Three-Body Model for Proton-Induced Reaction on ^{12}C

George L. Strobel*
Kernforschungsanlage Juelich, West Germany
(Received 19 June 1972)

The $(p,2p)$ and the (p,d) reaction cross sections for a ^{12}C target have been calculated using a three-body theory that avoids making the Born approximation. Real separable potentials were used to describe all interactions. The general shape and magnitude of the reaction cross section agree with the experimental data. Fine structure details of the experimental cross sections are not reproduced. Modifications to distorted-wave Born-approximation calculations are suggested.

I. INTRODUCTION

The recent mathematical impetus to three-body physics was provided by the work of Faddeev[1] and Lovelace.[2] Since then separable potentials have been used in numerous studies[3] to simplify or to at least approximately include part of a two-body interaction in a more complicated three-body problem. Alt, Grassberger, and Sandhas[4] have provided a perturbation technique where an interaction is split into two parts – a separable part that is included exactly in a three-body model, and a residual part that can be included as a perturbation. If the residual second part is neglected then one has effectively utilized a separable potential. In analyzing neutron elastic scattering and breakup on a deuteron target, Aaron and Amado[5] did use separable potentials. A generally adequate fit to the data was obtained. For protons interacting with heavier nuclei the effective two-body interaction is probably not completely describable as separable or as real. With heavier nuclei treated as one body, one can use a complex optical model to describe the effective two-body interaction. However a real separable interaction does provide a starting point for calculations and this was used quite successfully by Shanley[6] in his study of deuteron scattering on the α particle.

We assume real separable interactions for a study of proton-induced reactions on a ^{12}C target. For the (p,d) reaction, the three bodies of the model are a neutron, a proton, and a ^{11}C nucleus. For the $(p,2p)$ reactions the three bodies are the two protons and the ^{11}B nucleus. In general, for a target of A nucleons, we have a core of $(A-1)$ nucleons which becomes the residual nucleus. The target nucleus is composed of the remaining nucleon coupled to the core. The structure of the core must be identical to the structure of the final nucleus, or else one has a four or more body problem to consider.

Previous calculations for these reactions have usually utilized[7,8] the distorted-wave Born approximation. The calculations can usually be successfully compared to experiment if one or more parameters of the optical potentials are permitted to vary, the rest being fixed by elastic scattering analyses. One would like to do the calculation without making the Born approximation. Off-shell effects as well are ignored in the distorted-wave calculations to date. Are such effects important? A three-body model permits inclusion of some off-shell effects. Whether a separable potential includes the correct off-shell effects or not is a question that is bypassed here. One previous calculation, by Bencze and Doleschall,[9] including possibly complex $l=0$ only separable interactions, obtained reasonable looking cross sections for a 10-MeV deuteron-stripping calculation on an unspecified heavy nucleus. Redish, Stephenson, and Lerner[10] have shown that off-shell effects are important in a plane-wave Born-approximation calculation based on a three-body model for the $(p,2p)$ reaction on a nucleus reminiscent of ^{12}C. Whether these effects remain in a distorted-wave calculation has not yet been determined. Here we do not make the Born approximation but solve the Faddeev equations with some simplifying assumptions.

Usually in applications of the Faddeev theory to three-nucleon calculations the assumption of a separable potential is the last approximation made. Here with one heavy body, the core, we further neglect exchange effects between the nucleons within the core and the nucleons outside the core. For a (d,p) reaction, such exchange effects have been shown to be small in a distorted-wave calculation.[11] Further the three-body equations are solved in an approximate manner, rather than "exactly." The approximation used appears reasonable. It is completely described and partially justified below in the next section. The data analyzed here are in the 30- to 46-MeV region. Intrinsic nucleon spin is ignored. The bound state of the target has relative orbital angular momentum one. Therefore nucleon-

core interactions are included in states of orbital angular momentum equal to 0, 1, and 2.

