Neutron-Proton Interaction in Mirror Nuclei

Joachim Jänecke*
Max-Planck-Institut für Kernphysik, Heidelberg, Germany†
(Received 24 April 1972)

Approximate values for the residual neutron-proton interaction in mirror nuclei are derived from binding energy data. The comparison shows no indications for significant symmetry-breaking charge-dependent nuclear effects.

The quantity

$$\begin{aligned} I_{np}(N, Z) &= B(N, Z) - B(N, Z-1) \\ &\quad + B(N-1, Z-1) - B(N-1, Z) \\ &= -[M(N, Z) - M(N, Z-1) \\ &\quad + M(N-1, Z-1) - M(N-1, Z)] \qquad (1) \end{aligned}$$

represents an approximate measure for the residual neutron-proton interaction in a nucleus characterized by N and Z. Here, $B(N, Z)$ and $M(N, Z)$ denote the binding energy and mass, respectively. General properties of I_{np} and related quantities (where the effect of the curvature of the essentially parabolic mass surface is eliminated) have been studied and discussed by a number of authors, such as Zeldes, Gronau, and Lev[1] and Basu and Banerjee.[2] More complete lists of references are given by these authors. Regularities concerning I_{np} have been established, the most obvious one being the dependence on whether A is even or odd. In particular, the empirical rule of Way,[3] which states that I_{np} is approximately equal to zero if $A = N + Z =$ odd, has been discussed by de-Shalit.[4]

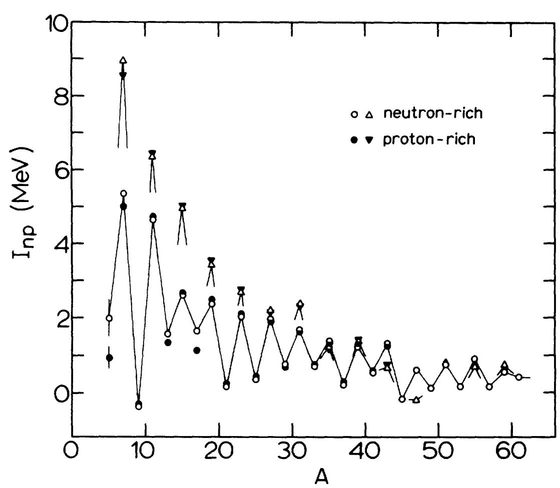

FIG. 1. Plot of I_{np} for the mirror nuclei as a function of A. The triangles denote the values obtained by using the (generally excited) $T=1$ states in the odd-odd self-conjugate nuclei. Experimental uncertainties are indicated if >50 keV.

In a recent communication Basu and Banerjee[5] studied the quantity I_{np} for the mirror nuclei. They compared I_{np} for the members of the isospin doublets and observed energy differences ranging from a few keV to more than 1 MeV. Shell effects seemed to be indicated, and the authors concluded that the departures from zero require the presence of symmetry-breaking charge-dependent nuclear effects.

Figure 1 shows a plot of the I_{np} values of the mirror nuclei obtained from the 1971 atomic-mass evaluation[6] as a function of mass number A. The $T=0$ states (generally the ground states) *and* the $T=1$ states of the odd-odd self-conjugate nuclei have been used (excitation energies from Ref. 7). The oscillatory behavior of I_{np} as a function of A has been recognized earlier.[5] It is easy to understand if one adopts an independent-particle picture where the nucleons move in a self-consistent single-particle field. The energetic position of the fourfold-degenerate Nilsson-like or Hartree-Fock single-particle levels, as well as the residual interactions, are assumed to vary slowly with A (see also Ref. 8). Figure 2 represents Eq. (1) based on this simple picture. Without specifying or discussing the important question of the various J and T couplings (there exist three types of pairing energies for nucleons with-

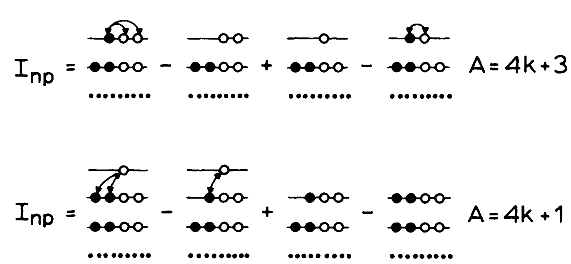

FIG. 2. Schematic representation of Eq. (1) based on an independent-particle model for nuclei with $T_z = +\frac{1}{2}$ and $A = 4k+3$ and $A = 4k+1$, respectively (k = integer). The dotted lines represent any number of completely filled orbitals.

