Elastic Pion-Nucleus Scattering for Studies of the Nuclear Surface[*]

L. S. Kisslinger

Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

and

R. L. Burman, J. H. Koch,[†] and M. M. Sternheim[‡]

University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544

(Received 15 May 1972)

The use of elastic $\pi^{\pm}$-nucleus scattering to explore the relative neutron-proton distributions in nuclei is studied theoretically. The large difference between π^{+}-N and π^{-}-N interactions in the nuclear surface leads to substantial effects. Neutron-radius determinations to 0.1 F appears feasible.

I. INTRODUCTION

One characteristic feature of the scattering of π mesons by nuclei in the energy region 0–300 MeV is a strong interaction at the nuclear surface. This arises from the strong p-wave π-nucleon scattering in this energy region. The second feature of great importance for our purposes is the difference between π^{+} and π^{-} interactions with neutrons at the energies in question. Since the dominant interaction is in an isospin $\frac{3}{2}$ state, the negative pions interact much more strongly with neutrons than do the positive pions.

In the present work we investigate the possibility of utilizing pion-nucleus elastic scattering to explore the relative neutron-proton distribution. It is these two properties of strong surface interactions and the difference between the pion interaction with neutrons in the different charge states which we shall exploit. Also, we survey some aspects of the sensitivity of π-nucleus elastic scattering to the pion optical potential in anticipation of experiments to be carried out during the first years of the "meson factories."

We carry out systematic calculations of the differential cross sections using an optical potential derived from the π-nucleon scattering amplitudes using multiple-scattering theory.[1] This potential[2] has been widely applied to elastic[3-6] and inelastic[7] π-meson scattering and to π-mesonic atoms,[8] so that the parameters are now semiquantitatively known. The most important feature of the potential for present purposes is a strong dependence on the gradient of the nuclear density – a reflection of the p-wave character of the π-nucleon interaction mentioned above.

We look in some detail at the sensitivity of the angular distributions to the neutron distribution, taking the proton distribution from the analysis of electron-scattering experiments. Since the gradient term arises from the interaction through the $\Delta(1236)$, at energies such that the $\Delta(1236)$ dominates the π-nucleon reactions, the effective radii for π^{+} and π^{-} will be different if there is a neutron-rich surface region. This will lead to two main effects. First, the break from the Mott[9] cross section, historically the first measurement of the nuclear radius, will occur at different angles for π^{+} and π^{-} mesons of the same energy. Secondly, the the positions of the minima for π^{+} and π^{-} will be shifted.

Many years ago it was suggested that one could use the difference between the π^{+} and π^{-} interaction with neutrons and protons to explore the relative neutron-proton density at the surface.[10] At energies of 0.5–1.0 GeV, the pion-nucleon cross sections are such that both $\pi^{\pm}$ are strongly absorbed from the elastic channel in the interior of a large nucleus. In the surface region, the π^{+} mainly interact with neutrons and the π^{-} with protons. Thus, the ratio of absorption cross sections for π^{+} and π^{-} is sensitive to the properties of the surface region. An optical-model analysis[11] of the experiment[12] on ^{208}Pb at 700 MeV indicates that there is no evidence for a substantial difference in neutron and proton densities. A recent survey of possible similar experiments indicates that a measurement of reaction cross sections shows some sensitivity to the neutron distribution.[13]

Recently, studies of π-mesonic atoms using a potential with a gradient term have been carried out,[14] showing no systematic over-all evidence for a neutron-rich surface. However, at low energies the p-wave dominance is lost. That is why we feel that elastic scattering should be a much more effective tool. In fact, the strong dependence of the medium-energy differential cross section on the nuclear surface was one of the striking points in the earliest calculations.[2]

It should be noted that several other methods[15]

have been used to study differences in neutron and proton distributions, e.g., proton scattering, isobaric analog levels, neutron pickup reactions, and α-particle scattering. Recent experiments indicate that the neutron radius is not appreciably larger than the proton radius. Typically, their accuracy is claimed to be 0.1 or 0.2 F. As we shall show below, the potential accuracy for a $\pi^{\pm}$ scattering experiment is comparable. Nevertheless, considering the model dependence of all such analyses, it would seem very useful to have another independent way of obtaining this information.

