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[1]B. D. Day, Rev. Mod. Phys. 39, 719 (1967).

[2]R. Rajaraman and H. A. Bethe, Rev. Mod. Phys. 39, 745 (1967).

[3]B. H. Brandow, Rev. Mod. Phys. 39, 771 (1967).

[4]B. H. Brandow, Phys. Rev. 152, 863 (1966).

[5]G. A. Baker, Jr., J. Math. Phys. 11, 2302 (1970).

[6]See, for example, A. Kallio and B. D. Day, Nucl. Phys. A124, 177 (1969).

[7]K. A. Brueckner and K. S. Masterson, Phys. Rev. 128, 2267 (1962).

[8]G. A. Baker, Jr., J. L. Gammel, and B. J. Hill, Phys. Rev. 132, 1373 (1963).

[9]E. J. Irwin, Ph.D. thesis, Cornell University, 1963 (unpublished).

[10]H. S. Köhler, Nucl. Phys. A128, 273 (1969).

[11]G. A. Baker, Jr., Rev. Mod. Phys. 43, 479 (1971).

[12]G. A. Baker, Jr., M. F. Hind, and J. Kahane, Phys. Rev. C 2, 841 (1970).

[13]*The Padê Approximant in Theoretical Physics*, edited by G. A. Baker, Jr. and J. L. Gammel (Academic, New York, 1970).

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Off-Shell Effects in the ^{18}O and ^{18}F Shell-Model Spectra*

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The two-particle shell-model spectra of ^{18}O and ^{18}F are studied using effective interactions obtained from phase-shift-equivalent nucleon-nucleon potentials. The spectra depend significantly on the off-shell behavior of the potentials.

I. INTRODUCTION

Adequate knowledge of the two-nucleon interaction, both on-shell and off-shell, and a proper many-body theory to employ that knowledge are the ingredients for a microscopic description of nuclear phenomena. At present, neither of these essentials is reliably established. The effective interaction in a finite, numerically manageable, model space remains the central problem of a shell-model calculation. However, this problem is beyond the scope of this study. We retreat to the crude and even questionable[1] but common assumption that the effective interaction can be approximated by the sum of the bare reaction matrix G and the three-particle-one-hole (3p-1h) core-polarization correction. Within this limited model we want to examine the degree of sensitivity which the low-lying shell-model spectrum exhibits to changes in the off-shell behavior of the free two-nucleon interaction. Is there reason to expect sensitivity?

Using phase-shifts only, Elliott *et al.*[2] obtained nuclear-structure matrix elements in an oscillator basis which are quite suitable for shell-model calculations. For the most part the diagonal elements compare satisfactorily with those of the bare effective shell-model interaction based on realistic potentials. Since the phase-shift method employs no dependence on off-shell information, this close agreement suggests that for certain nuclei the G matrix and low-lying nuclear spectra calculated with it are rather insensitive to the off-shell part of the nucleon-nucleon interaction. However, this is a rather forced comparison, since no model space dependence is maintained by the phase-shift method.

Lynch and Kuo[3] examined the question of off-

shell effects in the ^{18}O and ^{18}F shell-model spectra by using different nucleon-nucleon potentials. They found only minor variations. However, they employed only local potentials, i.e., potentials of a similar type. In contrast, the ^{18}F spectra computed from the nonlocal Tabakin[4] and the local Hamada-Johnston potential[5] differ in the low-lying 1^+ states substantially[6] when the same core-excitation corrections are included in the effective interaction. On the other hand, comparing the results of these two potentials is meaningless for the purpose of revealing off-shell effects, since the Tabakin potential does not fit the $^3S_1$-$^3D_1$ two-body data.[7]

Hence, we conclude, these previous efforts do not enlighten us about the importance of the off-shell part of the nucleon-nucleon interaction for nuclear spectra.

In this study we examine off-shell effects in nuclear spectra by performing shell-model calculations for ^{18}O and ^{18}F using a variety of nonlocal potentials which are exactly phase-equivalent with the soft-core Reid potential.[8] Each nonlocal potential was obtained by Haftel and Tabakin[9] from the Reid potential by a unitary transformation of limited range. The range restriction was chosen so that asymptotically the transformed wave functions agreed with the Reid wave functions [one-pion exchange (OPE)]. These nonlocal potentials have been applied in calculations of nuclear matter[9] and ^{16}O.[10] We select for our calculations from Ref. 9 transformations 1 and 6, which act in the $^1S_0$ partial wave and transformations 8, 10, and 11, which change the $^3S_1$-$^3D_1$ partial wave. All untransformed partial waves remain identical with the Reid potential. Those particular transformations were selected because they dramatically alter the off-shell behavior of the Reid potential, though it is very unlikely that the particular nonlocalities generated by the unitary transformations have any theoretical basis. All calculations were performed with the same approximations for the effective interaction and with the same computational techniques.

