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PHYSICAL REVIEW C VOLUME 6, NUMBER 6 DECEMBER 1972

Systematic Distorted-Wave Born-Approximation Predictions for Two-Nucleon Transfers: Applications to (d,α) Experiments*

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(Received 12 June 1972)

In the case of angular-momentum-mismatch conventional distorted-wave Born-approximation (DWBA) calculations tend to give results which are strongly dependent on the optical-model parameters chosen and, to a lesser degree, on finite-range and nonlocality effects. We discuss reasons for this sensitivity and present systematic calculations for (d,α) reactions on nondeformed targets ranging from ^{48}Ti to ^{208}Pb. Satisfactory DWBA results could be obtained for the entire range of targets, provided that *all* potentials for the generation of scattered and bound wave functions were restrained to have nearly identical physically meaningful real well geometries and real depths of $V \approx nV_0$, where V_0 is the proton scattering potential and n is the number of nucleons in the projectile. The use of well geometries with $r_0 = 1.2$ fm, $a = 0.75$ fm, and retention of the basic DWBA requirement that the optical potentials should also correctly fit elastic scattering removes the familiar ambiguities for deuteron and α potentials. It is shown that with these parameter restrictions finite range effects are expected to be small so that a first-order correction procedure is adequate. Explicit calculations are compared with over 30 (d,α) angular distributions of known angular momentum transfer for experimental bombarding energies ranging from 12 to 17 MeV. Consistent agreement with experiment was obtained.

I. INTRODUCTION

Previous experience by many investigators has shown that distorted-wave Born-approximation (DWBA) calculations tend to predict transfer reactions well if, in a semiclassical view, conservation of angular momentum permits a (short range) interaction to take place at the nuclear surface. This situation is often referred to as "angular momentum matching" and leads to strongly structured angular distributions with a distinct dependence on L, the angular momentum of the transferred particle or cluster of particles. In such cases, the neglect of finite-range and nonlocality effects, or the use of a rather wide range of "reasonable" optical-model parameters for the generation of the scattered waves may have only a minor effect on the DWBA predictions. However, for any particular experiment, good angular momentum matching is rarely possible over a broad range of L transfers or excitation energies.

The problem of angular momentum mismatch can be especially severe for (d,α) reactions. The attempt to obtain "good" DWBA calculations for such data can become a time consuming one of trial and error, sometimes involving special assumptions for the nucleus studied. DWBA then tends to lose much of its predictive value and safe L-transfer assignments require additional knowledge of the final states studied, e.g., prior knowledge of their parity. It is well known that for angular momentum mismatch the reaction cross sections are significantly affected by contributions from the nuclear interior.[1,2] Thus details of the

form factor and of the distorted waves in the nuclear interior become very important. However, it is precisely here that our knowledge of the wave functions is inadequate. Elastic scattering for strongly absorbed particles does not determine the low partial waves well,[3] which are the ones most likely to penetrate into the nuclear interior. Furthermore, the conventional assumption of two-nucleon transfer theories[4-8] that the form factor is adequately represented by the sum of a few products of two unperturbed single-particle orbitals is not well established.

In this article, we examine two possible ways to improve two-nucleon transfer calculations. At first we discuss the implicit DWBA assumptions and then test the validity of a first-order finite-range correction[9] (evaluated in the WKB approximation). Other authors have made similar studies,[10-12] but we present a different evaluation of the higher-order corrections to the WKB result. Our results indicate that second-order corrections tend to be smaller than the first-order term and do not constitute a marked improvement over the WKB approximation. We then investigate the influence of the optical-model parameters on the reaction amplitudes, especially with regard to continuous parameter ambiguities, and find that these can be of decisive importance in DWBA predictions. The method of well matching as a tool for improving DWBA results is explored. As emphasized by Stock *et al.* for (He^3, α) reactions[12] the use of suitable optical and bound-state potential parameters which minimize the higher-order corrections to DWBA will at the same time minimize the finite-range correction. We develop a prescription and apply it to the analysis of two-nucleon transfer reactions, and in particular to the analysis of various (d, α) experiments, ranging from ^{48}Ti(d, α)^{46}Sc to ^{208}Pb(d, α)^{206}Tl.

II. DISTORTED-WAVE TWO-NUCLEON TRANSFER THEORY, GENERAL CONSIDERATIONS

A. Implications of the DWBA

Distorted-wave transfer theories consider reactions of the type $a+A \rightarrow b+B$, where a stands for the incident projectile and A for the target nucleus, while b denotes the outgoing (detected) particle and B the residual nucleus, which generally is left in some excited state. If the transferred nucleon or cluster of nucleons is called x we have

$$b = a + x,$$

$$B = A - x.$$

Customarily the nonexchange DWBA transition amplitude is written as[1]

$$T^{\mathrm{DWBA}} = N \int d\vec{r}_{aA} \int d\vec{r}_{bB} \chi_b^{(-)*}(\vec{r}_{bB}) \times \langle \phi_b \phi_B | V_{ax} | \phi_a \phi_A \rangle \chi_a^{(+)}(\vec{r}_{aA}), \quad (1)$$

where the wave functions χ are the distorted projectile waves and the functions ϕ are internal nuclear wave functions. V_{ax} is the interaction between the particle systems a and x. The form factor $F = \langle \phi_b \phi_B | V_{ax} | \phi_a \phi_A \rangle$ contains integrals over all internal variables specified to describe the wave functions and the interaction. N is a numerical constant determined by the particular reaction under discussion. Equation (1) will be the starting point for our discussion of finite-range effects as it has been in previous work.[9-18]

It seems necessary to emphasize at the outset that the simplification achieved in Eq. (1) by taking explicit account only of the interaction V_{ax} does put certain restrictions on the potentials U_a, U_b, and V_x which are used to generate the scattered waves χ_a, χ_b and, ultimately, a form factor $F_x(R)$. Some commonly accepted conditions are that U_a and U_b should reproduce the elastic scattering cross sections of the projectiles a and b and that V_x give the correct separation energy for the cluster x. While these boundary conditions for $r \rightarrow \infty$ agree with our physical intuition and experience and serve well to limit the multitude of options for U_a, U_b, and V_x, they do not specify these potentials uniquely, nor is it clear that a minor violation of these conditions will significantly alter T^{DWBA}. On the other hand it has been found[4, 14, 2] empirically that genuine optical-model ambiguities in U_a and U_b can have profound effects on T^{DWBA} for two-nucleon transfers. Hence, stricter conditions seem to be needed for the use of Eq. (1).

The theoretical discussion by Stock, Bock, David, Duhm, and Tamura[12] on (^{3}He, α) DWBA matrix elements is easily extended to other transfer reactions and gives a theoretical justification for restrictions on U_a, U_b, and V_x. Following Ref. 12 we find Eq. (1) as a simplification (the Born approximation) of the more general distorted-wave amplitude

$$T^{\mathrm{DW}} = N \left\langle \chi_b^{(-)} \phi_b \phi_B \left| \left(1 + \frac{V_{aB} + V_{Bx} - U_b}{E^{(+)} - H}\right)(V_{aB} + V_{ax} - U_a) \right| \phi_a \phi_A \chi_a^{(+)} \right\rangle, \quad (2)$$

where the V_{ij} are interaction potentials for the systems defined above. It is clear that Eq. (2) reduces to Eq. (1) for

$$\| V_{aB} + V_{Bx} - U_b \| = 0 \tag{3a}$$

and

$$\| V_{aB} - U_a \| = 0 . \tag{3b}$$

The potentials in Eq. (3a) introduce effects of intermediate states in the $(a+B)$ and $(x+B)$ systems into the Born matrix element. $\| V_{aB} - U_a \|$ would account for core excitations in the $(a+B)$ system. These effects are neglected and are not always small, but diagonal terms may be approximated by a proper choice of scattering potentials.

