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Two-Step Processes in Stripping Reactions*

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A formalism using a different approach has been developed for the inclusion of two-step processes in one-particle transfer reactions on deformed nuclei. Calculations for the ^{24}Mg$(d,p)^{25}$Mg and ^{182}W$(d,p)^{183}$W reactions show that the two-step processes are very important for the weakly excited and forbidden transitions; however, for the transitions that are strongly allowed in the one-step process, the effect is small. Calculations for the ^{24}Mg$(d,p)^{25}$Mg ($\frac{1}{2}^+$, 1.61 MeV) and for the ^{182}W$(d,p)^{183}$W ($\frac{1}{2}^-$, g.s.) reaction show that the two-step process via inelastic scattering in the entrance channel is about equal to the two-step process via inelastic scattering in the exit channel. Good fits with the experimental data are obtained.

I. INTRODUCTION

The single-step distorted-wave Born approximation (DWBA),[1,2] where it is assumed that the transition takes place directly from the incident-deuteron channel to the exit-proton channel, has been found successful in explaining many deuteron stripping reactions. However, as has been discussed by Ascuitto and Glendenning,[3] the assumptions on which the DWBA is based might sometimes fail. One case where this may arise is when the stripping reaction takes place between states such that one of which is not the parent of the other. Another situation arises when there are strongly enhanced inelastic transitions, as for the cases of deformed nuclei. It has been suggested by several authors[3-16] that if core excitation of the target or residual nucleus is accounted for some of these difficulties may be eliminated. These processes, known as two-step (or multi-step) processes have also been found important for two-particle transfer reactions.[17] The effects of the two-step processes are that the usual selection rules found in one-step process (the usual DWBA) can be violated and also that the two-step-transition amplitudes can interfere with the one-step-transition amplitude.

The two-step processes for one-particle transfer reactions have been studied by several authors.[3-16] One method of investigation, the coupled-channel method[3-10] involves a huge computational effort. In order to minimize this effort, several authors have employed a number of approximations to calculate the effects of the two-step processes. Iano and Austern[16] have used a perturbation technique to simplify these calculations. In this work, we also make use of a perturbation method to expand the transition amplitude in terms of the deformation parameter β. But on the whole our technique is different from that of Iano and Austern. We do not make the adiabatic approximation in which the ground and excited states in each channel are considered to be degenerate. However, to simplify the calculations, we make an on-the-shell approximation for the Green's function which appears in the intermediate state.

In Sec. II, we present the general formalism of the theory. Some of the approximations used in the present work are also presented in this section. In Sec. III, we work out the specific details in order to obtain the expressions for the transition amplitudes. Selection rules obtained in the present study are discussed at the end of this section.

In Sec. IV, we present a comparison of calculations using our formalism with those of the coupled-channel Born approximation (CCBA). Here it is shown that the proposed formalism does successfully reproduce the results of the more lengthy CCBA. In Sec. V, we present the results of our calculations for the ^{24}Mg$(d,p)^{25}$Mg reaction with 15-MeV incident energy deuterons[18,19] and for the ^{182}W-$(d,p)^{183}$W reaction with 12 MeV incident-energy deuterons.[20] Finally in Sec. VI, the conclusions drawn from the present study are presented.

II. FORMULATION OF THE PROBLEM

In this section, we present a general formulation for one-particle transfer reactions where the effects due to inelastic scattering in the entrance and exit channels are included. For the purpose of notation, the theory will be presented for the deuteron stripping reaction, $A(d,p)B$; however, the results can be extended to other stripping and to pickup reactions. Since the deuteron consists of one proton and one neutron, the entire system consists of $(A+2)$ nucleons. The Hamiltonian for this system can be written, neglecting any three-body interactions, as

$$H=\sum_{i=1}^{A+2}T_i+\sum_{i>j=1}^{A+2}V_{ij}$$
$$=H_T+T_n+T_p+V_p+V_n+V_{np}\,, \tag{2.1}$$

where H_T represents the Hamiltonian for the target nucleus, T_n and T_p are the kinetic energy operators for the neutron and the proton, respectively. V_n and V_p denote the interactions of the neutron and the proton with the target nucleus and finally V_{np} denotes the interaction of the neutron with the proton. We can rewrite the kinetic energy operators T_n+T_p as T_d+T_{pn}, where T_d represents the kinetic energy operator for the motion of the center of mass of the deuteron and T_{pn} is the kinetic energy operator for the motion of the neutron and proton about the center of mass of the deuteron. Also we replace V_n+V_p by V_{dT} which represents the interaction of the deuteron with the target nucleus. Using these definitions, we get

$$H=H_T+T_d+H_d+V_{dT}\,, \tag{2.2}$$

where $H_d=T_{pn}+V_{np}$ is the Hamiltonian for the internal structure of the deuteron. Equation (2.2) is the Hamiltonian for the entrance channel which consists of an incident deuteron and the target nucleus. A similar expression for the exit channel consisting of an outgoing proton and the residual nucleus is given by

$$H=H_R+T_p'+V_p+V_{np}\,, \tag{2.3}$$

where H_R is the Hamiltonian for the residual nucleus and T_p' is the kinetic energy operator for the relative motion of the proton with respect to the residual nucleus.

Ignoring a small exchange ("knock-out") term,[21] the stripping amplitude is given by[22]

$$T_{fi}=\sqrt{N+1}\,\langle\eta_f^{(-)}\,|\,V_{np}\,|\,\Psi_i^{(+)}\rangle\,, \tag{2.4}$$

where N is the number of neutrons in the target nucleus and

$$\Psi_i^{(+)}=[1+(E^{(+)}-H)^{-1}V_d]$$
$$\times e^{i\vec{k}_d\cdot\vec{r}_d}\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\Psi_A(\xi)$$

and

$$\eta_f^{(-)}=[1+(E^{(-)}-T_p'-V_p-H_R)^{-1}V_p]$$
$$\times e^{i\vec{k}_p\cdot\vec{r}_p}S_{\mu_p}(\sigma_p)\Psi_B(\xi,n)\,, \tag{2.5}$$

where $\Psi_A(\xi)$ and $\Psi_B(\xi,n)$ are the wave functions of the target nucleus and the residual nucleus, respectively. The functions $S_{\mu_d}(\sigma_n,\sigma_p)$ and $S_{\mu}(\sigma_p)$ are the spin functions for the deuteron and the proton, and $\Psi_d(r_{pn})$ is the wave function for the deuteron. The symbol n in the wave function for the residual nucleus represents the spin and the radial coordinates of the captured neutron. The function $\Psi_i^{(+)}$ is an eigenfunction of H, the Hamiltonian of the entire system.

In the usual DWBA, the functions $\Psi_i^{(+)}$ and $\eta_f^{(-)}$ are approximated by

$$\Psi_i^{(+)}\simeq\Psi_A(\xi)\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\chi_i^{(+)}(\vec{r}_d)\,,$$

and

$$\eta_f^{(-)}\simeq\Psi_B(\xi,n)S_{\mu_p}(\sigma_p)\chi_i^{(-)}(\vec{r}_p)\,, \tag{2.6}$$

where $\chi_i^{(+)}(\vec{r}_d)$ describes the elastic scattering of the deuterons on the target nucleus and $\chi_f^{(-)}(\vec{r}_p)$ describes the elastic scattering of the protons on the residual nucleus by means of an optical potential. This approximation, where the many-body potentials are replaced by optical-model potentials, is based on the assumption that the most important process in a nuclear reaction is elastic scattering. But for certain nuclei, namely for deformed nuclei, it is observed experimentally that many nuclei induce strong inelastic transitions, often with a cross section at large angles, comparable to that of the elastic scattering. It has been suggested that one should include the inelastic scattering along with the elastic scattering in studying the stripping reactions (or more generally any one-particle transfer reaction) on deformed nuclei. It has been found from the study of inelastic scattering on deformed nuclei that the optical-model potentials assume deformed shapes that correspond to the deformation of the target nucleus.[23] These nonspherical potentials could have either a

static (rotational) or a dynamic (vibrational) deformation. Using the phenomenological theory of Bohr[24] the deformation of the nuclear surface is described by an angle-dependent nuclear radius, $R(\theta)=R_0[1+\sum_{LM}a_{LM}Y_{LM}(\theta',\phi')]$, which modifies the usual spherical optical-model potential. The set of angles (θ',ϕ') is referred to as the body-fixed system. For an axially symmetric nucleus and for a quadrupole deformation, the optical potential can be written as

$$V(\vec{r}) = V_0(r)+\beta Y_{20}(\theta',\phi')R_0\frac{d}{dr}V_0(r)$$
$$=U(r)+\Delta U(\vec{r}), \tag{2.7}$$

where we have taken $V_0(r)=U(r)$, the optical potential that describes the elastic scattering for a spherical nucleus of radius R_0 and the term ΔU is the nonspherical part.

