

# Single-Neutron-Transfer-Reaction Studies of $^{176}\text{Hf}$ and $^{180}\text{Hf}^\dagger$

J. I. Zaitz and R. K. Sheline

Florida State University, Tallahassee, Florida 32306

(Received 24 April 1972)

The strongly deformed even-even nuclei  $^{176}\text{Hf}$  and  $^{180}\text{Hf}$  have been studied using the reactions  $^{177}\text{Hf}(d,t)^{176}\text{Hf}$  and  $^{179}\text{Hf}(d,p)^{180}\text{Hf}$ . These reactions were performed at 12- and 14-MeV incident energy on the Florida State University super FN tandem Van de Graaff accelerator with the outgoing particles analyzed by means of a Browne-Buechner-type broad-range magnetic spectrograph. 25 states associated with two-quasiparticle rotational bands have been identified or tentatively identified in these nuclei; included are the intrinsic band heads  $6^+[514\uparrow + 512\uparrow]$ ,  $3^+[514\uparrow - 521\uparrow]$ ,  $4^+[514\uparrow + 521\uparrow]$ , and  $6^+[514\uparrow + 523\uparrow]$  in  $^{176}\text{Hf}$  and the intrinsic band heads  $4^- [624\uparrow - 510\uparrow]$ ,  $5^- [624\uparrow + 510\uparrow]$ ,  $6^- [624\uparrow + 512\uparrow]$ , and  $8^- [624\uparrow + 503\uparrow]$  in  $^{180}\text{Hf}$ . Coriolis-mixing calculations have been an important factor in making the state assignments.

## I. INTRODUCTION

Spectroscopic analysis of the data from neutron-transfer reactions on deformed even-even nuclei has been an important means for investigating the structure of deformed odd- $A$  nuclei and indeed has played a significant role in establishing the validity of the unified nuclear model. The spectroscopic method associated with neutron-transfer reactions has also been applied to the study of deformed even-even nuclei but the more complicated nature of the states being populated has limited the number of such investigations.

In the present research the nuclear structure of two even-even deformed nuclei,  $^{176}\text{Hf}$  and  $^{180}\text{Hf}$ , is investigated by studying the spectra from the reactions  $^{177}\text{Hf}(d,t)^{176}\text{Hf}$  and  $^{179}\text{Hf}(d,p)^{180}\text{Hf}$ . These reactions were initiated with 12- and 14-MeV deuterons from the Florida State University super FN tandem Van de Graaff accelerator. The outgoing tritons and protons were analyzed by means of a Browne-Buechner-type broad-range magnetic spectrograph.

The states which are populated in the present study are ground-state rotational band members, two-quasineutron states and their corresponding rotational band members, and large two-quasineutron components of vibrational states. Aiding in the identification of these states are studies of other hafnium isotopes<sup>1,2</sup> which have previously been performed in this laboratory.

## II. THEORETICAL CONSIDERATIONS

The most useful tools in analyzing the data from  $(d,p)$  and  $(d,t)$  reactions on deformed targets are the rotational formula, which predicts the energy spacing of rotational band members and the cross-section formula, which gives the probabilities for populating states of a single-particle nature in the residual nucleus. The rotational formula is described by the relation

$$E = \frac{\hbar^2}{2\mathcal{J}} I(I+1),$$

where  $\mathcal{J}$  is the effective moment of inertia about an axis perpendicular to the nuclear symmetry axis, and  $I$  is the total angular momentum of the nucleus. The cross-section formula is given by the relation<sup>3</sup>

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{2I_f + 1}{2I_t + 1} \sum_i S_i \phi_i(\theta),$$

with  $I_t$  the spin of the target,  $I_f$  the spin of the residual nucleus,  $S_i$  a spectroscopic factor containing all the information on nuclear structure, and  $\phi_i(\theta)$  the intrinsic single-particle cross section. The spectroscopic factor is defined by the relation

$$S_i = \sum_j \beta_{ji}^2$$

with the reduced-width amplitude  $\beta_{ji}$  defined by the form of the overlap integral of the initial and

final nuclear wave functions;

$$\int d\xi \Psi_{I_f M_f}(\xi, x_N) \Psi_{I_i M_i}(\xi) = \sum_{j1m} \beta_{j1} \phi_{j1m}(x_N) C(I_i j I_f; M_i m M_f),$$

$\xi$  representing the coordinates of the target nucleons,  $x_N$  representing the coordinates of the captured particle, and  $\phi_{j1m}(x_N)$  representing the bound-state wave function of the captured particle. For single-neutron-

transfer reactions on odd-neutron targets

$$\Psi_{I_i M_i} = \left( \frac{2I_i + 1}{16\pi^2} \right)^{1/2} \sum_{J_{ni}} C_{J_{ni}}^{\Omega_{ni}} [D_{M_i K_i}^{I_i} \chi_{J_{ni}}^{\Omega_{ni}}(1) + (-1)^{I_i - J_{ni}} D_{M_i - K_i}^{I_i} \chi_{J_{ni}}^{\Omega_{ni}}(1)],$$

$$\Psi_{I_f K_f} = \frac{1}{\sqrt{2}} \left( \frac{2I_f + 1}{16\pi^2} \right)^{1/2} \sum_{J_{n1}} \sum_{J_{n2}} C_{J_{n1}}^{\Omega_{n1}} C_{J_{n2}}^{\Omega_{n2}} \{ D_{M_f K_f}^{I_f} [\chi_{J_{n1}}^{\Omega_{n1}}(1) \chi_{J_{n2}}^{\Omega_{n2}}(2) - \chi_{J_{n2}}^{\Omega_{n2}}(1) \chi_{J_{n1}}^{\Omega_{n1}}(2)]$$

$$+ (-1)^{I_f - J_{n1} - J_{n2}} D_{M_f - K_f}^{I_f} [\chi_{J_{n1}}^{-\Omega_{n1}}(1) \chi_{J_{n2}}^{-\Omega_{n2}}(2) - \chi_{J_{n2}}^{-\Omega_{n2}}(1) \chi_{J_{n1}}^{-\Omega_{n1}}(2)] \},$$

and it follows that<sup>4</sup>

$$\beta_{ji} = \frac{1}{\sqrt{2}} \left( \frac{2I_i + 1}{2I_f + 1} \right)^{1/2} \{ C_{J_{n2}}^{\Omega_{n2}} [C(I_i J_{n2} I_f; K_i \Omega_{n2} K_f) \delta_{\Omega_{n1} \Omega_{ni}} + (-1)^{I_i - 1/2} \pi_{ni} C(I_i J_{n2} I_f; -K_i \Omega_{n2} K_f) \delta_{\Omega_{n1} - \Omega_{ni}}]$$

$$- C_{J_{n1}}^{\Omega_{n1}} [C(I_i J_{n1} I_f; K_i \Omega_{n1} K_f) \delta_{\Omega_{n2} \Omega_{ni}} + (-1)^{I_i - 1/2} \pi_{ni} C(I_i J_{n1} I_f; -K_i \Omega_{n1} K_f) \delta_{\Omega_{n2} - \Omega_{ni}}] \} \langle f | i \rangle$$

with  $\langle f | i \rangle$  the overlap integral of the initial and final vibrational wave functions. In  $(d, p)$  and  $(d, t)$  reactions the nucleus tends to stay in its vibrational ground state<sup>3</sup> and  $\langle f | i \rangle$  is taken to be unity. With this assumption and with the predictions of the pairing model the cross-section formula has the form

$$\left( \frac{d\sigma}{d\Omega}(\theta) \right)_{I_f K_f} = \frac{1}{2} \sum_i \sum_j (C_j^{\Omega_j})^2 [C(I_i j I_f; K_i \Omega_j K_f)]^2 \phi_i(\theta) \left\{ \frac{U^2}{V^2} \right\},$$

the unoccupation probability  $U^2$  corresponding to stripping reactions and the occupation probability  $V^2$  corresponding to pickup reactions. The only angular dependence in this formula is in the intrinsic single-particle cross section  $\phi_i(\theta)$ . This cross section is usually calculated using standard distorted-wave Born-approximation (DWBA) theory. In the present work the zero-range DWBA codes DWUCK and TARO have been used for the  $(d, t)$  and  $(d, p)$  calculations, respectively.