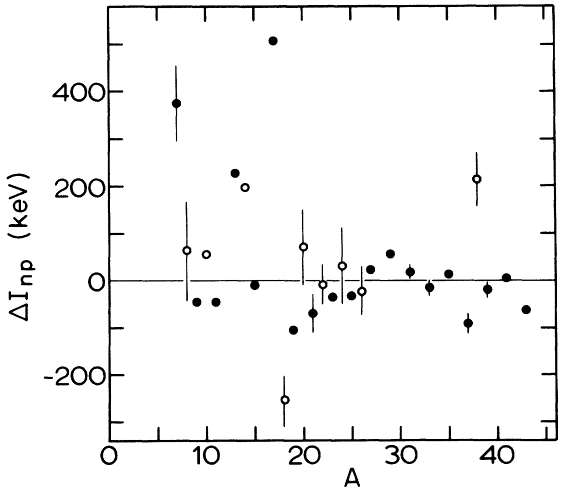

FIG. 3. Plot of the difference ΔI_{np} for the nuclei with $T_z = \pm\frac{1}{2}$ (filled circles) and $T_z = \pm 1$ (open circles) as a function of A. Experimental uncertainties are indicated if >15 keV.

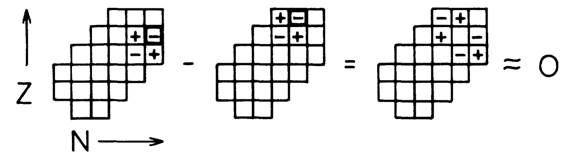

FIG. 4. Schematic representation of the (charge-symmetric) Garvey-Kelson nuclidic mass relationship (Refs. 10, 11) (GK-S) and its derivation from the approximate equality of I_{np} for the mirror nuclei. The plus and minus signs represent the masses, positive or negative, of the respective nuclei.

in the same orbital and four types for nucleons in different orbitals; see for example Ref. 9), Fig. 2 clearly shows that for nuclei with $A = 4k + 3$ (k = integer) the residual n-p interaction involves neutrons and protons within the same orbital, while for $A = 4k + 1$ it does not. We therefore expect $I_{np}(A = 4k + 3) > I_{np}(A = 4k + 1)$ for neighboring mirror nuclei.

Figure 3 shows the differences ΔI_{np} for $T_z = +\frac{1}{2}$ and $T_z = -\frac{1}{2}$ (filled circles) as a function of A. Only for $A = 7$, 13, and 17 does the difference deviate markedly from zero. (The value for $A = 5$ is not shown because it involves four unbound states.) These deviations, however, are easy to understand as a binding energy effect. The respective mass differences involve the nuclei ^{5}He, ^{5}Li, ^{6}Be, ^{12}N, and ^{16}F, where either a proton pair or the odd proton (or neutron) is not or only weakly bound. The result is a Coulomb perturbation in the wave function which leads to an energy shift (Thomas-Ehrman shift). All other energy differences are small. They range from about −100 to +50 keV with a slight preference for negative values, but otherwise no systematic behavior. Also shown in Fig. 3 are the differences ΔI_{np} for $T_z = +1$ and $T_z = -1$ (open circles). These differences have generally larger experimental uncertainties, and they are affected much more by the above-mentioned binding energy effect, which enters ΔI_{np} with positive *and* negative sign. The presence of this effect is nicely confirmed by the fact that the deviations of ΔI_{np} from zero are about the same for $A = 13$ and $A = 14$. The weakly bound proton in ^{12}N affects both values in the same way. The result that ΔI_{np} is small whenever Coulomb perturbations of the wave function are presumably small suggests that, contrary to Basu and Banerjee,[5] symmetry-breaking charge-dependent nuclear effects cannot be strong.

The same conclusions have actually been arrived at earlier. The approximate equality of I_{np} for the mirror nuclei leads to one of the Garvey-Kelson nuclidic mass relationships,[10,11] as can be seen from Fig. 4. These authors[10,11] already discussed the residuals and concluded that charge symmetry of nuclear forces must be satisfied to a high degree. It is worthwhile adding that a detailed study of the quantity I_{np} for *all* nuclei can lead to a deeper understanding of the other Garvey-Kelson nuclidic mass relationships.[11,12]

Thanks are due H. A. Weidenmüller for useful comments and the reading of the manuscript.

*On sabbatical leave from the University of Michigan, Ann Arbor, Michigan.

†Work supported in part by the U. S. Atomic Energy Commission.

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