If the interactions V_{dT} and V_P are replaced by the corresponding expansion (2.7) in the expressions (2.5) for the wave functions, the expression

for the transition amplitude becomes

$$T_{fi}=\langle\chi_f^{(-)}\Psi_B(\xi,n)S_{\mu_p}(\sigma_p)[1+\Delta U_p(E^{(+)}-H_f)^{-1}]|\,V_{np}\,|[1+(E^{(+)}-H_i)^{-1}\Delta U_d]\Psi_A(\xi)\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\chi_i^{(+)}\rangle\,, \tag{2.8}$$

where $H_i=H$, the Hamiltonian for the entire system and $H_f=H-V_{np}$. The functions $\chi_i^{(+)}$ and $\chi_f^{(-)}$ are the distorted waves calculated by using the optical-model potentials U_d and U_p, respectively.

The Green's function $(E^{(+)}-H_i)^{-1}$ in Eq. (2.8) can be expanded in powers of ΔU_d by using the iterative expansion

$$(E^{(+)}-H_i)^{-1}=(E^{(+)}-H_i+\Delta U_d)^{-1}+(E^{(+)}-H_i+\Delta U_d)^{-1}\Delta U_d(E^{(+)}-H_i)^{-1}$$
$$=(E^{(+)}-H_i+\Delta U_d)^{-1}[1+\Delta U_d(E^{(+)}-H_i+\Delta U_d)^{-1}+\cdots]. \tag{2.9}$$

Since we expect a small contribution from the higher-order terms, we approximate the Green's function $(E-H_i)^{-1}$ by the first-order term in Eq. (2.9). This greatly simplifies the calculation. A similar approximation is made for the Green's function $(E^{(-)}-H_f)^{-1}$. With these approximations, the transition amplitude becomes

$$T_{fi}=T^0+T_i^{(1)}+T_f^{(1)}+T^{(2)}, \tag{2.10}$$

where

$$T^0=\langle\chi_f^{(-)}\Psi_B(\xi,n)S_{\mu_p}(\sigma_p)\,|\,V_{np}\,|\,\Psi_A(\xi)\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\chi_i^{(+)}\rangle\,,$$
$$T_i^{(1)}=\langle\chi_f^{(-)}\Psi_B(\xi,n)S_{\mu_p}(\sigma_p)\,|\,V_{np}\,|\,(E^{(+)}-H_i+\Delta U_d)^{-1}\Delta U_d\Psi_A(\xi)\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\chi_i^{(+)}\rangle\,,$$
$$T_f^{(1)}=\langle\chi_f^{(-)}\Psi_B(\xi,n)S_{\mu_p}(\sigma_p)\Delta U_p(E^{(+)}-H_f-\Delta U_p)^{-1}|\,V_{np}\,|\,\Psi_A(\xi)\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\chi_i^{(+)}\rangle\,,$$

and

$$T^{(2)}=\langle\chi_f^{(-)}\Psi_B(\xi,n)S_{\mu_p}(\sigma_p)\Delta U_p(E^{(+)}-H_f-\Delta U_p)^{-1}|\,V_{np}\,|\,(E^{(+)}-H_i+\Delta U_d)^{-1}\Delta U_d\Psi_A(\xi)\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\chi_i^{(+)}\rangle\,. \tag{2.11}$$

The first expression in Eq. (2.11) is the zeroth-order term which is the usual DWBA transition amplitude. The next two expressions are the first-order terms in the deformation parameter β and the fourth expression is the second-order term in β. Notice that if we neglect the effects of the inelastic scattering, i.e., setting $\Delta U=0$, all the transition amplitudes excepting the zeroth-order amplitude become zero. This is exactly what is expected.

The Green's functions in Eq. (2.11) can be expanded by using the completeness relation[25] for the eigenstate of $H_i-\Delta U_d$,

$$1=\sum_n\Psi_n\tilde{\Psi}_n^{\dagger}+\int dE'\sum_\alpha\Psi^{(+)}(E',\alpha)\tilde{\Psi}^{(+)\dagger}(E',\alpha)\,, \tag{2.12}$$

yielding

$$G_i^{(+)}(E)=\sum_n\frac{\Psi_n\tilde{\Psi}_n^{\dagger}}{E-E_n}+\sum_\alpha\int_{E_\alpha}^{\infty}dE'\,\frac{\Psi^{(+)}(E',\alpha)\tilde{\Psi}^{(+)\dagger}(E',\alpha)}{E^{(+)}-E'}, \tag{2.13}$$

where the Green's function $G_i^{(+)}(E)=(E^{(+)}-H_i+\Delta U_d)^{-1}$ and the states Ψ_n and $\tilde{\Psi}_n$ refer to any discrete states. α refers to all the quantum numbers other than energy. The dual set of $\Psi^{(+)}$ and $\tilde{\Psi}^{(+)}$ is the biorthogonal set of wave functions which is introduced because H_i and H_f are non-Hermitian. These two wave functions are related by the relation[26]

$$\langle r\,|\,\tilde{\Psi}\rangle=\langle r\,|\,K\,|\,\Psi\rangle^*, \tag{2.14}$$

where K is the usual time-reversal operator. The operation of K is such that when K operates on the wave function $\Psi_{Ejm}(r)$, it gives[27]

$$K\Psi_{Ejm}=(-)^{j+m}\Psi_{Ej-m}. \tag{2.15}$$

The normalization of the functions $\Psi_i^{(+)}$ and $\tilde{\Psi}_i^{(+)}$ is given by

$$\langle\tilde{\Psi}^{(+)}(E,\alpha)\,|\,\Psi^{(+)}(E',\alpha')\rangle=\delta(E-E')\delta_{\alpha\alpha'}, \tag{2.16}$$

i.e., these functions are normalized on the energy scale. If we use the relation

$$\operatorname*{Lim}_{\epsilon\to 0^+}\int dx f(x)(x-x_0+i\epsilon)^{-1}=P\int dx f(x)(x-x_0)^{-1}-i\pi f(x_0)$$

in Eq. (2.13), we obtain

$$G_i^{(+)}(E)=\sum_n\frac{\Psi_n\tilde{\Psi}_n^{\dagger}}{E-E_n}+\sum_\alpha P\int_{E_\alpha}^{\infty}dE'\frac{\Psi^{(+)}(E',\alpha)\tilde{\Psi}^{(+)\dagger}(E',\alpha)}{E-E'}-i\pi\sum_\alpha\Psi^{(+)}(E,\alpha)\tilde{\Psi}^{(+)\dagger}(E,\alpha), \tag{2.17}$$

where P stands for the Cauchy principal-value integral. Now we make some physical approximations. In direct reactions, we do not expect any contribution from the bound states and we expect the scattering amplitudes to be slowly changing in energy. The principal-value term contributes to the background, mainly from resonances which are distant.[21] In our case we expect the most important contributions to be those due to the single-particle resonances, which are quenched by the use of an absorption potential,[21] so it is felt that these contributions will be small, but smoothly varying in energy. This can essentially be mocked up by the use of an optical-model potential in the intermediate channels. What we are saying is that all the processes are taking place on the energy shell. Calculations done by Bledsoe and Tamura[28] for the charge-exchange reactions show that the contribution of the principal-value term is small as compared to the $-i\pi$ term in most of the cases examined. In a study of neutron-resonance reactions Dover and Dietrich[29] showed that such a principal-value term was small if it were not in the vicinity of a single-particle resonance. However, a calculation of the Green's function which contains a non-Hermitian operator using only real-energy eigenfunctions may be incorrect. The complete spectrum may contain complex eigenvalues which may lie close to the real-energy axis. Thus in order to completely determine the Green's function one must know the complete spectrum of the optical Hamiltonian. However, if the absorption is strong these complex poles should in general lie further away from the real axis and thus not complicate the problem. Use of the on-shell approximation, of course, does not admit such solutions, as you allow only the one real-energy ones. Calculations done up to now using only the real-energy spectrum must be suspect because of this reason.