States of orbital angular momentum of the "free particle" with respect to the interacting pair are included up through $L=10$ when calculating the reaction amplitude. For protons scattering on ^{12}C a central interaction is usually determined[12] for the optical model from a fit to the elastic scattering data. A spin-orbit potential is found from a fit to polarization data. The spin-orbit potential is of secondary importance at these energies unless one is studying polarization phenomena. In recent $(p, 2p)$ calculations[13] using a distorted-wave approximation, no spin-orbit potential was included. The nucleon core-state description is also simplified by neglecting spin. Where states were specified by $J=l\pm\frac{1}{2}$, $\pi=(-1)^l$, they are now completely specified by l, the relative orbital angular momentum. Further the $^1S_0$ neutron-proton interaction is similar, though somewhat weaker than in the $^3S_1+{}^3D_1$ deuteron state. For these reasons, and for calculational simplicity we neglect the spin of all three bodies in our model. We also permit the two nucleons to interact only in a state with $l=0$. We can of course calculate only unpolarized cross sections. For two protons, the $^1S_0$ state has $J=l=s=0$, and antisymmetry was maintained by the spin part of the two-nucleon wave functions. We neglect interactions in all other two-nucleon states. The deuteron is treated as a $l=0$ state, with $J=0$, with binding energy = 2.226 MeV. We describe the two-nucleon force by a single attractive separable potential which neglects the hard core interpreted from two-nucleon phase-shift analyses at 200 or more MeV. The hard core is important for three-nucleon bound-state calculations, but its effects have nearly always been neglected in reaction calculations involving deuterons, nucleons, and a residual heavy nucleus. In fact, a solely attractive δ function (which is also separable) is often used, and with good success in distorted-wave calculations of (p, d) reactions. The inclusion of the hard core can be done approximately with a separable potential but it has not been done here for reasons of shortening the calculation.

II. PARAMETER SELECTION AND THEORETICAL APPROXIMATION

We follow the three-body theory of Alt, Grassberger, and Sandhas.[4] We consider only separable interactions and work in momentum space. We define momenta p_i and q_i, where $i=1$, 2, or 3, and where p_i is the momentum of particle i in the over-all center of mass, and q_i is the relative momentum in the j-k two-body subsystem (see Fig. 1). Where (ijk) are cyclic permutations of (123) we have,

$$\vec{q}_k=(m_j\vec{p}_i-m_i\vec{p}_j)/(m_i+m_j)\,. \tag{1}$$

In the center of mass we have

$$\vec{p}_1+\vec{p}_2+\vec{p}_3=0\,. \tag{2}$$

We take particle 1 to be free initially, particle 2 to be an initially bound nucleon, and 3 to be the (heavy) core. Particles 2 and 3 are bound together initially to form the target. Where needed for clarity the initial momenta of particle 1 will be labeled by $\vec{p}_0$. The mass of particle i is labeled by m_i. Reduced masses are defined as

$$\mu_i=m_jm_k/(m_j+m_k)$$

and

$$n_i=m_i(m_j+m_k)/M\,,$$

where

$$M=m_1+m_2+m_3\,.$$

The two-body parameters we need can be related to the potential

$$\langle\vec{q}|\,V\,|\vec{q}\rangle=-\lambda\sum_{lm}\Gamma^*(\vec{q})\Gamma(\vec{q}')\,, \tag{3}$$

where

$$\Gamma(\vec{q})=Y_{lm}(\hat{q})g(q)\,. \tag{4}$$

The two-body t matrix $\hat{T}$ is given by

$$\hat{T}=-|\,l\rangle t_l\langle\,l|\,, \tag{5}$$

where

$$t_l^{-1}=1/\lambda+\int_0^\infty q^2\,dq\,g^2(q)/(E+i\epsilon-q^2/2\mu)\,. \tag{6}$$

The two-body Schrödinger equation for the radial part Φ of a bound-state wave function is, for a

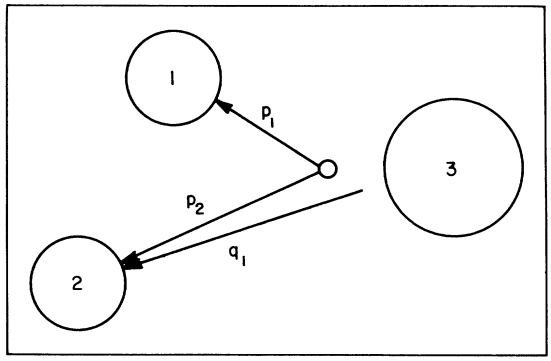

FIG. 1. Illustration of three-body kinematics. p_1 and p_2 are the momenta of particles 1 and 2 to the over-all center of mass.