The characteristic fingerprint pattern predicted by the cross-section and rotational formulas makes the identification of rotational bands built on two-quasiparticle states analogous to the identification of rotational bands built on single-particle states in deformed odd- $A$  nuclei. Complicating the problem in even-even nuclei is the fact that the two-quasiparticle states all occur in a region of the spectrum where strong mixing with vibrational excitations is a possibility. Also complicating the spectroscopic problem is the presence of Coriolis mixing between rotational states built on two-quasiparticle band heads with projection quantum numbers  $K$  differing by unity. Zaitz<sup>5</sup> gives the full Coriolis-coupling matrix element for two-quasiparticle band mixing. In instances such as the present analysis where the mixed two-quasiparticle states are composed of Nilsson orbits with different projection quantum numbers the simplified form below suffices:

$$\langle I M K \Omega_{n1} \Omega_{n2} | H_{Cor} | I' M' K' \Omega'_{n1} \Omega'_{n2} \rangle$$

$$= -\frac{1}{2} \frac{\hbar^2}{2\mathcal{J}} \{ \sum |C_{j_{n1}}^{\Omega_{n1}}|^2 C_{j_{n2}}^{\Omega_{n2}} C_{j_{n2}'}^{\Omega_{n2}'} [(I + K' + 1)^{1/2} (I - K')^{1/2} \delta_{KK'+1} \delta_{\Omega_{n2} \Omega'_{n2}+1} (j_{n2} + \Omega'_{n2} + 1)^{1/2} (j_{n2} - \Omega'_{n2})^{1/2}$$

$$+ (I - K' + 1)^{1/2} (I + K')^{1/2} \delta_{KK'-1} \delta_{\Omega_{n2} \Omega'_{n2}-1} (j_{n2} - \Omega'_{n2} + 1)^{1/2} (j_{n2} + \Omega'_{n2})^{1/2}]$$

$$+ \sum |C_{j_{n1}}^{\Omega_{n1}}|^2 C_{j_{n2}'}^{\Omega_{n2}'} C_{j_{n2}}^{\Omega_{n2}} [(I + K' + 1)^{1/2} (I - K')^{1/2} \delta_{KK'+1} \delta_{\Omega_{n2} \Omega'_{n2}+1} (j_{n2} + \Omega'_{n2} + 1)^{1/2} (j_{n2} - \Omega'_{n2})^{1/2}$$

$$+ (I - K' + 1)^{1/2} (I + K')^{1/2} \delta_{KK'-1} \delta_{\Omega_{n2} \Omega'_{n2}-1} (j_{n2} - \Omega'_{n2} + 1)^{1/2} (j_{n2} + \Omega'_{n2})^{1/2}] \}.$$

### III. EXPERIMENTAL PROCEDURES

The reactions  $^{177}\text{Hf}(d, t)^{176}\text{Hf}$  and  $^{179}\text{Hf}(d, p)^{180}\text{Hf}$  were performed in the tandem accelerator laboratory at Florida State University. Each reaction was studied at several different angles to sort out impurity peaks by observing their kinematic shift

in the triton and proton spectra and to better fix excitation energies. The emerging protons and tritons were analyzed by a Browne-Buechner-type broad-range magnetic spectrograph, the particles impinging on a set of four nuclear track emulsion plates which were placed along the spectrograph's focal curve. The developed plates were counted

in 0.5-mm strips by a binocular microscope equipped with a special stage for accurately measuring distances. The histograms of tracks per 0.5-mm strip were fitted with sums of Gaussian peaks by the computer code STRILDE.<sup>6</sup> This very useful code determines the centroids and areas of the fitted peaks, the energies and  $Q$  values to which the peak centroids correspond, and, from the peak areas and the focal-curve dependence of the solid angle of the spectrograph, the relative cross sections of the different peaks.

The targets used in the present experiment were prepared both by the technique of vacuum evaporation and through the use of an electromagnetic isotope separator. Both of these processes made use of 50- $\mu\text{g}/\text{cm}^2$  carbon backings to support the target material. An enriched sample of  $\text{HfO}_2$  was obtained from Oak Ridge National Laboratory for use in the evaporation. For the isotope separation it was found that  $\text{HfF}_4$  produced the strongest and most workable hafnium beams.

#### IV. EXPERIMENTAL RESULTS AND INTERPRETATIONS

##### A. $^{176}\text{Hf}$

The reaction  $^{177}\text{Hf}(d,t)^{176}\text{Hf}$  was studied at reaction angles of 35 and 85° using an evaporated target with a 92% isotopic enrichment and at reaction angles of 55 and 75° using a target which was made on the Florida State University isotope separator. The reactions at 35 and 85° were initiated with 14-MeV deuterons while the reactions at the other

angles were initiated with 12-MeV deuterons. A sample spectrum is presented in Fig. 1. A listing of the averaged excitation energies and the corresponding relative intensities of the states populated in  $^{176}\text{Hf}$  is given in Table I. The intensities are normalized to the strongest peak at each respective angle.

The  $Q$  value for the reaction  $^{177}\text{Hf}(d,t)^{176}\text{Hf}$  was determined through the identification of three impurity states (the 617-keV state of  $^{179}\text{Hf}$  and the 1638- and 2023-keV states of  $^{178}\text{Hf}$ ) in the evaporated target spectra and the knowledge of the  $Q$  values of these states. This was done by using the computer code SAM<sup>7</sup> to recalculate the incident energies and the spectrograph fields while the experimental angles and the positions of the three Hf impurity peaks in the spectra were held fixed. By rerunning the program STRILDE with the recalculated energy and field values the  $Q$  value was measured to be  $-127 \pm 11$  keV.

The analysis of the spectra obtained in the present study is aided by Rickey's<sup>1</sup> work on  $^{177}\text{Hf}$ . By performing both the  $(d,p)$  and  $(d,t)$  reactions leading to this nucleus Rickey was able to experimentally determine the occupation probabilities of the neutron states near the Fermi surface. These occupation probabilities are important in studying the relative strengths of the different rotational bands populated in the present  $(d,t)$  reaction.

The DWBA cross sections used in the present analysis were obtained by running the computer code DWUCK. The optical-model parameters used in the code are those of Perey<sup>8</sup> and were found by

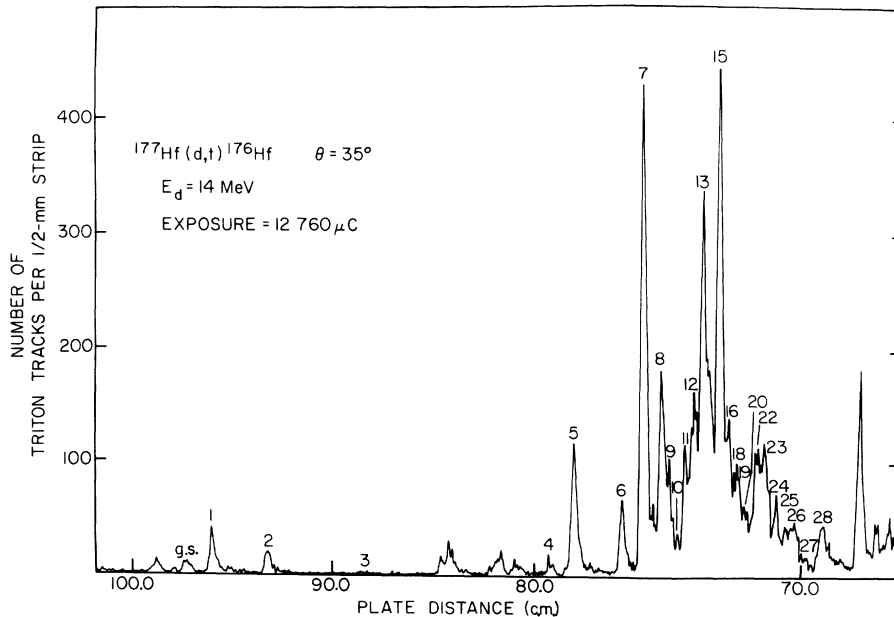


FIG. 1. Triton spectrum from the reaction  $^{177}\text{Hf}(d,t)^{176}\text{Hf}$  at 35°.

