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# First $0^+$ Excited State of $^{16}O$ in the $\alpha$-Particle Model of Light Nuclei*

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The pole approximation that Narodetskii introduced into the theory of four-particle scattering is here applied to $^{16}O$, which is assumed to be composed of four structureless $\alpha$ particles. The binding energy of the excited $0^+$ state of $^{16}O$ was calculated by using an Irving variational wave functions for the ground state of $^{12}C$ and taking both Yukawa and exponential $\alpha\alpha$ potentials. In the light of earlier theoretical results on $^{12}C$, comparing the present results with experiment suggests that at least for calculation of the bound states, the $\alpha$-particle model of $^{16}O$ is reasonable.

The $\alpha$-particle model of light nuclei has a long history.[1] In its simplest form, it is assumed that the $A=4n$ nucleus is composed of spinless and structureless $\alpha$ particles. While it was easy to check this theory for $^8Be$, the situation for $^{12}C$, $^{16}O$, etc. was much poorer. However, the success of the three-body theory of Faddeev,[2] Lovelace,[3] and Amado, Aaron, and Yam[4] provided the means to investigate the three-$\alpha$ model of $^{12}C$. Several authors[5] discuss this questions from different points of view. An almost common conclusion of all these papers is that, although the numerical results are not in very good agreement with experiment, this model nevertheless is reasonable. The purpose of this paper is to take the natural step, namely to discuss the question of the four-$\alpha$ model of $^{16}O$.

Although the formal extension of the Faddeev theory to $N>3$ has been carried out successfully by Yakubovskii,[6] its practical value was in serious doubt. The bound-state wave function of the four-body systems is the solution of 18 coupled integral equations in three vector variables. However, in a recent paper, Narodetskii[7] took an important step forward, by using the pole approximation for the three-body amplitude. He applied it to the four-nucleon problem,[8] in which spin and isospin must be taken into account. We are interested in his results for identical spinless particles which can be summarized as follows: (1) Let $g(\vec{k},\vec{p})$ be the properly symmetrized ground state wave function of the three particles in momentum space, with binding energy $\gamma^2$, and let $g_{00}(u^2)$ $(u^2=k^2+p^2)$ be its zero-order component in the harmonic polynomial expansion.[9] In first approximation Narodetskii takes $g(\vec{k},\vec{p})\approx g_{00}(u^2)$; (2) suppose that for $E\approx-\gamma^2$, where $E$ is the c.m. energy of the four-body system, the three-body $T$ matrix can be represented by its pole term; (3) suppose that Eq. (1) below has a solution with a parameter $\chi^2\gtrsim\gamma^2$. Then, as described in Ref. 7, one can construct from this solution a four-body $s$-wave bound state with binding energy $\chi^2$. The relevant integral equation is

$$C(q,\chi^2)=\int_0^\infty K(q,p,\chi^2)C(p,\chi^2)dp, \tag{1}$$

where, with $f(u^2)=(u^2+\gamma^2)g_{00}(u^2)$, the kernel $K$ is

$$K(q,p,\chi^2)=\frac{81.3\pi^2}{\sqrt{2}}\frac{pq}{[(\chi^2+q^2-\gamma^2)(\chi^2+p^2-\gamma^2)]^{1/2}}\times\int_0^\infty k^2dk\int_{-1}^1 dx\frac{f(u_1^2)f(u_2^2)}{k^2+\frac{9}{8}p^2+\frac{9}{8}q^2+\frac{3}{4}pqx+\chi^2}, \tag{2}$$

$$u_1^2=k^2+\tfrac{9}{8}p^2+\tfrac{1}{8}q^2+\tfrac{3}{4}pqx,\quad u_2^2=k^2+\tfrac{1}{8}p^2+\tfrac{9}{8}q^2+\tfrac{3}{4}pqx.$$

A look at the level scheme[10] of $^{12}C$ encourages us to use the pole approximation for $T_{3\alpha}$ (the three-$\alpha$ $T$ operator) for energies near the $^{12}C$ ground state, which is located 7.28 MeV below the three-$\alpha$ threshold. Indeed, the other singularities of $T_{3\alpha}$ come from the $2^+$ excited state (at excitation energy 4.439 MeV), from cuts and resonances above the three-$\alpha$ threshold, and from left cuts,[11] and none of these singularities are at energies close to the $0^+$ ground state. Next, we see from the level scheme[12] of $^{16}O$ that we have, in fact, something to look for – namely, the $0^+$ excited state, which is only 1.111 MeV below the $^{12}C+\alpha$ threshold.