II. CALCULATIONAL TECHNIQUE

In computing the spectra of ^{18}O and ^{18}F we restrict the model space to oscillator states with $0s_{1/2}$, $0p_{3/2}$, and $0p_{1/2}$ shells filled and two particles distributed among the valence levels $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$. Experimental single-particle energies are employed with $\epsilon(0d_{5/2}, 1s_{1/2}, 0d_{3/2})$ = (0.0, 0.87, 5.08) MeV, respectively. The residual shell-model interaction is taken to be the reaction matrix $G(\omega)$ corrected by the 3p-1h core-polarization contribution. Relative center-of-mass (RCM) matrix elements of $G(\omega)$ are calculated according to the method of Sauer,[11] which is especially suited for nonlocal potentials. An "angle-averaged" Pauli operator appropriate for shell-model calculations is used. All intermediate states, even the valence states when allowed by the shell-model Pauli operator, are assumed to have zero single-particle potential energy. For the oscillator energy we take $\hbar\Omega = 14.02$ MeV. The available energy ω in the bare reaction matrix is -10 MeV. The RCM matrix elements characteristic for the phase-equivalent potentials employed here are listed in Table I. Untabulated matrix elements for the other partial waves are those of the Reid potential.[11] For the c.m. variables, only a dependence on the combination $2N+L$ of the oscillator quantum numbers is maintained.

TABLE I. RCM G matrix elements in MeV. The matrix elements are compared with those of the Reid potential, as well as with those of the Tabakin potential, $\hbar\Omega = 14.02$ MeV and $\omega = -10$ MeV.

	l	l'	n	n'	N	L	Reid	1	6	8	10	11	Tabakin
								$T=0$					
$^3S_1$	0	0	0	0	2	0	−9.52			−9.52	−9.52	−9.47	−10.43
	0	0	1	1	1	0	−6.73			−4.46	−6.70	−6.58	−8.15
	0	0	2	2	0	0	−3.32			2.47	−3.27	−3.07	−6.20
$^3(S\text{-}D)_1$	0	2	1	0	1	0	−2.91			−2.92	−3.24	−4.46	−2.26
	0	2	2	1	0	0	−3.31			−3.43	−4.00	−5.76	−2.34
$^3D_1$	2	2	0	0	1	0	1.31			1.30	3.63	9.77	3.95
	2	2	1	1	0	0	1.46			1.42	5.91	12.91	3.69
								$T=1$					
$^1S_0$	0	0	0	0	2	0	−6.76	−4.92	−6.52				−7.05
	0	0	1	1	1	0	−4.55	−4.47	−3.25				−4.62
	0	0	2	2	0	0	−2.05	−2.82	0.84				−2.52

In the core-polarization correction of the effective interaction we include all excitations of a $0p$-shell core particle up to the $3p$, $2f$, and $1h$ oscillator orbitals. For a $0s$ core particle we include all excitations up to the $3s$, $2d$, and $1g$ orbitals. This allows for all intermediate particle-hole states up to an excitation energy of $6\hbar\Omega$, where the highest angular momentum orbital of a particle is restricted to be an h orbital. These states are taken to be pure oscillator states even though there is no single-particle potential in these states. Of course these oscillator states are not eigenstates of the single-particle Hamiltonian assumed for the computation of the reaction matrix, but no serious error is likely to be introduced other than that of truncation of the intermediate-state sum.

To reproduce the full off-shell effect we introduce an improvement to the customary approximations in these calculations. In the reaction matrices needed for the core-polarization correction the starting energy is shifted by $1\hbar\Omega$ ($\omega = -25$ MeV) or $2\hbar\Omega$ ($\omega = -40$ MeV) depending on whether a $0p$-state core particle or a $0s$-state core particle is excited. This is explained in the Appendix and its importance is demonstrated by the results (Sec. III A).

III. RESULTS AND DISCUSSION

We shall present the results on three levels. First, we discuss the changes of important RCM reaction matrix elements due to unitary transformations. Second, we describe the resulting ^{18}O and ^{18}F spectra, and third, we discuss the results and compare them with parallel calculations of ^{16}O.

