tion energies, had suggested that a constant limiting angular momentum, as suggested by the liquid-drop model, had been observed. In view of the present results, it would appear that such is not the case. However, in the reaction $^{12}C+^{27}Al$, the limiting angular momentum of $36\hbar$ at 180-MeV projectile energy is close to the $40\hbar$ limit calculated for ^{39}K and should projectiles of suitable energy become available, it would obviously be of considerable interest to perform similar experiments at higher excitation energies.

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$^{91}Zr(p, t)$ Reaction and the Level Structure of ^{89}Zr†

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The $^{91}Zr(p,t)$ reaction has been studied at a proton energy of 31 MeV. Tritons were detected in a magnetic spectrograph yielding a resolution of 18 keV. Angular distributions for levels up to 2.75 MeV of excitation in ^{89}Zr are compared to two-nucleon-transfer distorted-wave Born-approximation calculations and analyzed in terms of the mixed L transfers allowed from a nonzero-spin target. Two types of neutron configurations are observed to be populated in ^{89}Zr by this reaction: single-hole states and one-particle, two-hole states.

I. INTRODUCTION

Much of the existing data on two-neutron-transfer reactions such as (p,t) and (t,p) are concentrated on studies with even-target nuclei. One reason for this is the simplification introduced by the selection rules governing these reactions.[1] For a (p,t) or (t,p) reaction from a 0^+ initial state, only the natural-parity states of the final nucleus will be populated in the direct one-step transfer process. In addition, each final state will be populated with a unique L-transfer value. Thus the determination of the L value for a given final-state angular distribution gives directly both the spin and parity of the level observed.

The situation for (p,t) or (t,p) from nonzero-spin targets will usually be more complex. In general, more than one L transfer will contribute for a given final state and the observed angular distribution will be a composite of all the allowed L transfers. One purpose of this paper is to show that the present capability of calculating reliable two-neutron-transfer angular distributions makes the analysis of such mixed-transition data feasible.

The ^{91}Zr(p,t) reaction was chosen to illustrate the more complex case for nonzero-spin targets, since it leads to levels in ^{89}Zr which have been studied in some detail by other means.[2-6] In particular, the single-neutron-pickup reaction from the closed $N=50$ shell nucleus ^{90}Zr indicates that the ground and first three excited states in ^{89}Zr are due principally to a very simple neutron-hole structure. That is, the ground state $\frac{9}{2}^+$ corresponds to a neutron hole in the $1g_{9/2}$ orbital, and the 0.588-MeV $\frac{1}{2}^-$, 1.095-MeV $\frac{3}{2}^-$, and 1.452-MeV $\frac{5}{2}^-$ states arise from exciting this neutron hole into the $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ orbitals, respectively. In Sec. III we will show that such a simple structure for the neutron part of the wave functions for these levels leads to a rather simple weighting of the allowed L transfers for the (p,t) reaction. These low-lying levels can thus serve as a check on our ability to fit mixed L-transfer angular distributions.

In Sec. IV we will use the calculated angular distributions to analyze the higher excited states of ^{89}Zr and attempt to remove some of the previous uncertainties in spin-parity assignments. Some previously reported ^{90}Zr(p,d) data[2] are reanalyzed to provide additional information on some levels of ^{89}Zr.

II. EXPERIMENTAL RESULTS

The target was a rolled self-supported foil enriched to 91.5% of ^{91}Zr. The thickness of the foil was 0.25 mg/cm^2 and the major isotopic impurity was 5.0% of ^{90}Zr. The (p,t) reaction was studied with 31-MeV protons from the Oak Ridge isochronous cyclotron and the tritons were detected with nuclear emulsions in the broad-range spectrograph facility. Typical beam current on target was 0.5 μA and the observed experimental resolution was about 18 keV. An example of the triton spectra is shown in Fig. 1. Angular distributions were obtained over a range of angles from 8 to 65°.

III. DISTORTED-WAVE ANALYSIS

The principal reason for choosing a proton-bombarding energy of 31 MeV was the previous observation that, for this region of nuclei, the (p,t) angular distributions exhibit more structure and more distinctive shapes at this energy[7] than at 38 MeV.[8] A reasonably high energy is required to shift the intense deuteron groups from competing (p,d) reactions out of the region of the spectrograph focal plane where the triton groups of interest are located. At the 31-MeV proton energy, we observe triton groups corresponding to about 3 MeV of excitation in ^{89}Zr free from any deuteron interference. The previously reported work on ^{90}Zr(p,t) at 31 MeV showed that the observed angular distributions for different L transfers could be well reproduced by zero-range distorted-wave Born-approximation calculations (DWBA).[5] The present work uses the computer code JULIE[9] to perform these calculations.