Rickey to account for the angular distribution of states of known  $l$  value in  $^{177}\text{Hf}$ . The parameters are presented in Table II.

The nucleus  $^{177}\text{Hf}$  has 72 protons and 105 neutrons. The odd neutron populates the  $\frac{7}{2}^- [514]$  Nilsson orbital so that in the  $(d, t)$  reaction the two-quasiparticle states populated are those in which the  $\frac{7}{2}^- [514]$  neutron orbital is coupled to another Nilsson neutron state. The ground state is populated when the  $\frac{7}{2}^- [514]$  neutron is the particle picked up in the  $(d, t)$  reaction.

Previous experimental studies<sup>9</sup> of the  $^{176}\text{Hf}$  level scheme give the energies of the  $0^+$ ,  $2^+$ ,  $4^+$ , and  $6^+$  members of the ground-state rotational band as 0, 88.36, 290.3, and 596 keV. In the present work these states are identified at energies of 0, 89, 291, and 598 keV. Theory indicates that only the  $2^+$  and  $4^+$  members of this band are appreciably populated in the  $(d, t)$  reaction and indeed these are the only ground-state members seen in all four experimental spectra. The  $6^+$  member is especially weak with predicted strengths relative

to the  $4^+$  state of 0.166 at  $35^\circ$  and 0.172 at  $85^\circ$ . These strengths amount to peaks of a very few counts, but nevertheless peaks definitely seen in the  $35^\circ$  and  $85^\circ$  spectra.

Nilsson systematics from  $^{177}\text{Hf}^1$  suggests that the lowest-lying two-quasiparticle state in the present spectrum should be the  $8^- [514\uparrow + 624\uparrow]$  state. The fact that this is a particle state and that 97% of the strength of this state lies in the  $l=6$ ,  $\frac{13}{2}^+$  member of the rotational band results in an extremely small cross section. Neither this state nor the corresponding  $1^- [514\uparrow - 624\uparrow]$  state is identified in the present work.

The first hole state in Rickey's  $^{177}\text{Hf}$  spectrum is the  $\frac{5}{2}^- [512]$  state and the first two-quasiparticle state seen in the present spectrum is the corresponding  $6^+ [514\uparrow + 512\uparrow]$  state. The  $6^+$  and  $7^+$  members of the  $6^+ [514\uparrow + 512\uparrow]$  rotational band are both strongly populated in the  $(d, t)$  reaction. (See Table III for a listing of theoretical relative cross sections at  $35^\circ$ .) In addition both of these states have virtually the same cross section. At

TABLE I. Energies and relative cross sections for levels in  $^{176}\text{Hf}$  populated in the  $(d, t)$  reaction.

Peak No.	Excitation energy (keV)	$\sigma^a$ (keV)	Relative differential cross section			
			$35^\circ$	$55^\circ$	$75^\circ$	$85^\circ$
g.s.	0	1.5	0.024	...	...	0.021
1	89	1.4	0.088	0.023	0.092	0.133
2	291	2.2	0.054	0.069	0.114	0.155
3	598	1.3	0.002	...	...	0.038
4	1245	1.0	0.021	0.022	...	0.098
5	1333	2.7	0.220	0.206	0.207	0.365
6	1508	... <sup>b</sup>	0.128	0.210	0.172	0.354
7	1580	1.7	0.872	1.000	0.989	0.870
8	1678	1.5	0.402	0.307	1.000 <sup>c</sup>	0.502
9	1706	2.1	0.192	0.182	0.276	0.346
10	1726	3.6	0.023	0.199	0.068	0.128
11	1761	3.8	0.259	0.070	0.168	0.518
12	1797	4.7	0.376	...	0.229	0.700
13	1830	2.1	0.682	0.471	0.480	0.767
14	1856	2.3	0.368	0.171	0.318	0.782
15	1888	2.8	1.000	0.665	0.872	1.000
16	1924	1.9	0.264	0.171	0.160	0.517
17	1940	2.5	0.076	...	0.123	...
18	1958	3.0	0.221	0.064	0.904	0.379
19	1986	2.7	0.143	0.042	0.091	0.256
20	2020	4.4	0.161	0.129	0.081	0.522
21	2040	1.7	0.152	0.156	0.151	0.250
22	2062	2.0	0.232	0.033	0.157	0.402
23	2080	1.0	0.102	0.035	0.035	...
24	2107	3.4	0.142	0.068	0.183	0.471
25	2144	4.3	0.106	0.048	0.047	0.310
26	2177	4.5	0.116	0.065	0.038	0.198
27	2219	0.5	0.035	...	...	0.095
28	2276	3.3	0.118	...	0.049	0.235

<sup>a</sup>  $\sigma$  represents the standard deviation of the energy excitations at different angles.

<sup>b</sup> Excitation reference.

<sup>c</sup>  $^{13}\text{C}$  peak enhances 1678-keV state at  $75^\circ$ .

TABLE II. Optical-model parameters used in the  $^{177}\text{Hf}(d, t)^{176}\text{Hf}$  study.

Particle	$V_S$	$W_S$	$r_{0S}$	$a_{0S}$	$4W_D$	$r'_{0i}$	$a'_i$	$r_{\text{Coul}}$
Deuteron	77	0	1.24	0.84	45	1.37	0.93	1.25
Triton	100	14	1.07	0.854	0	1.70	0.73	1.40

1333 and 1508 keV are two strong states of approximately equal strength whose energy separation gives a moment of inertia parameter of 12.50 keV if they are assigned as the  $6^+$  and  $7^+$  members of the  $6^+[514\uparrow + 512\uparrow]$  band. This assignment is made and with a good deal of confidence since the 1333- and 1508-keV states are the first strong states in the spectrum. The  $8^+$  and  $9^+$  members of the  $6^+[514\uparrow + 512\uparrow]$  band are predicted to have strengths relative to the band head of 0.020 and 0.003 at  $35^\circ$ . By referring to Fig. 1 it is easily seen that states with this strength are too weak to be seen in the dense region of the spectrum which follows the 1508-keV state.

The strongest states in the  $^{177}\text{Hf}(d, t)^{176}\text{Hf}$  spectrum are predicted to be the band heads of the  $3^+[514\uparrow - 521\uparrow]$  and  $4^+[514\uparrow + 521\uparrow]$  rotational bands. These band heads should have a relative strength of 0.901 at  $35^\circ$ . The present  $35^\circ$  triton spectrum has two giant states at 1580 and 1888 keV whose relative strength is 0.913. This agreement between experiment and theory makes the assignment of the 1580- and 1888-keV states as the  $3^+[514\uparrow - 521\uparrow]$  and  $4^+[514\uparrow + 521\uparrow]$  two-quasiparticle states very plausible. The 308-keV ener-

gy separation of these states is similar to the 350-keV energy separation of the known set of two-quasiparticle states formed from the same single-particle states in  $^{178}\text{Hf}$ .<sup>2</sup>

The rotational bands built on the  $3^+[514\uparrow - 521\uparrow]$  and  $4^+[514\uparrow + 521\uparrow]$  states are Coriolis mixed. The results of the mixing calculations and the effects of the mixing on the cross sections of the states in the two bands are shown in Table IV. It should be mentioned that the mixing calculations have been done by considering the experimental energy positions (which are discussed below) as the correct perturbed energy positions of the states.