Inspection of Eqs. (2) and (1) suggests that T^{DWBA} as obtained from Eq. (1) is dependent on the choice of optical-model potentials U_a and U_b and that at least for the discrete optical-model ambiguities different choices could lead to predictions of Eq. (1) which differ drastically. The empirically preferred parameter families,[4,14,2] characterized by

$$V_n \approx n V_{\mathrm{nucleon}} , \tag{4}$$

where n is the number of nucleons in the projectile, are in fact those which come closest to obeying Eqs. (3a) and (3b). Hence we have empirical and theoretical reasons to use for DWBA calculations only those optical-model potentials for which at least the volume integrals for Eqs. (3) would approximately vanish. For projectiles of up to four nucleons there is always one optical-model parameter family of this type.[19-22] Even if it could be argued that a different family of (e.g., α-) parameters produces somewhat superior fits to *elastic* scattering it would not seem advantageous to use them in calculations based on Eq. (1). If Eq. (3b) could be satisfied, we would get the approximate well-matching condition

$$U_a + V_{Bx} - U_b \approx 0 , \tag{3c}$$

where for single-nucleon or simple cluster transfer $V_{Bx} = V_x$, the potential well for the bound state.

B. Finite-Range Calculations

Dickens, Drisko, Perey, and Satchler have shown for single-nucleon transfers that a finite-range correction in the WKB approximation reproduces the relative angular distributions predicted by a more exact finite-range calculation.[13] However, the value of a corresponding approach for two-nucleon transfer reactions is still subject to debate and a detailed discussion of this point is in order. In particular, we must answer two questions: (i) Assuming that the approximations inherent in Eq. (1) are reasonable is it possible and meaningful to derive a finite-range correction factor for two-nucleon transfers similar in form to that used for single-nucleon transfers; and (ii) are the remaining higher-order corrections small compared to this first-order WKB term? In studying question (i) we generally employ approximations and ideas previously used by other authors.[9-11] The answer to question (ii) is not easily deduced from earlier work and results from a new approach.

For practical calculations ϕ_A in Eq. (1) is expressed as a core ϕ_{A-2} coupled to the wave functions ϕ_x^{JM} of the two transferred nucleons. In general there are a number of two-nucleon configurations in ϕ_A that can couple to JM. We label these configurations by $\gamma \equiv (n_1 l_1 j_1, n_2 l_2 j_2)$; all have different spectroscopic amplitudes. After recoupling ϕ_A in this manner one can integrate over all coordinates of ϕ_B and obtain

$$T_{JM}^{\mathrm{DWBA}} = N \int d\vec{r}_{aA} \int d\vec{r}_{bB} \chi^{(-)*}(\vec{r}_{bB}) \times \sum_{\gamma} \kappa_\gamma \langle \phi_{a+x} | V_{ax} | \phi_a \phi_{x\gamma}^{JM} \rangle \chi_a^{(+)}(\vec{r}_{aA}) . \tag{5}$$

This transition amplitude contains a sum over (much simpler) matrix elements which only involve the incident and transferred nucleons. The constants κ_γ account for the two-nucleon angular momentum coupling coefficients and the overlaps of the residual nucleus ϕ_B with the core wave functions ϕ_{A-2}^{γ}. The summation over γ can be performed after the integrations so that the terms of Eq. (5) are formally identical to those for single-nucleon transfer.

If the transferred cluster $\phi_{x\gamma}$ is a two-particle system, its wave function is taken to be a coupled pair of single-nucleon Woods-Saxon wave functions.[4,6] This two-particle wave function can be expressed, via harmonic-oscillator expansion techniques, in terms of center-of-mass and relative coordinates.[4,7]

For simplicity the wave function ϕ_b and the potential V_{ax} are assumed to have Gaussian shapes in their respective variables. The particle a is assumed to have no internal structure. These simplifying assumptions are frequently made in two-nucleon transfer calculations[9,10,11] and have the great advantage of permitting us to retain a more convenient form for the reaction amplitude than accurate wave functions and potentials would. A Gaussian wave function may not be a bad approximation for α particles although it is probably less desirable for tritons and ^{3}He. Neglect of the internal structure of projectile a is clearly justified if ϕ_a is a proton or neutron, but the neglect of the internal structure of the deuteron for the (d, α) reaction is an oversimplification, probably at least

as serious as the Gaussian approximations above. It is conceivable that these approximations may be improved by using size parameters for the "composite" projectiles and by interaction potentials that differ somewhat from those obtained from a best fit to free triton or α wave functions or to a realistic nucleon-nucleon interaction potential.

The six-dimensional integral in Eq. (5) can be reformulated by the introduction of a suitably chosen space vector $\vec{\rho}$ which can be integrated out after a Taylor expansion of the functions $\chi(\vec{r},\vec{\rho})$. The necessary mathematical manipulations and approximations have been discussed by Bencze and Zimanyi,[9] and others.[10,11] The two-nucleon form factor F is rewritten in center-of-mass and relative coordinates. After integration over the relative form-factor coordinates, Eq. (5) finally is reduced to

$$T = N' \int d\vec{r}\, \chi_b^{(-)*}(\vec{r}) \exp\left(\frac{C_a \nabla_a^2 + C_b \nabla_b^2 + C_x \nabla_x^2}{4\epsilon^2}\right) \times F_x(\vec{r})\, \chi_a^{(+)}\left(\frac{m_B}{m_A}\vec{r}\right), \tag{6}$$

where $\vec{r} \equiv \vec{r}_{bB}$; the C's are constants depending on the masses,[10] ϵ is a constant depending on the range of the potential V_{ax} and the size parameter of particle b. The subscripts i on the ∇_i^2 operators indicate the wave function on which they operate and N' is a new numerical constant, F_x is a function of the center-of-mass coordinate of the two particles transferred which is similar to, but not identical with the zero-range microscopic form factor F_x'. In fact the finite-range correction in F_x can be reproduced by a change in the size parameter η for the heavier projectile (except for normalization).

Comparing Eqs. (5) and (6) we see that the six-dimensional integral has been reduced to a three-dimensional one. However, this three-dimensional integral contains an infinite number of derivatives so that its exact evaluation may be no easier than evaluating Eq. (1) directly. The virtue of representation (6) is that it lends itself easily to approximations. The most natural of these is the WKB approximation where the ∇_i^2 operators are replaced, by means of the Schrödinger equation,

$$\nabla_i^2 = \frac{2m}{\hbar^2}(V_i - E_i). \tag{7}$$

This results in a finite-range correction factor of the form

$$W = \exp\left(\frac{C_a \nabla_a^2 + C_b \nabla_b^2 + C_x \nabla_x^2}{4\epsilon^2}\right) \approx \exp\left(\frac{2}{\hbar^2}\frac{m_x m_a}{m_b}\frac{1}{4\epsilon^2}(V_a + V_x - V_b + S_x)\right), \tag{8a}$$

where the C coefficients have been evaluated explicitly, m_x, m_a, m_b denote the masses of the respective particles, and $S_x = E_b - E_a - E_x$. It should be noted here that had one chosen a Hulthén form for the potential $V_{ax}(r_{ax})$, the equation corresponding to Eq. (8a) would be[15]

$$W' \approx \left(1 - \frac{2}{\hbar^2}\frac{m_x m_a}{m_b} R^2 (V_a + V_x - V_b + S_x)\right)^{-1}, \tag{8b}$$

where R^{-2} corresponds to $4\epsilon^2$ in Eq. (8a). These two functions are very similar as can be seen by a series expansion of W and W', provided the series converges rapidly. However, Eq. (8b), a frequently used correction,[1] has the property of becoming singular for certain values of the potentials in conjunction with a large enough value of R.