separable potential:

$$(q^2/2\mu + BE)\Phi(q) = \lambda g(q)\int_0^\infty k^2\,dk\,g(k)\Phi(k) = \lambda \hat{N} g(q)\,, \tag{7}$$

where g can be normalized so that $\hat{N} = 1/\lambda$. This normalization is determined from

$$\int_0^\infty q^2\,dq\,\Phi^2(q) = 1 = \int g^2(q) q^2\,dq/(q^2/2\mu + \mathrm{BE})^2\,. \tag{8}$$

The binding energy of the bound state is indicated above by BE. The potential-strength parameter λ is given by

$$1/\lambda = \int_0^\infty q^2\,dq\,g^2(q)/(q^2/2\mu + \mathrm{BE})\,. \tag{9}$$

We take for the form factor g (which is related to the Fourier transform of the wave function)

$$g(q) = N q^l\, e^{-bq^2}\,. \tag{10}$$

The inverse Fourier transform of g is

$$\tilde{g}(r) \propto r^l\, e^{-(r/a)^2}\,, \tag{11}$$

where b and a are related by $b = a^2/4$. We set $a = 1.2A^{1/3}$ fm, where A is the mass of the core in atomic mass units, then

$$b = 0.36A^{2/3}\,.$$

This allows the form factor a nonlocality, or allows the wave function to have, roughly the expected size of a nucleus. The q^l factor provides the correct threshold behavior for the wave function. For $l=0$, and for small q

$$g(q) \propto 1/(1/b + q^2)\,.$$

For ^{12}C, $b^{-1/2}$ is about 5 MeV$^{1/2}$ which is of a similar magnitude as used by Bencze and Doleschall.[9] For l values which are fitted to a resonance rather than a bound state, λ can be determined from a principal value form of Eq. (9). Its value is dependent on the normalization N which is to be determined by Eq. (8). For resonances this is not possible to determine and we follow Lovelace,[2] and choose the normalization N as appropriate for a wave function with 2-MeV binding energy. The $l=0$ wave function should have a node in it according to the shell model, for a $2s_{1/2}$ level. The wave function we use has no nodes; its general shape is that of a $1s$ wave function rather than a $2s$ wave function. It is assumed that this feature of the $l=0$ wave function is not critical for calculating the stripping or breakup reaction from an $l=1$ bound state. The parameters used in the calculation are listed in Table I.

The three-body equations we use are now discussed. Alt, Grassberger, and Sandhas[4] define the three-body transition amplitude as

$$T^{\beta 1}_{nm} - Z^{\beta 1}_{nm} = \sum_{\gamma r} Z^{\beta\gamma}_{nr}\, t^\gamma_r\, T^{\gamma 1}_{rm}\,. \tag{12}$$

Here $\alpha\beta\gamma$ denote which particle is free, while nmr denote the quantum numbers of the interacting pair. One expresses the momentum-space matrix element of this operator equation, expands in partial waves, and couples to states of definite total J and parity. One then has a set of integral equations, as you must integrate over the intermediate momenta on the right-hand side of Eq. (12). This integral is replaced by a sum and the integral done by N-point Gaussian quadrature. Then for a given value of J and parity, one has the matrix of equations

$$T^{11}_{i0} = 0 - (Z^{12}_{ij}\, t^2_j\, T^2_{j0} + Z^{13}_{ij}\, t^3_j\, T^{31}_{j0})\,, \tag{13}$$

$$T^{21}_{i0} = Z^{21}_{i0} - (Z^{21}_{ij}\, t^1_j\, T^1_{j0} + Z^{23}_{ij}\, t^3_j\, T^{31}_{j0})\,, \tag{14}$$

$$T^{31}_{i0} = Z^{31}_{i0} - (Z^{31}_{ij}\, t^1_j\, T^1_{j0} + Z^{32}_{ij}\, t^2_j\, T^{21}_{j0})\,. \tag{15}$$

There is a sum over j on the right-hand side of Eqs. (9)–(11) that is to be understood. Now i (and j or k) is an arbitrary index label that takes on all possible values of the set $\{l\,Lp\}_\alpha$, where l previously included in the label n, m, or r, is the orbital angular momenta for the interacting pair when particle α is free. L is the orbital angular momenta of the free particle α with respect to the over-all center of mass, and p is representative of one of the N points that Gaussian quadrature uses to approximate the integration over intermediate momenta. L is restricted in that it must be chosen in compliance with conservation of angular momentum and parity selection rules.