With these approximations, the first-order terms in Eq. (2.11) become

$$T_i^{(1)}=-i\pi\sum_\alpha\langle\chi_f^{(-)}\Psi_B(\xi,n)S_{\mu_p}(\sigma_p)\,|\,V_{np}\,|\,\Psi^{(+)}(E_1,\alpha)\rangle\langle\tilde{\Psi}^{(+)}(E_1,\alpha)\,|\,\Delta U_d\,|\,\Psi_A(\xi)\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\chi_i^{(+)}\rangle,$$

and

$$T_f^{(1)}=-i\pi\sum_\gamma\langle\chi_f^{(-)}\Psi_B(\xi,n)S_{\mu_p}(\sigma_p)\,|\,\Delta U_p\,|\,\Psi^{(+)}(E_2,\gamma)\rangle\langle\tilde{\Psi}^{(+)}(E_2,\gamma)\,|\,V_{np}\,|\,\Psi_A(\xi)\psi_d(r_{np})S_{\mu_d}(\sigma_n,\sigma_p)\chi_i^{(+)}\rangle. \tag{2.18}$$

In this equation, the energy E_1 is given by

$$(H_i-\Delta U_d)\Psi(E_1,\alpha)=E_1\Psi(E_1,\alpha).$$

Since H_i is the complete Hamiltonian, we have

$$E_1=E-\Delta E_1, \tag{2.19}$$

where E is the total energy of the system which includes the kinetic energy E_{inc} of the incident beam of deuterons, ΔE_1 is the energy needed to excite the target nucleus from the ground state to some excited state, so that we are left with $(E_{\text{inc}}-\Delta E_1)$ energy for the deuterons (in the center-of-mass system) before stripping takes place. This is the energy we use in calculating the function $\Psi^{(+)}(E_1,\alpha)$ for the intermediate channel. In the work by Iano and Austern[16] the excitation energy ΔE_1 was neglected. In a similar way, the proton energy E_p to be used in calculating the wave function $\Psi^{(+)}(E_2,\gamma)$ in Eq. (2.18) is given by

$$E_p=E_{\text{out}}\pm\Delta E_2, \tag{2.20}$$

where E_{out} is the energy of the outgoing protons and ΔE_2 is the energy difference of the two levels in the residual nucleus. In Fig. 1, we show A and B as the ground states of the target and the residual nucleus,

respectively. A_1, B_1, and B_2 are the excited states of these nuclei. Suppose we are studying the angular distribution of protons corresponding to nuclear stripping to the level B_1 in the residual nucleus. The transition amplitude T^0 in Eqs. (2.10) and (2.11) gives rise to the transition shown by path 1. This is the one-step process. Paths 2 and 3 are included in the transition amplitudes $T_f^{(1)}$ and $T_i^{(1)}$, respectively. These processes need two steps in going from A to B_1 and are the two-step processes. In transitions via path 2, if stripping takes place to a lower-lying state as compared to B_1 (the ground state in the case of Fig. 1) before the outgoing proton excites the residual nucleus to the level B_1, we use a plus sign in Eq. (2.20). Otherwise, as in the transition $A \to B_2 \to B_1$, we use a minus sign in Eq. (2.20).

For the second-order effects, some of the paths available are labeled as 5 and 6. These paths need three steps and are given by the transition amplitude $T^{(2)}$ of Eq. (2.11). The contribution from these processes is expected to be small and so they are neglected in the present study.

III. CALCULATION OF TRANSITION AMPLITUDES

An expression for the zero-order transition amplitude will not be given here as it is the quite familiar expression for the DWBA transition matrix. The calculation of this amplitude is made with the use of the DWBA computer program DWUCK.[30] The calculation is made in zero range and the appropriate zero-range factor D_0 is taken to have the value 1.65×10^2 MeV fm$^{3/2}$.[31]

The first-order amplitudes given by Eq. (2.18) can be easily derived if one determines the product of the two transition amplitudes appearing in this equation by use of standard DWBA techniques. For the intermediate states the dual set of eigenfunctions $\Psi_{E\alpha}^{(+)}$ and $\tilde{\Psi}_{E\alpha}^{(+)}$ are taken to be

$$\Psi^{(+)}_{A',l'j'm'}(\vec{k},\vec{r}) = \left[\frac{2\mu k}{\pi\hbar^2}\right]^{1/2} \frac{\phi^{(+)}_{l'j'}(kr)}{kr} \mathcal{Y}^m_{l'sj'}(\hat{r})\Psi_{J'_AM'_A}(\xi) \tag{3.1}$$

and

$$\tilde{\Psi}^{(+)}_{A',l'j'm'}(\vec{k},\vec{r}) = \left[\frac{2\mu k}{\pi\hbar^2}\right]^{1/2} \frac{\tilde{\phi}^{(+)}_{l'j'}(kr)}{kr} \mathcal{Y}^m_{l'sj'}(\hat{r})\Psi_{J'_AM'_A}(\xi)\,,$$

where the quantum numbers J'_A and M'_A are represented by the symbol A' on the left-hand side. These functions have the normalization property

$$\langle\tilde{\Psi}^{(+)}_{Aljm}\,|\,\Psi^{(+)}_{A',l'j'm'}\rangle = \delta(E-E')\delta_{ll'}\delta_{jj'}\delta_{mm'}\delta_{J_AJ'_A}\delta_{M_AM'_A}. \tag{3.2}$$

In order to explicitly show the spin dependence in Eq. (3.2) we can write

$$\Psi^{(+)}_{A',l'j'm'}(\vec{k},\vec{r}) = \sum_{m_s}\eta^{(+)}_{l'j's'm'm_s}(\vec{k},\vec{r})S_{s'm_s}\Psi_{J'_AM'_A}(\xi) \tag{3.3a}$$

and

$$\tilde{\Psi}^{(+)}_{A',l'j'm'}(\vec{k},\vec{r}) = \sum_{m_s}\tilde{\eta}^{(+)}_{l'j's'm'm_s}(\vec{k},\vec{r})S_{s'm_s}\Psi_{J'_AM'_A}(\xi)\,, \tag{3.3b}$$

which define the functions $\eta^{(+)}$ and $\tilde{\eta}^{(+)}$.

If we use the relation

$$\tilde{\phi}^{(+)}_{lj}(kr) = e^{+2i\delta^*_{lj}}\phi^{(-)}_{lj}(kr)\,,$$

Eq. (3.3b) becomes

$$\Psi^{(+)}_{A',l'j'm'}(\vec{k},\vec{r}) = \sum_{m_s}e^{+2i\delta^*_{lj}}\eta^{(-)}_{l'j's'm'm_s}(\vec{k},\vec{r})S_{s'm_s}\Psi_{J'_AM'_A}(\xi)\,. \tag{3.4}$$

The expression given by Eq. (2.8) for $T_i^{(1)}$ can then be rewritten as

$$T_i^{(1)} = -i\pi\sum\exp(-2i\delta_{l'_dj'_d})T_i^{st}\times T_i^{\text{inel}}\,, \tag{3.5}$$

where the sum is over l'_d, j'_d, J'_A, m'_d, m_1, m_2, m_3, m_4, and M'_A, and

$$T_i^{\text{inel}} = \int d\vec{r}'_d\int d\vec{r}_d\eta^{(-)*}_{l'_dj'_ds'_dm'_dm_1}(\vec{k}'_d,\vec{r}'_d)\langle J'_AM'_AS'_dm_1\,|\,\Delta U_d\,|\,J_AM_AS_dm_2\rangle\chi^{(+)}_{m_2m_d}(\vec{k}_d,\vec{r}_d) \tag{3.6}$$

and

$$T_i^{st} = J\int d\vec{r}_p\int d\vec{r}'_d\,\chi^{(-)*}_{m_3m_p}(\vec{k}_p,\vec{r}_p)\langle J_BM_BS_pm_3\,|\,V_{np}\,|\,J'_AM'_AS'_dm_4\rangle\eta^{(+)}_{l'_dj'_ds'_dm'_dm_4}(\vec{k}'_d,\vec{r}'_d)\,. \tag{3.7}$$

Here the symbol J denotes the Jacobian for the transformation to relative coordinates.