The evaluation of Eq. (8a) or (8b) is trivial for single-nucleon transfers, but while ∇_x^2 is well defined it is not immediately obvious how V_x is to be chosen in the two-nucleon transfer case. Ideally, V_x should be the potential in a Schrödinger equation which has F_x as an eigenfunction, i.e.,

$$\left(\frac{-\hbar^2}{2m_x}\nabla_x^2 + V_x\right) F_x = E_x F_x. \tag{9}$$

It has been reported in a number of previous publications[2,4,14] that F_x, the center-of-mass motion of the microscopic dinucleon, usually has a very good

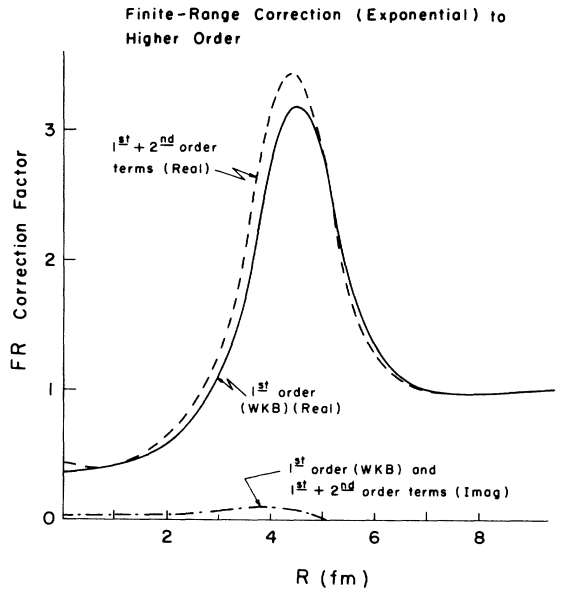

FIG. 1. Radial finite-range correction functions for $^{52}Cr(d,\alpha)^{50}V$ (Ref. 16) with parameters of Table I. Solid line: first-order (WKB) correction. Dashed line: first- plus second-order corrections. Dash-dotted line: imaginary part of both the first-order and first- plus second-order finite-range correction. The term S_x which gives rise to a multiplicative constant has not been included.

overlap (90% to 99.9%) with the wave function F_x' of a mass-two particle of the same quantum numbers, generated by a real potential $V_x(R)$ which is almost identical with the real part of the deuteron optical-model potential. I.e., usually the function $F_x(R)$ can be represented with good accuracy as the solution of Eq. (9). $F_x(R) = \alpha F_x'(R)$ with $\alpha < 1$. Conversely, V_x can be deduced from F_x. This point is important for the existence of a close similarity of the finite-range correction for one- and two-nucleon transfers. The equivalence in *shape* of cluster and microscopic form factor no longer exists if the latter shows severe cancellations from coherent contributions or comparable contributions from different harmonic-oscillator shells. V_x has been deduced from F_x, for instance by the microscopic form-factor search code MIFF.[14, 16] If such a code is not available it is possible to approximate V_x by using the sum of the two single-nucleon bound-state potentials used in calculating ϕ_x.

The solid line in Fig. 1 shows the real part of Eq. (8a) for a previous (d, α) analysis.[16] The pronounced effect of this correction on the predicted angular distributions can be seen from Fig. 2 where zero-range and "finite-range" calculations are compared. While the WKB correction here is large enough to raise doubts regarding its usefulness, calculations for other targets with the same finite range (FR) parameter [$R = 0.4$ in Eq. (8b)] had given successful, but much smaller corrections ($0.7 \leq W' \leq 1.5$) and were viewed as a useful empirical way to take approximate account of finite-range effects.[14] With correction factors large compared to unity it is necessary to recheck the validity of the WKB approximation, e.g., by estimating higher terms for Eq. (8a). We chose this particularly unfavorable case (V^{50}) for a numerical estimate of second-order terms.

In order to obtain some insight into the nature of the higher-order corrections one can use the Baker-Housdorff theorem, for operators A, B:

$$e^A e^B = \exp(A + B + \tfrac{1}{2}[A, B] + \cdots) . \tag{10}$$

Setting $A = C\nabla^2$ and $B = C(2m/\hbar^2)H$, where $H = -(\hbar^2/2m)\nabla^2 + V$, yields

$$e^{C\nabla^2} \exp\left(C\frac{2m}{\hbar^2}H\right) = \exp\left\{C\left(\nabla^2 + \frac{2m}{\hbar^2}H\right) + \tfrac{1}{2}C^2\left[\nabla^2, \frac{2m}{\hbar^2}H\right] + \cdots\right\} . \tag{11}$$

Making use of Eq. (7) and evaluating the commutator, we get

$$e^{C\nabla^2} \exp\left(C\frac{2m}{\hbar^2}H\right) = \exp\left\{C\frac{2m}{\hbar^2}\left[V + \tfrac{1}{2}C(\nabla^2 V + 2\vec{\nabla}V\cdot\vec{\nabla}) + \cdots\right]\right\} .$$

Hence

$$e^{C\nabla^2} = \exp\left\{C\frac{2m}{\hbar^2}\left[V + \tfrac{1}{2}C(\nabla^2 V + 2\vec{\nabla}V\cdot\vec{\nabla}) + \cdots\right] - C\frac{2m}{\hbar^2}H\right\} . \tag{12}$$

Now since χ is an eigenfunction of H with energy E,

$$\exp\left(-C\frac{2m}{\hbar^2}H\right)\chi = \exp\left(-C\frac{2m}{\hbar^2}E\right)\chi \tag{13}$$

so that

$$e^{C\nabla^2}\chi = \exp\left\{C\frac{2m}{\hbar^2}\left[V - E + \tfrac{1}{2}C(\nabla^2 V + 2\vec{\nabla}V\cdot\vec{\nabla}) + \cdots\right]\right\}\chi . \tag{14}$$

When this is done for all appropriate terms in Eq. (6), we get

$$\exp\left(\frac{C_a\nabla_a^2 + C_b\nabla_b^2 + C_x\nabla_x^2}{4\epsilon^2}\right) = \exp\left\{\frac{2\tau}{\hbar^2}\left[S_x + (V_a + V_x - V_b) + \tfrac{1}{2}\tau\left(\frac{\nabla^2 V_a}{m_a} + \frac{\nabla^2 V_b}{m_b} + \frac{\nabla^2 V_x}{m_x}\right)\right.\right.$$
$$\left.\left. + \tau\left(\frac{\vec{\nabla}V_a\cdot\vec{\nabla}_a}{m_a} + \frac{\vec{\nabla}V_b\cdot\vec{\nabla}_b}{m_b} + \frac{\vec{\nabla}V_x\cdot\vec{\nabla}_x}{m_x}\right) + \cdots\right]\right\} , \tag{15}$$

where

$$\tau = \frac{m_x m_a}{m_b} \frac{1}{4\epsilon^2}$$

and m_a etc. are the reduced masses. The advantages of this second-order approximation are that it treats scattering and bound-state potentials equally and that the higher-order corrections which partially cancel are retained in the exponent. Approximations which are made in the expanded form of the exponential,[10] for example,

$$e^{\nabla^2}\chi = (1 + \nabla^2 + \tfrac{1}{2}\nabla^4 + \cdots)\chi \qquad (16)$$

risk the danger of not including enough or not even the largest terms. Moreover, it is seen in Eq. (15) that, since the expansion parameter τ is typically less than one, the convergence of the series in the exponent seems assured. For the case of (d, α), $\tau = 0.4$ fm^2 amu.[10, 14, 16] The terms involving $\nabla^2 V$ in Eq. (15) can be evaluated analytically. An estimate of the $\vec{\nabla} V \cdot \vec{\nabla}$ terms in first order shows that – as for the WKB term – the corrections peak at the surface, but they show cancellations and configuration dependence and are hard to evaluate in general. The $\vec{\nabla} V \cdot \vec{\nabla}$ corrections are of the same order of magnitude as the ∇^2 terms, but are not included in Fig. 1. With all V_i taken as Woods-Saxon or derivative Woods-Saxon potentials the dashed line in Fig. 1 shows a plot of the real part of Eq. (15) including all ∇^2 terms in the exponent. We see that the change from the first-order result is not very large and could perhaps be simulated by readjustment of the parameter ϵ^2.