The rank of this matrix of equations, Eqs. (13)–(15) depends on J and parity. If $J = 10$ and parity = plus, for $\alpha = 1$ or 2 (treating neutron-core interactions as occurring in the same states as the proton-core interactions occur), by assumption l

TABLE I. Separable potential parameters.

System	BE	λ	N	l
units	(MeV)	(1/MeV)	(MeV fm$^{1/2}$)	($\hbar$)
n-^{11}C	18.1	0.30	50	1
	2	0.13	25	0
	0	5.0	10	2
p-^{11}C	5	0.62	26	1
	0	0.12	25	0
	−5	4.8	10	2
p-^{11}B	16	0.32	47	1
	0	0.12	25	0
	−5	4.8	10	2
p-p	0	0.07	28	0

can be 0, 1, or 2 only. Then for $l=0$, L_α must be 10 to provide the correct J. For $l=1$, L_α can be 9 or 11 to provide the correct J and parity. For $l=2$, L_α can be 8, 10, or 12 and satisfy the J and parity requirements. For $\alpha=3$, the core is free and the two nucleons are assumed to interact in $l=0$ states only. Then $L_3=10$ only.

For the $J=10$ parity = odd state, T^{31} is zero as Z^{31} vanishes for this case (see Appendix for details of Z^{31} calculation).

But for the 10^+ state, as Eqs. (13)–(15) stand, one must calculate and invert a complex matrix of rank $13N$ or a real matrix of rank $26N$ to solve for the transition amplitudes. For some small J values of course, the rank is less. To shorten such a calculation, we consider the following. Substitution of Eq. (13) into (14) and (15), results in,

$$T^{21}_{i0}=Z^{21}_{i0}+Z^{21}_{ij}\,t^1_j\,Z^{12}_{jk}\,t^2_k\,T^{21}_{k0}+\sigma(T^{31})\,. \tag{14'}$$

In this form, Eq. (14′) and a corresponding one for Eq. (15), one would have a matrix of rank of at least $7N$ which could be inverted to determine T^{21} and T^{31}. Eq. (13) would then be solved for T^{11}, once T^{21} and T^{31} were known. The calculation of each element of Eq. (14′), is more complicated than for Eqs. (13)–(15), but there are less elements to calculate and a smaller matrix to invert. Now T^{21}_{k0} appearing in Eq. (14′) is considered. The discrete quantum numbers (l_2 and L_2) of the element labeled k of T^{21}_{k0} are restricted to the discrete quantum numbers of the element labeled i of T^{21}_{i0}. This restriction is brought about by assuming the ratio of the mass of a nucleon to the mass of the core tends to zero in the calculation of Z^{12}_{jk}. See the Appendix for details of this calculation.

Thus if N, the number of points in the Gaussian quadrature summation were 1, the element k of T^{21}_{k0} would be identical to the element i and (14′) could be solved for T^{21} as

$$T^{21}_{i0}=\frac{Z^{21}_{i0}}{1-Z^{21}_{ij}\,t^1_j\,Z^{12}_{ji}\,t^2_i}+\sigma(T^{31})\,. \tag{16}$$

Rather than truncate N to so unrealistically small a value (one) we approximate the momentum dependence of T^{21}_{k0} in the right-hand side of Eq. (14′) by

$$T^{21}_{k0}\approx CZ^{21}_{k0}\,, \tag{17}$$

where C is an arbitrary (possibly complex) constant, but independent of momentum.

This approximation for the momentum dependence is at least partially correct as the left most factor of each term in Eq. (14′) has Z^{21}_{i0} as a factor, so a perturbative solution of T^2_{i0} would have Z^{21}_{i0} as a factor. Even the momentum dependence asymptomatically of Z^{23} is similar to Z^{21}. When this approximation is substituted back into Eq. (14′) one obtains

$$T^{21}_{i0}=A(Z^{21}_{i0})+\sigma(T^{31})\,, \tag{18}$$

where

$$A^{-1}=1-[(Z^{21}_{ij}\,t^1_j\,Z^{12}_{jk}\,t^2_k\,Z^{21}_{k0})/Z^{21}_{i0}]\,. \tag{18'}$$

Eqs. (18) and (13) can be substituted into (15) resulting in an equation for T^{31} alone. With these approximations, the resulting real matrix which must be inverted to determine T^{31} has a rank of only $2N$. The calculations reported on here use this approximation.