The relationship between the output of the JULIE code and the experimental cross section has been discussed previously in some detail.[8] For the purposes of this paper, we will use an abbreviated expression appropriate to the two-neutron-pickup

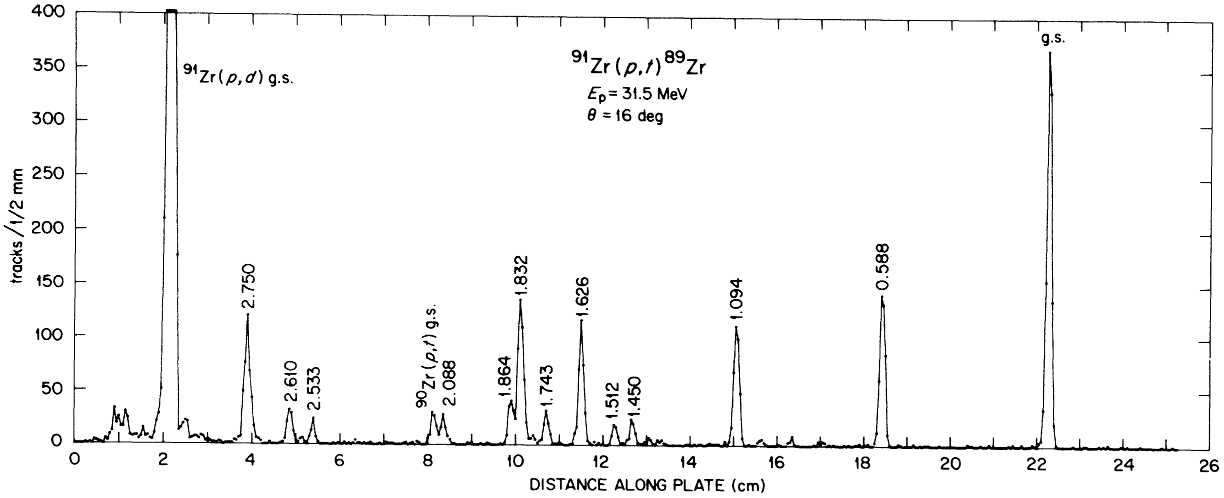

FIG. 1. A triton spectrum from the ^{91}Zr$(p,t)^{89}$Zr reaction observed at a scattering angle of 16°.

reaction from a nonzero-spin target nucleus:

$$\sigma_{\text{exp}}(\theta) = 2D_0{}^2\mathcal{E}\sum_L \Big|\sum_{j_1 j_2} B(j_1 j_2 J)\tilde{\beta}_{\text{JULIE}}(j_1 j_2 J, \theta)\Big|^2 . \tag{1}$$

In this expression $\tilde{\beta}(j_1 j_2 J, \theta)$ is the complex transition amplitude computed by the code JULIE, $B(j_1 j_2 J)$ is the nuclear two-particle spectroscopic amplitude, $D_0{}^2$ is the normalization factor introduced by the zero-range approximation, and $\mathcal{E}$ is the "enhancement factor." This last term is used to systematize comparison of experiment with the DWBA predictions. For realistic choices of the configurations contributing to a given transition, the extracted enhancement factor should be unity. For more simple model assumptions, the enhancement factor indicates the degree to which an assumed configuration could account for the observed transition strength.

As discussed in Sec. I, the single-neutron-pickup data leading to ^{89}Zr indicate that the lowest levels arise primarily from creating a single hole in the neutron orbitals filled at $N = 50$. Thus, the neutron structure for these levels can be written as

$$|^{89}\text{Zr}; j'\rangle = |(l_{j'})^{n-1}{}_{j'}\rangle , \tag{2}$$

where $l_{j'}$ indicates any of the orbitals filled at $N = 50$ and n is the total occupation number of that orbital.

For the calculations to be performed in this paper, the ground state of ^{91}Zr will be assumed to have a neutron structure represented by a single $2d_{5/2}$ neutron outside a closed $N = 50$ core:

$$\begin{aligned}|^{91}\text{Zr}; \tfrac{5}{2}\rangle &= |[(N)^{50}{}_0(d_{5/2})]_{5/2}\rangle \\ &= \sum_l |[(l_j)^n{}_0(d_{5/2})]_{5/2}\rangle . \end{aligned}\tag{3}$$

The assumption of such a simple structure is supported by the single-neutron-pickup results.[2] The promotion of a zero-coupled neutron pair across the closed-shell energy gap at $N = 50$ was found to occur with a probability of only about 3% and coupling of the $d_{5/2}$ neutron to excited states of the proton configuration accounted for less than 4% of the ^{91}Zr ground-state wave function. Since other types of excitations will have higher basic state energies than these, additional admixtures should be very small and the simple configuration written above should dominate the neutron part of the ^{91}Zr ground-state wave function.

With the above assumption about ^{91}Zr, each of the hole states in ^{89}Zr will be populated in the (p,t) reaction by removing the $d_{5/2}$ valence neutron and a neutron from one specific orbital in the $N = 50$ core. Thus, for this case, there is no sum over different $j_1 j_2$ pairs and the cross section expression simplifies to

$$\sigma_{\text{exp}}(\theta) = 2D_0{}^2\mathcal{E}\sum_L B^2(j_1 j_2 J)\tilde{\beta}^2_{\text{JULIE}}(j_1 j_2 J, \theta) . \tag{4}$$

The weighting for the different L-transfer contributions will be determined completely by the nuclear structure as expressed in the $B^2(j_1 j_2 J)$ coefficients.

