

For the reasons stated in Ref. 13 it proves convenient to include the possibility of a soft repulsive core, represented by the second term. We make two choices for the parameters b_1 and b_2. If we omit the repulsive term, we take b_1 to correspond to the exchange of two pions between the hyperon and the nucleon: $V_2 = 0$, $b_1 = 1.05$ fm. When we include the repulsive term, we assume: $V_2 = 145$ MeV, $b_2 = 0.82$ fm, $b_1 = 1.21$ fm. In both cases, V_1 is chosen to produce the correct $^5_\Lambda$He Λ-separation energy.

We also require the mass density of ^{4}He, $\rho(^4\text{He}, r)$. For simplicity we choose this to be of Gaussian form also

$$\rho(^4\text{He}, r) = \frac{4}{\pi^{3/2}a^3}e^{-r^2/a^2},$$

where a is chosen so that the calculated rms mass radius agrees with the rms point proton charge radius, $a^2 = \tfrac{2}{3}(r_{\text{ch}}^2 - r_p^2)$ with r_{ch} the ^{4}He charge radius (1.71 fm) and r_p the proton radius (0.8 fm).

Using these functional forms and the parameters indicated, we solved Eq. (1) numerically, varying the depth V_1 so as to reproduce the experimental Λ-separation energy for $^5_\Lambda$He, 3.08 MeV. The resulting parameter values were

$$V_1 = 38.2\ \text{MeV}, \quad \text{for } V_2 = 0,$$

$$V_1 = 82.7\ \text{MeV}, \quad \text{for } V_2 = 145\ \text{MeV}.$$

These parameter values were then used for the Λ-N potential throughout the computations described below.

For the ${}^4_\Lambda$H-${}^4_\Lambda$He case, we again chose the mass density to be Gaussian

$$\rho({}^3\text{H}, r) = \rho({}^3\text{He}, r) = \frac{3}{\pi^{3/2}\bar{a}^3} e^{-r^2/\bar{a}^2}.$$

One notes that the charge radii of ^{3}H and ^{3}He are not equal:

$${}^3\text{H}: \ r_{\text{ch}} = 1.70\ \text{fm}, \quad {}^3\text{He}: \ r_{\text{ch}} = 1.84\ \text{fm}.$$

Thus the charge radius contains both isoscalar and isovector components. The mass radius, on the other hand, is an isotopic scalar. We, therefore, set the mass radius equal to the isoscalar part of r_{ch} (after proton size effects have been removed); i.e.,

$$\bar{a}^2 = \tfrac{2}{3}\{\tfrac{2}{3}[r_{\text{ch}}^2({}^3\text{He}) - r_p^2] + \tfrac{1}{3}[r_{\text{ch}}^2({}^3\text{H}) - r_p^2]\}.$$

If we now use the potential depths V_1 given above to solve for the Λ-separation energies of the ${}^4_\Lambda$H-${}^4_\Lambda$He hypernuclei by means of Eq. (1), we find that neither potential set yields a bound state. It is necessary to increase the well depth to $V_1 = 53$ MeV (for $V_2 = 0$) and $V_1 = 93$ MeV (for $V_2 = 145$ MeV) to obtain bound states with eigenvalue 2 MeV.

TABLE I. Values of U_1 and W_1 which reproduce the Λ-separation energy in ${}^4_\Lambda$H, 2.02 MeV. P_Σ is the amount of Σ admixture. ΔB_C is the Coulomb energy difference with $\Delta m_\Sigma = 78$ MeV. ΔB_m is the "Coulomb energy difference" due to mass splittings of the $\Sigma^{\pm,0}$. ∇B is the total Coulomb energy difference. The second part has a soft-core repulsive potential.

No core: $V_1 = 38.2$ MeV, $V_2 = U_2 = W_2 = 0$, $b_1 = 1.05$ MeV

U_1 (MeV)	W_1 (MeV)	ΔB_C (MeV)	P_Σ (MeV)	ΔB_m (MeV)	ΔB (MeV)
38.2	−11.0	0.04	0.04	0.04	0.08
36.0	−1.0	0.05	0.04	0.06	0.11
30.0	29.0	0.07	0.07	0.08	0.15
25.0	48.0	0.10	0.09	0.11	0.21
20.0	64.0	0.15	0.14	0.15	0.30
15.0	77.0	0.25	0.24	0.23	0.48

Soft core: $V_1 = 82.7$ MeV, $V_2 = U_2 = W_2 = 145$ MeV, $b_1 = 1.21$ fm, $b_2 = 0.82$ fm

U_1 (MeV)	W_1 (MeV)	ΔB_C (MeV)	P_Σ (MeV)	ΔB_m (MeV)	ΔB (MeV)
82.7	30.0	0.005	0.01	0.04	0.05
40.0	96.0	0.15	0.14	0.08	0.23
100.0	−126.0	0.003	0.005	0.02	0.02

TABLE II. Value of E_Λ, the ${}^4_\Lambda$H separation energy as a function of W_1 for various values of U_1.