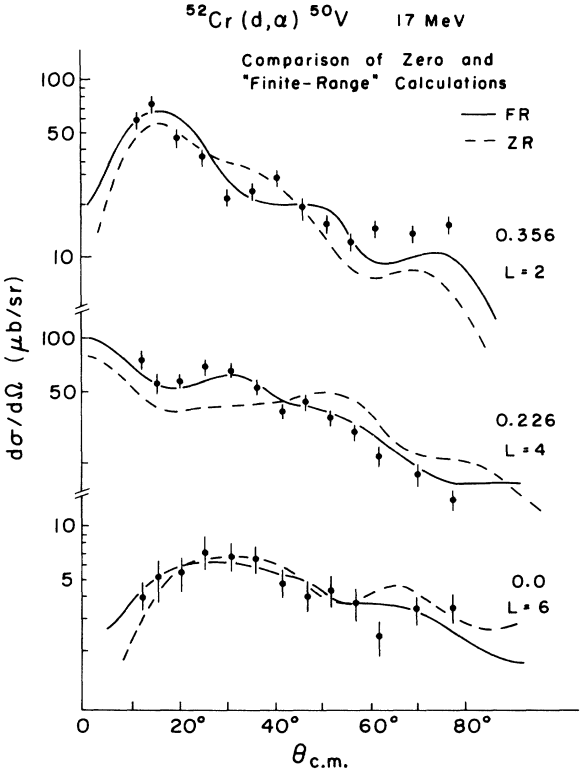

FIG. 2. Comparison of ^{52}Cr(d, α) data (Ref. 16) with calculations using parameters of Table I. The dashed line is a zero-range (cluster) calculation. The solid line employs the first-order FR correction of Fig. 1.

We have neglected, in the above evaluation the finite-range effects which modify the function F_x of Eq. (6). It has been reported by Park *et al.*[17] that this correction is highly configuration-dependent. Its principal effect seems to be a further reduction of the interior contributions while not significantly altering the reaction cross sections. It thus appears that the essential part of the finite-range correction is found in the first-order WKB approximation term, and is given by Eq. (8a) or (8b). Although the additional corrections of Eq. (15) or Ref. 17 may be helpful, one probably is already close to the point where the errors introduced by the various initial approximations, needed to reduce the transition amplitude to manageable form, are as significant as the corrections made.

Inspection of Eq. (8a) immediately suggests a relation between the optical-model potentials V_a, V_b and the "bound-state" potential V_x which would minimize finite-range corrections, namely,

$$V_a(r) + V_x(r) - V_b(r) \approx 0 . \qquad (17)$$

This equation happens to be identical with condition (3c) which assured the usefulness of Eq. (1). In practice this equation does not hold simultaneously for its real and imaginary parts; however, Fig. 1 as well as numerical results for (d, α) transitions show that the imaginary terms of Eq. (8a) have a rather minor effect on the transition amplitude, even if $|W|$ is large. We therefore introduce a less stringent condition which is easier to satisfy and which we call the "well matching condition":

$$\mathrm{Re}[V_a(r) + V_x(r) - V_b(r)] \approx 0 . \qquad (18)$$

If Eq. (18) is obeyed for all values of r the r dependence in Fig. 1 essentially disappears and W, in first order, reduces to a multiplicative constant.

Just as Eq. (3b), Eq. (18) selects the nV_{nucleon} parameter family among the discrete optical-model ambiguities for complex projectiles. The new emphasis in this study concerns the radial dependence of the potentials V. It can be seen from Table I that for the initial ^{52}Cr$(d, \alpha)^{50}$V analysis[16] Eq. (4) was approximately obeyed, nevertheless, the corrections shown in Fig. 1 became very large

TABLE I. Optical-model parameters found in the literature. All energies are given in MeV, all lengths in fm.

Target	Energy	V	r_0	a	$4W_D$	r_i	a_i	χ^2	Reference
				Deuteron parameters					
^{48}Ti	21.5	106.0	0.997	0.916	73.3	1.40	0.643	9.3	21
^{52}Cr	17.0	91.76	1.147	0.705	48.11	1.33	0.771	1.0 [a]	31
^{58}Ni	15.0	98.6	1.105	0.709	65.96	1.170	0.831	1.1	21
Zn	11.8	103.3	1.083	0.835	76.7	1.354	0.715	0.46	21
Zr	15.0	98.1	1.127	0.848	59.48	1.394	0.655	0.71	21
Pb	15.0	113.4	0.930	1.175	51.2	1.50	0.623	1.8	21
				α parameters					
Target	Energy	V	r_0	a	W_{vol}	r_i	a_i	χ^2	Reference
Ti [b]	19.47	183.7	1.4	0.564	26.6	1.4	0.564	c	23
Cu	24.7	157.8	1.443	0.544	21.9	1.443	0.544	1.09	22
^{89}Y	20.0	177.2	1.443	0.514	19.84	1.459	0.445	0.66	17
^{209}Bi	24.7	177.3	1.342	0.569	15.6	1.342	0.569	0.144	22

[a] Fit included a spin-orbit term.
[b] Average fits for ^{48}Ti, ^{50}Ti, and ^{58}Ni.
[c] Not given.

at the surface, because the α well used had a significantly larger radius than the deuteron and cluster wells. Not until the elastic α data were reanalyzed with a six-parameter well for which the real parameters r_0 and a_0 were held to the deuteron well geometry (Table II) was it possible to make finite-range corrections small. At this point zero-range (d, α) calculations also became useful and gave good agreement with experiment.

Recently, Bayman reported exact finite-range calculations for ^{40,48}Ca$(t,p)^{42,50}$Ca, which gave a good representation of the experimental data.[18,23] He also found that zero-range curves for the same data not only agreed with the shapes of the finite-range angular distributions for various L values, but also with their relative cross sections. This unexpected success led to speculation that finite-range effects in (t,p) may be small for angular-distribution shapes in other cases as well.[23]

In connection with our discussion above it is perhaps interesting to point out two aspects of Ref. 18: The successful finite-range and zero-range fits happened to employ identical geometries ($r_0 = 1.25$, $a = 0.65$) for *all* potentials in the calculation, a situation where according to our Eqs. (8) and (18) zero-range curves should be good. However, also shown in Ref. 18 is a prior set of finite-range calculations which differed from the success-

TABLE II. Optical-model parameters obtained by refitting data of Table I for angles below 120° with a fixed well geometry. $\chi^2 = 1$ is equivalent to an average 10% deviation per point.

Nuclide	Energy	V	r_0	a	$4W_D$	r_i	a_i	χ^2
				Deuteron parameters				
^{48}Ti	21.5	78.6	1.20	0.75	65.0	1.347	0.734	3.5
^{52}Cr	17.0	82.3	1.20	0.75	71.4	1.37	0.664	3.7
^{58}Ni	15.0	84.0	1.20	0.75	84.1	1.260	0.679	0.57
Zn	11.8	89.6	1.20	0.75	88.8	1.300	0.700	1.0
Zr	15.0	90.0	1.20	0.75	66.3	1.300	0.700	0.29
Pb	15.0	89.9	1.20	0.75	51.7	1.405	0.912	0.42
				α parameters				
Nuclide	Energy	V	r_0	a	W_{vol}	r_i	a_i	χ^2
^{48}Ti	19.47	180.7	1.20	0.75	13.0	1.760	0.564	1.9
^{50}Ti	19.47	179.8	1.20	0.75	13.5	1.747	0.572	1.7
^{58}Ni	19.47	188.1	1.20	0.75	14.5	1.735	0.604	0.63
Cu	24.7	209.4	1.20	0.75	14.2	1.727	0.532	2.8
^{89}Y	20.0	181.3	1.20	0.75	15.0	1.700	0.600	0.71
^{209}Bi	24.7	194.4	1.20	0.75	21.9	1.400	0.600	1.0

ful one only in the choice of the proton potential geometry ($r_0 = 1.17$, $a = 0.75$). This other finite-range calculation not only differs from a corresponding zero-range curve (as we would have expected), but it also disagrees significantly with experiment (Ref. 18, p. 11). This disagreement may signify that seemingly minor details of optical parameters, i.e., their continuous ambiguities, can have unexpectedly large effects for transfer calculations which cannot be compensated for even by an exact finite-range treatment.

Apparently, the importance of well matching far exceeds the scope of finite-range effects. The arguments in Sec. IIA and in Ref. 12 let us understand this conclusion in principle. The numerical studies discussed in Sec. IIB have shown rather persuasively that a refined evaluation of Eq. (1) for two-nucleon transfers[10, 11, 18] is meaningful only to the degree that the conditions (3) are met, be it by a fortunate choice of optical-model parameters or by a conscious restriction of their discrete and continuous ambiguities [Eq. (18)].