The results of the mixing calculations give the  $4^+$  member of the  $3^+$  band a predicted strength relative to the band head of 0.478 at  $35^\circ$ . A state is observed at 1678 keV with a strength relative to the  $3^+$  band head of 0.435 at  $35^\circ$ . Since this is the only state within over 200 keV with such a comparable cross section, and since the energy position of this state gives a reasonable moment of inertia parameter of 12.25 keV, it is assigned as the  $4^+$  member of the  $3^+[514\uparrow + 521\uparrow]$  band.

The  $5^+$  member of the  $3^+$  band is identified as the level at 1797 keV. The strength of this state relative to the  $3^+$  band head is 0.164 at  $75^\circ$ , in good agreement with the theoretical value for the relative strength at this angle of 0.186. At  $35^\circ$  and  $85^\circ$ , however, this state is seen with strengths two and four times that of the expected value. This is due to the presence in this vicinity of the 560-keV state of  $^{177}\text{Hf}$ , the strongest state in the  $(d, t)$  reaction leading to this nucleus.

TABLE III. Relative theoretical cross sections of levels in  $^{176}\text{Hf}$  populated in the  $(d, t)$  reaction at  $35^\circ$ .

Intrinsic state	Occupation <sup>a</sup> probability	Level									
		0	1	2	3	4	5	6	7	8	9
$3^+[514\uparrow - 510\uparrow]$					5.98	4.12	1.23	b			
$4^+[514\uparrow + 510\uparrow]$	0.08					7.26	3.75				
$8^-[514\uparrow + 624\uparrow]$											
$1^-[514\uparrow - 624\uparrow]$	0.29										
$0^+[514\uparrow - 514\uparrow]$	0.57	0.206		0.590		0.891		0.148		0.001	
$6^+[514\uparrow + 512\uparrow]$								15.68	15.66		
$1^+[514\uparrow - 512\uparrow]$	0.70		2.65	7.79	9.94	7.16	3.16				
$3^+[514\uparrow - 521\uparrow]$	0.90				60.00 <sup>c</sup>	20.02	7.64	2.90			
$4^+[514\uparrow + 521\uparrow]$						75.40	13.29	5.92	1.41		
$0^-[514\uparrow - 633\uparrow]$	0.99										
$7^-[514\uparrow + 633\uparrow]$										1.83	
$1^+[514\uparrow - 523\uparrow]$			2.18	3.72	4.34	3.57	2.02				
$6^+[514\uparrow + 523\uparrow]$	1.0							8.81	6.36	1.48	
$6^-[514\uparrow + 642\uparrow]$								1.16	1.09	2.17	1.75
$1^-[514\uparrow - 642\uparrow]$	1.0					1.09	1.53	1.69	1.26		
$5^+[514\uparrow + 521\uparrow]$							30.91	15.79	7.84		
$2^+[514\uparrow - 521\uparrow]$	1.0			23.12	16.42	14.41	8.95	3.24			

<sup>a</sup> Occupation probabilities are those of Rickey, Jr., and Sheline (Ref. 1).<sup>b</sup> Cross sections less than 1.0 are omitted with the exception of the ground-state band.<sup>c</sup> Cross sections do not include Coriolis mixing.

The  $6^+$  member of the  $3^+[514\downarrow - 521\downarrow]$  rotational band is expected at 1944 keV with a strength relative to the  $3^+$  band head of 0.041. A state is seen at 1940 keV with a strength of 0.084 relative to the  $3^+$  band head. Since this state is the only weak state in this region of the spectrum, it is tentatively assigned as the  $6^+$  member of the  $3^+[514\downarrow - 521\downarrow]$  band.

As the  $3^+[514\downarrow - 521\downarrow]$  state has a compressed rotational band due to Coriolis coupling, the  $4^+[514\downarrow + 521\downarrow]$  band is expected to have an expanded rotational structure. The  $5^+$  member of this band is seen at 2062 keV, an energy that yields a moment of inertia parameter of 17.4 keV. The strength of this state relative to the band head is 0.234 at  $35^\circ$ . This compares favorably with the theoretical relative strength at this angle of 0.206. Energetically the 2040-keV state is also a candidate for the  $5^+$  member of the  $4^+[514\downarrow + 521\downarrow]$  band, but the 2062-keV state has been chosen because of cross-section considerations.

The  $6^+$  member of the  $4^+[514\downarrow + 521\downarrow]$  band is tentatively identified as the level at 2276 keV. The strength of this state relative to the  $4^+$  band head is 0.122 at  $35^\circ$ . This compares well with the theoretical relative strength at this angle of 0.096.

The other strong states expected to be seen in the  $^{177}\text{Hf}(d, t)^{176}\text{Hf}$  spectrum are those hole states with large enough strengths in the low  $l$  value members of their rotational bands to have sizable cross sections. There are surprisingly few of these states, even though the occupation probability becomes 0.99 for the  $\frac{7}{2}^+[633]$  state, the hole state beneath the  $\frac{1}{2}^-[521]$  state, and 1.0 for all deeper hole states. The  $\frac{7}{2}^+[633]$  and  $\frac{5}{2}^+[642]$  states have 93% and 88% of their strengths in the  $l=6$ ,  $\frac{13}{2}^+$  members of their rotational bands and the  $\frac{5}{2}^-[523]$  state has 78% of its strength in the  $l=5$ ,  $\frac{9}{2}^-$  member of its rotational band. The  $6^+[514\downarrow + 523\downarrow]$  state, or the higher-energy configuration of the  $\frac{7}{2}^-[514]$  and  $\frac{5}{2}^-[523]$  states, does have a strength of approximately 50% of the  $6^+[514\downarrow + 512\downarrow]$  state and should be seen in the present

$(d, t)$  spectrum. This state is tentatively identified at 1986 keV with the  $7^+$  member of its rotational band tentatively identified at 2177 keV. The theoretical relative cross sections of these states are 1.39 at  $35^\circ$  and 1.25 at  $85^\circ$ . The experimental values for these relative cross sections are 1.22 at  $35^\circ$  and 1.28 at  $85^\circ$ . The moment of inertia parameter for this rotational band is 13.8 keV.

The other hole state which should be seen in the present  $(d, t)$  spectrum is the  $5^+[514\downarrow + 521\downarrow]$  state. This state and the first two members of its rotational band are strongly populated in the  $(d, t)$  reaction and should account for some of the large unidentified states which appear above 1850 keV in the present spectrum. There are several candidates for the  $5^+$  and  $6^+$  members of the  $5^+[514\downarrow + 521\downarrow]$  rotational band, but none of the combinations produces a reasonable assignment for the  $7^+$  state. The most reasonable assignments for the  $5^+$  and  $6^+$  members of the  $5^+[514\downarrow + 521\downarrow]$  rotational band are the 1856- and 2020-keV states. These states have relative cross sections of 2.26 at  $35^\circ$  and 1.49 at  $85^\circ$ . This compares favorably with the predicted relative cross sections at these angles of 1.96 and 1.31. The moment-of-inertia parameter for this band is 13.6 keV and the  $7^+$  state should hence be seen at 2211 keV. A state is seen at 2219 keV with strengths relative to the  $5^+$  band head of 0.026 at  $35^\circ$  and 0.124 at  $85^\circ$ . This compares unsatisfactorily with the theoretical values for these strengths of 0.254 and 0.383, respectively.