No core			Soft core		
U_1 (MeV)	W_1 (MeV)	E_Λ (MeV)	U_1 (MeV)	W_1 (MeV)	E_Λ (MeV)
36.0	−10.0	1.78	82.7	20.0	1.89
	1.0	2.02		30.0	2.03
	10	2.27		40.0	2.20
25.0	40.0	1.68	40.0	90.0	1.73
	48.0	2.00		96.0	2.02
	60.0	2.73		100.0	2.28

In the Σ coupled model given by Eq. (2) we must choose potential parameters for the Λ-Σ coupling and the Σ-N potentials. Again we assume the Gaussian forms,

$$V_{\Lambda\Sigma N}(\vec{r} - \vec{r}\,') = -U_1 \exp[-(\vec{r} - \vec{r}\,')^2/b_1^2] + U_2 \exp[-(\vec{r} - \vec{r}\,')^2/b_2^2],$$

$$V_{\Sigma N}(\vec{r} - \vec{r}\,') = -W_1 \exp[-(\vec{r} - \vec{r}\,')/b_1^2] + W_2 \exp[-(\vec{r} - \vec{r}\,')^2/b_2^2].$$

As with $V_{\Lambda N}$ we investigate the case with no core ($U_2 = W_2 = 0$) and the case with a soft core ($U_2 = W_2 = 145$ MeV) keeping the corresponding values for the ranges b_1 and b_2.

Finally we choose the Λ-Σ mass difference to be the mean of the charged Σ mass differences, $\Delta m_\Sigma = 78$ MeV.

The coupled equations, Eq. (2), were then solved numerically for various values of the parameters U_1 and W_1. The results are shown in Tables I-III. In Table I we indicate the sets of parameters U_1 and W_1 which reproduce the Λ-separation energy for ${}^4_\Lambda$H, 2.02 MeV. The fraction P_Σ of the Σ admixed into the Λ-Σ state and the Coulomb energy differences ΔB_C is also given for each set of parameters. Without the coupling term Eq. (3) produces no bound state at all, as we have noted previously. Yet with a coupling which gives only a small (4%) Σ admixture one can reproduce the de-

TABLE III. Values of E_Λ, the ${}^4_\Lambda$H separation energy as a function of U_1 for various values of W_1.

No core			Soft core		
U_1 (MeV)	W_1 (MeV)	E_Λ (MeV)	U_1 (MeV)	W_1 (MeV)	E_Λ (MeV)
20.0	48.0	1.27	75.0	30.0	1.36
25.0		2.00	82.7		2.03
30.0		2.93	90		2.94
30.0	1.0	1.36			
36.0		2.02			
40.0		2.57			

sired 2.02-MeV separation energy. It is clear that even moderate Λ-Σ coupling is significant in the $^4_\Lambda$H-$^4_\Lambda$He system. Table II indicates the sensitivity of the separation energy E_Λ to the Σ-N potential for several choices of U_1. Table III indicates the sensitivity of the separation energy E_Λ to the Λ-Σ coupling potential for several values of W_1. As might be expected, the results are much more sensitive to the Λ-Σ coupling term U_1 than to the Σ-N potential W_1 because the effect of the latter is, essentially, inhibited by the large mass difference.