A. Off-Shell Effects in Reaction Matrix Elements

The unitarity transformations yield sizable changes in the RCM matrix elements as demonstrated in Table I. Variations are much stronger than believed possible in view of the calculations of Lynch and Kuo[3] and of Elliott *et al.*[2]

A strong state dependence of the changes is to be noted; e.g., transformation 1 removes attraction from the diagonal $n=0$ and $n=1$ $^1S_0$ matrix elements and adds some to $n=2$. A somewhat contrary trend occurs with transformation 6, which induces an especially strong repulsive shift from Reid in the $n=2$ $^1S_0$ element. In the $^3S_1$-$^3D_1$ partial wave, the transformation 8 (10 and 11) only changes the $l_1=0$ ($l_1=2$) components of the deuteron and of the two-nucleon scattering wave functions, $\langle r l_1 | \Psi^+(k) l_2 \rangle$, where k is the momentum of relative motion. As a consequence, only the $l_1=0$ ($l_1=2$) half-shell elements $\langle k_1 l_1 | T(k_2^2 + i0) | k_2 l_2 \rangle$ of the free-nucleon transition matrix $T(\omega)$ are changed. According to the dispersion integral,[12] which connects the off-shell $T(\omega)$ to its half-shell elements and to the bound-state pole, the $^3D_1$-$^3D_1$

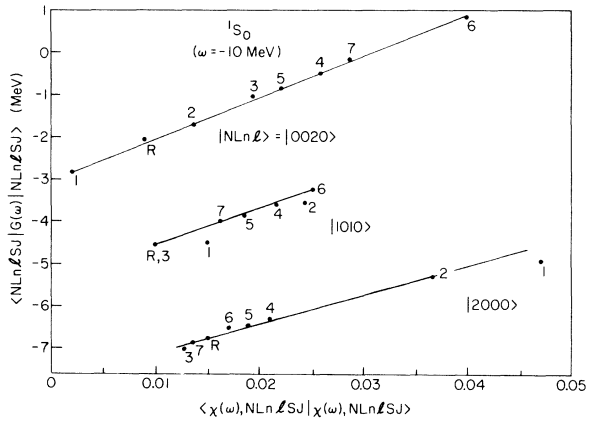

FIG. 1. $^1S_0$ G matrix. Diagonal matrix elements are plotted versus the corresponding wound integral for three states of relative motion $|NLnl\rangle$ with the same oscillator energy. The solid lines are visual guides and are used to extract the coefficients given in the text. The labels correspond to the unitary transformations of Ref. 9. These results are for a starting energy of −10 MeV.

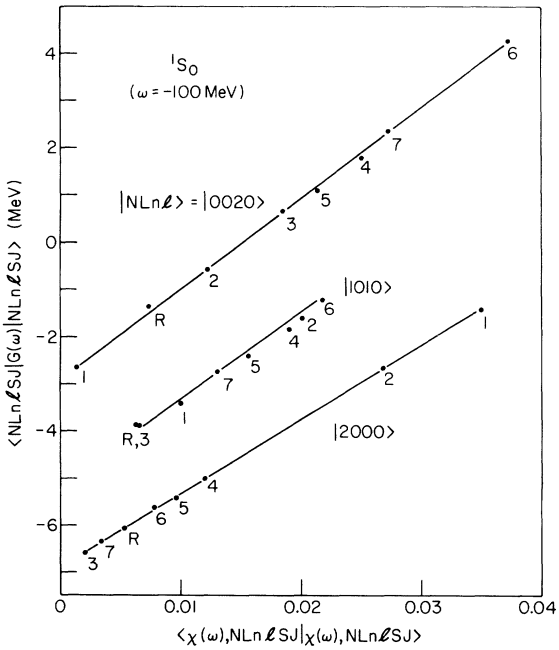

FIG. 2. $^1S_0$ G matrix. These results should be compared with those in Fig. 1, since the only change here is a choice of −100 MeV for the starting energy. See caption to Fig. 1.

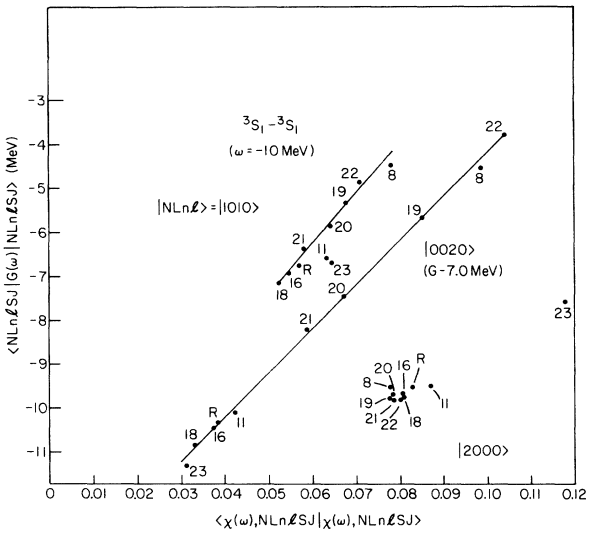

FIG. 3. $^3S_1$-$^3S_1$ component of the G matrix. These results are for a starting energy of −10 MeV. Note the shift in scale for the diagonal elements of the state $|NLnl\rangle = |0020\rangle$. See caption to Fig. 1.