Other studies of the  $^{176}\text{Hf}$  level scheme involve studies of the decays and the activities produced by the decays<sup>10-15</sup> of  $^{176}\text{Lu}$ ,  $^{176m}\text{Lu}$ , and  $^{176}\text{Ta}$  as well as studies of the reactions  $^{177}\text{Hf}(\gamma, n)$ <sup>16</sup> and  $^{176}\text{Lu}(p, n\gamma)$ .<sup>17</sup> The only states identified in the present study (other than members of the ground-state band) which have previously been observed are the  $6^+$  and  $7^+$  members of the  $6^+[514\downarrow + 512\downarrow]$  rotational band. These states were observed at 1335 and 1508 keV in the  $^{176}\text{Lu}(p, n\gamma)$  work of Borggreen *et al.* Because of the low population of

TABLE IV. Results of a Coriolis band mixing calculation for the two bands in  $^{176}\text{Hf}$  resulting from the  $\frac{1}{2}^-[521]$  orbital. The quantity  $b$  is the matrix element of the Coriolis term  $H_{\text{RPC}}$  of the rotational Hamiltonian. The superscript 0 refers to unmixed quantities.

$I$	$K$	$b$ (keV)	Energies (keV)		Wave function		Relative intensities at $35^\circ$		
			$E^0$	$E_{\text{Cor}}$	$a_K=3$	$a_K=4$	$I^0$	$I_{\text{Cor}}$	$I_{\text{exp}}$
3	3	...	1540	1590	1.000	0.000	1.000	1.000	1.000
4	3	39	1695	1688	0.982	-0.187	0.334	0.478	0.435
5	3	58	1816	1803	0.975	-0.221	0.127	0.119	...
6	3	75	1968	1950	0.974	-0.229	0.048	0.041	0.084
4	4	39	1891	1898	0.187	0.982	1.255	1.110	1.095
5	4	58	2059	2072	0.221	0.975	0.222	0.229	0.257
6	4	75	2268	2286	0.229	0.974	0.099	0.106	0.133

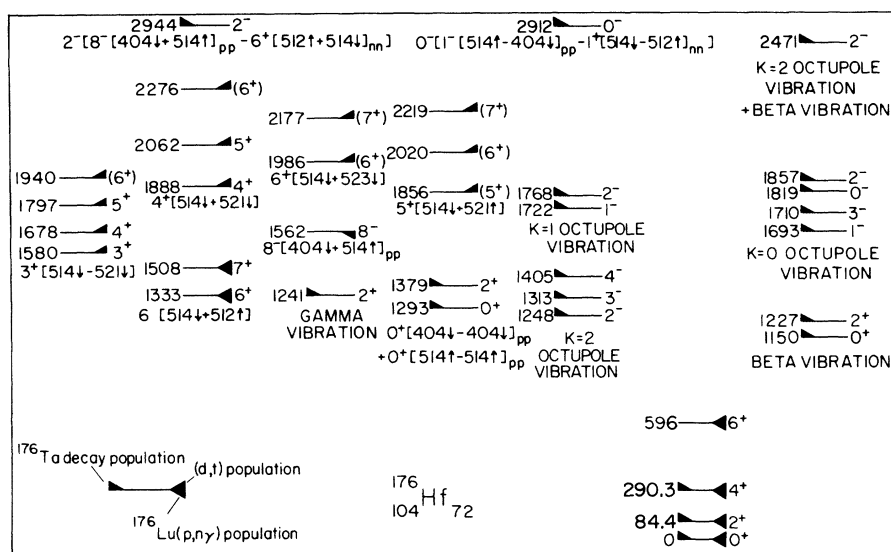


FIG. 2. Level scheme for  $^{176}\text{Hf}$  showing assignments of rotational bands. States which have not been given wave function assignments are not shown. Energies for members of the ground-state band are from Ref. 9.

the ground-state band members in the  $^{177}\text{Hf}(d, t)$  reaction, the  $7^+$  state at 1508 keV serves as an excitation reference for all two-quasiparticle assignments in the present work.

Figure 2 presents a level scheme of all the known assigned states in  $^{176}\text{Hf}$ . Included are the  $8^- [404+ + 514+]$  two-quasiproton state identified by Borggreen *et al.*<sup>17</sup> and the states recently investigated by Bernthal, Rasmussen, and Hollander<sup>15</sup> from Bernthal's study<sup>14</sup> of the electron capture decay of  $^{176}\text{Ta}$ . These latter states include  $(2, 2, -)$ ,  $(0, 1, -)$ , and  $(1, 1, -)$  octupole vibrations at energies of 1248, 1643, and 1722 keV, and  $(0, 0, +)$  states at energies of 1150 and 1293 keV. The energies for the  $(2, 2, -)$  and  $(0, 1, -)$  octupole vibra-

tions are in reasonable agreement with the calculations of Neergard and Vogel,<sup>18</sup> calculations which have proved extremely successful in predicting the energies of octupole vibrations in  $^{178}\text{Hf}$ .<sup>19</sup> The  $(1, 1, -)$  state, however, appears 79 keV above the  $(0, 1, -)$  octupole state while the Neergard-Vogel prediction is that this state lies 90 keV below the  $(0, 1, -)$  state.

Rotational band members are seen on the  $(2, 2, -)$ ,  $(0, 1, -)$ , and  $(1, 1, -)$  octupole states. These are indicated in Fig. 2. Of particular interest are the  $0^-$  and  $2^-$  states at 1819 and 1857 keV. Bernthal, Rasmussen, and Hollander suggest that these states are the remaining fragments of a  $0^-$  two-quasiparticle band whose odd members heavily

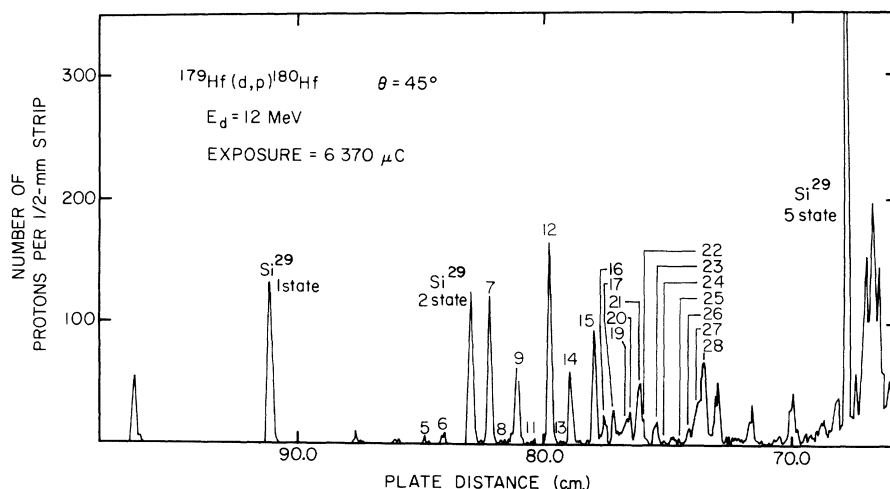


FIG. 3. Proton spectrum from the reaction  $^{179}\text{Hf}(d, p)^{180}\text{Hf}$  at  $45^\circ$ .

contribute to the  $(0, 1, -)$  octupole vibrational band.

The  $0^+$  states at 1150 and 1293 keV are also interesting to consider. In most even-even rare-earth nuclei the lowest  $0^+$  state is a  $\beta$  vibration. However, a calculation by Mikoshiba *et al.*<sup>20</sup> which takes into account the mixing between the quadrupole and pairing vibrations predicts that in  $^{176}\text{Hf}$  the lowest  $0^+$  state is primarily a pairing vibration. The evidence for this prediction could come from a  $(t, p)$  reaction. The Bernthal electron capture work allows only speculation about the nature of the  $0^+$  states. Bernthal, Rasmussen, and Hollander tentatively identify the 1150-keV state as a  $\beta$  vibration and the 1293-keV state as a mixture of the  $\frac{7}{2}^+[404]$  and  $\frac{9}{2}^- [514]$  proton pairs.