We return now to the point mentioned in the previous section concerning the effect on the Λ-Σ coupling in $^5_\Lambda$He of the large threshold (20 MeV) for excitation of ^{4}He. If we now solve Eq. (3) for the $^5_\Lambda$He Λ-separation energy using $\Delta m = 100$ MeV, rather than 78 MeV, we find that nonzero U_1 and W_1 produce not negligible, but rather quite large contributions to the Λ-separation energy. For $V_1 = 38.2$ MeV and $U_1 = W_1 = 0$ this energy is 3.08 MeV. Choosing from Table I the values $U_1 = 25$ MeV and $W_1 = 48$ MeV, we then obtain $E_\Lambda = 10.5$ MeV. One must realize that the addition of an extra nucleon plus the compactness of the ^{4}He core both serve to increase the effective Λ-core potential; in $^5_\Lambda$He it is almost twice as large as it is in $^4_\Lambda$He. Hence, the suppression of the Λ-Σ coupling in $^5_\Lambda$He, if indeed this coupling is suppressed (as we have assumed), must arise from the nature of the ^{4}He state rather than simply its large excitation energy. That is, the absence of low-lying observed excited states in ^{4}He capable of coupling with the Σ means such states are at sufficiently high excitation that these wave functions are sufficiently diffuse or complicated as to reduce the coupling. This is the assumption that one is really making in neglecting the Σ in $^5_\Lambda$He.

The Coulomb energies in Table I have the sign required to account for the experimental difference in E_Λ between $^4_\Lambda$H and $^4_\Lambda$He. But in general the values of ΔB_C are somewhat too small to account for the quoted difference of 0.29 MeV. However, this value is experimentally uncertain, and there appears to be sufficient freedom in the choice of potential well depths U_1 and W_1 to give the desired value. Whether such depths are reasonable is another matter. In any case, there is a second effect, hitherto neglected, which contributes in the correct manner to this energy difference, namely the mass splitting within the Σ isotriplet state.

The mass differences of the Σ triplet are

$$\Delta m_{\Sigma^+} = 73.8 \text{ MeV}, \quad \Delta m_{\Sigma^0} = 76.9 \text{ MeV},$$
$$\Delta m_{\Sigma^-} = 81.7 \text{ MeV}.$$

The $^4_\Lambda$He hypernucleus contains components of the Σ^0 and Σ^+, while the $^4_\Lambda$H hypernucleus contains the Σ^0 and Σ^-. Hence the mass difference parameters used in Eq. (3) are different for the two components of the isodoublet. It is shown in the Appendix that the mass difference appropriate to each case is given by

$$^4_\Lambda\text{He:}\quad \Delta m_\Sigma = \tfrac{2}{3}\Delta m_{\Sigma^+} + \tfrac{1}{3}\Delta m_{\Sigma^0} = 74.8 \text{ MeV},$$

$$^4_\Lambda\text{H:}\quad \Delta m_\Sigma = \tfrac{2}{3}\Delta m_{\Sigma^-} + \tfrac{1}{3}\Delta m_{\Sigma^0} = 80.1 \text{ MeV}.$$

Use of these mass differences in Eq. (3) will, of course, lead to different eigenvalues, which we label as $E(^4_\Lambda\text{He})$ and $E(^4_\Lambda\text{H})$. The quantity ΔB_m in Table I is then the energy difference, $\Delta B_m = E(^4_\Lambda\text{He}) - E(^4_\Lambda\text{H})$. Note that ΔB_m has the same sign as ΔB_C and therefore contributes constructively to the energy splitting between the components of the isodoublet. It is also comparable in magnitude to ΔB_C in most cases. The sum of the two terms is listed in Table I under the column ΔB_Λ, where

$$\Delta B_\Lambda = \Delta B_C + \Delta B_m.$$

In view of the experimental uncertainties in the measured ΔB_Λ, the agreement in most cases is quite acceptable.

IV. EXCITED STATES AND HYPERNUCLEAR SPINS

There is some evidence[12,25] that the angular momentum of the ground state of ($^4_\Lambda$He-$^4_\Lambda$H) is $J = 0$. We cannot judge the reliability of that conclusion. If one were to give up that "fact" then something of a simplification results in our understanding of the ground and excited state of this system.[27] In any case, as one can infer the spin dependence of the Λ-N interaction from the spin of the ground and excited state, we wish to indicate the delicacy of

TABLE IV. Values of the spin-flip excited state $E(J=0)$ and ΔE, the difference in energy between the ground and excited state under the assumptions of Sec. IV.

No core: $U_1 = \frac{4}{5}U_1$				
U_1 (MeV)	W_1 (MeV)	U_1 (MeV)	$E(j=0)$ (MeV)	ΔE (MeV)
38.2	−11.0	30.6	1.24	0.78
36.0	−1.0	28.8	1.22	0.80
30.0	29.0	24.0	1.27	0.75
25.0	48.0	20.0	1.27	0.75
20.0	64.0	16.0	1.31	0.71
15.0	77.0	12.0	1.43	0.59

that conclusion relative to inclusion of the Σ admixture.