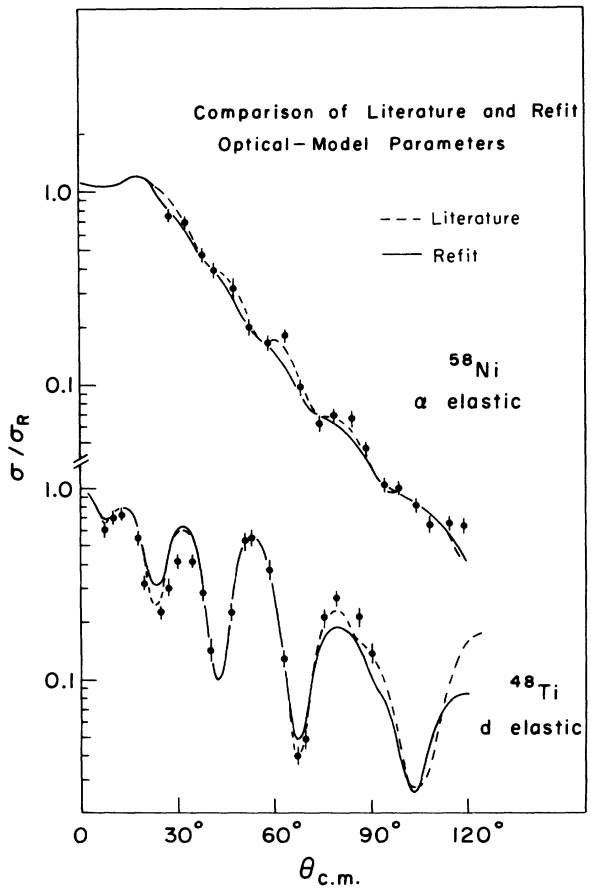

FIG. 3. Comparison of elastic deuteron and α-scattering data with the original best fit (dashed line) and the fits provided by the restricted parameters of Table II.

III. (d,α) TRANSFER CALCULATIONS

A. Continuous Optical Parameter Ambiguities

The complex scattering potentials used for the generation of distorted waves are characterized by four to nine parameters which are generally chosen to reproduce elastic scattering cross sections for the projectiles and energies in question. Among many possible variations of such potentials one tends to prefer those whose parameters are simply related to observable "macroscopic" properties of the scattering nucleus. The Woods-Saxon shape generally chosen for the real wells is given by

$$V(r) = -V_0 f(r, r_0, a) = -V_0\left[1 + \exp\left(\frac{r - r_0 A^{1/3}}{a}\right)\right]^{-1} \quad (19)$$

and bears a close resemblance to the shape of the measured charge and (presumably) mass distribution of nuclei. For proton and neutron scattering the parameter values $r_0 \approx 1.2$ fm and $a \approx 0.7$ fm can be kept constant for a wide range of target nuclei and bombarding energies,[20] while the real potential depth V_0 is relatively stable (50 ± 5 MeV for nucleons). Its value can be understood from nuclear matter considerations, and its variation with bombarding energy and target isospin is well predicted by empirical rules.[20] Imaginary terms used in the optical potential are of the form

$$W(r) = i\left(-W' f(r, r_0, a) + 4\, W_D a' \frac{d}{dr} f(r, r_0', a')\right) \quad (20)$$

and account for the partial absorption of incident waves into reaction channels. The somewhat larger empirical values for r_0' and a' can be understood qualitatively from the Pauli principle. Finally, known Coulomb terms are added, and spin-orbit terms for nucleon-nucleus scattering are also predictable with little ambiguity. A versatile nucleon scattering potential is given by

$$V = -V_0 f(r, r_0, a) + V_{so}\, \lambda_\pi^2 \vec{l} \cdot \vec{\sigma} \frac{1}{r}\frac{d}{dr}[f(r, r_{so}, a_{so})] + V_{Coul}(r) + i\left(-W' f(r, r_0', a') + 4 W_D a' \frac{d}{dr} f(r, r_0', a')\right). \quad (21)$$

Potential (21) was used by Becchetti and Greenlees in their global fits to proton scattering[20] and was very successful for $A > 40$ and $10 \leq E \leq 50$ MeV. It must be noted, however, that even the restriction to 8 parameters and a fit of a large amount of data did not eliminate all parameter ambiguities. It was found[20] that similarly successful potential

sets can be constructed as r_0 is varied from $r_0 = 1.12$ to $r_0 = 1.22$.

Complex projectiles (deuterons, tritons, α particles, etc.) have been studied less exhaustively and although many excellent optical-model fits exist[21, 22, 19] no unique prescription for the best well geometries or the values V_0, V_s, W', and W_D has emerged. Since these projectiles are more strongly absorbed, optical parameter ambiguities are much larger. Published best-fit radius parameters have been as small as $r_0 = 0.9$ fm for deuteron scattering and as large as $r_0 = 1.45$ fm for α scattering. There is little prospect that elastic scattering analyses alone will eliminate the frequently encountered continuous parameter ambiguities, which are roughly characterized by the relation

$$V_0 r^n \approx \text{const}. \tag{22}$$

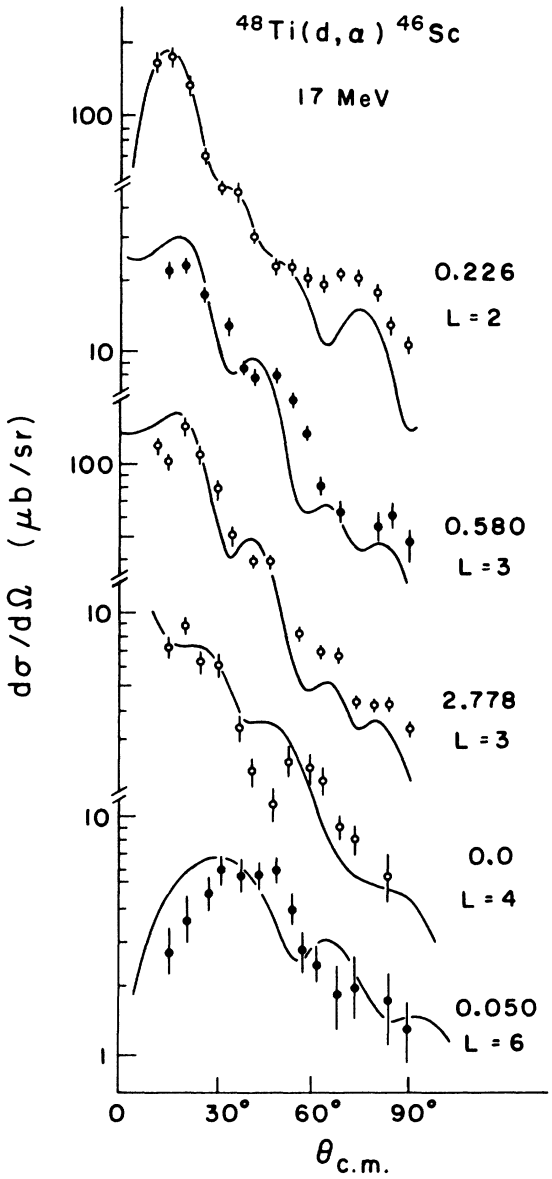

FIG. 4. Zero-range DWBA predictions as described in the text for 17-MeV ^{48}Ti(d,α) data of Ref. 24.