To evaluate the two-particle spectroscopic amplitudes, it is convenient to rewrite the wave function for ^{91}Zr given by Eq. (3) in the equivalent form

$$|^{91}\text{Zr}; \tfrac{5}{2}\rangle = \sum_l |\{[(l_j)^{n-1}{}_j(l_j)]_0(d_{5/2})\}_{5/2}\rangle .$$

This is then regrouped to give

$$|^{91}\text{Zr}; \tfrac{5}{2}\rangle = \sum_l\sum_J U(j, j, \tfrac{5}{2}, \tfrac{5}{2}; 0, J) \times |\{(l_j)^{n-1}{}_j[(l_j)(d_{5/2})]_J\}_{5/2}\rangle , \tag{5}$$

where U is the normalized Racah coefficient.

We write the two-particle spectroscopic amplitude as,

$$B(j_1 j_2 J) = \sqrt{\Pi}\,\langle^{89}\text{Zr}; j'|^{91}\text{Zr}; \tfrac{5}{2}\rangle ,$$

where Π is the total number of identical neutron pairs in the $j_1 j_2$ orbitals and the removal of a neutron pair is understood in the overlap integral. Inserting expressions (2) and (5) gives,

$$B^2(j, \tfrac{5}{2}, J) = \Pi\sum_J U^2\langle(l_{j'})^{n-1}{}_{j'}|\{(l_j)^{n-1}{}_j[(l_j)(d_{5/2})]_J\}_{5/2}\rangle^2 . \tag{6}$$

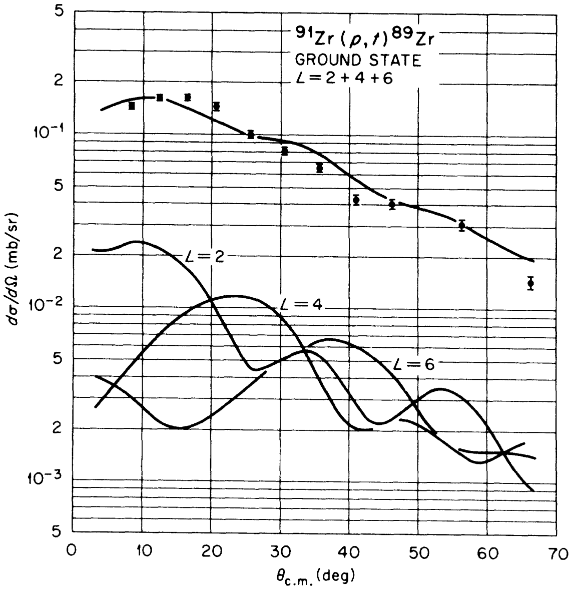

FIG. 2. Experimental and calculated angular distributions for the ^{91}Zr(p,t) reaction to the $\frac{9}{2}^+$ ground state of ^{89}Zr showing the relative contributions predicted for the allowed L transfers.

Since there is only a single $d_{5/2}$ neutron, the number of identical pairs formed with any filled l_j orbital will be $2j+1$. The overlap integral reduces to $\delta_{jj'}$ and the recoupling coefficient has a particularly simple form,[10]

$$U^2(j,j,\tfrac{5}{2},\tfrac{5}{2};0,J)=\frac{2J+1}{6(2j+1)}.$$

Combining the above, expression (6) reduces to

$$B^2(j,\tfrac{5}{2},J)=\frac{2J+1}{6}$$

independent of the orbital j.

Remembering that the neutron pair is assumed to have $S=0$ in the (p,t) reaction, the L of the transferred pair is equivalent to the J of the transferred pair and the expression for the cross section now reduces to

$$\sigma_{\exp}(\theta)=2D_0{}^2\mathcal{E}\sum_L\left(\frac{2L+1}{6}\right)\tilde{\beta}^2_{\mathrm{JULIE}}(j_1j_2J=L,\theta). \tag{7}$$

Thus, for the particular cases considered here, the predicted DWBA cross sections for the L transfers allowed will contribute to the total cross section weighted by a $2L+1$ factor.

The application of Eq. (7) to the ground-state transition is shown in Fig. 2. The $\frac{9}{2}^+$ ground state of ^{89}Zr is assumed to be populated by the removal of the $d_{5/2}$ and one $g_{9/2}$ neutron from ^{91}Zr. The allowed L values for the (p,t) reaction are thus 2, 4, and 6. The lower portion of the figure shows the result of the JULIE calculation for each L transfer weighted by $(2L+1)/6$. Combining the three curves with the relative intensities as shown results in the composite angular distribution shown normalized to the experimental data in the upper portion of the figure. Although individual L-transfer distributions possess significant angular structure, the sum curve is rather featureless. This predicted shape is, however, in good agreement with the experimental results. The magnitude, and hence the enhancement factor obtained, will be discussed in the following section.