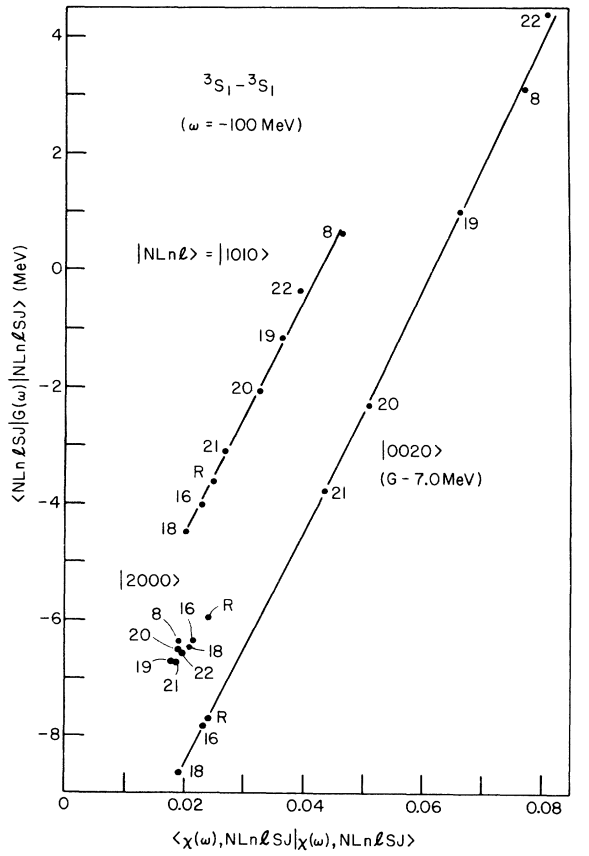

FIG. 4. $^3S_1$-$^3S_1$ component of the G matrix. These results should be compared with those in Fig. 3, since the only difference here is a choice of −100 MeV for the starting energy.

($^3S_1$-$^3S_1$) part of $T(\omega)$ remains unaltered. Since the reaction matrix $G(\omega)$ differs from $T(\omega)$ by the Pauli correction only, the negligible shifts in its $^3D_1$-$^3D_1$ ($^3S_1$-$^3S_1$) elements arise solely through the Pauli correction.[11] However, the changes found in the other matrix elements can be quite dramatic. The trend of the variations are the same for the G matrices with starting energies $\omega = -25$ and -40 MeV.

In order to categorize these variations it is convenient to compare relevant G matrix elements with the wound

$$\langle \chi(\omega), NLnlSJ | \chi(\omega), NLnlSJ \rangle = -\langle NLnlSJ | \partial G(\omega)/\partial\omega | NLnlSJ \rangle ,$$

where NL and nl are the oscillator quantum numbers of c.m. and relative motion, respectively, S and J denote the spin and total angular momentum of the partial wave considered, and ω is the starting energy. Figures 1–4 display the relationship found in the $^1S_0$ and $^3S_1$-$^3S_1$ partial waves for

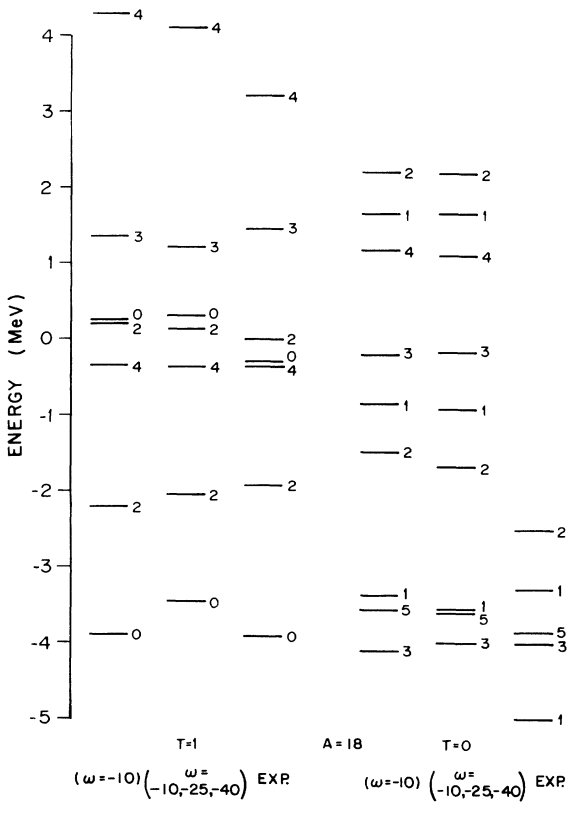

FIG. 5. $T = 1$ spectra of ^{18}O and $T = 0$ spectra of ^{18}F. The spectra are calculated with the Reid potential using different prescriptions for the starting energy in the reaction matrices for core polarization. The results are compared with the experimental spectra.