In Sec. II of the paper, we briefly review the theory of pion-nucleus scattering and study the model dependence of the calculations. In Sec. III we present calculations selected to guide experimental efforts with regard to the most appropriate energies and the resolution needed to extract useful information about the relative proton and neutron radial distribution.

II. REVIEW OF THEORY OF PION-NUCLEUS SCATTERING

In this work we employ a π-nucleus optical potential derived from experimental π-nucleon scattering via multiple-scattering theory. Although in principle an optical potential can be found which can account for the elastic scattering, there are a number of fundamental problems in obtaining this potential.

First, the potential is obtained from an expansion in the number of collisions. Since we use the impulse approximation and restrict ourselves to the first term in this expansion, the only nuclear property which enters is the single-particle distribution function. The higher-order terms, which introduce nuclear correlations, are neglected. Thus, our conclusions about relative neutron- and proton-density distributions are in error to the extent that correlations are important.[16] Although it is known that these higher-order terms affect the shape of the potential,[17] we expect that the resulting differences in the relative neutron vs proton radii will be small. Another modification of the optical potential which is quite dependent on nuclear correlations is the effect of pion absorption. We do not expect this to be important here, and do not explicitly include this "true" absorption.

A second fundamental uncertainty follows from the need for off-shell π-nucleon information in constructing the π-nucleus potential. In our calculations of the effect of different neutron and proton distributions we use the off-shell extrapolation originally introduced for the pion optical potential. The form of the potential and parameters are reviewed in Part A of this section. In Part B, we

try to assess the importance of this off-shell uncertainty for the present work.

A. Description of the Optical Potential

The first-order optical potential in momentum space is[1]

$$\langle \vec{K}'|v|\vec{K}\rangle = \sum_i \langle \vec{K}'|t_i|\vec{K}\rangle \rho_i(\vec{K}' - \vec{K}) , \qquad (1)$$

where the sum is over the A target nucleons, ρ_i is the distribution function for the ith nucleon, t_i is the scattering operator for the incident pion and the ith nucleon, and $\vec{K}$ and $\vec{K}'$ are the incident and outgoing pion momenta.

At low energies the pion-nucleon t matrix can be written as

$$\langle \vec{K}'|t|\vec{K}\rangle = a_0 + a_1 \vec{K} \cdot \vec{K}' , \qquad (2)$$

where a_0 and a_1 are slowly varying functions of energy. Assuming that Eq. (2) holds for all values of the momenta (on and off shell), taking a_0 and a_1 to be constants, neglecting binding effects, and using the impulse approximation for the π-nucleon scattering in the nucleus, one obtains from Eq. (1) the potential for π^+ mesons[2,3]

$$v(r) = -Z[\, b_0 p_0^2 \rho_p(r) + b_1 \vec{p} \cdot \rho_p(r)\vec{p}\,]/2E_\pi$$
$$- N[\, b_0' p_0^2 \rho_n(r) + b_1' \vec{p} \cdot \rho_n(r)\, \vec{p}\,]/2E_\pi . \qquad (3)$$

In Eq. (3),

$$b_l = (4\pi/p_0^2)f_l(\pi^+ p, \text{ lab}) ,$$
$$b_l' = (4\pi/p_0^2)f_l(\pi^+ n, \text{ lab}) , \qquad l = 0, 1 , \qquad (4)$$

where p_0 and E_π are the pion lab momentum and total energy and $\vec{p}$ is the momentum operator $-i\vec{\nabla}$. The distributions of protons and neutrons, ρ_p and ρ_n, will be assumed to be of the Woods-Saxon form

$$\rho_i = \rho_0\{1 + \exp[(r - R_i)/a_i]\}^{-1} , \qquad i = p, n ,$$

with normalization

$$\int \rho_i \, d^3x = 1 .$$

The f_l are the spin-averaged partial-wave amplitudes. For the π^- optical potential, the b_l and b_l' are interchanged. A table of b's obtained from pion-nucleon phase shifts averaged over the nuclear Fermi momentum is given in Ref. 13.

With b's determined from π-nucleon phase shifts and electron scattering density parameters, this model successfully predicts the qualitative features of π^--carbon elastic[5] and inelastic scattering[7] over the kinetic energy range 120 to 280 MeV, the region dominated by the $\Delta(1236)$. Similar results are obtained at lower energies.[3,4,6] However, below 50 MeV or so, the fit is good only if b_0 is appreciably adjusted.[3,6] One apparent reason for

TABLE I. Sensitivity of 100-MeV $\pi^\pm$-C minima to the form of the optical potential [see Eq. (7)].