In order to have tractable numerical calculations, we must give the functional form of $g(\vec{k}, \vec{p})$, the three-$\alpha$ wave function in momentum space, corresponding to the $^{12}C$ binding energy. We have chosen an Irving[13] wave function in configuration space, namely

$$\psi = N^{1/2}\exp[-\alpha(r_{12}^2+r_{13}^2+r_{23}^2)^{1/2}] = N^{1/2}\exp(-\sqrt{\tfrac{3}{4}}\alpha\rho), \quad (3)$$

where $\alpha$ is a variational parameter, $N$ is a normalization factor, and

$$\rho^2 = \eta^2 + \xi^2,$$

with

$$\vec{\eta} = (\vec{r}_1 - \vec{r}_2)/\sqrt{2}, \quad \vec{\xi} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6}.$$

Now going back to momentum space, one has

$$g(\vec{k}, \vec{p}) = g_{00}(u^2) = (1/u^2)\int_0^\infty \rho^3\psi(\rho)J_2(u\rho)d\rho. \quad (4)$$

Thus,

$$f(u^2) = \frac{15\alpha^4}{4}\left(\frac{3}{10\pi^3}\right)^{1/2}(u^2+\gamma^2)(u^2+\tfrac{3}{4}\alpha^2)^{-7/2}. \quad (5)$$

To find the variational parameter $\alpha$, one has to minimize the energy expectation:

$$E(\alpha) = \langle\psi(\alpha)|H_0 + 3V_{\alpha\alpha}|\psi(\alpha)\rangle. \quad (6)$$

For $V_{\alpha\alpha}(r)$ we assumed both the Yukawa form

$$V_{\alpha\alpha}(r) = -Ae^{-\kappa r}/\kappa r$$

and also the exponential form

$$V_{\alpha\alpha}(r) = -Ae^{-\kappa r}.$$

These forms, besides being an oversimplification of the real world, suffer from three drawbacks: (1) They contain no hard core; (2) they do not include the Coulomb force; (3) one could recover neither the $^8Be$ resonance energy (from the solution of the radial Schrödinger equation) nor the $^{12}C$ ground-state energy [by finding the minimum of $E(\alpha)$ in Eq. (6)]. Nevertheless, it can be seen from Table I that, within the framework of binding energies, these choices are not too bad. The parameters $A$ and $\kappa$ should be chosen in such a way that the physical properties of $^8Be$ and $^{12}C$ will be reproduced approximately well. Among these properties, the most important are binding energies and form factors. Since the vital part of the input to Eq. (2) is the pole term at the $^{12}C$ binding energy, we found it necessary to choose $A$ and $\kappa$ so that $E(^{12}C)_{th} \approx E(^{12}C)_{exp}$. It turns out, however, that this choice leads to a loosely bound $^8Be$, in slight disagreement with experiment.

The kernel $K(q, p)$ in Eq. (2) was calculated by using a double Gaussian quadrature with 24 mesh points in each quadrature. The eigenvalues of $K$ were traced as a function of $\chi^2$ in the range $\chi^2 \gtrsim \gamma^2$, in which we used $\gamma^2 = -[E(\alpha)]_{min}$ instead of the experimental value. A solution of the equation $\lambda_2(\chi_0^2) = 1$ is therefore the energy of the first excited state of $^{16}O$ in this model. The present results (summarized in Table I) show the existence of such a state.

In conclusion, we see that the combination of the pole approximation, the variational calculation, and the $\alpha$-particle model of $^{16}O$ provides a tractable way to calculate the energy of the first $0^+$ excited state of $^{16}O$. The results deviate from experiment to about the same degree as do those of Ref. 5. It is therefore suggested that within this range of energies, the $\alpha$-particle model is quite reasonable. Whether that is true also for scattering calculations remains to be seen.