states of relative motion that contribute to the bare effective interaction for the $A=18$ system. The comparison is performed for several additional interactions[9] which will not be used for a computation of the spectra and for two values of ω in order to properly determine the trends. The approximately linear relationship between G elements and the wound at a fixed ω is to be noted. A similar situation was observed in the nuclear-matter calculations[9] and, by analogy, we propose a phenomenological relationship for finite nuclei

$$\begin{aligned}\langle NLnlSJ|G(\omega)|NLnlSJ\rangle &= C[\langle NLnl|K_{\text{c.m.}}+K_{\text{rel}}-\omega|NLnl\rangle] \\ &\times\langle\chi(\omega),NLnlSJ|\chi(\omega),NLnlSJ\rangle+D \\ &= C[\tfrac{1}{2}(e_{NL}+e_{nl})-\omega] \\ &\times\langle\chi(\omega),NLnlSJ|\chi(\omega),NLnlSJ\rangle+D\,,\end{aligned}$$

where $K_{\text{c.m.}}$ and K_{rel} are the kinetic energy operators of the c.m. and relative motion and

$$e_{NL}=(2N+L+\tfrac{3}{2})\hbar\Omega\,,$$

$$e_{nl}=(2n+l+\tfrac{3}{2})\hbar\Omega\,.$$

Here we have allowed that the net result of the approximations can be roughly combined into two potential-independent constants C and D. The parameter C may depend on ω and on the partial wave, while D may depend in addition upon the state. We do not pretend that the arguments presented in the case of nuclear matter for the relationship between the G matrix and the wound are valid here. Specifically, Haftel and Tabakin[9] assumed the state independence of the defect wave function for momenta below the Fermi momentum. However, Figs. 1–4 exhibit a strong state dependence of the wound integral. Therefore our intention is to employ the above relationship on a phenomenological basis, where the ω dependence of C is to be determined numerically.

TABLE II. The shift in the calculated ground-state energy of ^{18}O due to the inclusion of core polarization in the effective interaction.

Force	Shift (MeV)
Reid	−0.82
1	−0.71
6	−0.87
8	−0.35
10	−0.78
11	−0.73
1+11	−0.62
6+8	−0.48
Tabakin	−1.71

In Figs. 1 and 3 we display diagonal G matrix elements and wound integrals for a fixed energy

$$\tfrac{1}{2}(e_{NL}+e_{nl})-\omega=59\ \text{MeV}\,,$$

while in Figs. 2 and 4 the energy is

$$\tfrac{1}{2}(e_{NL}+e_{nl})-\omega=149\ \text{MeV}\,.$$

From these results we obtain directly

$$C(^1S_0,\ \omega=-10\ \text{MeV}) = C(^1S_0,\ \omega=-100\ \text{MeV})=1.00\pm0.05\,,$$

$$C(^3S_1\text{-}^3S_1,\ \omega=-10\ \text{MeV})=1.70\pm0.05\,,$$

$$C(^3S_1\text{-}^3S_1,\ \omega=-100\ \text{MeV})=1.33\pm0.05\,,$$

which are quite analogous to the results obtained in nuclear matter for the relationship between binding energy and total wound. There also the approximate formula works quite well ($C\approx1$) for the $^1S_0$ partial wave, and in the $^3S_1$-$^3D_1$ a multiplicative correction ($C\neq1$) is necessary because of the strong second-order tensor force.[9]

The lack of spread in the lowest state of relative motion for the $^3S_1$-$^3S_1$ partial wave may be attributed to the constraint that these unitary transformations do not alter the fit to certain deuteron data. Transformation 22 borders on an unacceptable fit, while 23 is far from acceptable[9] and this corresponds with the grouping in Figs. 3 and 4.

Since we have seen large shifts in G matrix elements and a starting-energy dependence to the shifts, it appears important to maintain this ω dependence in our calculations in order to reproduce the proper off-shell effects. Thus the starting energy in the reaction matrices needed for the core-polarization correction is shifted by $1\hbar\Omega$ ($\omega=-25$ MeV) or $2\hbar\Omega$ ($\omega=-40$ MeV) depending on whether a $0p$-state core particle or a $0s$-state core particle is excited, as explained in the Appendix. This is to be contrasted with the usual practice of using the same starting energy in G for all higher-order diagrams in the expansion of the effective shell-model interaction. For the Reid potential we see a sizable effect in the results of Fig. 5 which arises from the inclusion of the proper ω dependence. The results differ from those of an ordinary calculation which employs $\omega=-10$ MeV in the bare and core-polarization diagrams.