Now t_i has a pole at an energy corresponding to a bound-state two-body binding energy. These poles are avoided by following a technique utilized by Hetherington and Schick.[14] Here the intermediate momentum p is treated as a complex variable so p is replaced by

$$\vec{c}=\vec{p}e^{-i\Phi}\,.$$

The maximum angle Φ can be is 45°. If Φ is less than 45° one can replace the integral dp by contour integration in the complex p plane, by the integral dc. The restriction on Φ is necessary to insure that the contribution to the contour integral vanishes at infinity. The value of Φ used in calculations was 22.5°. Once T^{11}, T^{21}, and T^{31} have been determined for a complex momenta, Eqs. (13)–(15) are used to determine them for real momenta.

One can write $p_i=Ux_i$, where x_i are the set of dimensionless points used for Gaussian quadrature for the integral;

$$\int_0^\infty x^2e^{-x^2}f(x)\,dx=\sum_{i=1}^{N}W_i f(x_i)\,.$$

U corresponds to the momentum units or scale and was chosen so that the maximum variation of $f(x)$ occurred between the middle two Gaussian points. Cross sections for a test case varied by about 10% as N, the number of points was varied between 8 and 16. Therefore the calculations reported in the next section were done with $N=10$, as the variation in the relative calculated cross sections was about only 2% as N varied from 10 to 16.

For breakup reactions, the amplitude[15] is calculated from the off-shell T matrices already calculated for two-body final states as

$$A=-\sum_{\alpha r}\Gamma^\alpha_r(q_i)t^\alpha_r(p_i)\langle p_i\,|\,T^{\alpha 1}\,|\,p_0\rangle\,. \tag{19}$$

When one specifies p_1 and p_2, the momenta of the two outgoing bodies, p_i and q_i are then specified by Eqs. (1) and (2). The sum indicated in Eq. (19) goes over each state r for which an interaction has been assumed and over all three values of α.

$t_\tau^\alpha(p_i)$ is calculated analytically by a principal value method as p_i is now real. It is never needed for a bound-state energy.

III. APPLICATIONS

The first reaction studied is the $^{12}C(p,d)^{11}C$ reaction at 30 MeV. Data over the full angular region are available from Chant, Fisher, and Scott.[16] The three-body model assumes ^{11}C is an inert core, ^{12}C is a neutron bound to this core in a state of orbital angular momentum $l=1$ with 18.1-MeV binding energy. The $l=0$ state was assumed to have a binding energy of 2 MeV, and the $l=2$ state a binding energy of 0 MeV.

The proton-core interaction assumed the $l=1$ state had a binding energy of 5 MeV, the $l=0$ state had zero binding energy and the $l=2$ state a resonance at 5 MeV excitation. These energies are obtained by averaging over E_j, $j=l\pm\frac{1}{2}$ where the corresponding experimental levels are identifiable as single particle levels. The parameters used are shown in Table I. The calculated (p,d) cross section is shown in Fig. 2. The over-all agreement with experiment is good. The calculated peak at forward angles is wider than the experimental peak. The cross section at larger angles is less than experimentally seen but the maxima and minima seem to be reproduced. Having systematically neglected spin, what is calculated is a totally unpolarized cross section, the same as what is measured. An excellent fit to the experimental data can be obtained[16] using the distorted-wave Born approximation (DWBA). However using optical-model parameters which fitted elastic scattering for 30-MeV deuterons resulted in an approximately isotopic DWBA cross section. The excellent fit results when the deuteron optical-potential parameters are varied, most notably, the absorptive part of the potential is increased from the value which fitted elastic scattering data. This contrasts to the three-body-model cross section shown in which the calculated three-body cross section can be said to have no free parameters. By this one means that all the parameters were fitted to some other relevant experimental data. Nonetheless some parameters could be changed. The nucleon-nucleus interaction strengths λ could be taken as complex[9] or chosen to reproduce elastic scattering phase shifts including absorptive effects. This was not done here. The interaction strengths were taken as real and fitted to bound-state data where possible. The energy at which a fit to the scattering phase shifts should be made is not clear; possibly the initial energy is the most appropriate. If the separable potential reproduced the phase shifts at all energies, the energy utilized for the fit would not matter. Such is not believed to be the case, but we have no calculations to report on this.