λ	π^- minimum (deg)	π^+ minimum (deg)	Difference (deg)
0	67.1	68.5	1.4
0.25	64.9	66.3	1.4
0.5	64.1	65.5	1.4
0.75	64.7	66.1	1.4
1.0	65.6	67.1	1.5

this is the accidental, almost exact, cancellation of the low-energy s-wave phase shifts contributing to b_0. Moreover, corrections to the expression (1) for the optical potential discussed above play an important role at low energies.[18] Because these cannot, at present, be reliably estimated, this limits our ability to utilize low-energy pion scattering as a nuclear probe.

B. Model Dependence—Local Model

Of the various approximations discussed above which have been used to derive the optical potentials of Part A, the nature of the off-shell extra-

polation might be most important. The gradient form for the potential, which gives the strong surface dependence being utilized in this work, follows only with the ansatz that the form $a_0 + a_1 \vec{\mathrm{K}} \cdot \vec{\mathrm{K}}'$ holds off shell. An important question here is the sensitivity of the results to that ansatz.

One argument against a critical dependence on the off-energy-shell behavior can be found in the comparison of the optical potential with the Glauber approximation. In the latter, the strongest assumption is that of high-energy small-angle scattering; the form for the off-shell behavior does not directly enter. In spite of the great differences in these two formulations, there is qualitative agreement in the prediction for π-^{12}C scattering over a wide range of energies.[19] Also, a somewhat different approximate multiple-scattering formulation developed by Gibbs gives similar results.[20] Thus, it seems that the qualitative features of the elastic scattering, such as the positions of the forward maxima and minima, are not strongly model-dependent.

In order to make these observations more nearly quantitative, we consider a quite different off-mass-shell extrapolation than that used in Part A. Note that on the energy shell one can rewrite
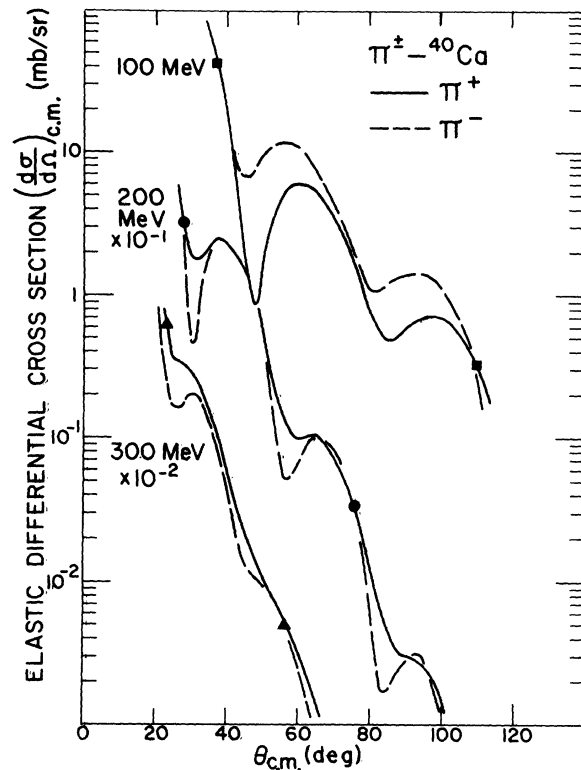

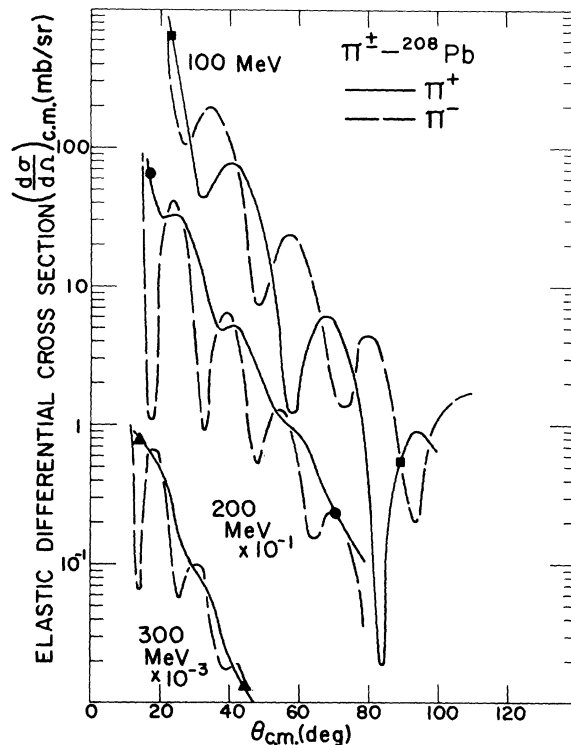FIG. 1. Pion elastic scattering from ^{40}Ca with equal neutron and proton densities.