In brief, we want to keep the correct dependence in the present study for two reasons. First, for those partial waves ($^3S_1$-$^3D_1$ and $^1S_0$) of the Reid potential in which off-shell variations are generated the reaction matrix is strongly ω dependent. The unitary transformations can dramatically change this ω dependence, as reflected by dramatic changes in the wound. These changes in the

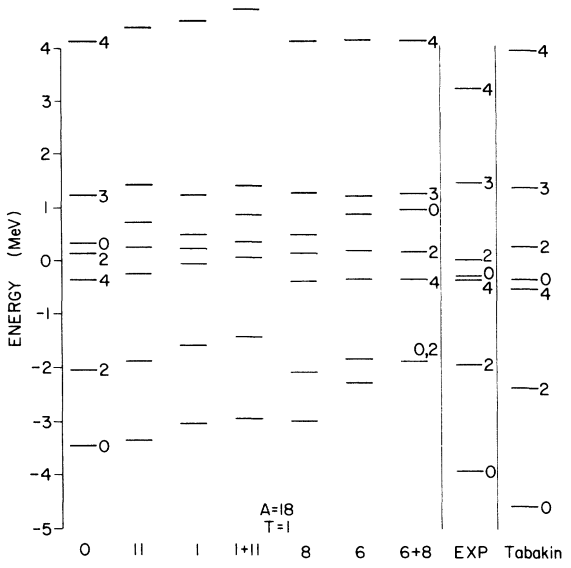

FIG. 6. $T=1$ spectra for ^{18}O. Results for the Reid potential (force 0) and various nonlocal potentials, labeled as in Ref. 9 and phase equivalent with the Reid potential, are presented. Also shown are the experimental spectrum and that of the Tabakin potential.

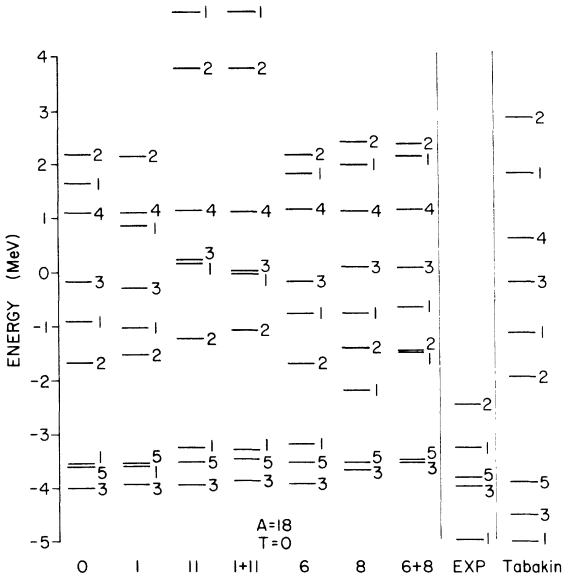

FIG. 7. $T=0$ spectra for ^{18}F. See caption to Fig. 6.

wound can be read off from Figs. 1–4 and in a picturesque way from the configuration-space defect wave functions displayed in Ref. 10. The unitary transformations that increase the wound shift the G matrix to greater repulsion. But choosing a more negative starting energy generally shifts the G matrix to greater repulsion and by an amount which increases with increasing wound. Thus, the full shift in nuclear properties induced by unitary transformations can only be seen with the proper choice of ω.

Second, the core-polarization correction of the ^{18}O ground state is especially sensitive to the $^3S_1$-$^3S_1$ matrix elements, which usually exhibit the strongest ω dependence. This is shown in Table II. Interactions with the same $^3S_1$-$^3S_1$ matrix elements as the Reid potential but substantial differences in other partial waves yield comparable shifts. Force 8 has less attractive $^3S_1$-$^3S_1$ matrix elements than the Reid potential, while those for the Tabakin potential are more attractive, and this pattern correlates well with the ^{18}O ground-state shift due to core-polarization.

Through the next two subsections the sensitivity of the calculated $A=18$ spectra to these off-shell variations will be examined. It will be shown that the spectral behavior is accounted for by the strong state-dependent changes seen above in the reaction matrix elements, particularly in relative S states.

