PHYSICAL REVIEW C VOLUME 6, NUMBER 2 AUGUST 1972

Alpha-Cluster Structure of Light Self-Conjugate 4*n* Nuclei in Their Ground States

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(Received 24 March 1972)

From phenomenological considerations, a mass formula is derived to calculate the interaction energy among the last two neutrons and last two protons in a nucleus. This interaction energy is the intra-α-cluster energy of the last α cluster in the nucleus. Then from a proper analysis of these intra-α-cluster energies, a separation of the intra- and inter-α-cluster energies out of the total binding energy of the nucleus is made. Clear ideas about the sizes of the α clusters relative to the size of the free α particle and also about the degree of α clustering in each α nucleus are obtained. Positive evidence supporting the additive nature of the α-α-cluster interaction is found. Finally, the intra-α-cluster interaction energies are compared with the *nn*-, *pp*-, and *np*-interaction energies in the same even-even self-conjugate nuclei in order to explore the similarities between the nature of two-body and four-body interactions.

I. INTRODUCTION

The α-particle model of nuclei had its origin in Gamow's assumption of preformed α particles inside the nucleus in connection with his successful quantum-mechanical explanation of α emission. In the study of nuclear structure, light self-conjugate 4*n* nuclei, called α nuclei, have always been associated with the α-particle model. This association began from the simple fact that an α particle is an exceptionally stable nucleus which is emitted by naturally radioactive nuclei on the one hand and the α nuclei, on the other hand, which are more stable than their neighbors, can be easily thought of as composed of an integral number of α particles as stable substructures. Reviews of the α-particle model of these nuclei are given by Blatt and Weisskopf,[1] Rosenfeld[2] and more recently, in a less elaborate way, by Afzal *et al.*[3] The celebrated "resonating-group" formalism, introduced by Wheeler,[4] is the most realistic approach to the α structure of nuclei. Following Wheeler, Wildermuth[5] proposed a generalized cluster model of nuclei of which the α-cluster model is a particular case. These two above-mentioned formalisms are the most often adopted approaches to the study of α structure of nuclei. Almost simultaneously with Wheeler, quite an altogether different approach in this field was introduced by Wefelmeier.[6] He assumed that these so-called α nuclei were composed of structureless, rigid α particle as their stable constituents. Though initially his model yielded some encouraging results in the form of constancy of average α-α bond energy, this naïve model is more or less discredited by the present improved understanding of nuclear forces. But this model in its entirety[7,8] or some of its basic assumptions[9,10] are still used in the α-cluster structure investigations of these α nuclei. A recent trend in this field is to investigate the presence or otherwise of α-particle-type density localizations in these nuclei. This approach is based on the fact that the presence of α structures in α nuclei presupposes a nonuniform density distribution in these nuclei, unlike in non-α nuclei. Hartree-Fock (HF) calculations by Eichler and Faessler[11] and molecular orbital-model calculations by Abe, Hiura, and Tanaka[12] actually indicate α-particle-type density localizations in α nuclei. But the applicability of this model is limited only to the first few α nuclei. An up-to-date survey or the subsequent references quoted by us will reveal that these α-particle models, whatever their version, are more successful in explaining the excited states of these α nuclei in terms of their rotational states than their ground-state properties.[13-16] These models offer an equally satisfactory explanation of the results of $(\alpha, 2\alpha)$, (p, α), and $(p, p\alpha)$ reactions involving one of these nuclei as a target and an α particle as an ejectile in terms of direct or quasielastic knockout mechanisms.[17-20] Lithium-induced reactions[21,22] in these nuclei are also satisfactorily analyzed in terms of transfer of an α cluster from the lithium to the target. In spite of all this success of the different versions of the α-particle model in the excited states, the fact remains that nucleons in the excited states of a nucleus are in highly deformed configurations, depending on the degree of excitation, and become less compact and hence more prone to α clustering than nucleons in the ground state of the nucleus. α clustering is a phase which frequently appears in the excited state of nucleons. Ikeda, Takigawa, and

Horiuchi[23] have clearly shown how in the excited state these nuclei undergo systematic change into molecule-like structure. Attempts[9, 24, 25] to investigate the ground-state properties of α nuclei on the basis of an α-particle model do not always yield the desired results. All this points only too clearly to the fact that the status of these so-called α nuclei in their ground states is not yet well defined in the framework of the α-particle model. This leaves enough scope for further investigation into the ground-state α structure of these light $4n$ nuclei.