FIG. 2. Pion elastic scattering from ^{208}Pb with equal neutron and proton densities.

Eq. (2) as

$$\langle \vec{k}' | t | \vec{k} \rangle = a_0 - \tfrac{1}{2} a_1 [(\vec{k} - \vec{k}')^2 - k^2 - k'^2]$$

$$= [a_0 + \tfrac{1}{2} a_1 (k^2 + k'^2)] - \tfrac{1}{2} a_1 (\vec{k} - \vec{k}')^2$$

$$= a_0' - \tfrac{1}{2} a_1 (\vec{k} - \vec{k}')^2 , \tag{5}$$

where a_0' and a_1 are relatively slowly varying functions of energy. Taking a_0' and a_1 as constants, and using Eq. (3), one obtains a local optical potential[21]

$$v_L(r) \propto a_0' \rho(r) + \tfrac{1}{2} a_1 \nabla^2 \rho(r) . \tag{6}$$

Thus, choosing a different form for the off-shell extrapolation of the t matrix produces what seems to be a quite different optical potential. Recently, this model has been used in a number of calculations.[22,23] For the present study consider a linear combination of these two models. Defining

$$v_\lambda = (1 - \lambda) v + \lambda v_L , \tag{7}$$

a family of optical models is obtained with members which are identical in the Born approximation but which differ in higher order. Calculations of the differential cross sections for $\pi^\pm$-C scattering at 100 MeV have been carried out for various values of λ. It is found that the effects of choosing

a variety of off-shell forms as represented by various values of the parameter λ are relatively unimportant. In particular, the positions of the minima and maxima are not changed much. Of special importance to us here is the insensitivity of the relative π^+ and π^- minima to λ, as seen in Table I. Other off-shell extrapolations can be defined, such as the recent one by Landau and Tabakin[24] with a separable effective potential, but we do not expect qualitative changes in the positions of the minima.

One reason for this is that apparently at these energies large-angle scattering arises mainly from a number of small-angle deflections. Therefore, the off-shell properties do not enter in as important a way as they do, for example, in a large-angle double-scattering term. We thus proceed to the study of the dependence of the elastic scattering on the difference between the proton and neutron distributions using the potential of Part A with some confidence in the correctness of the qualitative conclusions which can be drawn.

III. RESULTS AND CONCLUSIONS

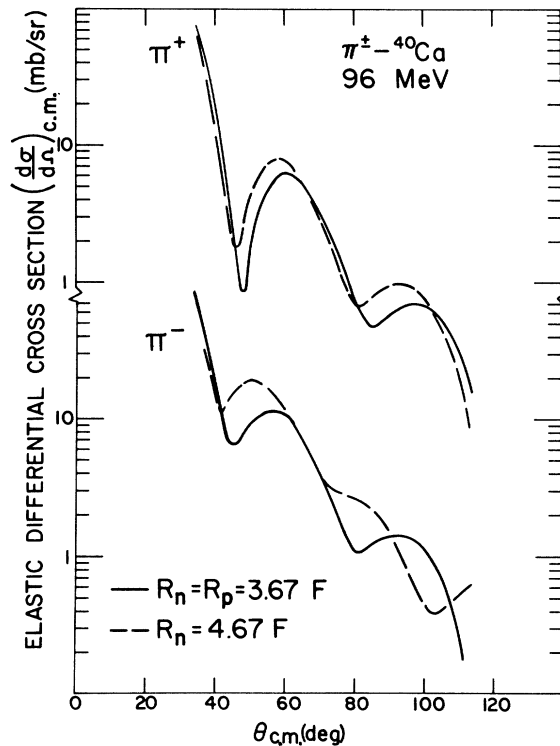In this section we shall give sample theoretical results for π^+ and π^- scattering at various ener-