B. Off-Shell Effects in the Shell-Model Spectra

Figures 6 and 7 display the calculated $T=1$ spectra of ^{18}O and the $T=0$ spectra of ^{18}F, respectively. The results of force 10 are not presented. They are similar to those of force 11 in that all low-lying levels deviate by no more than 0.15 MeV with three exceptions. For force 10 the first excited 0^+ state of ^{18}O is more bound by 0.28 MeV, and the second and third 1^+ states of ^{18}F are more bound by 0.72 and 1.90 MeV, respectively, than for force 11. For comparison we present the ex-

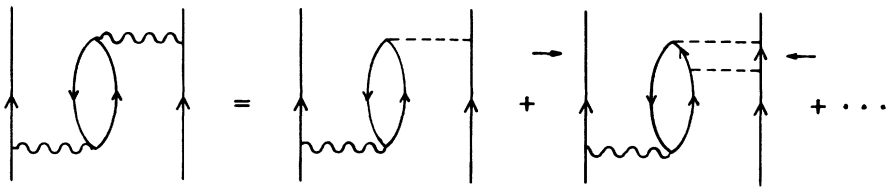

FIG. 8. An expansion of the core-polarization contribution to the effective interaction to indicate how the proper starting energy is selected. Wavy lines represent G interactions and dashed lines represent the bare potential.

perimental spectra and the spectra resulting from a calculation with the same technical apparatus using the Tabakin potential. We note in passing that the spectra for the Reid and the Tabakin potentials show even larger discrepancies than those mentioned in Sec. I for the Hamada-Johnston and the Tabakin potentials.

For ^{18}O the differences between the $T=1$ spectra resulting from the untransformed Reid potential (force 0) and those resulting from forces 8, 10, and 11 are generated solely through core-polarization, since these transformations affect only the $^3S_1$-$^3D_1$ states. The changes are small. The particular strong differences in the RCM matrix elements between forces 8 and 11 do not significantly alter the ^{18}O spectra. For ^{18}F forces 1 and 6 affect the $T=0$ spectra only via core polarization. Except for the third 1^+ state in the results of force 1 the level shifts encountered are comparable to those obtained in ^{18}O with forces 11 and 8.

Transformations 1 and 6 (8 and 11) affect both the bare and the core-polarization contribution to the effective interaction for ^{18}O (^{18}F). The resulting spectra appear with significant shifts usually beyond those obtained with those forces that affect core polarization alone. When the transformations are combined to yield the forces labeled 1+11 and 6+8, the spectra are further shifted from the Reid spectra. We especially note that the combined force 6+8 makes the ground state and the first excited state of ^{18}O degenerate. For the Reid potential these levels are 1.40 MeV apart. In ^{18}F force 6+8 makes the first 1^+ state and the first 2^+ state almost degenerate which were separated by 2.06 MeV for the Reid potential. In ^{18}F forces 1, 11, and 1+11 shift the third 1^+ level over a range of 4 MeV, while leaving the lowest 1^+ relatively unchanged. On the other hand, forces 6, 8, and 6+8 shift the lowest 1^+ level over a range of 2 MeV, while leaving the third 1^+ level relatively unchanged. The other states display varying degrees of sensitivity.

C. Discussion and Comparison with ^{16}O Results

Since we use a limited model[1] for the effective interaction, agreement with experiment would mean little and should not be expected. In fact, the agreement is poor for the Reid potential, especially in ^{18}F. Furthermore, the off-shell changes studied here push the low-lying levels up in energy, i.e., normally further away from their experimental positions, in both ^{18}O and ^{18}F. Increased repulsion is also a consequence of these transformed potentials in nuclear matter[9] and ^{16}O.[10] However, the detailed trends have little else in common. A comparison of our results with those for the binding energy of ^{16}O serves as an example: The loss of energy per particle in ^{16}O is minor for the forces 6, 8, and 6+8 (0.31, 0.06, and 0.38 MeV, respectively[13]). The loss is substantial for the forces 1, 11, and 1+11 (2.00, 0.98, and 2.76 MeV, respectively). In contrast, the ^{18}O and ^{18}F spectra are severely affected by the transformations 6, 8, and 6+8, but are rather stable under 1, 11, and 1+11. The reason is: The changes in the reaction matrix are strongly state dependent, and the ^{16}O ground state and the shell-model states are sensitive to different parts of the reaction matrix; ^{16}O especially to $n=0$, ^{18}O and ^{18}F especially to $n=1$ and 2 elements. In addition, even the different shell-model states tend to exploit the components of the effective interaction with varying weights. Thus, off-shell effects in the spectra do not simply show up as a displacement of a whole group of levels. The shifts of levels occurs selectively. E.g., the violent changes in the $^3D_1$-$^3D_1$ matrix elements for forces 10 and 11 are almost unfelt in the low-lying spectra, whereas the comparatively smaller changes in the S waves of forces 6 and 8 have a devastating effect on the same levels. We conclude strong sensitivity of the low-lying ^{18}O and ^{18}F spectra with respect to the off-shell behavior of the nucleon-nucleon interaction in the relative S waves. This conclusion contrasts with the implications of the works by Lynch and Kuo[3] and Elliott *et al.*[2]

It should be recalled that the strong sensitivity of the shell-model spectra is observed in this paper for a specific set of phase-equivalent potentials. Not all of these phase-equivalent potentials might turn out physically interesting and reasonable. Much more work is necessary to narrow the present off-shell possibilities in the nucleon-nucleon interaction.