If we insert Eqs. (3.6) and (3.7) into (3.5) and if we perform all the necessary angular momentum algebra we arrive at the final expression for $T_i^{(1)}$

$$T_i^{(1)} = -iD_0(k_d'/E_d' k_p k_d)\sum i^{l_d-l_p-l-L+j+J}\ \exp(-2i\delta_{l_d' j_d'})\hat{l}_d\hat{s}_d(\hat{l}_d')^2(\hat{j}_d')^2\hat{s}_d'\hat{J}_A'\hat{L}\hat{J}\hat{l}_p\hat{j}_p\hat{l}\hat{j}(l_d s_d 0 m_d | j_d m_d)$$
$$\times(l_d' L 0 0 | l_d 0)(l_p s_p 0 m_p | j_p m_p)(l_p l 0 0 | l_d' 0)\begin{Bmatrix} l_d' & s_d' & j_d' \\ L & S & J \\ l_d & s_d & j_d \end{Bmatrix}\begin{Bmatrix} l_p & s_p & j_p \\ l & s & j \\ l_d' & s_d' & j_d' \end{Bmatrix} B_{LSJ} A_{BA'}(lsj)$$
$$\times I^{LSJ}_{l_d' j_d' l_d j_d}(B/A) I^{lsj}_{l_p j_p l_d' j_d'} F(j_d, J_A, j_d', J_A', j_p, J_B, m_d, m_p, M_A, M_B) d^{j_p}_{m_p m_d - M_B + M_A}(\theta)\,, \tag{3.8}$$

where $\hat{j} = (2j+1)^{1/2}$, $(j_1 j_2 m_1 m_2 | jm)$ is the Clebsch-Gordan coefficient coupling j_1 and j_2 and

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \\ j_7 & j_8 & j_9 \end{Bmatrix}$$

is the usual nine-j coefficient. The sum is over l_d, j_d, l_d', j_d', L, S, J, J_A', l, s, j, l_p, and j_p. The quantity F is given by

$$F = \sum_{j'}(2j'+1)\ (-)^{-j'}(J_A j' M_A M_B - M_A | J_B M_B)(j_p j' m_d - M_B + M_A M_B - M_A | j_d m_d)$$
$$\times W(J_A J J_B j; J_A' j') W(j_p j j_d J; j_d' j')\,,$$

where $W(j_1 j_2 j_3 j_4; j_5 j_6)$ is a Racah coefficient.

If we use the rotational model to describe the target, intermediate, and residual nuclei, we have

$$B_{LSJ} = (\hat{J}_A/\hat{J}_A')\beta_L(J_A L K_A 0 | J_A' K_A)\delta_{S0}\delta_{LJ}$$

for an even-even target nucleus. This is due to the fact that S, the spin transfer, is zero for inelastic scattering. K_A is the projection of the angular momentum in the body-fixed axes of the target nucleus and we assume that the inelastic scattering occurs within the ground-state band. We also have

$$A_{BA'}(lsj) = g(\hat{J}_A'/\hat{J}_B)\langle\phi_f | \phi_i\rangle C_{lj}(J_A' j \mp K_A K_B \pm K_A | J_B K_B)\,,$$

where $g = \sqrt{2}$ if either K_A or $K_B = 0$ and unity otherwise.[32] The overlap integral $\langle\phi_f | \phi_i\rangle$ between the wave function ϕ_i describing the zero-point oscillation of the intermediate nucleus and ϕ_f, describing the zero-point oscillation of the residual nucleus, will be assumed to be unity in the present calculation.

The radial integrals are given by

$$I^{LSJ}_{l_d' j_d' l_d j_d} = \int dr\ \phi^{(-)*}_{l_d' j_d'}(k_d' r) F_{LSJ}(r) \phi^{(+)}_{l_d j_d}(k_d r)\,, \tag{3.9}$$

where the form factor F_{LSJ} is given by

$$F_{LSJ} = -R_0 \frac{dU}{dr}\,,$$

for $L = J = 2$ and $S = 0$ transitions. R_0 is the radius of the Woods-Saxon potential. We also have

$$I^{lsj}_{l_p j_p l_d' j_d'} = \int dr\ \phi^{(-)*}_{l_p j_p}\left(\frac{A}{B} k_p r\right) U_{lj}(r) \phi^{(+)}_{l_d' j_d'}(k_d r)\,, \tag{3.10}$$

where $U_{lj}(r)$ is the radial wave function of the bound transferred nucleon. The quantity A/B is the ratio of the target mass to the residual nucleus mass.

The quantities $d^j_{m',m}(\theta)$ are defined in terms of the rotation matrices

$$d^j_{m',m}(\theta) = D^j_{m',m}(0, \theta, 0)\,.$$

In a similar way it is possible to obtain an expression for $T_f^{(1)}$. This will not be presented here. There are further simplifications of Eq. (3.8) if the target is an even-even axially symmetric deformed nucleus. In this case we have $J_A = M_A = 0$ so that the quantity F as well as the entire expression is simplified owing to the relations

$$W(0 J J_B j; J_A' j') = (1/\hat{J}_A\hat{J}_B)\delta_{j' J_B}\delta_{J_A' J}$$

and

$$(0 j' 0 M_B | J_B M_B) = \delta_{j' J_B}\,.$$

Selection rules for the transition amplitude are easily obtained upon examination of Eq. (3.8). Similar selection rules also apply to $T_f^{(1)}$ with the appropriate changes. From the various vector-cou-

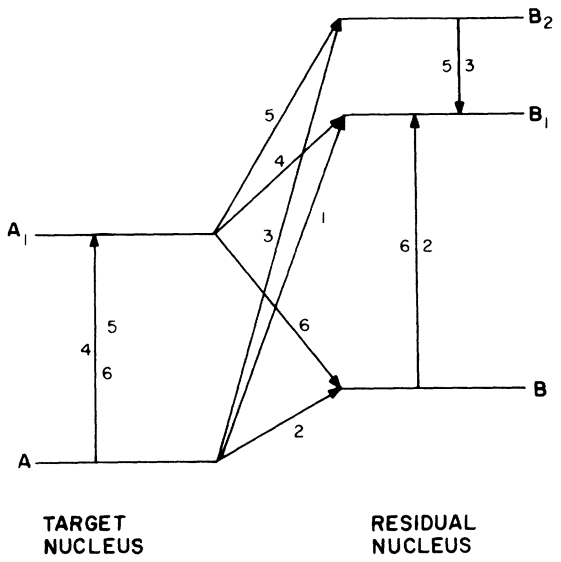

FIG. 1. Various multi-step processes to reach a state B_1 of the residual nucleus from the ground state A of the target nucleus are shown.

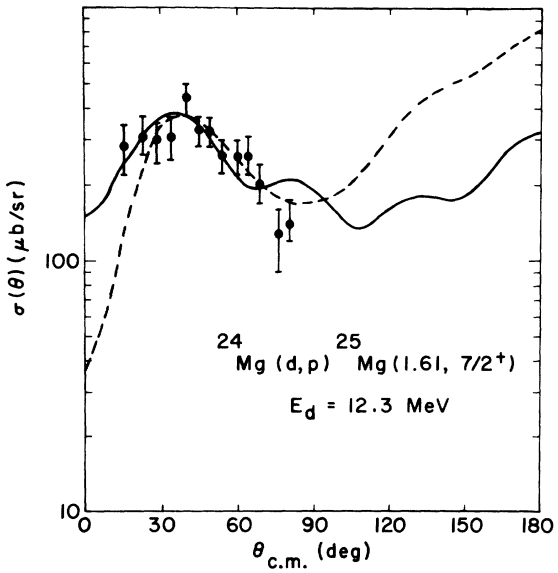

FIG. 2. The dashed curve is taken from the CCBA calculations done by Mackintosh (Ref. 10) and the solid curve is from the present study using the DWBA approach. The deformations assumed are $\beta_2 = 0.4$ in the entrance channel and $\beta_2 = 0.3$ in the exit channel.

pling coefficients we have

$$\vec{j}' = \vec{J}_B - \vec{J}_A ,$$

$$\vec{L} = \vec{J}'_A - \vec{J}_A ,$$

$$\vec{j} = \vec{J}_B - \vec{J}'_A ,$$

and

$$\vec{j}' = \vec{L} + \vec{j} , \tag{3.11}$$

where J_A and J_B are the ground-state spin of the target nucleus and the final-state spin of the residual nucleus, respectively, and J'_A is the spin of the target nucleus in an excited state. L is the angular momentum transfer in the inelastic scattering which excites the target nucleus from its ground state to an excited state and j is the total angular momentum transfer in the stripping reaction. j' is the over-all angular momentum transfer needed for the entire process.