The present paper is devoted to a study of the α-cluster structure of the light self-conjugate $4n$ nuclei in their ground states, starting with no assumptions whatsoever. From some phenomenological considerations, a mass formula is derived to calculate the interaction energy among the last two neutrons and last two protons in a nucleus. This interaction energy is none other than the intra-α-cluster energy of the last α cluster in the nucleus. Then from a proper analysis of these intra-α-cluster energies, a separation of the total intra- and total inter-α-cluster energies out of the total binding energy of the nucleus is made, a clear idea about the sizes of the α clusters relative to the size of the free α particle is obtained, and some remarks about the additive nature of α-α interactions are made. Finally, the intra-α-cluster interaction energies are compared with the nn, pp, and np interaction energies in the same even-even self-conjugate nuclei in order to explore the similarities between the nature of the two-body and four-body interactions.

2. PHENOMENOLOGICAL DERIVATION OF THE FORMULA FOR α-CLUSTERING ENERGY CALCULATION

To obtain the intra-α-cluster energy E_{cl}^{α}, a hypothetical process in the following sequences is invoked. In the first sequence, the last two neutrons and the last two protons are emitted from the nucleus of mass $M(A; N, Z)$. This sequence involves an expenditure of energy equal to the simultaneous separation energy of the last two neutrons and last two protons [i.e., an amount of energy equal to the mutual interaction energy of the last four nucleons plus the sum of the interactions of each nucleon with the core $M(A\text{-}4; N\text{-}2, Z\text{-}2)$]. The mutual interaction energy of the last four nucleons is the intra-α-cluster energy E_{cl}^{α}, and the sum of the interactions of each nucleon with the core is obviously equal to 2 times the sum of $E_s(n)$ and $E_s(p)$, where $E_s(n)$ and $E_s(p)$ are the neutron separation energy from the nucleus $M(A\text{-}3; N\text{-}1, Z\text{-}2)$ and the proton separation energy from the nucleus $M(A\text{-}3; N\text{-}2, Z\text{-}1)$, respectively. The results of the first sequence can therefore be written as

$$2E_s(n) + 2E_s(p) + E_{\mathrm{cl}}^{\alpha} = M(A; N, Z) - [M(A-4; N-2, Z-2) + 2n^1 + 2\mathrm{H}^1] \cdots . \tag{1}$$

In order to eliminate the contributions $2[E_s(n) + E_s(p)]$ in the first sequence, the emitted nucleons are fused with the core $M(A\text{-}4; N\text{-}2, Z\text{-}2)$, each nucleon separately with the core $M(A\text{-}4; N\text{-}2, Z\text{-}2)$ in the second sequence. The results of these fusions can be expressed as follows:

$$-2E_s(n) = 2[M(A-4; N-2, Z-2) + n^1 - M(A-3; N-1, Z-2)] \cdots \tag{2}$$

and

$$-2E_s(p) = 2[M(A-4; N-2, Z-2) + \mathrm{H}^1 - M(A-3; N-2, Z-1)] \cdots , \tag{3}$$

where n^1 and H^1 are the neutron and proton masses, respectively. The sum total of this process in the above-mentioned sequences, i.e., the combination of Eqs. (1)–(3), leaves one with only E_{cl}^{α}, the intra-α-cluster energy. Then we obtain the following expression for E_{cl}^{α}:

$$\begin{aligned} E_{\mathrm{cl}}^{\alpha} &= M(A; N, Z) + 3M(A-4; N-2, Z-2) \\ &\quad - 2[M(A-3; N-1, Z-2) + M(A-3; N-2, Z-1)] \\ &= B(A; N, Z) + 3B(A-4; N-2, Z-2) \\ &\quad - 2[B(A-3; N-1, Z-2) + B(A-3; N-2, Z-1)] \cdots . \end{aligned} \tag{4}$$

Here $M(A; N, Z)$ and $B(A; N, Z)$ are the total mass and total binding energy, resepctively, of a nucleus with mass number A, neutron number N, and proton number Z. When the formula (4) is applied to the case of He^4, E_{cl}^{α} comes out -28.296 MeV, the total binding energy of a free α particle. This is as it should be, since E_{cl}^{α} in the ground state of a free α particle is the same as its total binding energy. This is a check on the formula (4). Formula (4) reduces to the formula for I_{np}, the np interaction,[26] if in the above-mentioned hypothetical process, the last neutron and last proton are only considered in place of the last two neutrons and last two protons. This gives another check on the formula (4).