FIG. 3. Pion elastic scattering from ^{40}Ca for a proton radius $R_p = 3.67$ F and two values of the neutron radius, $R_n = 3.67$ F and $R_n = 4.67$ F.
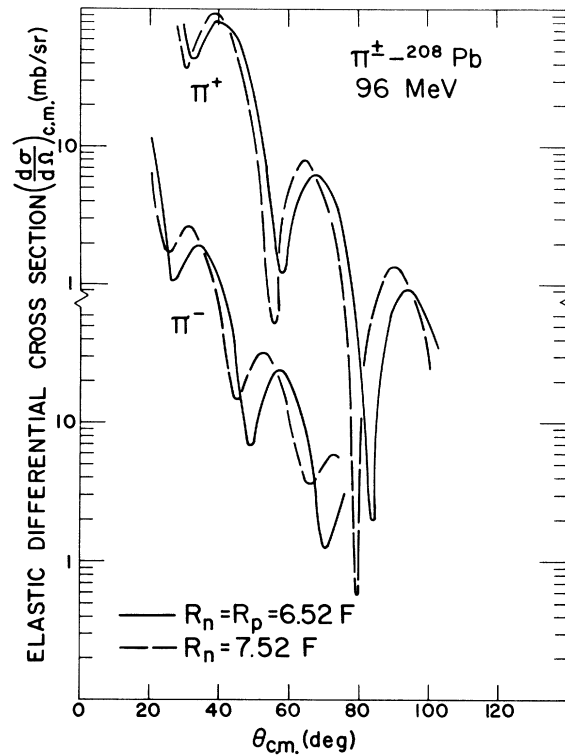

FIG. 4. Pion elastic scattering from ^{208}Pb for a proton radius $R_p = 6.52$ F and two values of the neutron radius, $R_n = 6.52$ F and $R_n = 7.52$ F.

TABLE II. Positions of the first and second minima for two neutron radii. Three dots indicate the presence of a shoulder rather than a minimum.

T_π (MeV)	π^+-^{40}Ca $R_n = 3.67$ F	4.67 F	π^--^{40}Ca 3.67 F	4.67 F	π^+-^{208}Pb 6.52 F	7.52 F	π^--^{208}Pb 6.52 F	7.52 F
96	48.0	45.8	45.8	42.0	32.3	30.8	27.8	26.3
	85.5	81.8	81.8	...	57.8	54.8	48.8	45.0
193	31.5	30.8	30.0	27.8	...	...	18.0	16.5
	...	...	57.0	51.0	...	...	33.0	30.0
309	...	...	25.5	22.5	...	...	14.3	12.8
	...	...	...	...	...	...	25.5	24.0

gies from several nuclei. They were obtained by solving the Klein-Gordon equation with the ABACUS-M code.[21] The potential of Eq. (3) plus a Coulomb potential was assumed. These results are not intended to serve for a comparison with experiment – present or future. They survey the qualitative features of the angular distributions with special regard to the sensitivity to the one-particle distributions. The main purpose is to help in the planning of optimum experiments.

Theoretical differential cross sections for π^+ and π^- scattering on ^{40}Ca and ^{208}Pb, at 100, 200, and 300 MeV, are shown in Figs. 1 and 2. The formalism of Sec. III A has been used, with equal neutron and proton densities. It should be noted that, given the present stage of pion-nucleus-scattering experiments, our calculations beyond the second minimum are meant to be suggestive only. Since the diffraction patterns are most pronounced at 100 MeV in these model calculations, we shall use results at this energy in our discussion.

Some sample results for two different neutron radii are given in Figs. 3 and 4. Two curves are shown for π^+ and π^-, with neutron radii R_n equal

TABLE III. Positions of the first and second minima for two neutron radii with the π^+ kinetic energy adjusted to compensate for Coulomb repulsion. Theoretical model parameters are those for the given π^- energies.