A similar treatment of the angular distribution for the first three excited states is shown in Figs. 3–5. In these cases the core neutron picked up is assumed to be from the $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ orbital, respectively. For the 0.588-MeV $\frac{1}{2}^-$ level, only $L=3$ is allowed and Fig. 3 shows the agreement with the experimentally observed angular distribution. For the 1.094-MeV $\frac{3}{2}^-$ level, both $L=1$ and $L=3$ are allowed and the weighted contribution again produces a rather featureless shape. As shown in Fig. 4, the data are in good agreement with the predicted angular distribution. Although both the ground state and 1.094-MeV state are seen to have little angular structure, there are slight differences particularly at forward angles that are reasonably well accounted for by the calculations. The 1.450-MeV $\frac{5}{2}^-$ level has $L=1$, 3, and 5 all allowed but both the reaction dynamics and the $2L+1$ weighting enhance the $L=5$ transfer sufficiently to make this component dominate the composite angular distribution. As shown in Fig. 5, the data for the 1.450-MeV level agree well with this prediction.

In the next section a similar DWBA analysis will be used to suggest spin assignments for some of the higher-lying levels. The parameters used in the DWBA calculations are shown in Table I. The proton parameters have been taken from the work of Becchetti and Greenlees[11] and the triton parameters interpolated from the results of Flynn *et al.*[12] The bound-state wave functions for the transferred neutrons were calculated for a Woods-Saxon well by an oscillator wave-function expansion technique.[13] The Woods-Saxon well employed had a radius parameter $r_0=1.25$ and a diffusivity $a=0.65$. A Thomas-type spin-orbit term was included with this same geometry and a strength $\lambda=25$. Single-particle energies were chosen so that the total binding energy of the two neutrons matched the experimental two-neutron separation energy.

For the purpose of extracting enhancement factors in the following section, a value of $D_0{}^2=22$ will be used. This constant was determined empirically in the previous study at 38 MeV and is consistent with our initial work at 31 MeV which employed a different bound-state potential geometry. The ^{90}Zr(p,t) reaction was studied at both energies and the same enhancement factors were derived from the DWBA analysis.

TABLE I. Optical-model parameters used in the DWBA calculations.

	Proton	Triton
V (MeV)	55.2	170.0
r_0 (fm)	1.12	1.16
a (fm)	0.78	0.752
W (MeV)	4.23	21.5
W_D (MeV)	3.93	0.0
r_0' (fm)	1.32	1.498
a' (fm)	0.59	0.817
r_c (fm)	1.20	1.25
V_s (MeV)	6.2	0.0
r_s (fm)	0.98	0.0
a_s (fm)	0.75	0.0

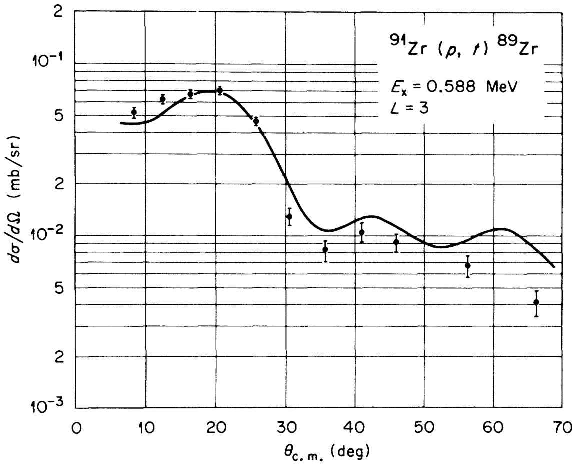

FIG. 3. Experimental and calculated angular distributions for the ^{91}Zr(p,t) reaction to the $\frac{1}{2}^-$ first excited state of ^{89}Zr.

IV. DISCUSSION

A. Low-Lying Levels

The transitions to the ground and first three excited states have been treated in the previous section by interpreting these levels as single-neutron hole states. It was seen that such an interpretation leads to predicted angular distributions that agree well with those observed in our experiment.

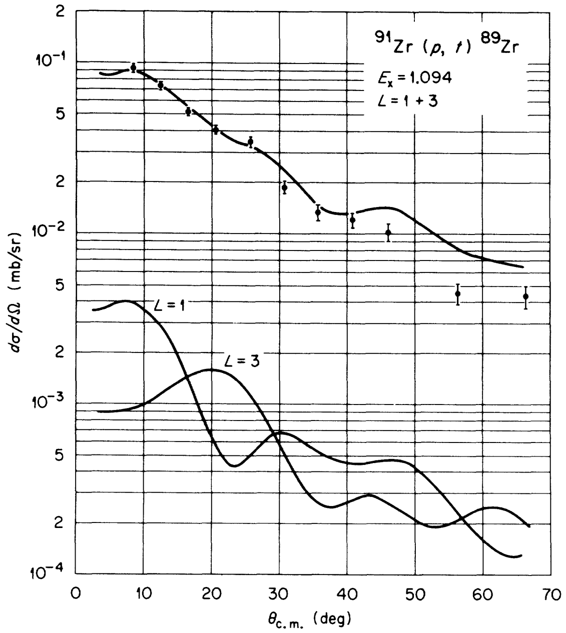

FIG. 4. Experimental and calculated angular distributions for the ^{91}Zr(p,t) reaction to the $\frac{3}{2}^-$ second excited state of ^{89}Zr showing the relative contributions predicted for the allowed L transfers.

The results for these and higher-lying levels are summarized in Table II along with enhancement factors derived from comparison with the DWBA analysis.