The ground-state configuration for the nucleus  $^{176}\text{Ta}$  consists of a proton in the  $\frac{7}{2}^+[404]$  orbital and a neutron in the  $\frac{5}{2}^- [512]$  orbital. It is suggested by Bernthal, Rasmussen, and Hollander that the low  $\log(ft)$  transitions from  $^{176}\text{Ta}$  to the five states above 2900 keV in the Bernthal  $^{176}\text{Hf}$  spec-

trum proceed by the Gamow-Teller transformation of the  $\frac{9}{2}^- [514]$  proton to the  $\frac{7}{2}^- [514]$  neutron. The  $(2, 2, -)$  state at 2944 keV thus consists primarily of the  $2^- \{8^- [404\uparrow + 514\uparrow]_{pp} - 6^+ [512\uparrow + 514\uparrow]_{nn}\}$  four-quasiparticle state while the  $(0, 0, -)$  state at 2912 keV consists primarily of the  $0^- \{1^- [514\uparrow - 404\uparrow]_{pp} - 1^+ [514\uparrow - 512\uparrow]_{nn}\}$  four-quasiparticle state.

### B. $^{180}\text{Hf}$

The even-even nucleus  $^{180}\text{Hf}$  has been studied by the reaction  $^{179}\text{Hf}(d, p)^{180}\text{Hf}$ . This reaction was carried out at angles of 25, 35, 45, 55, and 65° using 12-MeV deuterons incident on a separated target. A sample spectrum is presented in Fig. 3. The numbered peaks in the spectrum correspond to states which have been observed at three or more angles and have been designated as states in  $^{180}\text{Hf}$ . A listing of these states with the averaged excitation energies alongside the corresponding

TABLE V. Energies and relative cross sections for levels in  $^{180}\text{Hf}$  populated in the  $(d, p)$  reaction.

Peak No.	Excitation energy (keV)	$\sigma$ (keV)	Relative differential cross section				
			25°	35°	45°	55°	65°
g.s.	0	... <sup>a</sup>	...	...	...	0.004	0.017
1	95	...	...	...	...	...	0.010
2	301	...	...	...	...	...	0.039
3	640	...	...	...	...	...	0.013
4	1090	1.0	...	0.117	...	...	0.021
5	1146	1.1	0.224	0.142	0.004	...	...
6	1197	2.8	...	0.346 <sup>c</sup>	0.039	...	0.016
7	1371	... <sup>b</sup>	1.122	0.907	0.610	0.603	1.000
8	1408	2.2	...	0.391	0.023	...	0.079
9	1480	1.0	0.489	0.738	0.389	...	0.652
10	1508	2.2	0.183	0.499	...	...	0.405
11	1525	3.6	...	0.244	0.003	...	0.090
12	1609	1.2	1.000	1.000	1.000	1.000	2.231 <sup>c</sup>
13	1650	4.1	0.376	0.006	0.024	0.088	...
14	1689	1.7	0.530	0.321	0.414	0.544	0.648
15	1787	1.5	2.132 <sup>d</sup>	0.737	0.555	0.895	1.105
16	1824	2.3	0.355	0.303	0.171	0.091	0.710
17	1861	2.7	0.535	0.821 <sup>d</sup>	0.212	0.465	0.524
18	1889	2.5	...	0.150	0.073	...	0.079
19	1908	4.5	0.409	...	0.122	0.097	0.204
20	1926	3.8	...	0.143	0.141	0.138	0.410
21	1971	3.2	0.440	0.336	0.379	0.679	0.442
22	2023	2.0	...	...	0.134	0.189	0.127
23	2074	2.4	...	...	0.151	0.201	0.412
24	2143	4.0	...	...	0.064	0.087	0.286
25	2170	3.6	...	...	0.036	0.096	0.245
26	2213	3.3	0.316	...	0.113	0.124	...
27	2238	3.1	...	0.344	0.232	0.238	0.795
28	2271	2.5	...	0.395	0.485	0.493	0.745

<sup>a</sup> The ground-state band is seen only at 65°.

<sup>b</sup> Excitation reference.

<sup>c</sup> Strength augmented by silicon impurity.

<sup>d</sup> Strength augmented by sulfur impurity.



TABLE VI. Optical-model parameters used in the  $^{179}\text{Hf}(d, p)^{180}\text{Hf}$  study.

Particle	$V_S$	$W_S$	$r_{0S}$	$a_{0S}$	$4W_D$	$r_{0i}$	$a_i$	$r_{\text{Coul}}$
Deuteron	104	0	1.15	0.81	48	1.34	0.68	1.15
Proton	55	0	1.25	0.65	60	1.25	0.47	1.25

relative intensities is presented in Table V. Also in Table V are the energies and relative intensities observed for the members of the ground-state band at  $65^\circ$ . The fact that this band is only observed at this angle can perhaps be accounted for by the theoretically predicted low cross section for populating this band and the long exposure of the  $65^\circ$  run.

The  $(d, p)$  spectra obtained in the present work are characterized by a quintet of strong states at energies of 1371, 1480, 1609, 1689, and 1787 keV. It is against this background that the migration from angle to angle of the first five states of  $^{29}\text{Si}$  populated in the reaction  $^{28}\text{Si}(d, p)^{29}\text{Si}$  is observed. The identification of these strong impurity peaks (which appear because of the high concentration of silicon in the vacuum pump oil) allows, through the use of the computer code SAM,<sup>7</sup> a fixing of the experimental parameters and a determination of the  $Q$  value for the reaction  $^{179}\text{Hf}(d, p)^{180}\text{Hf}$ . The  $Q$  value of  $5167 \pm 7$  keV obtained compares reasonably well with the  $Q$  value of  $5158.5 \pm 3$  keV determined by Namenson, Jackson, and Smither.<sup>21</sup>

The analysis of the  $^{180}\text{Hf}$  spectrum is aided by Rickey's work on the odd- $A$  hafnium isotopes.<sup>1</sup> Experimentally determined values of the occupation probabilities are available and the optical-model parameters used in the DWBA calculations are

those used by Rickey in the study of the reaction  $^{180}\text{Hf}(d, p)^{181}\text{Hf}$ . These parameters were obtained from the systematic study of Perey<sup>8</sup> and are presented in Table VI. The code TARO was used in the calculations.

The nucleus  $^{180}\text{Hf}$  has 72 protons and 108 neutrons. The two-quasiparticle states expected to be populated in the present work are those in which the  $\frac{5}{2}^+ [624]$  neutron, the ground-state neutron of  $^{179}\text{Hf}$ , couples with a neutron in a neighboring Nilsson orbital.

From a study of the systematics of  $^{179}\text{Hf}^1$  the first two-quasineutron state expected is the  $8^- [624\uparrow + 514\uparrow]$  state. Previous researchers have identified a state of spin  $8^-$  at 1142 keV, but have assigned it as the  $8^- [404\uparrow + 514\uparrow]$  two-quasiproton state.<sup>17</sup> The fact that a level is observed at 1146 keV might indicate that this state is being populated in the  $(d, p)$  reaction. This would indicate a strong admixture of the  $8^- [624\uparrow + 514\uparrow]$  two-quasineutron state and a situation analogous to that in  $^{178}\text{Hf}$  where the  $8^- [404\uparrow + 514\uparrow]_{pp}$  and  $8^- [624\uparrow + 514\uparrow]_{nn}$  states are strongly mixed.<sup>2</sup> The  $8^-$  state with the larger two-neutron configuration cannot be identified in the present work but suggested possible energies for this state are 1197, 1408, 1508, and 1525 keV.