Similar selection rules can be obtained for the various partial-wave angular momenta, namely

$$\vec{l}'_d = \vec{l}_d - \vec{L} ,$$

$$\vec{l}_p = \vec{l}'_d - \vec{l} ,$$

and

$$\vec{j}' = \vec{j}_d - \vec{j}_p , \tag{3.12}$$

where j_d and j_p are given by

$$\vec{j}_d = \vec{l}_d + \vec{s}_d$$

and

$$\vec{j}_p = \vec{l}_p + \vec{s}_p , \tag{3.13}$$

where l_d and l_p are the partial waves for the incoming and the outgoing particles and l'_d is for the intermediate state.

For the special case, when $J_A = 0$, i.e., for an even-even target nucleus, we have

$$j' = J_B ,$$

$$L = J'_A ,$$

and

$$\vec{j} = \vec{j}' - \vec{L} .$$

For $L = 2$, we find that the allowed angular momentum transfer for the stripping reactions is given by

$$J_B + 2 \geq j \geq |J_B - 2| .$$

IV. COMPARISON WITH THE COUPLED-CHANNEL BORN APPROXIMATION

Since the method proposed here for calculating two-step processes in transfer reactions provides a very quick way to calculate cross sections it was decided that in order to test the reliability of our results that a comparison would be made with our results and those obtained from the slower and presumably more accurate CCBA.

We compare our results with those obtained by Mackintosh[10] for deuteron stripping to the $\frac{7}{2}^+$ member of the ground-state $\frac{5}{2}^+$ rotational band of ^{25}Mg for incident-energy deuterons of 12.3 MeV. We chose this case because Mackintosh provides a set of optical parameters which can be used in

DWBA calculations and which are in some sense equivalent to the potentials used in the corresponding coupled-channel analysis. While these sets of parameters are not completely equivalent we can make a somewhat more reliable comparison of the results obtained by the two methods.

Before the results of the two calculations are presented and compared we must say something about the mechanics of the calculation. A standard optical potential including a spin-orbit term is used to calculate the radial integrals given in Eqs. (3.9) and (3.10). These are calculated by the computer program DWUCK.[30] The bound-state form factors used in the radial integrals are also calculated by DWUCK and are calculated by using a spherical Woods-Saxon well. In a more exact calculation, a deformed well should be used for these form factors. The values of the C_{lj} coefficients are taken from Chi.[33] The signs of these coefficients depend on the choice of phases employed for the calculation of the radial function for the bound particles. The phases of the radial wave functions used by DWUCK differ from those by a factor

$$P = (-)^{(N-l)/2}, \tag{4.1}$$

where N is the principal quantum number and l is the angular momentum of the bound particle. Since we add the transition amplitudes in a coherent fashion, proper use of this phase convention is extremely important.

Mackintosh does his calculation using a deformed form factor, i.e., he determines the single-particle orbital by using a superposition of harmonic-oscillator states in order to solve the equation containing the deformed Woods-Saxon well. As was mentioned above we use a spherical well. Iano, Penny, and Drisko[34] have performed a calculation for the ^{24}Mg$(d,p)^{25}$Mg reaction leading to the $\frac{5}{2}^+$ ground state in ^{25}Mg with 10-MeV incident energy deuterons. They show that the results for the differential cross section, obtained when a form factor calculated with a deformed well, were smaller than those obtained when a spherical well was used. The angular distributions, however, had the same shape. Thus, we should expect our cross sections to be somewhat larger than those obtained by Mackintosh.

The results of the two calculations and the experimental data are shown in Fig. 2. Mackintosh uses a normalization factor of 25.19 to obtain agreement with the experimental data. A normalization factor of 19 is needed for our calculations.

While there are some differences in the shapes of the two curves and in the normalization the agreement is really quite good considering the differences in the bound-state wave functions and the always present ambiguity that exists for potentials found in coupled-channel and in optical-model calculations. As discussed above, our use of a spherical well probably increases the magnitude of our calculations. This would imply the use of a smaller normalization factor. Thus, the formulation presented here seems to be able to reproduce the results of the more complicated CCBA.

V. RESULTS AND DISCUSSION

In this section, we shall present and discuss the results of our calculations for the stripping reactions ^{24}Mg$(d,p)^{25}$Mg leading to the $\frac{5}{2}^+$ ground state and to the $\frac{7}{2}^+$ state at 1.61 MeV with 15-MeV incident deuterons[18,19] and the ^{182}W$(d,p)^{183}$W reaction leading to the $\frac{1}{2}^-$ ground state and the first $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states with 12-MeV incident energy deuterons.[20]

We chose these two reactions because we could study the effects of two-step processes for a case where a direct transition is forbidden, namely the $\frac{7}{2}^+$ state in ^{25}Mg, and a case where the transition is allowed, but the transition is weak, namely the $\frac{1}{2}^-$ ground state transition in ^{183}W. In the latter case the transition cannot be explained by the allowed one-step transition.[20]

A. Calculations for the Reaction ^{24}Mg$(d,p)^{25}$Mg

In order to analyze this experiment, the optical-potential parameters for the elastic scattering of protons were taken from Blair *et al.*[35] These parameters are listed in Table I as parameter set PA. This set of proton parameters fit the 20-MeV proton elastic scattering and the polarization data.[35] We found that this set of parameters also fits the 17.5-MeV proton elastic scattering data.[36,37] The situation for the case of deuterons is not as clear, since there exists no published experimental work for the elastic scattering of 15-MeV deuterons on ^{24}Mg. Fortunately, there are two experiments available for inelastic scattering at this energy. Two sets of optical-model parameters, used by Iano and Austern[16] and listed in Table I, were used to see which one fits the available experimental data for the inelastic scattering. As shown in Fig. 3, there is unfortunately a discrepancy in the data obtained by Haffner[38] and by Blair and Hamburger.[39] As seen from this figure, apart from the forward angles, both sets seem to fit the differential cross section very well. The shallow potential[40] DA gives a larger contribution at the forward angles, which fits the data obtained by Haffner, whereas the deep potential[41] DB gives a smaller contribution at the forward angles giving better agreement with Blair and Hamburger's data. However, because it is now assumed that the real deuteron

TABLE I. Optical-model parameter sets used in the ^{24}Mg$(d,p)^{25}$Mg analysis. The potential is defined as

$$U(r) = V_c - V_0 f(x_0) - iW f(x_i) + 4i W_D \frac{d}{dx_D} f(x_D) + 2V_{so}(r) \frac{1}{r} \frac{d}{dr} f(x_D) \vec{l} \cdot \vec{\sigma} ,$$

where

$$x_j = (r - r_j A^{1/3})/a_j \quad \text{and} \quad f(x_j) = (e^{x_j} + 1)^{-1} .$$

Channel	Parameter set	V (MeV)	r_0 (fm)	a_0 (fm)	W (MeV)	r_i (fm)	a_i (fm)	W_D (MeV)	r_D (fm)	a_D (fm)	V_{so} (MeV)	r_s (fm)	a_s (fm)	r_c (fm)
Deuteron	DA	50.0	1.50	0.59	16.0	1.5	0.59	0.0			8.0	1.5	0.59	1.5
	DB	101.3	1.00	0.90	0.0			28.9	1.443	0.50	0.0			1.30
Proton	PA	46.7	1.24	0.65	0.0			8.3	1.28	0.50	5.5	0.92	0.50	1.25
Bound neutron		Adjusted to give right binding energy	1.25	0.65										