T_{π^+} (MeV)	T_{π^-} (MeV)	Nucleus	$R_n = R_p$ π^+	π^-	$R_n = R_p + 1.0$ F π^+	π^-
65	57	^{40}Ca	61.5	60.8	57.9	...
			101.3	101.3	99.0	96.0
104	96	^{40}Ca	45.0	45.8	43.0	42.0
			82.0	81.8	77.5	...
157	150	^{40}Ca	34.5	34.5	33.2	31.5
			63.8	63.8	61.4	57.8
85	57	^{208}Pb	35.8	37.0	33.5	37.5
			63.0	63.0	59.3	62.0
125	96	^{208}Pb	28.2	27.5	26.2	26.0
			48.8	48.8	46.5	45.5

to, and 1.0 F larger than, the proton radius R_p determined from electron-scattering experiments.[25] In each case, a thickness parameter $a_n = 0.5$ F has been used. Calculations in which both R_n and a_n are varied show that the main sensitivity is to the root-mean-square radius, R_{rms}; variation of R_n by 1.0 F, with a_n fixed at 0.5 F, corresponds to a change in R_{rms} of about 0.7 F. It is apparent from these figures that the change in the neutron radius shifts the positions of the minima, and that the effect differs for π^+ and π^- scattering. Note, incidently, that there is a tendency for the π^+ minima to be much shallower than the π^- minima, especially at higher energies. This is a result of the superposition of the strong amplitude with a Coulomb amplitude which is repulsive for π^+ and attractive for π^-. Presumably, a significant part of this effect arises from the gradient term, which has an important repulsive component at the surface.

In Table II the positions of the first two minima for Figs. 3 and 4, and minima for energies at and above the resonance, are given. (A blank indicates the presence of a shoulder rather than a true minimum.) The over-all qualitative feature is that the π^- minima decrease with increasing neutron radius R_n at about twice the rate as do π^+ minima. This verifies our expectation that the π^- mesons will be more sensitive to the neutron distribution than the π^+. This can be understood from the potentials (3) and (4) for the π^+ and cor-

TABLE IV. Positions of the first four minima in 100-MeV $\pi^\pm$ scattering on ^{208}Pb for optical-model parameters corresponding to pion kinetic energies of 81.9, 100, and 118.1 MeV. Equal neutron and proton radii are used.

100-MeV parameters π^+	π^-	81.9-MeV parameters π^+	π^-	118.1-MeV parameters π^+	π^-
31.5	27.0	31.5	27.0	31.5	27.0
57.0	48.0	55.5	48.0	57.0	48.0
82.5	69.0	81.0	70.5	82.5	70.5
106.5	91.5	106.5	91.5	106.5	91.5

responding ones for π^-. If there is a neutron ex-
cess, in the pure neutron region the parameter
b_1 is much larger for π^- than π^+, since an iso-
spin-$\frac{3}{2}$ phase shift is dominant. It is in this sur-
face region that the gradient potential is strong,
leading to different effective radii for π^+ and π^-.

Note that quite often there is a 0.3° change in
the first π^- minimum per 0.1-F change in the neu-
tron radius, and a 0.6° change in the second mini-
mum. This suggests that careful experiments at
one energy will be able to detect a radius differ-
ence of the order of about 0.2 F. Systematic mea-
surements at a number of energies could perhaps
yield a sensitivity of 0.1 F.

The presence of the Coulomb interaction is a
confusing element in this analysis. For π^+ and π^-
beams at the same energy, the effective kinetic
energy for positive and negative pions at the nu-
clear surface is different. This suggests that one
might obtain similar differential cross sections
for the two charged mesons by running the π^+ beam
at higher energy to compensate for the Coulomb
repulsion. The results in Table III confirm this
conjecture. Note that the positions of the theoreti-
cal minima are approximately the same for ener-
gies given in the table if $R_p = R_n$.

One must keep in mind, however, that the dif-
ferential cross section arises from a coherent
superposition of the strong and Coulomb ampli-
tudes. As a result, the difference between the π^+
and the π^- energies for the situation with approxi-
mately equal minima is not equal to the Coulomb
energy difference at the surface. Thus the values
of the effective π^+ and π^- wave numbers k are not

necessarily equal. For example, if one compares
the 85-MeV π^+ and 57-MeV π^- cross sections, it
is seen (Table III) that the positions of the π^+ min-
ima decrease with increasing neutron radius fast-
er than that of the π^- minima, contrary to expec-
tation for equal wave number. In other cases the
situation is reversed. It is clear that the compli-
cations due to Coulomb effects cannot be easily
removed. Here we would like to emphasize the
need for systematic studies at a variety of ener-
gies. Only by carefully fitting accurate experi-
ments for both π^+ and π^- will one be able to ex-
tract the optical parameters from which the nu-
clear densities are determined.