Another parameter which we shall require for the complete analysis of these α nuclei is the core interaction on the α cluster. This parameter, which actually gives the binding energy of an α cluster to the nucleus or strength of the coupling

TABLE I. Intra- and inter-α-cluster energies, degrees of α clustering, etc. of the α nuclei.

α nuclei	E^{α}_{cl} (MeV)	Cumulative E^{α}_{cl} (MeV)	E_{core} (MeV)	Cumulative E_{core} (MeV)	Cumulative E^{α}_{cl} + cumulative E_{core} (MeV)	Cumulative E^{α}_{cl} divided by total binding energy	Energy per bond (MeV)
He^4	−28.296	−28.296	0	0	−28.296	1.000	
Be^8	−34.047	−62.343	+5.846	+5.846	−56.497	1.104	+5.846 (1)
C^{12}	−32.705	−95.048	−2.959	+2.887	−92.161	1.031	+0.962 (3)
O^{16}	−21.676	−116.724	−13.781	−10.894	−127.618	0.914	−1.815 (6)
Ne^{20}	−23.539	−140.263	−9.486	−20.380	−160.643	0.873	−2.547 (8)
							−2.264 (9)
Mg^{24}	−19.228	−159.491	−18.383	−38.763	−198.254	0.804	−3.230 (12)
Si^{28}	−19.046	−178.537	−19.231	−57.994	−236.531	0.754	−3.866 (15)
S^{32}	−12.805	−191.342	−22.438	−80.432	−271.774	0.704	−4.233 (19)
Ar^{36}	−13.076	−204.418	−21.863	−102.295	−306.713	0.666	−4.649 (22)
Ca^{40}	−14.019	−218.437	−21.317	−123.612	−342.049	0.638	−4.944 (25)
Ti^{44}	−14.629	−233.066	−18.901	−142.513	−375.579	0.620	

of the α cluster to the core, is also of great importance in the analysis of $(\alpha, 2\alpha)$, (p, α), and $(p, p\alpha)$ reactions, or rotational bands, where the binding of the α cluster to the nucleus is assumed to be negligible. As mentioned earlier, α clustering is caused by the E^{α}_{cl} of the last two neutrons and last two protons. E^{α}_{cl}, in fact, arises from the interactions of these nucleons in the configurations to which these nucleons have been forced by the core nucleons. Now if E is the separation energy of the last two neutrons and last two protons, E_{core} can be found from the following relation:

$$E = E_{core} + E^{\alpha}_{cl} \cdots . \tag{5}$$

With the help of the 1964 atomic-mass table,[27] first E^{α}_{cl} and then the corresponding E_{core} are computed for all the α nuclei from relations (4) and (5).

3. DISCUSSIONS

Column 1 of Table I gives the intra-α-clustering energy of the last α clusters of all the α nuclei from He^4 to Ti^{44}. Values of the E^{α}_{cl}'s show a remarkable feature in that the E^{α}_{cl}'s are a minimum at and a maximum just after a shell and subshell closure except at Ca^{40} (Fig. 1). These shell-structure features in α clustering are quite unexpected in view of the fact that the shell model in its simplest form does not allow for correlation between nucleons of differing spin and isotopic spin. But Perring and Skyrme[28] have actually shown that the shell-model wave functions for the ground state of a $4n$ nucleus will automatically give rise to α-particle structure as a consequence of the exclusion principle and the symmetry properties of the individual orbitals. Absence of shell structure in E^{α}_{cl} around Ca^{40} is to be attributed to the increasing Coulomb energy. Viewed in terms of core interaction (column 4), one finds that the core-interaction is very small on the four nucleons outside the closed shell so that these nucleons are subject to a less stringent Pauli principle and are almost free to have a greater degree of α clustering. Opposite to this argument is the small α-clustering energy expected in α nuclei with closed shells or subshells. α clustering is, according to "resonating-group" and "cluster-structure" formalisms, one of the many transient types of clusterings that are continuously appearing and disappearing inside the nucleus. But the magnitude of the α-clustering energy in the light α nuclei suggests that the lifetime of the α clusters is large enough to impress on these nuclei an over-all permanent α-cluster structure. Hauge, Williams, and Duffey[10] come to the same conclusion from their study of these nuclei in the phenomenological classical α-particle model.