The potential depths determined in Sec. III are, of course, combinations of spin singlet and triplet Λ-N and Σ-N interactions. We have assumed that the singlet and triplet potentials are central and have the same form and ranges, differing only in their depths. The relation of the effective depths V_1, U_1, and W_1 to the singlet and triplet depths $V_1^{(S)}$, $V_1^{(S)}$, $V_1^{(T)}$, etc., depends on the spin J of the hypernucleus. We note that

$$J=0\begin{cases} V_1=\frac{2}{3}V_1^{(T)}+\frac{1}{3}V_1^{(S)}, \\ U_1=\frac{2}{3}U_1^{(T)}+\frac{1}{3}U_1^{(S)}, \\ W_1=\frac{2}{3}W_1^{(T)}+\frac{1}{3}W_1^{(S)}, \end{cases}$$
$$J=1\begin{cases} V_1=\frac{5}{6}V_1^{(T)}+\frac{1}{6}V_1^{(S)}, \\ U_1=\frac{5}{6}U_1^{(T)}+\frac{1}{6}U_1^{(S)}, \\ W_1=\frac{5}{6}W_1^{(T)}+\frac{1}{6}W_1^{(S)}. \end{cases} \qquad (5)$$

From the summary given by Alexander,[28] equality of the Λ-N triplet and singlet scattering lengths appears to be consistent with the experimental data, although the errors are quite large. Neglecting coupling to the Σ channel we assume that $V_1^{(S)}=V_1^{(T)}$. Further, analysis of the $\Sigma^- + p \rightarrow \Lambda + n$ reaction near threshold[29] indicates that the process goes predominantly through the triplet state, and hence it is consistent with an assumption of $U_1^{(S)}=0$. This, of course, produces an effective spin-dependent interaction.

It is clear that the above model, combined with these (perhaps rather drastic) assumptions about the hyperon-nucleon potentials, requires the ground-state spin of ${}^4_\Lambda$He or ${}^4_\Lambda$H to be $J=1$. One can understand this from Eq. (5): $U_1^{(S)}=0$ implies that for the $J=1$ state U_1 is stronger than for the $J=0$ state. This unfortunately, disagrees with other analyses of the ${}^4_\Lambda$H-${}^4_\Lambda$He spin, in particular the angular distributions in the hypernuclear decay which conclude that the ground-state spin is $J=0$.[12,30] (We cannot resolve this disagreement.) If, however, we assume that the ground state does have $J=1$, and we use Eq. (5) to generate an effective $\Lambda\Sigma N$ potential coupling term $\overline{U}_1$ for the $J=0$ state, then we obtain from Eq. (3) the eigenvalues given in Table IV. Here, ΔE is the excitation energy of the $J=0$ excited state relative to the $J=1$ ground state at $E=2.02$ MeV. An excited state in ${}^4_\Lambda$He has, in fact, been reported with an excitation energy of approximately 1 MeV.[27] On the basis of this model, the state would represent a spin-flip excitation, decaying to the ground state via an $M1$ transition. Of course it is neither likely that the ground state is $J=1$ etc., nor that the potentials are as simple as we have assumed from the imperfect scattering data. Furthermore, it is always dangerous to infer that the phenomenological potentials for bound-state calculations have a simple relationship with the free potentials due to spin- and isospin-dependent polarization corrections.

V. CONCLUSIONS

The calculations reported above are not sufficiently realistic to even attempt to represent the final word on this subject. Rather they are intended as a (primarily qualitative) guide. However, within the rather broad experimental uncertainties prevalent in hypernuclear work (both in scattering data and bound-state energies) they are consistent with most of the data, and at the least indicate the importance of a hitherto neglected mechanism relevant to these hypernuclei.

Our conclusions may be summarized as follows:

(1) The Λ-Σ coupling produces a significant effect in the binding energies of ${}^4_\Lambda$H and ${}^4_\Lambda$He. If one assumes that this coupling is suppressed ${}^5_\Lambda$He, then it is possible to obtain the Λ-separation energies for both systems with reasonable potentials. This suppression, however, arises not from the large excitation energy of ^{4}He, but rather from the odd parity of the $T=1$ resonant ^{4}He states and the absence of low-lying even-parity $T=1$ states in ^{4}He. It is clear that one must eventually consider virtual excitations of the ^{4}He core (in addition to the virtual excitation of the Λ) as well as possible resonant states in ^{3}H and ^{3}He. One such state to be considered is the 0^+ state in ^{4}He at 20.2-MeV excitation.