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TABLE I. Potential constants, the variational parameter, and the calculated binding energies for the ground states of $^8Be$ and $^{12}C$ and for the $0^+$ excited state of $^{16}O$. The energies, which are given relative to the $A/4$-$\alpha$ threshold, are to be compared with the experimental values (Refs. 10 and 12) $E(^8Be) = -0.095$ MeV, $E(^{12}C) = 7.28$ MeV, and $E(^{16}O^*) = 8.385$ MeV.

| | Yukawa $V_{\alpha\alpha}(r) = -Ae^{-\kappa r}/\kappa r$ | Exponential $V_{\alpha\alpha}(r) = -Ae^{-\kappa r}$ |
|---|---|---|
| $A$ (MeV) | 9.7 | 11.7 |
| $1/\kappa$ (fm) | 1.7 | 2.4 |
| $\alpha$ (fm$^{-1}$) | 0.89 | 0.46 |
| $E(^8Be)$ (MeV) | 3.15 | 1.74 |
| $E(^{12}C)$ (MeV) | 6.726 | 7.271 |
| $E(^{16}O^*)$ (MeV) | 6.935 | 8.314 |

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# Some Properties of the Lowest $T = 2$ States in $^{16}O^\dagger$

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A state at 22.722± 0.005-MeV excitation in $^{16}O$, with a width of 12.5± 2.5 keV, has been observed as a resonance in the $^{14}N+d$ and $^{12}C+\alpha$ reactions. The level is assigned $J^\pi = 0^+$ on the basis of excitation functions for the reaction $^{12}C(\alpha, \alpha_2)^{12}C$ measured at various angles and is presumed to be the $T = 2$ state reported at this energy. Several partial widths are obtained.

## I. INTRODUCTION

The lowest $T = 2$ state of $^{16}O$, first observed in the reaction $^{18}O(p, t)^{16}O$,[1] has recently been assigned an accurate excitation energy, 22.717 ±0.008 MeV, from studies using the reaction $^{14}C(^3He, n)^{16}O$.[2] Although decay of this state has not been observed, the dominant modes are expected to be neutron, proton, and $\alpha$ decay via lower isospin impurities in the predominantly $T = 2$ state. These "forbidden" channels should be of greater importance than diproton decay, which is isospin-allowed but, with a $Q$ value of 383 keV, is expected to contribute little width (≃0.01 eV from Ref. 2). If the state can be excited via such forbidden channels, information on its decay properties may be obtained. Since many channels are open because of the state's high excitation energy, such information may be extensive enough to lead to detailed knowledge of the state's isospin impurities. In the present work, the $T = 2$ state was observed as a narrow resonance in charged-particle yields from the $^{14}N+d$ and $^{12}C+\alpha$ reactions. These measurements are discussed in Secs. II and III, and they are interpreted in Sec. IV.

## II. $^{14}N+d$ MEASUREMENTS

A deuteron energy of 2.265 ± 0.009 MeV is expected to produce the $T = 2$ state in the $^{14}N+d$ reaction. We used the Stanford 3-MeV Van de Graaff accelerator to produce a beam of 20 $\mu$A in a differentially pumped gas target with an effective length of 2 cm. The target chamber was filled with natural nitrogen at a pressure of 0.6 Torr, corresponding to a deuteron energy loss of 0.4 keV. Two surface-barrier detectors subtending solid angles of $4 \times 10^{-3}$ sr could be positioned at laboratory angles between 20 and 160°. An initial set of measurements covered the energy region 2.25 to 2.28 MeV in 1-keV steps, with the detectors at $\theta_{lab} = 90$ and 160°; a second set covered 2.250 to 2.305 MeV in 2.5- and 5-keV steps with $\theta_{lab} = 125$ and 160°. A sample spectrum is shown in Fig. 1. Since current integration in the gas target was not entirely reliable (±10% variations), measurements were made for ≃6000 $\mu$C, and the yields of various