It is evident from column 2 that none of the values of E^{α}_{cl} is equal to the value (−28.29 MeV) of E^{α}_{cl} for the free α particle. This clearly indicates that the α clusters inside the α nuclei each have a size different from that of the free α particle. In

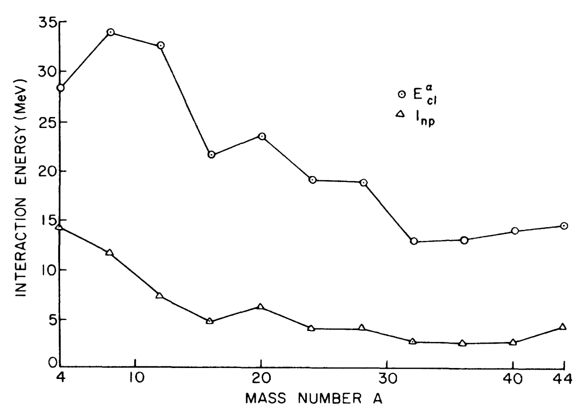

FIG. 1. Shows plots of I_{np} and E^{α}_{cl} of α nuclei against mass-number A.

Be^8 and C^{12} which contain super α clusters, the cluster size is more compact and, in all other nuclei, it is less compact and hence more extended than the free α size. This shows that the α clusters inside the α nuclei are not structureless and rigid, contrary to Wefelmeier's assumptions. This may be explained in the following way. The clustering nucleons in the free α particle, besides being free from any core interaction, belong to the $1s_{1/2}$ state; but the nucleons forming a particular cluster in any other nucleus, besides being subject to a core interaction, belong to a shell-model angular-momentum state which is different from the $1s_{1/2}$ state. A plot of the E_{cl}^{α}'s against A will reveal that the E_{cl}^{α}'s fall more or less on the same straight line for clustering nucleons belonging to the state with the same value of J. This is more evidence of how the J value of the state (and shell structure) to which the clustering nucleons belong influences E_{cl}^{α} and thus determines the cluster size. Irregularity in the specific bond distances and distortion in the supposed regular geometrical spacings of α clusters will be a natural consequence of the variation in the sizes of the clusters.

Columns 3 and 5 contain the cumulative E_{cl}^{α} and cumulative E_{core}, respectively. The sum of these two cumulative quantities is shown in column 6. Surprisingly, though interestingly, the sum of the cumulative E_{cl}^{α} and corresponding E_{core} comes out to be just equal to the total binding energy of the corresponding α nucleus. This finding removes a difficult hurdle to the understanding of α structure of the α nuclei. This, at the same time, reveals that the cumulative E_{cl}^{α} and the corresponding cumulative E_{core} are none other than the total intra-α-cluster energy and the total inter-α-cluster energy, respectively, of the corresponding α nucleus. Thus the total intra-α-cluster energy and total inter-α-cluster energy of an α nucleus become clearly separated out of the total binding energy. This separation had previously always been effected on the erroneous assumption of the presence of rigid structureless α particles inside the nucleus. This separation, in its turn, reveals one of the fundamental laws of interaction between α-α clusters. The interaction between two α clusters is additive by nature. For example, one finds that the interaction of an α cluster with its core in an α nucleus does not affect the interaction of the preceding α cluster with its core in the preceding α nucleus. In other words, the interaction between a pair of α clusters is independent of the presence of other α clusters. The additive nature of α-α interactions was expected, in analogy to van der Waal forces between atoms in molecular physics, since the very inception of the α-particle model, but could not be established beyond doubt for lack of direct evidence. Herzenberg,[7,29-31] however, has shown in a series of papers that the direct and the polarization components of α-α forces are additive in the nonoverlapping region, the direct component being predominant.