(2) The Λ-separation energy difference in the ${}^4_\Lambda$H-${}^4_\Lambda$He doublet results from the virtual admixture of the Σ state. The splitting is due in part to Coulomb effects (due to the $T=1$ isospin of the Σ and its resulting charge density) and in part to the mass splitting within the Σ triplet.

(3) Within a number of extreme assumptions about the Λ-N potential spin dependence, the ground-state spin of ${}^4_\Lambda$He can be inferred to be $J=1$, in contradiction with the previously assigned value of $J=0$. An excited state at approximately 0.7-MeV excitation is then predicted theoretically to represent the $J=0$ state. As we have indicated in Sec. IV, we do not have any evidence that this is correct but perhaps a more modern look at the evidence would not be inappropriate.

If one accepts the basic implication of this work, that the Σ admixture can qualitatively account for all the previous difficulties in explaining ${}^5_\Lambda$He and (${}^4_\Lambda$He-${}^4_\Lambda$H), one is still left with the possibility that an intrinsic spin dependence in the force may still be important. Moreover, if the Σ is really admixed as we have described, then its omission in calculations of related phenomena may lead to misleading conclusions. A case that comes immediately to mind is in Λ scattering from ^{4}He recently studied by Gibson and Weiss[31] and Londergan and Dalitz.[32] For the *P*-wave phase shifts, the arguments previously applied to Σ suppression in $^5_\Lambda$He no longer hold and conclusions drawn there about the Λ-*N* force would have to be reconsidered. A calculation of this effect is in progress.

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APPENDIX

The effective Λ-Σ mass differences in $^4_\Lambda$H and $^4_\Lambda$He can be determined by considering Eq. (2) of the text where we have written the wave functions for the two hypernuclear states:

$$|^4_\Lambda\mathrm{He}\rangle = \Phi_\Lambda |^3\mathrm{He}\rangle - \sqrt{\tfrac{1}{3}}\Phi_{\Sigma^0}|^3\mathrm{He}\rangle + \sqrt{\tfrac{2}{3}}\Phi_{\Sigma^+}|^3\mathrm{H}\rangle ,$$

$$|^4_\Lambda\mathrm{H}\rangle = \Phi_\Lambda |^3\mathrm{H}\rangle + \sqrt{\tfrac{1}{3}}\Phi_{\Sigma^0}|^3\mathrm{H}\rangle - \sqrt{\tfrac{2}{3}}\Phi_{\Sigma^-}|^3\mathrm{He}\rangle .$$

We define a mass splitting operator ΔM, such that

$$\Delta M \Phi_\Lambda = 0 ,$$

$$\Delta M \Phi_{\Sigma^{0,\pm}} = \Delta m_{\Sigma^{0,\pm}} \Phi_{\Sigma^{0,\pm}} ,$$

where $\Delta m_{\Sigma^{0,\pm}}$ are the Σ-Λ mass differences for each member of the Σ triplet,

$$\Delta m_{\Sigma^+} = 73.8 \text{ MeV} , \quad \Delta m_{\Sigma^0} = 76.9 \text{ MeV} ,$$

$$\Delta m_{\Sigma^-} = 81.7 \text{ MeV} .$$

As is well known, Eq. (3) of the text is obtained from a variational principle, minimizing the expectation value of the Hamiltonian with respect to variations in Φ_Λ and Φ_Σ subject to the normalization constraint. The Hamiltonian contains a term ΔM, which contributes to the expectation value

$$^4_\Lambda\mathrm{He}:\ \tfrac{1}{3}\langle\Phi_{\Sigma^0}|\Delta M|\Phi_{\Sigma^0}\rangle + \tfrac{2}{3}\langle\Phi_{\Sigma^+}|\Delta M|\Phi_{\Sigma^+}\rangle = \tfrac{1}{3}\Delta m_0\langle\Phi_{\Sigma^0}|\Phi_{\Sigma^0}\rangle + \tfrac{2}{3}\Delta m_+\langle\Phi_{\Sigma^+}|\Phi_{\Sigma^+}\rangle ,$$

$$^4_\Lambda\mathrm{H}:\ \tfrac{1}{3}\langle\Phi_{\Sigma^0}|\Delta M|\Phi_{\Sigma^0}\rangle + \tfrac{2}{3}\langle\Phi_{\Sigma^-}|\Delta M|\Phi_{\Sigma^-}\rangle = \tfrac{1}{3}\Delta m_0\langle\Phi_{\Sigma^0}|\Phi_{\Sigma^0}\rangle + \tfrac{2}{3}\Delta m_-\langle\Phi_{\Sigma^-}|\Phi_{\Sigma^-}\rangle .$$