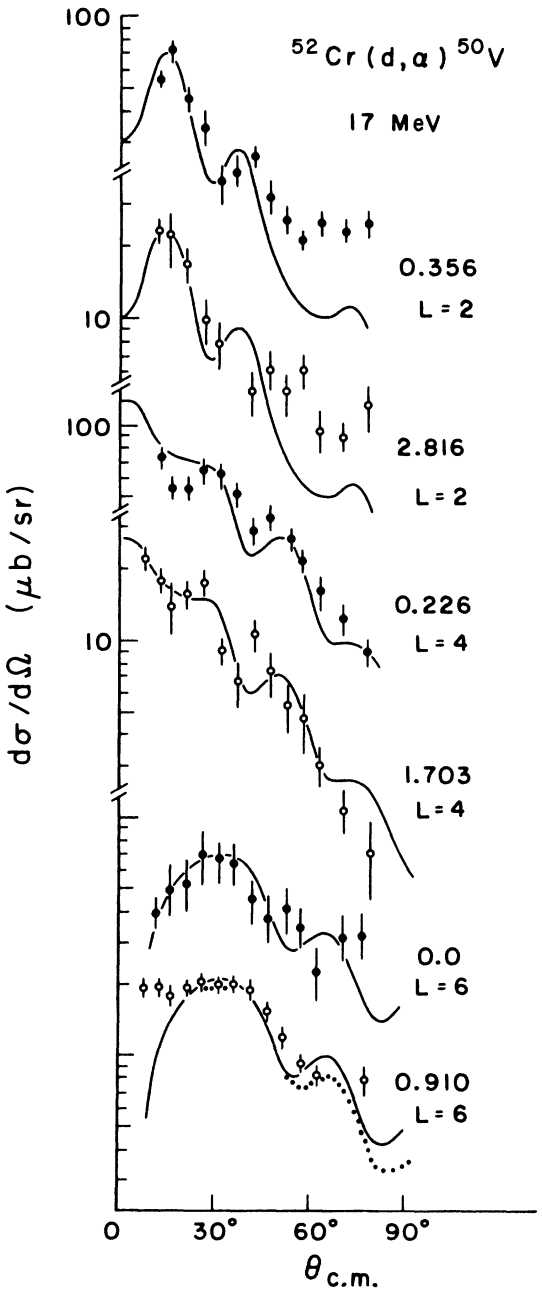

FIG. 5. Zero-range DWBA predictions as described in the text for the 17-MeV ^{52}Cr(d,α) data of Ref. 16. The dotted line is the microscopic calculation used for Table III.

On the other hand, reaction calculations in the DWBA formalism are often extremely sensitive to the scattering potentials chosen for the complex projectiles. Also, the well-matching condition [Eq. (18)] discussed in Sec. II drastically limits permissible ambiguities in DWBA calculations.

Parameters in the literature seem to exclude good well matching in (^{3}He, α) and (d, α) calculations unless parameter values are chosen which give inferior elastic fits.[12] We found, however, that the large radii for published α scattering potentials are largely a result of the tendency to analyze α scattering with a four-parameter model in order to get unique fits. Our reanalysis of published data shows that r_0 is poorly determined by

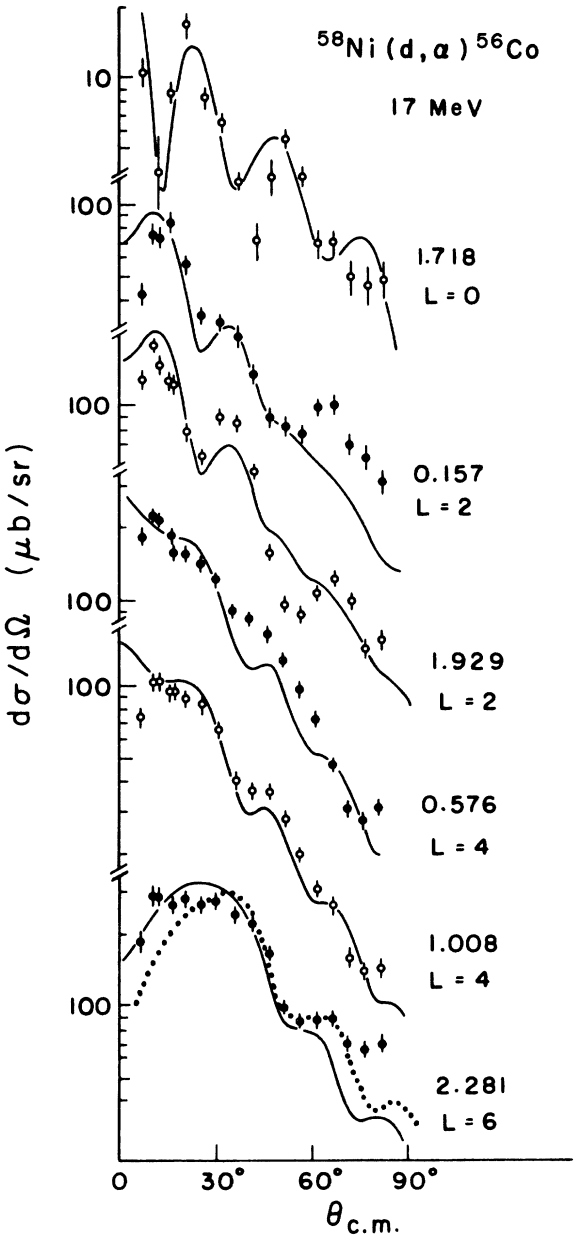

FIG. 6. Zero-range DWBA predictions for the 17-MeV ^{58}Ni(d,α) data of Ref. 25. The dotted line is the microscopic calculation used for Table III.

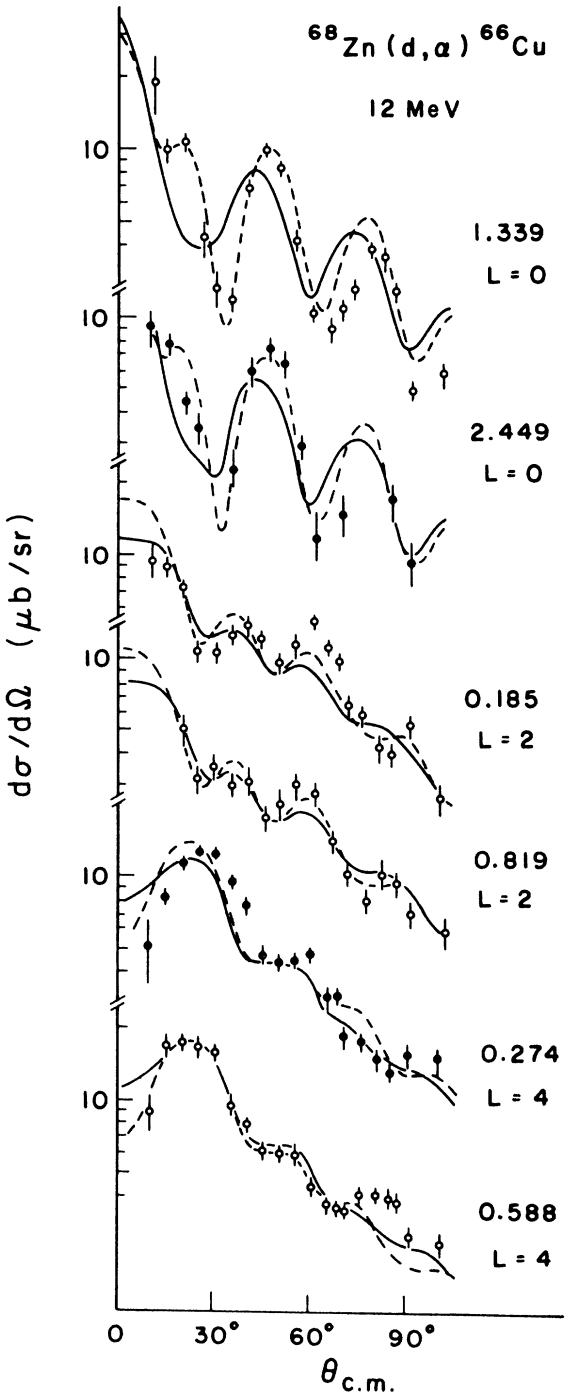

FIG. 7. Zero-range DWBA predictions for the 12-MeV ^{66}Zn(d,α) data of Ref. 14. The dashed lines are calculations including the WKB finite-range correction [Eq. (8a)].

elastic α scattering. It is possible to set $r_0 = 1.2$, $a = 0.75$ for the real α well and get fits comparable to those in the literature if a six-parameter well is used. The imaginary parameters r_0' remain very large, but this does not violate our intuition that *real* potential wells should be closely related to the nuclear charge and mass distribution. Since absorbed α particles carry more angular momentum than lighter projectiles of like energy it would seem reasonable to find them strongly absorbed at larger radii.