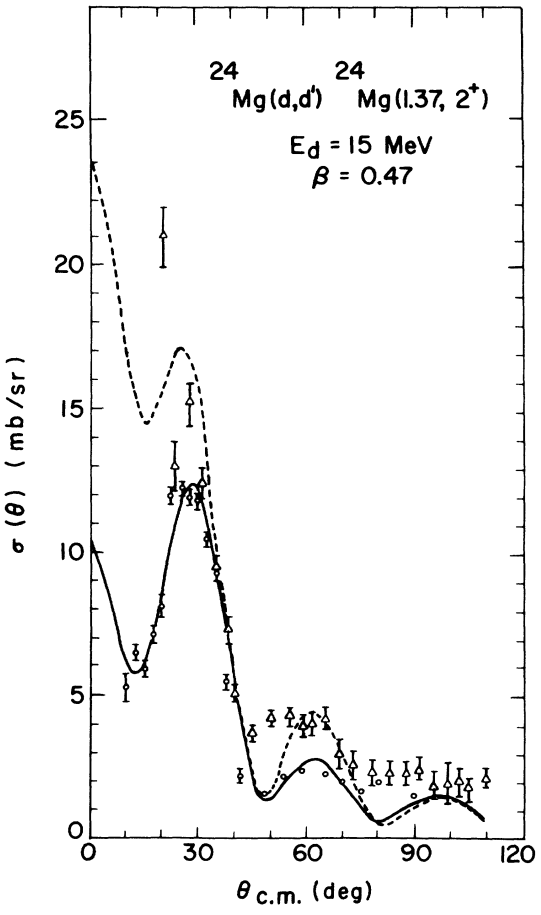

FIG. 3. The dashed line is the result of the calculations using the shallow-well potential set DA of Table I and the solid line is obtained by using the deep-well potential set DB of Table I. The triangles and circles represent the experimental data obtained by Haffner (Ref. 38) and by Blair and Hamburger (Ref. 39), respectively.

optical potential should have a strength approximately twice that of the nucleons strength, the deep well DB was used.

If the inelastic scattering takes place after stripping to the $\frac{5}{2}^+$ ground state of ^{25}Mg, then the angular momentum transfers allowed by the selection rules are $l=2$ and $j=\frac{5}{2}$ for the stripping and $L=2$ for the inelastic scattering to excite the $\frac{5}{2}^+$ ground state to the $\frac{7}{2}^+$ state. If the inelastic scattering takes place before the stripping, we need $L=2$ to excite the target nucleus from the 0^+ ground state to the 2^+ first excited state so the values of j transfer needed to reach the $\frac{7}{2}^+$ state in ^{25}Mg are $\frac{7}{2}-2 \leq j \leq \frac{7}{2}+2$. But $j=\frac{3}{2}$ transfer is not allowed since $K=\frac{5}{2}$ for this band.

In Fig. 4 we present the results of the calculation for stripping to the $\frac{7}{2}^+$ state of ^{25}Mg. These results are for $\beta=0.47$. The same value of β is used by Schultz *et al.*[8] and by Braunschweig, Tamura, and Udagawa.[9] In the calculations the contribution from the $l=4$, $j=\frac{7}{2}$ transfer with inelastic scattering in the entrance channel was extremely small and is not shown in the figure. For this reason, we feel that the contributions from the higher l and j transfers allowed by the selection rules for the $0^+ \rightarrow 2^+ \rightarrow \frac{7}{2}^+$ path can be neglected. The contribution from the entrance and exit channels are for the most part equal. The fit to the experimental data is good.

Figure 5 shows a comparison of the results obtained by Iano and Austern[16] and those obtained here. The comparison should not be taken too seriously as Iano and Austern use a shallow potential well for the deuteron optical parameter; however, since the authors also quote a set of deep-well parameters, presumably the shallow-well parameters gave better agreement with experimental data.

It can be seen on investigating Fig. 5 that our results are for the most part greater than those of Iano and Austern and agree much better with the data.

Figure 6 illustrates the effects of two-step processes on the differential cross section in the allowed direct transition to the $\frac{5}{2}^+$ state of ^{25}Mg. In our formalism, the only two-step process that will contribute will be the $l=2$ and $j=\frac{5}{2}$ transfer with inelastic scattering in the entrance channel, as we do not include interband transitions. Except for the extreme backward angles the effects are small, thus indicating that higher-order terms in the perturbation expansion of the Green's operator which appears in the transition amplitude are indeed small when compared with the two leading terms.

B. Calculations for the Reaction $^{182}W(d,p)^{183}W$

Calculations were carried out for this reaction for 12-MeV incident energy deuterons. Of particular interest is the stripping to the $\frac{1}{2}^-$ ground state of ^{183}W. This state is the band head for the (510) $\frac{1}{2}^-$ band. Conventional DWBA calculations have not been able to reproduce the data; however, the small value of the coefficient $C_{1\,1/2}$ for this state would imply that two-step processes could be important for this reaction.

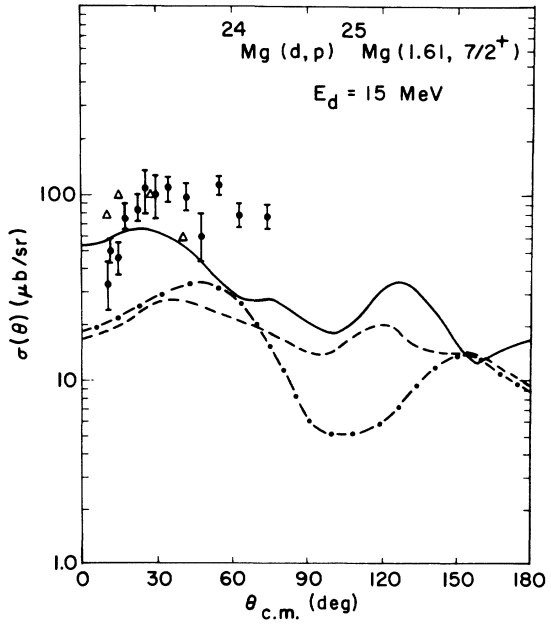

FIG. 4. These results are obtained for the parameter sets DB and PA of Table I and are plotted for $\beta=0.47$. The dash-dotted and dashed curves are for the $l=2$ and $j=\frac{5}{2}$ transfers with the inelastic scattering in the entrance and the exit channels, respectively. The solid curve is obtained by adding these contributions coherently. The circles and triangles represent the experimental data obtained by Hamburger and Blair (Ref. 18) and by Cujec (Ref. 19), respectively.

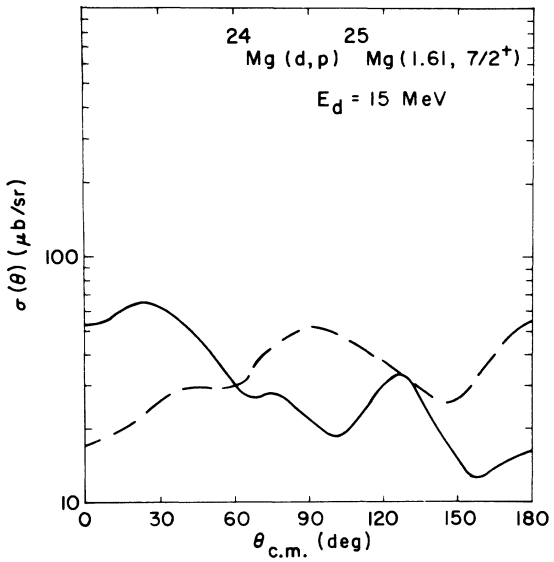

FIG. 5. Comparison of the present calculations (solid curve) with those of Iano and Austern (Ref. 16) (broken curve). Parameter sets used in the present calculations are not the same as used by Iano and Austern.