Isolation of the inter- and intra-α-cluster energies from each other helps answer another vaguely understood question about the α nuclei. One finds from column 7 that the total intra-α-cluster energy forms 110% for Be^8 to 62% for Ti^{44} of the total binding energy of the nucleus. This, at the same time, gives an idea about the degree of average α clustering in the respective nucleus. This is in contrast to the previously held belief that the intra-α-cluster energy of each of the α nuclei forms about 90% of the binding energy. More than 100% average α clustering in Be^8 and C^{12} arises from the fact that these two nuclei are composed of super-α clusters, each such cluster with an α-clustering energy greater than that of a free α particle for which the degree of α clustering is 100%. The binding energy of a free α particle is not the maximum limit to the clustering energy between two neutrons and two protons inside a nucleus.

If one considers the α nuclei as composed of particles arranged in regular geometrical configurations,[6,7] or as an aggregate of rigid α particles,[8] one fails to find any constancy in the average α-bond energy (column 8) or in the average α-particle interaction energy from the values of the cumulative E_{core} contained in column 5 of Table I. The number in parentheses in column 8 gives the total number of bonds linking the α particles arranged in the particular geometrical configuration assumed for the corresponding α nucleus.

Be^8 is the lightest simple α nucleus, which decays into two α particles. This makes the study of Be^8 together with the α-α scattering results the basis of understanding the α-α interactions and, for that matter, interactions between α clusters of α nuclei. In fact, literature[3,25,32,33] connected with this particular study is voluminous. Our study of Be^8 reveals that it is composed of two super-α clusters with sizes corresponding to -28.29 and -34.04 MeV and with an inter-α-cluster interaction of $+5.84$ MeV. This repulsive inter-α-cluster interaction explains its instability against α decay. He^4 is an exceptionally stable closed-shell nucleus. So the fifth to eighth nucleons in Be^8 in the $1p_{3/2}$ state, being almost free from the Pauli principle, cluster into an α particle. But the α-α potential is definitely not attractive enough to hold the clusters together. As a result, a polarization force[31] comes into play and distorts the clusters to minimize the energy of the α-α system. This process brings the clusters into an overlapping region, giving rise to a repulsive soft core, due to the Pauli principle, of effective value +5.84 MeV which is responsible for the decay of Be^8 into two α particles. This picture of Be^8 fits well with the latest knowledge of the α-α interaction[3] obtained through α-α scattering and fundamental studies.

C^{12} is the lightest stable α nucleus. There is much experimental evidence[15, 16, 19, 34] in favor of its being a 3α system in its excited states. Igo, Hansen, and Gooding[18] have long ago come to the conclusion from the results of $(\alpha, 2\alpha)$ reactions in C^{12} that α clustering in carbon is nearly 100% complete. Theoretical studies by different authors[11, 12, 35] of the ground-state properties of C^{12} are more or less in agreement with the assumption that C^{12} is a 3α system in its ground state. Analysis[36] of electron scattering from C^{12} also bears this out; but this is contradicted by Klim[37] on the basis of his analysis of the ground state of C^{12} via local phenomenological α-α potentials. Our study of the ground state of C^{12} indicates that it consists of three α clusters with sizes commensurate with −28.29, −34.04, and −32.70 MeV and with a total residual interaction of +2.88 MeV. This shows that α clustering in the ground state of C^{12} is more than 100% complete and C^{12} is, because of its repulsive residual interaction, seriously underbound as was noted also by Faessler, Schmid, and Plastino[38] and Abul-magd.[9] For C^{12} to spontaneously decay by α emission, the last α cluster must come out first; but it is bound to the nucleus by −2.959 MeV. This gives C^{12}, in spite of a total repulsive residual interaction, a stability against spontaneous α decay. Theoretical study of C^{12} as a 3α system will yield excellent ground-state results provided the appropriate sizes of the clusters and consequent variations in inter-α-cluster distances are given due consideration. This important aspect of the study of the ground state of C^{12} is receiving its proper consideration in recent theoretical investigations.[39, 40]