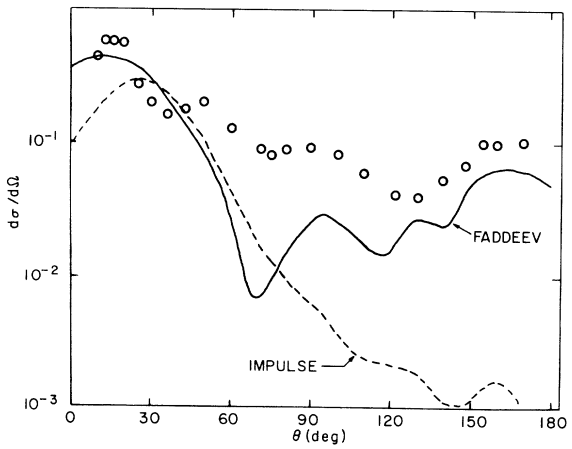

FIG. 2. The $^{12}C(p,d)^{11}C$ cross section calculated in a three-body model for 30-MeV protons.

This calculation has described ^{12}C as one neutron bound to an inert ^{11}C nucleus. If one assumes this calculation has fitted the experimental data and that ^{12}C more correctly has four neutrons in a $p_{3/2}$ level that can independently be picked up to form the final-state deuteron, then one can infer a spectroscopic factor of $\frac{1}{4}$ for the neutron-^{11}C description of this nucleus. Also in Fig. 2 is a dashed curve labeled impulse where the amplitude was calculated keeping only the inhomegenous term in the Faddeev equations. The result agrees moderately well at forward angles with the full Faddeev calculations. At larger angles the impulse calculation is too small, compared to the Faddeev result, indicating the necessity of obtaining a solution to the full three-body equations at this energy.

For the $^{12}C(p,2p)^{11}B$ reaction, the core is taken as ^{11}B. The proton-core interaction in the $l=1$ state is assumed to have a binding energy of 16 MeV; for $l=0$, a binding energy of zero; and for $l=2$, a resonance energy of 5 MeV. The experimental data of Richie and Wright[17] are shown in Fig. 3. The final two protons are detected at equal angles with respect to the incident beam and with equal energies. The momenta of final-state protons and the initial proton momenta all lie in the same plane.

The three-body calculation for the $(p,2p)$ cross section is shown as a solid line in Fig. 3. The general shape and magnitude of the experimental data are reproduced. In particular, the forward peaking of the data is reproduced. The secondary peak of the experimental data at 70° is not reproduced. The magnitude of the calculated cross section is not normalized to the data, but as calculated over-

estimates somewhat the cross section at forward angles. The calculated three-body cross section had no free parameters in it. The parameters selected however have the same uncertainties as discussed for the $^{12}C(p,d)^{11}C$ reaction. Letting the interaction strength λ for the proton-core interaction become complex and the more proper inclusion of spin are the most obvious next steps in making this three-body theory more realistic. However, the calculation as done has resulted in gratifying agreement with experiment.

A calculation was done, labeled impulse in Fig. 3, where the off-shell T matrices were replaced by the Born terms; that is $T^{\alpha 1}$ was replaced by $Z^{\alpha 1}$ in Eq. (19). Such a substitution resulted in a cross section similar to the three-body calculated cross section but lower by a factor of 3 to 4 for angles less than 70°.

The cross section was also calculated including only the $\alpha = 3$ term in Eq. (19). This cross section corresponds to the case where the incident proton and the bound proton interact, and then propagate off shell at forward angles, as a diproton. Subsequently they interact in a final interaction decaying into the two final-state protons. This calculated cross section is labeled FSI in Fig. 3, symbolic for including only the two-proton final interactions. This cross section is quite close to the full three-body cross section except for angles between 80 and 150° where it is too small and the other amplitudes dominate the cross section. This cross section is similar in spirit to a distorted-wave Born-approximation calculation for the $(p, 2p)$ reaction. Including only the $i = 3$ term of Eq. (19), one can