A related question is the Coulomb correction
due to the energy dependence of the optical param-
eters, b_i of Eqs. (3) and (4). Since the effective
π^+ and π^- kinetic energies are different in the
region of strong interaction, there is some uncer-
tainty in the magnitude of these parameters.
(Note that the two-body amplitude for scattering
in the medium is needed.) This effect is tested by
calculating the differential cross sections with
parameters that correspond to an energy change
equal to the Coulomb energy at the nuclear radi-
us. Typical results are shown in Table IV. In
the table the first four minima are given for π^+
and π^- scattering at 100 MeV from ^{208}Pb. In the
first two columns are the results for Fig. 4, with
$R_n = R_p$. The other columns give the results of
using optical parameters for the kinetic energy
expected at the nuclear surface (81.9 MeV for π^+
and 118.1 MeV for π^-). The effects are very small.

Finally, there is another feature of the angular
distribution which one might use for obtaining in-
formation about the nuclear-density distribution.
Historically, the earliest measurement of the nu-
clear radius was done by determining the angle of
the break in the Coulomb cross section for α-nu-
cleus scattering, and by using the classical Cou-
lomb orbit to determine the distance of closest
approach (the radius) at that angle.[9] We have in-
vestigated this property for pion-nucleus scatter-
ing. A sample result is given in Fig. 5. The ra-
tio of the theoretical cross section to the Ruther-
ford cross section is plotted. Note that there is
some sensitivity of the break to the neutron ra-
dius. Unfortunately, the effect is most prominent
at low energy, where the break occurs at a larger
angle, and where there is a greater change in the
position of the break for a given change in the neu-
tron radius. The theoretical basis of the model,
especially the relevance of the theoretical parame-
ters, is questionable at energies low enough to
make experiments practical. However, when the
low-energy experiments are done and analyzed,
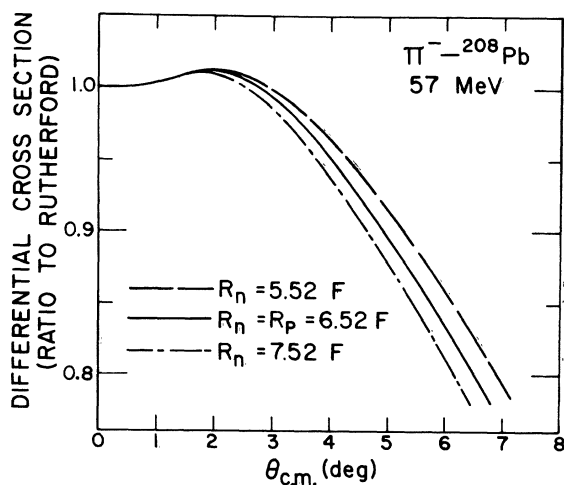it will be interesting to study this phenomenon.

FIG. 5. Ratio of the cross section to the Rutherford
cross section for small-angle π^- elastic scattering from
^{208}Pb at 57 MeV. The proton radius $R_p = 6.52$ F, while
the neutron radius $R_n = 5.52$ F (dashed line), 6.52 F
(solid line), and 7.52 F (dot-dashed line).

In conclusion, it is expected that a careful analysis of systematic experiments at various energies, with the energy and angular resolution expected at the new "meson factories," will allow the neutron vs proton radius to be determined to about 0.1 F.

This conclusion is based on the expectation that with reasonable parameters the optical model will provide accurate fits to the data.

---

[1]See, e.g., M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964), last chapter.

[2]L. S. Kisslinger, Phys. Rev. 98, 761 (1955).

[3]E. H. Auerbach, D. M. Fleming, and M. M. Sternheim, Phys. Rev. 163, 1683 (1967).

[4]E. H. Auerbach, D. M. Fleming, and M. M. Sternheim, Phys. Rev. 171, 1781 (1968).

[5]M. M. Sternheim and E. H. Auerbach, Phys. Rev. Letters 25, 1500 (1970); M. Krell and S. Barmo, Nucl. Phys. B20, 461 (1970).

[6]J. F. Marshall, M. E. Nordberg, Jr., and R. L. Burman, Phys. Rev. C 1, 1685 (1970).