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APPENDIX. EVALUATION OF THE CORE-POLARIZATION DIAGRAM

We have approximated the effective shell-model interaction as

$$V_{\text{eff}} = G(\omega = \langle E_V \rangle) + \left(G(\omega') \frac{Q_{3p1h}}{\langle E_V \rangle - e_{3p1h}} G(\omega') \right)_{\text{Linked}},$$

where $\langle E_V \rangle$ is the average binding energy of the two valence nucleons with respect to the ^{16}O core. We take -10 MeV for $\langle E_V \rangle$. The propagator in the core-polarization correction, G_{3p1h}, involves three particles and one hole. The kernel of each

G matrix in G_{3p1h} therefore propagates an extra passive valence particle and a passive hole. This is illustrated in Fig. 8, where one G matrix is expanded into the ladder of bare potential interactions. Consequently,

$$\omega' = \langle E_V \rangle - (\epsilon_V - \epsilon_h)$$
$$= -25 \text{ MeV} \quad \text{for the hole in a } p \text{ orbital}$$
$$= -40 \text{ MeV} \quad \text{for the hole in an } s \text{ orbital}.$$

In addition, a particularly simple and compact expression can be given for G_{3p1h} in the coupled representation. Let a, b, c, and d represent the quantum numbers of valence orbitals; we obtain

$$\langle abJT | G_{3p1h} | cdJT \rangle = \frac{-(-1)^{j_a+j_b+j_c+j_d}}{(1+\delta_{ab})^{1/2}(1+\delta_{cd})^{1/2}} \sum_{T'T''} (2T'+1)(2T''+1)(-1)^{T+T'+T''} \begin{pmatrix} T' & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & T'' & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & T \end{pmatrix}$$
$$\times \sum_{J'J''} (2J'+1)(2J''+1)(-1)^{J+J'+J''} \sum_{\text{ph}} (-1)^{j_p+j_h+1}$$
$$\times [X(abcd\text{ph}, JJ'J'', TT'T'') - (-1)^{j_a+j_b+J+T+1} X(bacd\text{ph}, JJ'J'', TT'T'')$$
$$- (-1)^{j_c+j_d+J+T+1} X(abdc\text{ph}, JJ'J'', TT'T'') + (-1)^{j_a+j_b+j_c+j_d} X(badc\text{ph}, JJ'J'', TT'T'')],$$

where

$$X(abcd\text{ph}, JJ'J'', TT'T'') = \langle ahJ'T' | G(\omega') | pdJ'T' \rangle \langle chJ''T'' | G(\omega') | pbJ''T'' \rangle \frac{(1+\delta_{bp})^{1/2}(1+\delta_{dp})^{1/2}}{\langle E_V \rangle - \epsilon_p + \epsilon_h - \epsilon_b - \epsilon_d} \begin{pmatrix} J' & j_p & j_d \\ j_h & J'' & j_c \\ j_a & j_b & J \end{pmatrix}.$$

The major geometrical factors have been combined into $9j$ coefficients. All two-particle matrix elements are properly normalized and antisymmetrized.

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[1]M. W. Kirson, Ann. Phys. (N.Y.) 66, 624 (1971).

[2]J. P. Elliott, A. D. Jackson, H. A. Mavromatis, E. A. Sanderson, and B. Singh, Nucl. Phys. A121, 241 (1968).

[3]R. P. Lynch and T. T. S. Kuo, Nucl. Phys. A95, 561 (1967).

[4]D. M. Clement and E. U. Baranger, Nucl. Phys. A108, 27 (1968).

[5]T. T. S. Kuo, Nucl. Phys. A103, 71 (1967).

[6]E. U. Baranger, in *Nuclear Structure and Nuclear Reactions, Proceedings of the International School of Physics "Enrico Fermi," Course XL,* edited by M. Jean and R. A. Ricci (Academic, New York, 1969), p. 643.

[7]P. Signell, Phys. Rev. C 2, 1171 (1970).

[8]R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).

[9]M. I. Haftel and F. Tabakin, Phys. Rev. C 3, 921 (1971).

[10]M. I. Haftel, E. Lambert, and P. U. Sauer, to be published.

[11]P. U. Sauer, Nucl. Phys. A150, 467 (1970).

[12]P. U. Sauer, Nucl. Phys. A170, 497 (1971).

[13]The ^{16}O binding energy is not given for the combination 6+8 in Ref. 10. It was calculated for the present paper. All results quoted here refer to self-consistent occupation probabilities.