The enhancement factors for the lowest four states are of order unity which suggests little collective enhancement and again supports the assumption that the single-hole structure dominates the particle-transfer reactions to these levels. If collective effects are, in fact, rather unimportant in the excitation of these "single-hole" states, then there should be a close correspondence between the (p,t) enhancement factors and spectroscopic factors from the ^{90}Zr(p,d) reaction leading to these same levels. The single-neutron-transfer strengths, derived from the data of Ref. 2, are included in Table II. The fractional fullness v^2 is used since it shows more directly, than the spectroscopic factor, the degree to which a given level possesses the pure single-hole character. Within the inaccuracies inherent in extracting (p,t) enhancement factors and (p,d) spectroscopic factors, a quite striking correspondence between the two experiments is apparent.

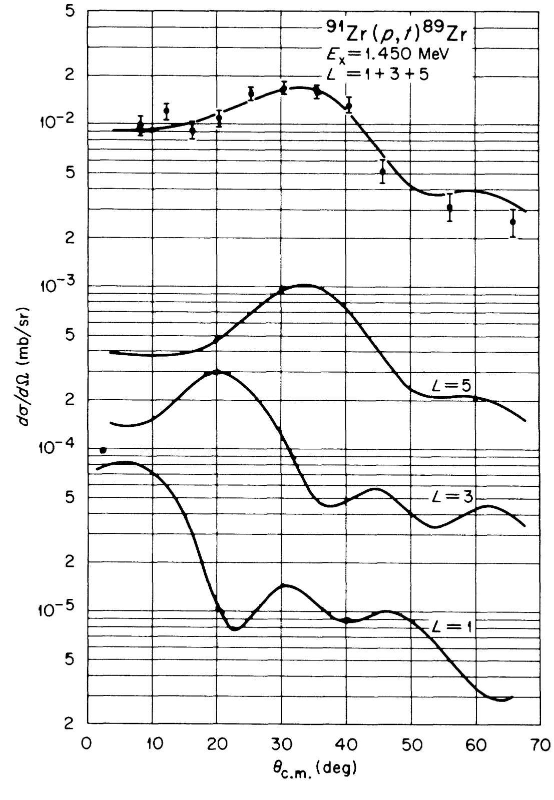

FIG. 5. Experimental and calculated angular distributions for the ^{91}Zr(p,t) reaction to the $\frac{5}{2}^-$ third excited state of ^{89}Zr showing the relative contributions predicted for the allowed L transfers.

B. $L = 0$ Transitions

In the region of excitation energy up to 2.75 MeV in ^{89}Zr, only two levels are observed to exhibit an angular distribution with $L = 0$ character and these are a rather closely spaced pair at 1.63 and 1.83 MeV. The angular distributions for these states are compared with the $L = 0$ DWBA predictions in Fig. 6. Since the ground state of ^{91}Zr is well established at $\frac{5}{2}^+$, the observation of these strong $L = 0$ transitions uniquely identifies these two states as $\frac{5}{2}^+$ levels. The enhancement factors derived for these transitions, as shown in Table II, indicate a strong coherent contribution to both levels.

In a previous study of the ^{90}Zr(p, t) reaction,[7] the ground state of ^{88}Zr was populated with a strongly coherent $L = 0$ transition. This transition was found to have an enhancement factor of 10.0, calculated on the same basis as the $L = 0$ enhancements listed here in Table II. This strong enhancement was attributed to the removal of a highly correlated neutron pair from the closed $N = 50$ shell exhausting nearly all of the $L = 0$ strength expected from the filled $1g_{9/2}$, $2p_{1/2}$, and $2p_{3/2}$ neutron orbitals. We would expect to observe this same $L = 0$ core pickup transfer as an excited state in the present reaction.

TABLE II. Levels in ^{89}Zr observed with the ^{91}Zr(p, t) reaction. Excitation energies are ±0.003 MeV. Also shown for comparison are the single-nucleon-transfer data of Ref. 2.

E (MeV)	L	J^π	Assumed transition	$\mathcal{E}$	ν^2	l_n
0.000	2+4+6	$\frac{9}{2}^+$	a	1.1	0.96	4
0.588	3	$\frac{1}{2}^-$	b	1.0	0.85	1
1.094	1+3	$\frac{3}{2}^-$	c	0.45	0.60	1
1.450	1+3+5	$\frac{5}{2}^-$	d	0.35	0.50	3
1.512	2+4+6	$\frac{9}{2}^+$	a	0.08	0.03	4
1.626	0	$\frac{5}{2}^+$	e	2.7	0.01	2
1.743	1+3	$\frac{3}{2}^-$	c	0.09	0.06	1
1.832	0	$\frac{5}{2}^+$	e	3.4	0.0005	(2)
1.864	1+3	$\frac{3}{2}^-$	c	0.14	0.10	1
2.088	2+4+6	$(\frac{9}{2}^+)$	a	0.09		
2.390	···	···				
2.533	(3)	$(\frac{1}{2}^-)$	b	0.14		
2.570	1+3	$(\frac{3}{2}^-)$	c	0.02		
2.610	2	$(\leq\frac{9}{2})^+$	e	0.35		
2.710	···	···				
2.725	···	···				
2.750	2	$(\leq\frac{9}{2})^+$	e	1.2		

[a] $[(g_{9/2})^{10}{}_0(d_{5/2})]_{5/2} \rightarrow (g_{9/2})^9{}_{9/2}$.