The first strongly excited two-quasiparticle state expected in the present spectrum is the  $4^- [624\uparrow + 510\uparrow]$  state. This state is identified at 1371 keV, the first state in the above-mentioned quintet. The other states in this quintet are identified as follows: the 1480-keV state, the  $5^-$  member of the  $4^- [624\uparrow + 510\uparrow]$  band, the 1609-keV state, the  $5^- [624\uparrow + 510\uparrow]$  two-quasiparticle state, the 1689-keV state, the  $6^- [624\uparrow + 512\uparrow]$  two-quasi-

TABLE VII. Relative theoretical cross sections of levels in  $^{180}\text{Hf}$  populated in the  $(d, p)$  reaction at  $65^\circ$ .

Intrinsic state	Occupation probability <sup>a</sup>	Level									
		0	1	2	3	4	5	6	7	8	9
$5^- [624\uparrow + 521\uparrow]$							12.38	2.20	b		
$4^- [624\uparrow + 521\uparrow]$	0.843					10.89	3.02	1.19			
$2^- [624\uparrow + 512\uparrow]$				1.24	2.21	2.02	1.15				
$7^- [624\uparrow + 512\uparrow]$	0.843								3.91	3.13	
$8^- [624\uparrow + 514\uparrow]$										2.52	1.65
$1^- [624\uparrow + 514\uparrow]$	0.635				1.09						
$0^+ [624\uparrow + 624\uparrow]$	0.618	0.041		0.206		0.776		0.671		0.136	
$4^- [624\uparrow + 510\uparrow]$						65.96	35.42	8.96	1.55		
$5^- [624\uparrow + 510\uparrow]$	0.133						76.41 <sup>c</sup>	31.50	3.87		
$6^- [624\uparrow + 512\uparrow]$								45.57	14.8	1.57	
$3^- [624\uparrow + 512\uparrow]$	0.100				28.62	19.45	8.92	2.83			
$1^- [624\uparrow + 503\uparrow]$			15.3	16.3	11.6	5.85	2.14				
$8^- [624\uparrow + 503\uparrow]$	0.060								51.10		

<sup>a</sup> Occupation probabilities, with the exception of that associated with the  $\frac{3}{2}^- [512]$  state (which is estimated), are those of Rickey, Jr., and Sheline (Ref. 1).

<sup>b</sup> Cross sections less than 1.0 are omitted with the exception of the ground-state band.

<sup>c</sup> Cross sections do not include Coriolis mixing.

TABLE VIII. Results of a Coriolis band mixing calculation for the  $5^-$  states at 1480 and 1609 keV and for the  $6^-$  states at 1689 and 1787 keV assigned in the present analysis of  $^{180}\text{Hf}$ . See caption for Table IV.

$I$	$K$	$b$ (keV)	Energies (keV)		Wave function		Relative intensities at $35^\circ$		
			$E^\circ$	$E_{\text{Cor}}$	$a_{K_{\text{lower}}}$	$a_{K_{\text{higher}}}$	$I^\circ$	$I_{\text{Cor}}$	$I_{\text{exp}}$
4	4	...	1371	1371	1.000	0	1.000	1.000	1.000
5	4	-12	1481	1480	0.995	0.098	0.515	0.641	0.813
5	5	-12	1608	1609	-0.098	0.995	1.155	1.031	1.102
6	5	-47	1755	1787	0.817	-0.576	0.451	0.731	0.813
6	6	-47	1721	1689	0.576	0.817	0.596	0.315	0.354

particle state and the 1785-keV state, the  $6^-$  member of the  $5^- [624\uparrow + 510\uparrow]$  rotational band. The  $6^-$  member of the  $4^- [624\uparrow + 510\uparrow]$  rotational band is expected at an energy of 1611 keV. This energy puts this state close to the giant state at 1609 keV and it cannot be identified in the spectrum.

The identification of the 1371- and 1609-keV states as the  $4^- [624\uparrow - 510\uparrow]$  and  $5^- [624\uparrow + 510\uparrow]$  intrinsic states is in conformity with the work of Namenson, Jackson, and Smither.<sup>21</sup> Namenson, Jackson, and Smither performed the reaction  $^{179}\text{Hf}(n, \gamma)^{180}\text{Hf}$  and observed eleven states (exclusive of members of the ground-state rotational band) at energies of 1152, 1180, 1187, 1193, 1260, 1288, 1371, 1430, 1468, 1541, and 1610 keV. These states should have spins 3, 4, 5, and 6, since the capture state in the  $(n, \gamma)$  reaction has spin 4 or 5 and the primary  $\gamma$  transition is electric dipole. The  $4^-$  and  $5^-$  states at 1371 and 1609 keV correspond to states seen by Namenson, Jackson, and Smither. Supporting the assignments being made is the fact that the 1371- and 1610-keV

states are very strongly populated in the  $(n, \gamma)$  reaction, a fact which suggests that they have negative parity.

The quintet of states identified represents the low-lying two-quasiparticle states which are expected to be strongly populated in the  $(d, p)$  reaction. One difficulty in making the assignments was the low strength of the 1689-keV state, the state assigned as the  $6^- [624\uparrow + 512\uparrow]$  intrinsic state. This low strength was accounted for by considering the mixing between this state and the state at 1785 keV, the state assigned as the  $6^-$  member of  $5^- [624\uparrow + 510\uparrow]$  band. The unperturbed theoretical strengths of the  $6^- [624\uparrow + 512\uparrow]$  state relative to the  $6^-$  member of the  $5^- [624\uparrow + 510\uparrow]$  band are 1.32, 1.73, and 1.48 at 35, 45, and  $65^\circ$ , respectively. (See Table VII for a listing of unmixed theoretical relative cross sections at  $65^\circ$ .) The Coriolis-mixing calculations alter these strengths considerably so that they become 0.574, 1.11, and 0.431. The experimental values for these relative strengths are 0.579, 0.740, and 0.433, a remarkable agree-

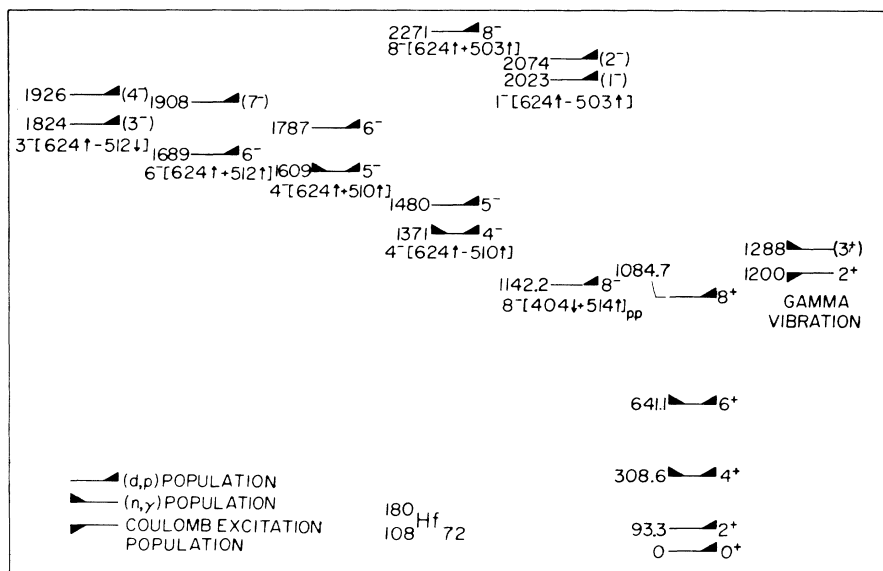


FIG. 4. Level scheme for  $^{180}\text{Hf}$  showing assignments of rotational bands. States which have not been given wave function assignments are not shown. The energies listed for the lowest six states are from Ref. 9.

ment at 35 and 65° with the values from the mixing calculations.

Coriolis-coupling calculations have also been performed for the mixing between the 5<sup>-</sup> state at 1480 keV and the 5<sup>-</sup> state at 1609 keV. The results of these calculations and the calculations discussed above appear in Table VIII.