Since $\Phi_{\Sigma^{0,\pm}}$ are written as $[\phi_\Sigma(r)/\sqrt{4\pi}r]$ multiplied by normalized spin and isospin factors, we have

$$\langle\Phi_{\Sigma^0}|\Phi_{\Sigma^0}\rangle = \langle\Phi_{\Sigma^+}|\Phi_{\Sigma^+}\rangle = \langle\Phi_{\Sigma^-}|\Phi_{\Sigma^-}\rangle = \int d^3r \frac{\Phi_\Sigma(r)}{4\pi r^2} .$$

Thus the variational principle applied to $^4_\Lambda$He directly leads to Eq. (2) with the mass parameters discussed at the end of Sec. III.

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High-Energy Neutron-Nuclei Total Cross Sections*

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Total cross sections for collisions between high-energy neutrons and nuclei are calculated by means of the Glauber approximation. Both Woods-Saxon and Gaussian density distributions are assumed for the nuclei. The two distributions yield results which may differ from each other by as much as 15%. For light nuclei harmonic-oscillator wave functions are used. The calculations are compared with measurements for neutron energies above 1 GeV. A simple explanation is given to show why the dependence of the cross sections on the mass number A is greater than $A^{2/3}$. Although the multiple scattering series for a mass-A nucleus contains A terms, it is shown that excellent accuracy is obtained by retaining only approximately $3A^{1/3}$ terms and a geometrical argument leading to this result is given. The ratios of the real to imaginary parts of the hadron-nuclei forward elastic scattering amplitudes are calculated and the decrease of their magnitudes with increasing mass number is explained. The neutron-nuclei data are consistent with little or no regeneration.

I. INTRODUCTION

Some years ago it was predicted[1,2] that double collisions (i.e., collisions with two target nucleons) are more probable than single collisions in high-energy hadron-deuteron scattering at angles away from the forward direction. Since that time a considerable number of experimental studies of hadron-deuteron scattering have been made, and analyses of the measurements have confirmed this prediction.[3-12] Recently a number of analyses of high-energy hadron-nucleus collisions have been based upon the diffraction approximation due to Glauber.[13] This approximation is most accurate for collisions involving small momentum transfers. Consequently it is not unreasonable to expect that high-energy hadron-nucleus total cross sections, which depend only upon the forward elastic scattering amplitudes via the optical theorem, could be calculated quite reliably for a given nuclear model. Such calculations have been carried out for a simple model in which nuclei are described by Gaussian density distributions.[14] Such a model, although quite unrealistic, is very useful because it leads to an analytic expression for the total cross sections which exhibits some qualitative features that are likely to reappear in more realistic calculations.[15] Alternative approaches for calculating neutron-nucleus cross sections are possible using, for example, the optical-model methods of Francis and Watson,[16] Bethe,[16] and of Kerman, McManus and Thaler.[16] The accuracy of the optical model is, however, less satisfactory for calculations of nucleus-nucleus cross sections. The present analysis can be extended to treat nucleus-nucleus collisions.[17]

We have performed analyses of total cross sections in which the explicit multiple scattering form of the Glauber approximation has been retained. We use both a model in which the nuclei have Gaussian density distributions and a model in which the nuclei have Woods-Saxon shapes for the density distributions.[18] (We have also performed the calculations for light nuclei using harmonic-oscillator wave functions.) The quantitative results obtained with the Gaussian and Woods-Saxon models differ by as much as 15%. This seemingly small difference is significant, since recent measurements have uncertainties which are much smaller than 15%. Nevertheless, over the current physical range of nuclei ($A \lesssim 240$) the quantitative results are not grossly sensitive to the nuclear model. Consequently our predictions could serve as a rather severe test of the basic theory. Alternatively, if we have confidence in the theory, our predictions could serve as a test of the reliability of total cross-section measurements. To the degree that the theory is sensitive