[b] $[(p_{1/2})^2{}_0(d_{5/2})]_{5/2} \rightarrow (p_{1/2})$.

[c] $[(p_{3/2})^4{}_0(d_{5/2})]_{5/2} \rightarrow (p_{3/2})^3{}_{3/2}$.

[d] $[(f_{5/2})^6{}_0(d_{5/2})]_{5/2} \rightarrow (f_{5/2})^5{}_{5/2}$.

[e] $(g_{9/2})^{10}{}_0 \rightarrow (g_{9/2})^8{}_J$.

The two $L = 0$ transitions observed in the present study have a "center of gravity" at a Q value of −12.4 MeV which is quite close to the −12.8-MeV ground-state Q value for the ^{90}Zr(p, t) reaction. Thus the association of this $L = 0$ strength with the correlated pickup from the $N = 50$ core seems quite clear. What is surprising is the splitting of this $L = 0$ strength into almost equal intensities to two closely spaced levels.

The existence of two $\frac{5}{2}^+$ states in this region of excitation is rather easy to understand. The coupling of a $1g_{9/2}$ neutron hole to states in ^{90}Zr (principally the first excited 2^+ level) is expected to give rise to a $\frac{5}{2}^+$ level in this region of excitation energy. This state would not be expected to be populated by the ^{91}Zr(p, t) reaction. Obviously, the state corresponding to removal of a zero-coupled neutron pair from the $N = 50$ core, leaving a single neutron in the $2d_{5/2}$ orbital, also falls at about this same energy. The data from this present experiment suggest that these two states

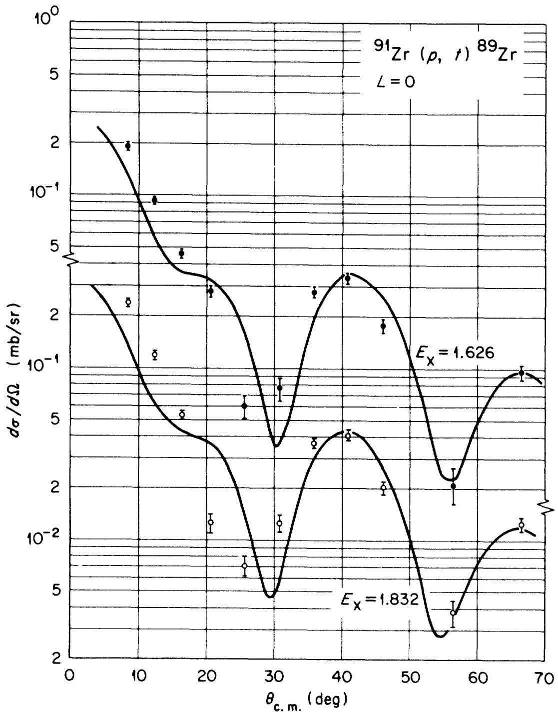

FIG. 6. Angular distributions for the 1.63- and 1.83-MeV states compared with $L = 0$ transfer DWBA predictions.

mix very strongly. There seems to be no special property of this core-pickup state that makes it retain its character in the presence of other states with the same spin and parity.

Another interesting feature of these two $L=0$ transitions is that the total strength adds to only about 60% of the core-pickup strength observed for the ^{90}Zr(p,t) reaction.[7] Thus, the decrease and fractionation of core-pickup strength noted previously for the (p,t) reaction on the $N=52$ nucleus, ^{92}Zr, is seen to occur even at $N=51$. In the present case, this decrease in $L=0$ strength suggests that there are probably $\frac{5}{2}^+$ levels above 2.75 MeV of excitation in ^{89}Zr that carry the remaining $L=0$ intensity. Below this excitation, we see no evidence of any significant $L=0$ components in the angular distributions of levels other than the two assigned $\frac{5}{2}^+$.