[7]G. W. Edwards and E. Rost, Phys. Rev. Letters 26, 785 (1971).

[8]M. Krell and T. E. O. Ericson, Nucl. Phys. B11, 521 (1969).

[9]E. Rutherford, Phil. Mag. 37, 537 (1919).

[10]O. Piccioni, private communication quoted in E. D. Courant, Phys. Rev. 94, 1081 (1954).

[11]E. H. Auerbach, H. M. Quereshi, and M. M. Sternheim, Phys. Rev. Letters 21, 162 (1968).

[12]A. Abashian, R. Cool, and J. W. Cronin, Phys. Rev. 104, 855 (1956).

[13]M. M. Sternheim and E. H. Auerbach, Phys. Rev. C 4, 1805 (1971).

[14]D. K. Anderson, D. A. Jenkins, and R. J. Powers, Phys. Rev. Letters 24, 71 (1970); observation of a possible neutron halo in ^{44}Ca is reported by R. Kunselman and G. A. Grin, Phys. Rev. Letters 24, 838 (1970).

[15]See, e.g., the following and references contained therein: G. W. Greenlees, W. Makofske, and G. J. Pyle, Phys. Rev. C 1, 1145 (1970); B. A. Shilyaev *et al.*, Yadern. Fiz. 13, 918 (1971) [transl.: Soviet J. Nucl. Phys. 13, 526 (1971)]; J. A. Nolen, Jr., J. P. Schiffer, and N. Williams, Phys. Letters 27B, 1 (1968); H. J. Körner and J. P. Schiffer, Phys. Rev. Letters 27, 1457 (1971); B. Tatischeff, I. Brissaud, and L. Bimbot, Phys. Rev.

C 5, 234 (1972); and H. C. Lee and R. Y. Cusson, Nucl. Phys. A170, 439 (1971).

[16]Note also that we use Woods-Saxon densities in this exploratory work. In the detailed calculations which will follow experiments, a more realistic distribution should be used. The gradient term is not very sensitive to the long neutron tail. This should be contrasted with the situation in K^-mesonic atoms in which the nature of the neutron tail plays a major role. However, the uncertainties in the theoretical interpretation for the K^- case prevents the K^- atomic results from being used for determining the relative neutron-proton radii at the present time; see, e.g., T. E. O. Ericson, in Proceedings of the Fourth International Conference on High Energy Physics and Nuclear Structure, Dubna, USSR, September 1971 (to be published); and J. H. Koch and M. M. Sternheim, Phys. Rev. Letters 28, 1061 (1972).

[17]J. S. Chalmers and A. M. Saperstein, Phys. Rev. 168, 1145 (1968); E. Kujawski, Phys. Rev. C 1, 1651 (1970); H. Feshbach, A. Gall, and J. Hüfner, Ann. Phys. (N.Y.) 66, 20 (1971).

[18]L. Moyer and D. S. Koltun, Phys. Rev. 182, 999 (1969).

[19]M. M. Sternheim, Phys. Rev. 135, B1364 (1964). K. Bjornenak *et al.*, Nucl. Phys. B22, 179 (1970); and T. Kohmura, Nucl. Phys. B36, 228 (1972); C. Schmit, Lettere Nuovo Cimento 1, 454 (1970); C. Wilkin, *ibid*. 1, 499 (1970).

[20]W. Gibbs, Phys. Rev. C 3, 1127 (1971); Phys. Rev. C 5, 755 (1972).

[21]This model was applied by one of us (M.M.S.) in 1967 (unpublished) to 80-MeV π^--^{12}C scattering with results similar to those obtained with Eq. (3). The model is included in the ABACUS-M code as distributed to several laboratories [E. H. Auerbach and M. M. Sternheim, Brookhaven National Laboratory Report No. BNL-12696, July 1968 (unpublished)].

[22]G. Fäldt, Phys. Rev. C 5, 400 (1972).

[23]H. K. Lee and H. McManus, Nucl. Phys. A167, 257 (1971).

[24]R. Landau and F. Tabakin, to be published.

[25]H. R. Collard, L. R. B. Elton, and R. Hofstadter, in *Landolt-Bornstein: Numerical Data and Functional Relationships; Nuclear Radii*, edited by K.-H. Hellwege (Springer, Berlin, 1967), New Series, Group I, Vol. 2.