Our choice of $r_0 = 1.2$ and $a = 0.75$ as a uniform well geometry is not arbitrary. It represents essentially an adoption of the largest acceptable proton scattering well[20] as a standard. This geometry allows very good fits for proton, triton, and ^{3}He scattering. $r_0 = 1.2$ is the lower limit for good α scattering fits and the upper limit for good deuteron scattering fits for lighter targets in the energy region $10 < E < 50$ MeV. A variation to $r_0 = 1.15$ or $r = 1.25$ would lead to some difficulties with α and deuteron scattering, respectively. This geometry is also quite close to the values $r_0 = 1.25$, $a = 0.65$ commonly used for the bound-state well used in stripping calculations and could be substituted for it.

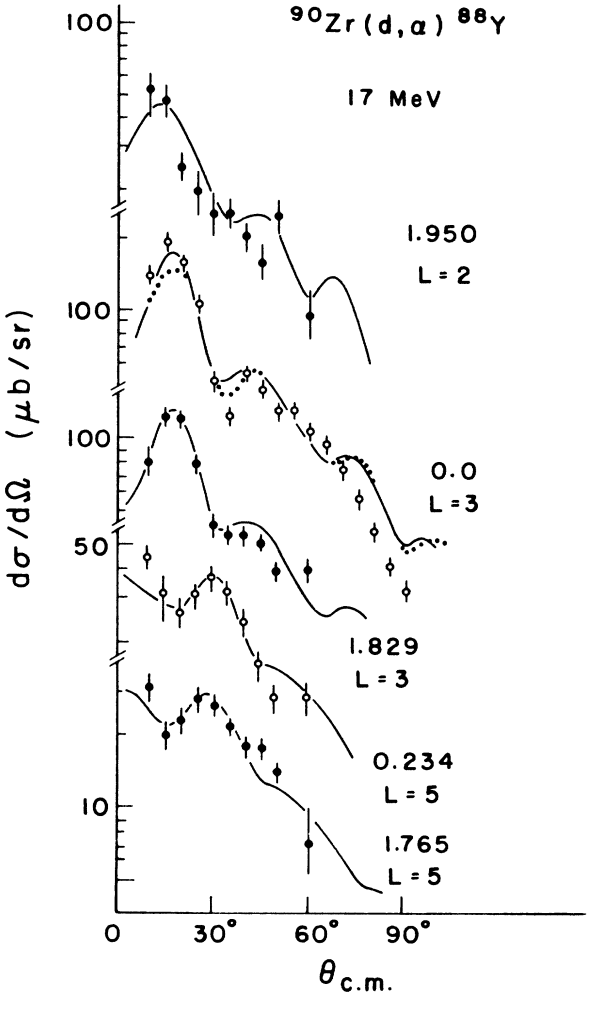

FIG. 8. Zero-range DWBA predictions for the 17-MeV ^{90}Zr(d,α) data of Ref. 26. The dotted line is the microscopic calculation used for Table III. The L assignments for the levels at 1.765 and 1.829 were unknown previously.

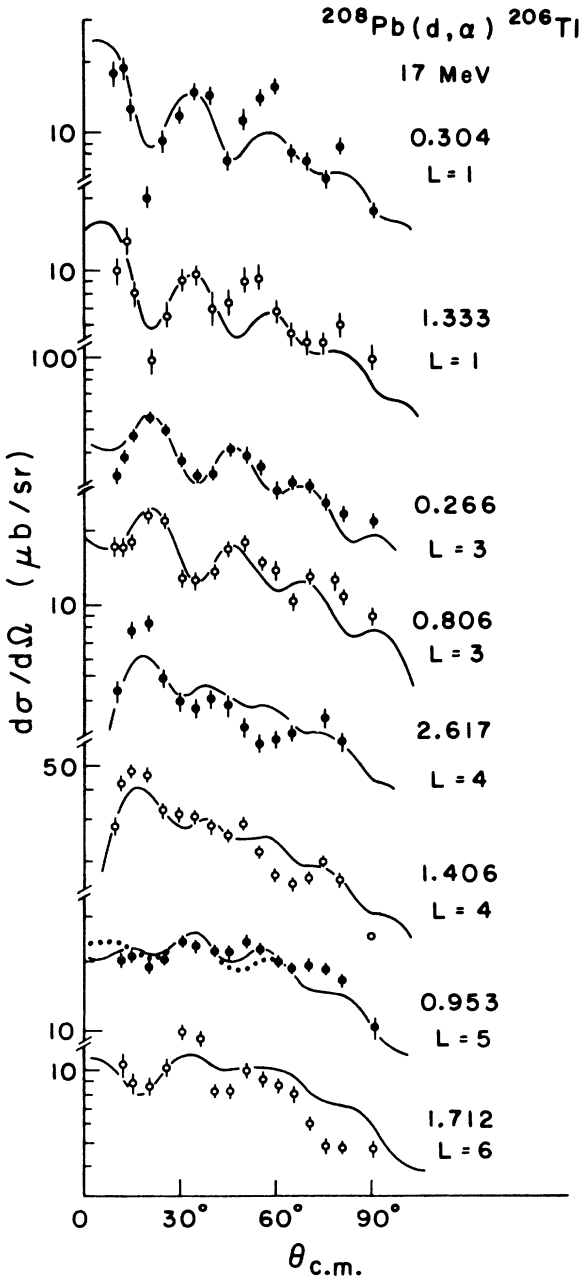

FIG. 9. Zero-range DWBA predictions for the 17-MeV ^{208}Pb(d,α) data of Ref. 2. The dotted curve is the microscopic calculation used for Table III.

B. Analysis of (d, α) Experiments

The (d, α) data analysed here have been chosen in order to cover a broad range of near-spherical nuclei. Most of the experiments have been published and the pertinent references (Refs. 2, 14, 16, 24, 25, and 26) are given in the figure caption. The choice of angular distributions to be studied was made primarily on the basis of a previously well-established L transfer. Hence, the intent of these new calculations is not to determine L values, but to check the reliability of a method for doing this by comparison with known results. Although, the original fits to the data appearing in these references were typically quite good, they were usually obtained by selecting by trial and error the "best" combination of various published optical parameters. Only in the case of the ^{58}Ni(d, α)-^{56}Co experiment[25] was the present well-matching method used.

Optical parameters for elastic deuteron and α scattering of the type used for (d, α) calculations are shown in Table I. The isotope on which the elastic scattering experiment was done and the bombarding energy are also indicated in Table I. The original optical-model fits were very good at forward angles, but deteriorated somewhat beyond 120°. We therefore felt justified in using these published parameters in place of the raw data points to regenerate the "experimental" elastic angular distributions up to 120° for the appropriate nucleus at the indicated energy. "Data points" were read in 4° intervals, beginning at 4° and ending at 120°, and were assigned an error of 10%. This information was then taken as input for the optical parameter search routine HUNTER.[27] In the subsequent searches with six-parameter wells, r_0 and a_0 were fixed at the values

$$r_0 = 1.2 \text{ fm}, \qquad a_0 = 0.75 \text{ fm}. \tag{23}$$

Care was taken to insure that the final depths V_0 fell within the appropriate discrete family. The success achieved in refitting the data can be judged by Fig. 3.

For angles below 120° optical-model fits for d and α scattering found in the literature tended to agree with the data to within about 10%. Table II shows results of our reanalysis with the real wells held constant. The χ^2 values give a measure of the quality of the new fits ($\chi^2 = 1$ means a deviation of 10% from the old curve). We note that the new values for V_0 tend to stay within ±6% of the expected value $n \times V_{\text{nucleon}}$. Although the values r_0' and a' were not restricted, they too, fluctuate little (by less than 10% over the range $48 \leq A \leq 209$). This uniformity is most desirable, particularly for the α parameters which will have to be used for neighboring (unstable) residual nuclei for which a direct measurement is impossible. In Table II the small χ^2 values for the parameters shown indicate that the restrictions (23) are not incompatible with obtaining good fits to both deuteron and α elastic scattering. It should be noted that when the angular distributions are relatively structureless the new parameters are not quite unique but are somewhat biased by the starting parameters given to the search code.