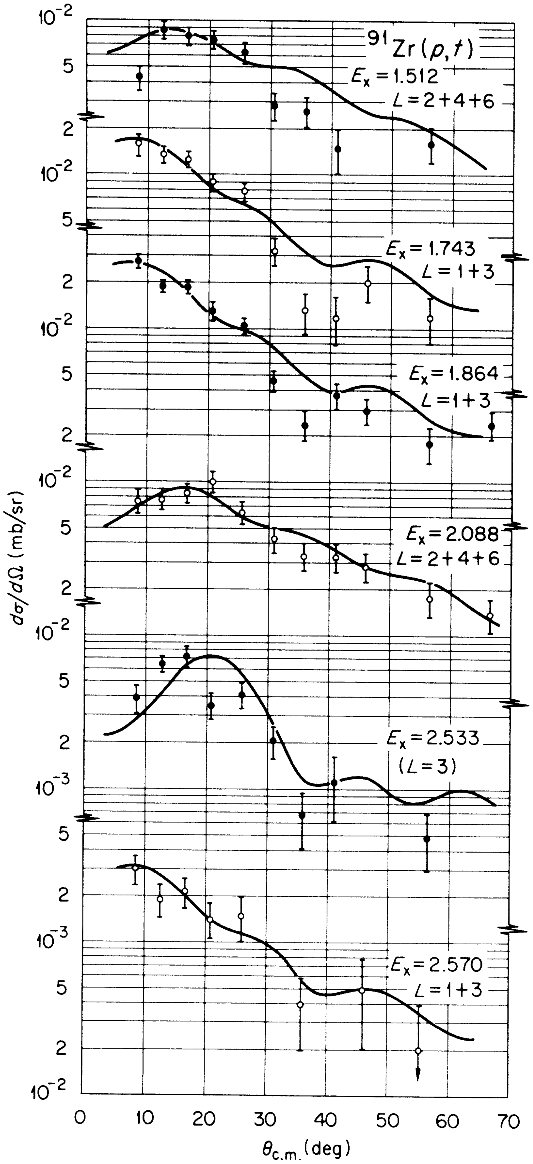

FIG. 7. Angular distributions for levels observed in the ^{91}Zr(p,t) reaction. The solid lines are the DWBA predictions for the L-transfer values indicated. The composite L-transfer predictions are obtained with the same relative weighting as shown in Figs. 2 and 4.

A surprising feature of these two $\frac{5}{2}^+$ levels is the disparity in strength seen for the single-neutron-transfer reaction. The 1.63-MeV level has been reported previously in a study of the ^{90}Zr(p,d) reaction[2] and assigned $\frac{5}{2}^+$ on the basis of an $l_n=2$ angular distribution. The population of this state was presumed to occur through a small admixture in the ^{90}Zr ground state of two-particle, two-hole excitation of neutrons from the $N=50$ core into the "empty" $2d_{5/2}$ orbital. The removal of one of these $d_{5/2}$ neutrons then leads to the same one-particle, two-hole configuration created by the ^{91}Zr(p,t) reaction. The small (p,d) spectroscopic strength observed for this level ($C^2S=0.05$) is consistent with such an interpretation. The 1.83-MeV $\frac{5}{2}^+$ level was, however, not reported in the (p,d) study.[2] The data from Ref. 2 have been reexamined for evidence of this state. In addition, ^{90}Zr(p,d) data taken concurrently with the ^{90}Zr(p,t) reaction study,[7] and having much improved counting statistics, were analyzed for the presence of the 1.83-MeV level. This 1.83-MeV state is excited by the (p,d) reaction, but it is a factor of 20 weaker than the 1.63-MeV $\frac{5}{2}^+$ level, and nearly obscured by the neighboring strong 1.86-MeV $\frac{3}{2}^-$ level. Thus, although the two $\frac{5}{2}^+$ levels are observed in the ^{91}Zr(p,t) reaction to be nearly equally populated, there is more than an order-of-magnitude difference in their population in the ^{90}Zr(p,d) reaction.

C. Higher-Lying Levels

The lowest four levels were discussed above and their angular distributions shown to be consistent with the (p,t) reaction populating these states through their single-neutron-hole character. To the extent that some of the higher levels observed may also contain small fragments of the single-hole strength, these may also exhibit similar angular distributions. The levels that appear to exhibit such behavior are shown in Fig. 7.

The 1.51-MeV $\frac{9}{2}^+$ level is reasonably well established by the observation of $l_n=4$ pickup in the (p,d) reaction. The experimental angular distribution seems to indicate that the $L=2$ contribution may be somewhat more dominant than the weighting given by Eq. (7). The remaining levels seem to be reproduced fairly well by the calculations although the cross sections are quite small and the statistics rather poor. The enhancement factors for these levels, up to 2 MeV, continue to scale remarkably well with the ν^2 extracted from the (p,d) reaction, as shown in Table II. This supports the assumption that these levels are being excited in the (p,t) reaction through their neutron single-hole component. Above 2 MeV of excitation the high-resolution (p,d) data are not available for comparison.

Two excited states exhibit angular distributions that are not fitted well by the composite DWBA predictions but appear to be almost pure $L=2$ in character. These distributions are shown in Fig. 8 along with the $L=2$ DWBA calculation. The center of gravity of this $L=2$ strength is at 2.72 MeV of excitation, almost exactly 1.0 MeV above the center of gravity of the $L=0$ strength discussed in Sec. IV B. In the ^{90}Zr(p,t) reaction, a strong $L=2$ transition was observed to a state in ^{88}Zr at 1.06 MeV. It thus seems reasonable to associate these levels at 2.61 and 2.75 MeV with two members of the multiplet created by the pickup of an $L=2$ coupled neutron pair from the $N=50$ core. This multiplet would be expected to show the same evidence for enhanced strength from coherent addition of several configurations as was observed for the 1.06-MeV level in ^{88}Zr. It is, of course, not possible to make spin-parity assignments for these levels on the basis of the angular distributions, but the fairly large enhancement factor for the 2.75-MeV level makes it a good candidate for a high-spin member of the multiplet.