Most of the other states identified in the present (*d, p*) spectrum are given tentative assignments. The 7<sup>-</sup> member of the 6<sup>-</sup>[624† + 512†] rotational band is tentatively identified as the level at 1908 keV. At 35° the strength of this state relative to the band head is 0.451, comparing well with the theoretical strength of 0.497. At 45 and 65° the agreement is not so good with the strength of this state only ~65% of the expected value.

The 3<sup>-</sup>[624† - 512†] two-quasiparticle state is tentatively identified at 1824 keV. The strength of this state relative to the 6<sup>-</sup>[624† + 512†] state is 0.95 at 35° and 1.19 at 65°. This agrees well with the theoretical ratios (reduced by Coriolis mixing) of 1.19 and 1.01. At 45° the agreement is not so good with a predicted relative strength of 0.742 and an experimental relative strength of 0.418.

The 4<sup>-</sup> member of the 3<sup>-</sup>[624† - 512†] band is tentatively identified at 1926 keV. The strengths of this state relative to the band head are 0.474, 0.831, and 0.581 at 35, 45, and 65°, respectively. This agrees reasonably well with the expected relative strengths at these angles of 0.621, 0.825, and 0.678.

The 1<sup>-</sup>[624† - 503†] and 8<sup>-</sup>[624† + 503†] two-quasiparticle states are identified (the former tentatively) as the levels at 2023 and 2271 keV. The population of the 8<sup>-</sup>[624† - 503†] state is predicted to be very strong and indeed at 35 and 65° it is seen with strengths of 0.461 and 0.745 of the 4<sup>-</sup>[624† - 510†] state at 1371 keV. This agrees well with the theoretical relative strengths at these angles of 0.577 and 0.788. At 45° the state is seen strongly but not with the very great strength that is predicted for this state at this angle. This results in a somewhat confusing situation since the strength at 45° relative to the corresponding

1<sup>-</sup>[624† - 503†] state is 3.26, in very good agreement with the theoretical value of 3.34.

The 2<sup>-</sup> member of the 1<sup>-</sup>[624† - 503†] band is tentatively identified at 2074 keV. The population strength of this state at 45° relative to the band head is 1.09, in good agreement with the predicted value of 1.06. The 3<sup>-</sup> member of 1<sup>-</sup>[624† - 503†] band is possibly seen at 2143 keV. The strength of this state relative to the 2<sup>-</sup> state is 0.425, at 45° and 0.696 at 65°. The theoretical relative strengths are 0.752 and 0.712 at these angles.

Another state which is possibly identifiable in the present spectrum is the state at 1197 keV. This state may be the  $\gamma$  vibration identified at 1200 keV in the recent Coulomb-excitation work of Varnell, Hamilton, and Robinson.<sup>22</sup> The calculations of Kern *et al.*<sup>23</sup> predict that the 3<sup>+</sup> member of this band will have a (*d, p*) population strength relative to the band head of 1.25 at 65° and 1.00 at 45°. Such a state is not observed in the present spectrum. However, the state seen by Namenson, Jackson, and Smither<sup>21</sup> at 1288 keV very probably is the 3<sup>+</sup> member of the  $\gamma$  vibrational band. This state is included in Fig. 4, a level scheme of the known assigned states in <sup>180</sup>Hf.

Recent calculations by Neergard and Vogel<sup>18</sup> suggest the existence of four octupole vibrational bands below an energy at 2.5 MeV in <sup>180</sup>Hf. It is interesting to note that the 3<sup>-</sup> state assigned at 1824 keV corresponds energetically to the (3, 3, -) octupole vibrational band head in the Neergard-Vogel calculations and it is suggested that this state may actually have a large component of the (3, 3, -) octupole vibration.

#### ACKNOWLEDGMENTS

The authors would like to thank Ron Mlekodaj, Roger Thompson, Lee Ponting, and Romedon Lasi-jo for their help in gathering data. Thanks are also due Bob Leonard for preparing the targets used in this research, to Ella Jean Wehunt and Susan Wozniak for their careful plate scanning, and to Dr. Roger Griffioen for his careful proof-reading of the final manuscript.

†Research supported in part by the U. S. Atomic Energy Commission under Contract No. AT-(40-1)-2434 with Florida State University. The Florida State University tandem Van de Graaff accelerator program was supported in part by the National Science Foundation Grant No. NSF-GP-25974.

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VOLUME 6, NUMBER 2

AUGUST 1972

## <sup>142</sup>Ce(*d*, *p*) and <sup>142</sup>Ce(*d*, *t*) Reactions and Isobaric Analog Resonances in <sup>142</sup>Ce(*p*, *p*<sub>0</sub>) and <sup>142</sup>Ce(*p*, *p*<sub>1</sub>)<sup>†</sup>

L. Lessard, S. Gales,\* and J. L. Foster, Jr.

*Laboratoire de Physique Nucléaire, Département de Physique, Université de Montréal,  
Montréal, Québec, Canada*

(Received 10 March 1972)

The reactions <sup>142</sup>Ce(*d*, *p*) and <sup>142</sup>Ce(*d*, *t*) at an incident deuteron energy of 12.6 MeV have been investigated with an energy resolution of about 16 keV, and excitation functions for the reactions <sup>142</sup>Ce(*p*, *p*<sub>0</sub>) and <sup>142</sup>Ce(*p*, *p*<sub>1</sub>) have been measured from 9.5- to 12-MeV incident proton energies. The analysis of the analog resonances observed in the excitation functions is compared with the results of the <sup>142</sup>Ce(*d*, *p*) experiment.

### I. INTRODUCTION

The shell-model structure of the 82-neutron nuclei is now rather well known from pickup and stripping reactions<sup>1-3</sup> and from the decay of isobaric analog resonances.<sup>4-6</sup> The modification of this structure as one goes away from the closed 82-neutron shell is also of interest. As part of an over-all study of (82+2)- and (82+3)-neutron nuclei we have investigated the reactions <sup>142</sup>Ce(*d*, *p*) and <sup>142</sup>Ce(*d*, *t*) and the analog resonances in the reactions <sup>142</sup>Ce(*p*, *p*<sub>0</sub>) and <sup>142</sup>Ce(*p*, *p*<sub>1</sub>).

The elastic scattering experiment supplements and serves as a comparison to the (*d*, *p*) results. In the analysis of the reaction <sup>141</sup>Ce(*p*, *p*<sub>1</sub>), results from the other three experiments are directly employed in an attempt to measure coupling of single neutron orbitals to the 2<sub>1</sub><sup>+</sup> level of <sup>142</sup>Ce in the wave functions of several states in <sup>143</sup>Ce.

The previous <sup>142</sup>Ce(*d*, *p*) and <sup>142</sup>Ce(*d*, *t*) measurements by Fulmer, McCarthy, and Cohen<sup>7</sup>

were made with natural cerium, 11% <sup>142</sup>Ce, and a resolution of about 50 keV. The analog of the first excited state of <sup>143</sup>Ce has been observed by Graw *et al.*<sup>8</sup>

### II. EXPERIMENTAL PROCEDURE

Proton and deuteron beams of the Université de Montréal Model EN tandem Van de Graaff accelerator were incident upon isotopically separated 92.7% <sup>142</sup>Ce and 7.3% <sup>140</sup>Ce targets. The excitation functions <sup>142</sup>Ce(*p*, *p*<sub>0</sub>) and <sup>142</sup>Ce(*p*, *p*<sub>1</sub>) were taken at proton laboratory energies from 9.6 to 12.0 MeV in steps of typically 10 keV. The scattered particles were detected by an array of four surface-barrier detectors cooled to -15°C at laboratory angles of 170, 160, 150, and 140°. The four counters were routed into two analog-to-digital converters (ADC) with direct access to the memory of an on-line CDC 3100 computer.

A 12.6-MeV incident deuteron beam, collimated