The optical parameters used in the calculations are given in Table II. The set DC for deuterons and the set PB for deuterons were the original set of parameters used by Erskine and Siemssen in their DWBA analysis of this reaction.[20] This set

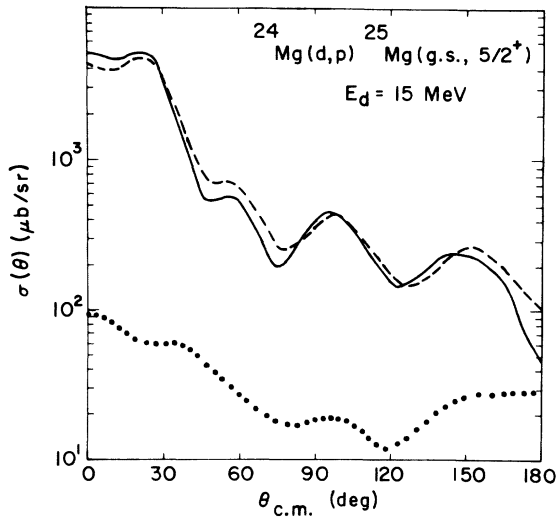

FIG. 6. These calculations are for the parameter sets DB and PA of Table I. The dashed curve is the contribution from the one-step process and the solid curve is obtained by adding the contributions of the two-step process ($0^+ \rightarrow 2^+ \rightarrow \frac{5}{2}^+$) with an $l=2$, $j=\frac{5}{2}$ transfer (the dotted curve) to the one-step process coherently.

TABLE II. Optical-model parameter sets used in the $^{182}W(d,p)^{183}W$ analysis.

Channel	Parameter set	V (MeV)	r_0 (fm)	a_0 (fm)	W_D (MeV)	r_D (fm)	a_D (fm)	V_{so} (MeV)	r_s (fm)	a_s (fm)	r_c (fm)
Deuteron	DC	104.0	1.15	0.81	13.5	1.34	0.68				1.15
	DD	77.0	1.30	0.79	20.4	1.37	0.67				1.30
Proton	PB	53.0	1.25	0.65	18.0	1.25	0.47	8.0	1.25	0.65	1.25
Bound neutron		Adjusted to give right binding energy	1.25	0.65							

of parameters was also used by Penny in his investigation of two-step processes in this reaction.[5,42] The set of deuteron optical parameters DD obtained by Siemssen and Erskine[43] in a new study of elastic and inelastic scattering of deuterons from ^{182}W give better fits to these data and as such should provide a better set of the parameters for the present study. Coulomb excitation was included in the calculation.

In the $0^+ \rightarrow 2^+ \rightarrow \frac{1}{2}^-$ two-step process, the only values of j transfer that are allowed are $\frac{3}{2}$ and $\frac{5}{2}$, which correspond to $l=1$ and $l=3$, respectively. If we allow the inelastic scattering in the exit channel, the stripping could take place between the 0^+ ground state of the target nucleus ^{182}W and the $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ excited states of the residual nucleus ^{183}W, which are members of the (510) $\frac{1}{2}^-$ band. The contribution from the two-step process via the scheme $0^+ \rightarrow \frac{7}{2}^- \rightarrow \frac{1}{2}^-$ is expected to be very small as the C_{lj} coefficient for the $\frac{7}{2}^-$ state is very small and also because it requires $L=4$ for the inelastic scattering. This process was not included in the calculation. The values of the C_{lj} coefficients are listed in Table III and are taken from the work of Chi.[33]

The value of β was taken to be 0.23. This value of β is taken from the study of inelastic scattering of deuterons on ^{182}W and from nuclear structure studies.[20] The results of the calculation using the set of parameters DC and PB are shown in Fig. 7. As seen from this figure the entrance and exit channels contribute almost equally. Also shown in this figure is the cross section for the direct transition. In this case some of the indirect transitions are comparable in magnitude to the direct transition. The final answer is obtained by taking a coherent superposition of all of the two-step and direct amplitudes. This curve has a much different shape than the direct transition and is also larger in magnitude.

Figure 8 shows the results obtained when the set of optical parameters DD and PB were used. There have been changes in shapes and magnitudes of the two-step processes. The shape of the direct transition has also been changed. The curve representing the total effect of all of the transitions has

TABLE III. Values of the C_{lj} coefficients used in the $^{182}W(d,p)^{183}W$ analysis. The C_{lj} values are taken from Ref. 33 and P is given by Eq. (4.1).

L	J	C_{lj}	P
1	$\frac{1}{2}$	+0.07	+
1	$\frac{3}{2}$	−0.65	+
3	$\frac{5}{2}$	+0.554	−

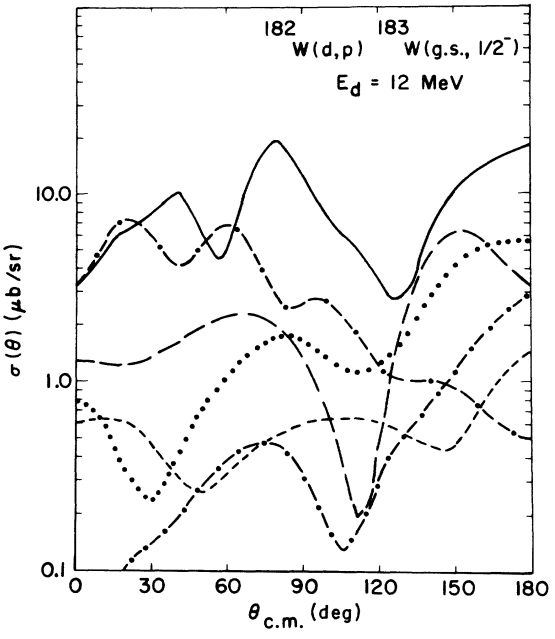

FIG. 7. The calculations are done for the parameter sets PB and DC of Table II. The broken and dash-dotted curves are for the $(0^+ \rightarrow 2^+ \rightarrow \frac{1}{2}^-)$ with the $l=1$, $j=\frac{3}{2}$ and $l=3$, $j=\frac{5}{2}$ transfers, respectively. The dotted curve is for the $(0^+ \rightarrow \frac{3}{2}^- \rightarrow \frac{1}{2}^-)$ with an $l=1$, $j=\frac{3}{2}$ transfer and the dashed curve for the $(0^+ \rightarrow \frac{5}{2}^- \rightarrow \frac{1}{2}^-)$ with an $l=3$, $j=\frac{5}{2}$ transfer. The double dash-dotted curve is the usual DWBA and the solid curve is obtained by adding all these contributions coherently.

again been obtained by taking a coherent superposition of the direct and two-step processes.

A comparison of the two theoretical calculations using the two different deuteron optical-parameter sets and the experimental data are shown in Fig. 9. Both curves give agreement with the experimental data over some portion of the angular range and are definite improvements over the one-step DWBA. Included in this figure are the results of Penny.[42] His results do not explain the experiment. This is probably due to the fact that he did not include the inelastic scattering effects in the exit channel.

Figure 9 also shows the effect of different sets of optical parameters on the cross section. There is no doubt that a better fit to the experimental data could be obtained by taking some compromise between the two sets of deuteron optical parameters DC and DD. This may be a reasonable thing to do in light of the ambiguity of the deuteron parameters for ^{182}W, however, this will not be done here but will be the subject of a future communication.

Figures 10 and 11 show the effects of two-step processes on the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ members of the ground-state band in ^{183}W. Here the effects are relatively small when compared to the transition to the $\frac{1}{2}^-$ state. This is due to the relatively large values of the C_{lj} coefficients of these states as shown in Table III.

VI. CONCLUSIONS

A formalism, which is essentially a perturbation approach, has been developed to include inelastic scattering in the entrance and exit channels in the study of nuclear stripping reactions. Although the formalism is for any spin of a target nucleus, applications presented here have been made only for spin-zero targets. The results of our calculations show that much better results are obtained with the inclusion of two-step processes.

A comparison has been given between our calculations and those obtained in the CCBA. The agreement between the two calculations is quite good.

For the ^{24}Mg$(d,p)^{25}$Mg ($\frac{7}{2}^+$, 1.61 MeV) reaction the fit to the experimental data for 15-MeV incident energy deuterons was quite good. Contributions from inelastic scattering in the entrance channel are comparable to those of the exit channel.