Since it is generally not possible to obtain optical parameters for the nucleus of interest at the energy of interest, one usually has to make do with elastic data on some neighboring nucleus at a neighboring energy. This can be a hidden source of trouble for reaction analyses especially when elastic parameters show no systematic trends with atomic number or energy so that extrapolation to the nucleus and energy of interest becomes questionable. With this qualification in mind, the optical parameters of Table II together with the bound deuteron well geometry of Eq. (23) were used to predict angular distributions corresponding to various L transfers for six different (d, α) experiments. The zero-range DWBA results generated by the code DWUCK[28] together with the data are shown in Figs. 4–9. In almost every case the fit to the data is as good as the original best-fit sets and in several cases it is better. The first-order finite-range correction was small so that the curves generated with finite range were not much different from the zero-range curves.

The curves generated with the microscopic form-factor calculations of DWUCK II[6, 29] are fairly close to the curves of Figs. 4–9 except for normalization so that these shapes are reasonably stable with respect to small alterations of the form factor. All of the curves of Figs. 4–9 contain the standard nonlocality corrections for the ingoing and outgoing particles with the parameters[30]

$$\beta_d = 0.54, \qquad \beta_\alpha = 0.2. \tag{24}$$

However, the shapes of the curves are insensitive to this correction. The single-nucleon binding energies were taken as $\frac{1}{2}$(the deuteron separation energy + 2.225 MeV).

There are situations where two different experimental shapes appear in the data, corresponding to the same L transfer, which cannot be resolved by admixtures of other L values, e.g., the two $L = 2$ angular distributions in the ^{58}Ni$(d, \alpha)^{56}$Co data. This is possibly due to a J-dependent effect or to a special configuration effect. In either case,

experience has shown that in the present DWBA formulation we cannot reproduce this variation. Hence, the best we can hope for, using the technique described in this article, is to predict an intermediate shape.

The problem of normalizing the calculations is, of course, highly dependent on how the form factor is treated and also on the optical parameters employed. Generally, it is desirable to have good theoretical wave functions for a variety of states and to extract some average normalization constant which best fits the cross-section magnitudes.[2] However, this is not always feasible so we have selected states which are expected to have relatively pure configurations, for example the 7^+ state in f-p shell nuclei. The states and their expected configuration are given in Table III. The normalization constant $N(d, \alpha)$ is found from the equation[2]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{expt}} = N(d, \alpha)\left(\frac{2S+1}{2}\,\frac{\sigma_{\text{DWUCK}}}{2J+1}\right), \tag{25}$$

where the spectroscopic amplitude for pickup from a state of pure configurations was taken as[7, 25]

$$\beta_{\gamma LSJ}(J, 0) = \left(\frac{n_\pi n_\nu (2J+1)}{(2j_\pi+1)(2j_\nu+1)}\right)^{1/2} \begin{bmatrix} l_\pi & \frac{1}{2} & j_\pi \\ l_\nu & \frac{1}{2} & j_\nu \\ L & S & J \end{bmatrix}. \tag{26}$$

π and ν refer to protons and neutrons, and n_π, and n_ν to the number of each in their respective orbitals which are characterized by (l_π, j_π) or (l_ν, j_ν). The square bracket is the LS-JJ transformation coefficient which is contained in the DWUCK II computer code. Since we are dealing with a subset of states with pure configurations, the square of this amplitude factors out of the DWUCK cross section and can be treated as a spectroscopic factor, as in single-nucleon transfers. The normalization constants $N(d, \alpha)$ extracted are given in Table III. The values of $N(d, \alpha)$ are comparable to those in Ref. 25, but much larger than those of Ref. 2, which used larger well parameters, and a significant FR correction. A more elaborate treatment of the form factors may remedy the fluctuations observed; however, there seems to be a tendency for lighter targets to require larger normalization factors. We attach little significance to the absolute values of $N(d, \alpha)$.

IV. SUMMARY AND CONCLUSIONS

Two weak points of conventional DWBA calculations for two-nucleon transfers, their sensitivity to optical-model parameter ambiguities, and to the commonly used zero-range approximation have been investigated. It has been shown that in some cases finite-range corrections (in the WKB approximation) are very large and their effect on differential cross sections is pronounced. A new method to estimate higher-order corrections is presented. Its application to ^{52}Cr$(d, \alpha)^{50}$V led to the conclusion that second-order terms are significantly smaller than the WKB term and roughly of the same functional form. For ^{52}Cr(d, α) they did not produce any significant changes in the calculated angular distributions. These results justify to some degree the semiempirical procedures used earlier[14, 2, 16, 29] to take account of finite-range effects in (d, α) reactions.

However, numerous calculations for (d, α) reactions on targets ranging from ^{48}Ti to ^{208}Pb showed that the use of finite-range corrections did not eliminate the sensitivity of DWBA predictions to the choice of optical-model and bound-state well parameters. Parameter sets which did not conform to the condition $V_n \approx nV_{\text{nucleon}}$, Eq. (4) failed to reproduce more than one or two L transfers correctly; whereas use of the "correct" fami-

TABLE III. DWUCK II normalization constants derived from microscopic calculations for states believed to have nearly pure configurations, as shown.

Residual nucleus	Excitation energy (MeV)	Assumed pure configuration	J^π	$L(d, \alpha)$	Normalization constant [a] $N(d, \alpha)$ FR	ZR
^{46}Sc	0.978	$(f_{7/2})_\pi (f_{7/2})_\nu$	7^+	6	3620	4070
^{50}V	0.910	$(f_{7/2})_\pi (f_{7/2})_\nu$	7^+	6	2510	2740
^{56}Co	2.281	$(f_{7/2})_\pi (f_{7/2})_\nu$	7^+	6	2130	2510
^{88}Y [b]	0.0	$(p_{1/2})_\pi (g_{9/2})_\nu$	4^-	3	2150	2600
^{206}Tl [c]	0.953	$(d_{3/2})_\pi (f_{5/2})_\nu$	4^-	5	1250	1150

[a] Normalizations based on the α size $r = 1.4$ fm. The single-particle well geometries were $r_0 = 1.25$, $a = 0.75$.

[b] The ^{88}Y ground state is almost certainly more complicated than this first-order shell-model wave function. Admixtures would tend to reduce the normalization constant.

[c] This configuration has the amplitude 0.966. (See Ref. 2.)

lies (among the discrete well-depth ambiguities) tended to give qualitative agreement for all sets of data. This agreement was considerably improved for the case of good "well matching" [Eq. (18)]. Since traditionally, the "best-fit" real well geometries for deuteron and α scattering were quite different ($r_0 \approx 1.1$ and 1.4, respectively) new optical-model analyses were made for a number of targets. The real well geometry was fixed on the basis of physical arguments at $r_0 = 1.20$ fm and $a_0 = 0.75$ fm. (These numbers also turned out to be the best compromise for elastic scattering on lighter targets where elastic deuterons tend to favor smaller radii, and α particles larger ones.) As shown in Table II very acceptable fits could be obtained by searching on the remaining (four) parameters.

The use of this uniform and physically meaningful real well geometry and use of the "correct" well depths (i.e., well matching) made the WKB–finite-range corrections small and sometimes insignificant, but more importantly it also led to unique DWBA predictions (Figs. 4–9) similar and sometimes superior to the best fits previously obtained. The approximations implied in the DWBA approach gave theoretical reasons for our prescription for two-nucleon transfer calculations [i.e., Eqs. (4), (18), and (23)] which minimizes corrections for zero-range DWBA calculations, eliminates optical-model parameter ambiguities, and in all cases tested ($48 < A < 208$) results in unique and very acceptable predictions for (d, α) differential cross sections.

It will be of interest to apply this prescription to data taken at higher energies. For low energies, very light targets and deformed nuclei the method used above is less likely to be adequate, since other reaction mechanisms become important. However, we expect that it is equally applicable to (p, t), $(p, {}^3\mathrm{He})$, and $({}^3\mathrm{He}, n)$ reactions at sufficiently high projectile energies. Precise well matching as described here may be of importance even for such "simple" reactions as (d, p).[32]

V. ACKNOWLEDGMENTS

The authors are grateful to R. F. Gibson for much help with computer programming, and to Dr. R. M. Drisko and Dr. B. F. Bayman for valuable comments on topics related to this paper. We are indebted to Dr. N. Austern for a critical reading of the manuscript.

*Work supported by the National Science Foundation.

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