Degrees of α clustering in O^{16} and Ne^{20} are 91 and 87%, respectively. These two nuclei in their excited states[13, 14, 21, 22, 41] behave as perfect 4α and 5α systems. O^{16} and Ne^{20} can, as their total intra-α-cluster energies (column 2) show, be looked upon on the average as 4α and 5α systems even in their ground states. Theoreticians[9−11, 36] are strongly divided in their opinion on the ground states of O^{16} and Ne^{20} being 4α and 5α systems.

Low degrees of α clustering in Mg^{24}, Si^{28}, and Ca^{40} are evidenced by the results of $(\alpha, 2\alpha)$ and (He^3, Be^7) reactions.[18, 42] As a matter of fact, for all nuclei from Mg^{24} onward, degrees of α clustering are small and the total E_{cl}^{α} for each of these nuclei (column 3) is not large enough to allow one to consider them, even on the average, as 6α, $7\alpha, \ldots,$ systems in their ground states. This is because degrees of orbital symmetry of the clustering nucleons in these nuclei are gradually reduced by the increasing Coulomb repulsive energy with increasing A, and the clustering nucleons, as a consequence, acquire a greater tendency of dissolving in the condensed nuclear matter than of clustering into α particles. This is amply demonstrated by the decreasing values of E_{cl}^{α} and increasing values of E_{core} with increasing A.

In the course of our investigation[26] into the charge-symmetry and charge-independence hypotheses of nuclear forces, we have already compared the nn-, pp-, and np-interaction energies in even-even self-conjugate nuclei, i.e., in α nuclei. When these last nn, pp, and np pairs of α nuclei are allowed to interact simultaneously with one another, one gets the α-like four-body correlation, i.e., α-clustering energy of these nuclei. As the interacting nucleons in an α cluster of an α nucleus are the same nucleons as the last nn, np, and pp pairs of the same nucleus belonging to the same angular momentum states, a reasonable common basis exists for comparing and contrasting the four-body interaction with the two-body nn-, pp-, and np-interaction energies. Figure 1 shows the plot of E_{cl}^{α} and I_{np} of these α nuclei against mass number A. Plots of P_p and P_n have been omitted as these are identical in nature to I_{np} (Fig. 5, Ref. 26). One can find quite clearly from Fig. 1 that E_{cl}^{α} and I_{np} have the same qualitative behavior. E_{cl}^{α} contains in it the same shell and subshell effect as I_{np} except at Ca^{40}, where there already exists an anomaly in the behavior of I_{np}, P_p, and P_n. Difference in quantitative behavior arises for obvious reasons. Similarity in the qualitative behavior of E_{cl}^{α} and I_{np} emphasizes the fact that a nuclear many-body system can be built out of the effective two-body interaction. It also stresses the fact that α clustering is not the result of any four-body forces operating among the clustering nucleons; rather it is the effect of the shell structure of the nucleus. E_{cl}^{α}'s for Be^8 and C^{12} are larger than the E_{cl}^{α} for a free α particle and out of line with the usual behavior pattern of E_{cl}^{α} *vis-a-vis* that of I_{np}. This anomaly may have its explanation in that the α clustering energies of the last four nucleons in Be^8 and C^{12} may not arise only out of the contributions of nn and pp interactions and interactions of all possible np pairs; it may be partly due to traces of four-body forces. Only further investigations into these two cases can clarify the picture.

4. CONCLUSION

The present investigation is not based on any assumptions, but it gives quite a clear picture of the α-cluster structure of the light self-conjugate $4n$ nuclei in their ground states and a clear understanding of the nature of the interaction between a pair of α clusters inside the nucleus. Against the background of this clear idea of the α nuclei and the definite knowledge of the additive nature of the α-α interaction, existing models can be better applied to an investigation of the α-particle structure of these nuclei in general and to a study of the α-α interaction and α-α scattering in particular.

5. ACKNOWLEDGMENT

Thanks are due Professor P. C. Bhattacharya for his interest in the work.

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