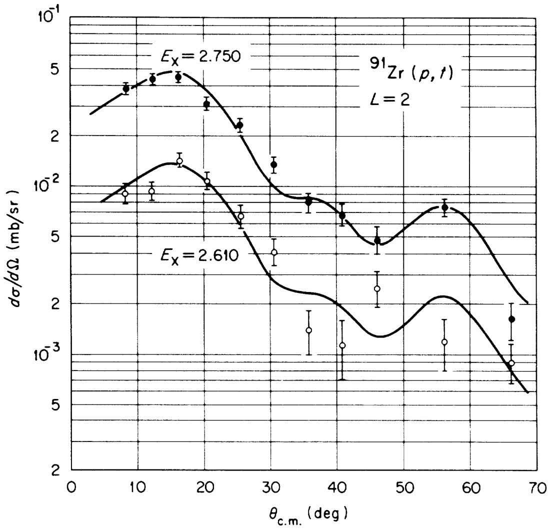

FIG. 8. Angular distributions for the 2.61- and 2.75-MeV states and their comparison with the DWBA predictions for $L=2$ transfer.

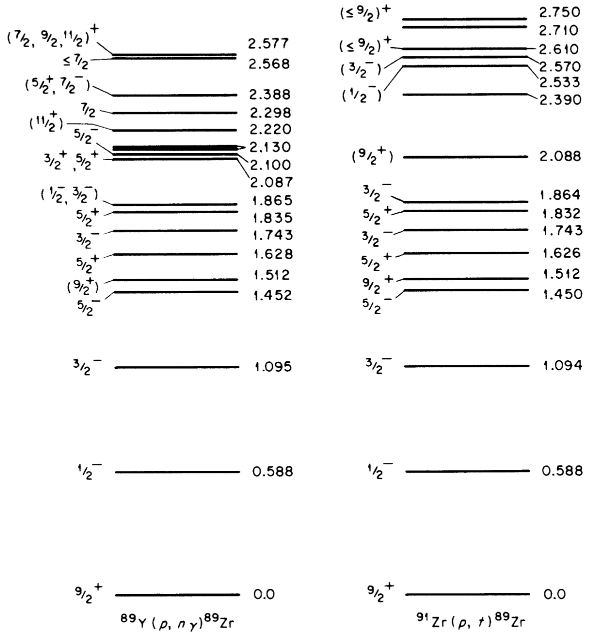

FIG. 9. Energy levels of ^{89}Zr observed in the present ^{91}Zr(p,t) study compared with recent results from Ref. 6 which summarize much of the previous data.

D. Comparison with Other Data

The energies and spin-parity assignments for levels in ^{89}Zr deduced from the present study are shown in Fig. 9. Also shown for comparison are results quoted in a recent study of the ^{89}Y$(p,n\gamma)$ reaction,[6] which are in essential agreement with previous $(p,n\gamma)$ work[3] and decay scheme studies.[4,5]

Besides providing a unique confirmation of the $\frac{5}{2}^+$ assignment to the levels at 1.63 and 1.83 MeV, the present work supports the $\frac{9}{2}^+$ assignment for the 1.51-MeV level and selects $\frac{3}{2}^-$ as the proper choice for the 1.86-MeV level. The present work favors assignment of $\frac{9}{2}^+$ to a level at 2.09 MeV. Although this level appears to be a singlet, this region is seen to be rather complex both from the $(p,n\gamma)$ results and a reexamination of the (p,d) data, and more than one level may be contributing to the measured angular distribution. No evidence is seen for the presence of $L=0$ transfer in this region which suggests none of these levels are $\frac{5}{2}^+$. The level at 2.39 MeV is populated very weakly in the present experiment. The data are consistent with a very weak $L=0$ transition (less than 1% of the 1.83-MeV level) which would favor the $\frac{5}{2}^+$ assignment, but no firm assignment is possible. Above this energy, there is no obvious correlation between the two sets of observed levels.

When this manuscript was essentially complete, a report[14] of a study of this same reaction at a proton energy of 51.7 MeV was received. There are several significant differences between that work and the results of the present experiment. Awaya *et al.*[14] are unable to obtain good fits to the angular distributions for the ground and first three excited states. Although they use a mixture of the allowed L transfers, it is not clear what weighting has been employed. They suggest that their failure to fit these angular distributions may indicate the presence of additional L transfers introduced by contributions from other than relative S-state motion of the transferred neutron pair. In the present study, the application of Eq. (7) was seen to provide good fits to the lower-energy data for these same states so that no additional mechanism was required to be invoked. None of the weak states reported in the present work, and analyzed in terms of this single-neutron-hole picture, were reported by Awaya *et al.* Apparently, this is due to their resolution being poorer by a factor of four (they report 9 levels up to 2.75 MeV of excitation, whereas the present work observed 17). In addition they report the $L=0$ intensity for the 1.63- and 1.83-MeV levels to sum to the previously measured $^{90}Zr(p,t)^{88}Zr$ ground-state transition intensity. This is contrary to the present results of Sec. IV B.

V. SUMMARY

The angular distributions for levels in ^{89}Zr populated in the $^{91}Zr(p,t)$ reaction have been interpreted in terms of the mixed L transfers allowed from a nonzero-spin target. Good fits were obtained, to the levels observed up to 2.75 MeV of excitation, by assuming the states were populated either through neutron configurations of single-hole character or one-particle, two-hole character. Intensities observed for the single-hole states were shown to closely resemble the single-neutron transfer strengths, while the one-particle, two-hole states could be related to the structure observed previously for ^{88}Zr.

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