The calculations for the ^{182}W$(d,p)^{183}$W ($\frac{1}{2}^-$, g.s.) reaction show that the effect of the two-step process is to change the shape of the angular distribu-

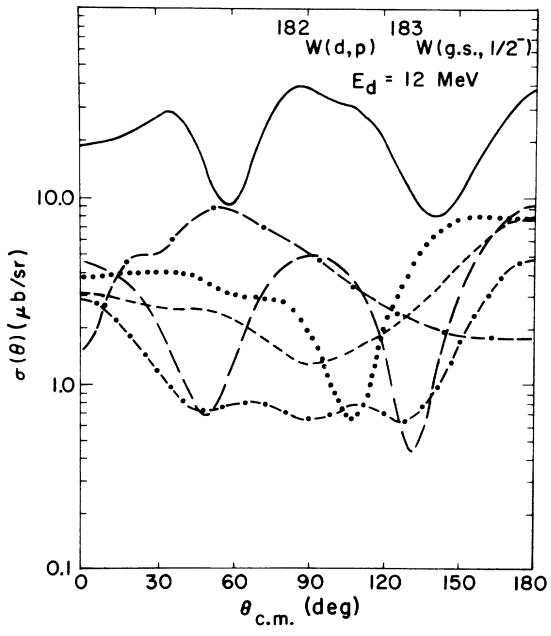

FIG. 8. These calculations are done for the parameter sets PB and DD of Table II. The dotted curve and the dashed curves are for the $(0^+ \to 2^+ \to \frac{1}{2}^-)$ with the $l=1$, $j=\frac{3}{2}$ and $l=3$, $j=\frac{5}{2}$ transfers, respectively. The broken curve is for the $(0^+ \to \frac{3}{2}^- \to \frac{1}{2}^-)$ with an $l=1$, $j=\frac{3}{2}$ transfer and the dash-dotted curve is for the $(0^+ \to \frac{5}{2}^- \to \frac{1}{2}^-)$ with an $l=3$, $j=\frac{5}{2}$ transfer. The double dash-dotted curve is for the usual DWBA and the solid curve is obtained by adding all these contributions coherently.

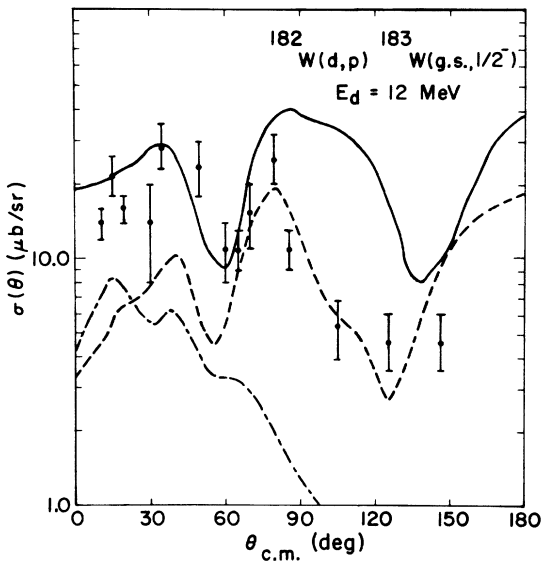

FIG. 9. The dashed line is obtained by using the parameter sets DC and PB, while the solid line is obtained by using the parameter sets DD and PB of Table II. See Figs. (7) and (8) for various contributions included in these calculations. For the sake of comparison, we have plotted the calculations obtained by Penny (Ref. 42) as a broken line. The experimental data are taken from Siemssen and Erskine (Ref. 20).

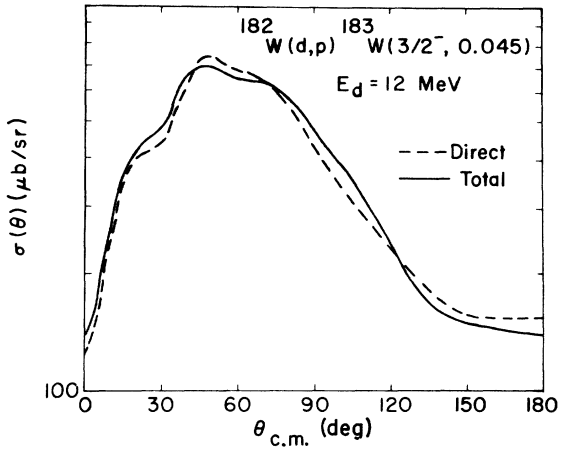

FIG. 10. The curves are obtained by using the parameter sets DC and PB of Table II. The dashed line is the usual DWBA. The two-step processes included in the solid curve are the $(0^+ \rightarrow 2^+ \rightarrow \frac{3}{2}^-)$ with the $l=1$, $j=\frac{3}{2}$ and $l=3$, $j=\frac{5}{2}$ transfers and the $(0^+ \rightarrow \frac{5}{2}^- \rightarrow \frac{3}{2}^-)$ with an $l=3$, $j=\frac{5}{2}$ transfer.

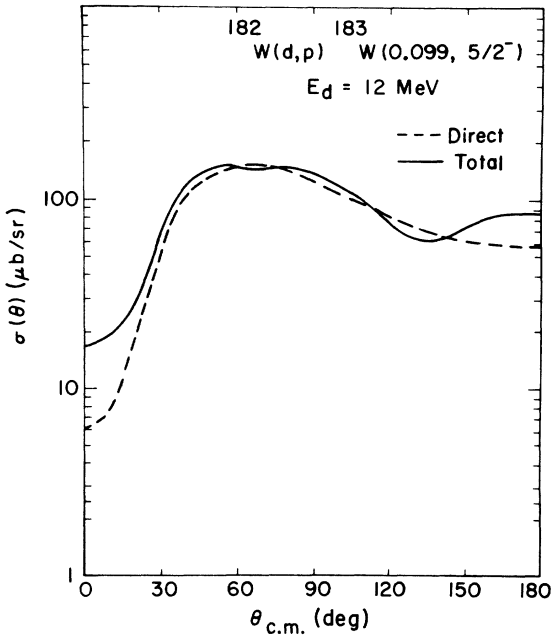

FIG. 11. See figure caption of Fig. 10. The two-step processes included in the solid curve are the $(0^+ \rightarrow 2^+ \rightarrow \frac{5}{2}^-)$ with $l=1$, $j=\frac{3}{2}$ and $l=3$, $j=\frac{5}{2}$ transfer and the $(0^+ \rightarrow \frac{3}{2}^- \rightarrow \frac{5}{2}^-)$ with an $l=1$, $j=\frac{3}{2}$ transfer.

tion and to increase the cross section at some angles by a factor of 4 to 15. We have reproduced the experimental distribution and also the magnitude of the cross section agrees with the experimental data. In the present study, we found that the contributions from the effects of inelastic scattering in the entrance channel and the exit channel are almost equal in importance. This is probably the reason for Penny's failure in fitting this reaction as he only included the effects of the inelastic scattering in the entrance channel.[42]

Our calculations for those reactions that are strongly allowed in the one-step process for both ^{24}Mg and ^{182}W show that the effect of the two-step processes is to change the cross section by 10–15% for ^{24}Mg and somewhat more at some angles for ^{182}W.

In short, we have shown that the inclusion of the two-step processes is very important for the calculations of the weakly excited and forbidden transitions as we can fairly well reproduce the angular distribution for the ^{24}Mg$(d,p)^{25}$Mg $(\frac{7}{2}^+, 1.61$ MeV) reaction and the ^{182}W$(d,p)^{182}$W $(\frac{1}{2}^-$, g.s.) reaction. This means that our formalism is successful for at least the cases we have studied. Our model is a simple one in the sense that we do not use coupled-channel methods to calculate the radial-wave functions. To calculate the radial integrals, we can use distorted-wave computer programs like DWUCK, which are readily available. Also, as we do not use coupled-channel methods, it is much faster to analyze experimental data by using the computer programs based on our model. However, the general validity of making an on-shell approximation cannot be proved analytically, but the results obtained here indicate that it does not seem to be a bad approximation. Even if this approximation is not completely accurate, it does provide an extremely fast and simple way of investigating the effects of two-step processes.

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