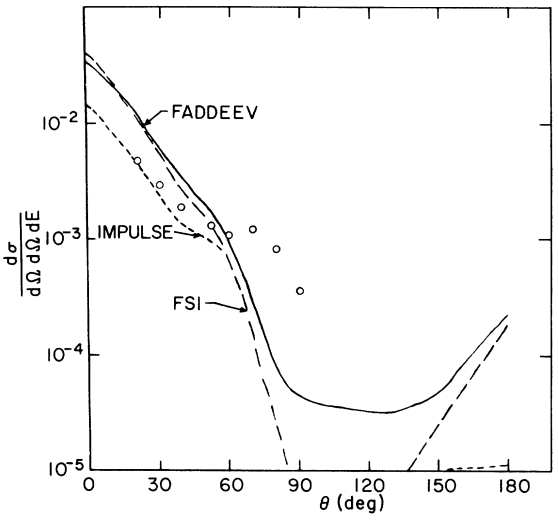

FIG. 3. The $^{12}C(p,2p)^{11}B$ symmetric coplanar cross section calculated in a three-body model for 46-MeV protons.

write for the amplitude:

$$A_3 = \Gamma_3(q_3) t_3(p_3) \langle \vec{p}_3 | T^{31} | \vec{p}_0 \rangle = \frac{\langle q_3 | \hat{T}_3(p_3) | q_3 \rangle}{\Gamma_3(q_3)} \langle \vec{p}_3 | T^{31} | \vec{p}_0 \rangle , \quad (20)$$

where $\hat{T}_3(p_3)$ is the two-proton t matrix (off shell) evaluated at the energy $p_3^2/2\mu_3$, where

$$p_3 = | -\vec{p}_1 - \vec{p}_2 | .$$

$\Gamma_3(q_3)$ is related to the two-proton $l = 0$ wave function in momentum space. The other factor is the off-shell amplitude for the (p,d) reaction where d is the diproton, moving forward with momentum p_3. For equal angles and equal energies for p_1 and p_2, $\vec{p}_3$ is parallel to $\vec{p}_0$.

The similarity of the FSI and the full three-body calculated cross sections suggest a modified DWBA picture of this reaction at this energy. A study of the major contributions to the angular structure in the three-body amplitude A_3, suggests the DWBA calculations for $(p, 2p)$ reactions might be improved by inclusion of an energy-dependent two-proton t matrix. Such an energy dependence would reflect the variation in the two-proton t matrix as the final energy in the two-proton subsystem varied. This factor is the source of most of the angular structure in the amplitude of Eq. (20).

If the off-shell dependence of the three-body amplitude T^{31} can be ignored in Eq. (20), a possible distorted-wave calculation for the $(p, 2p)$ reaction amplitude would consist of two factors. The first factor would be an (assumed) on-shell (p,d) forward-scattering amplitude where d means diproton bound with zero binding energy. The second factor would be the off-shell two-proton t matrix divided by Γ_3. Another possible modification of DWBA calculations for the $(p, 2p)$ reaction would be as follows. With a separable potential the two-proton t matrix energy dependence is completely contained in a single factor [see Eq. (6)]. The energy dependence of the two-proton t matrix, at least as predicted by a separable potential, could then be easily included in existing distorted-wave codes which currently utilize energy-independent two-proton t matrices. The close agreement in Fig. 3, of the FSI cross section to the full cross section, suggests that the breakup cross section is not very sensitive to the approximation [Eq. (17)] made for T^{21}_{k0}. This is so because the cross section at forward angles is dominated by the T^{31} amplitude. However, we point out, the approximation does influence the calculation of the T^{31} amplitude, but perhaps only in a minor fashion. The complete effect of this approximation can be best judged from a calculation where it is not made. The intractibility of such a calculation prevents it from being done currently.

IV. CONCLUSION

The $^{12}C(p,d)^{11}C$ and the $^{12}C(p,2p)^{11}B$ reactions have been calculated in a three-body theory. Nucleon-core interactions were included in $l=0$, 1, and 2 states of relative orbital angular momentum. Intrinsic spin was ignored. The cross section calculated for the (p,d) reaction tends to agree with experiment at 30 MeV, but is too small at intermediate angles.

The $(p,2p)$ calculated cross section for 46-MeV protons reproduces the general trend of the experimental data, particularly the peak at forward scattering angles. A major conclusion is that a three-body calculation, even with such simplified separable forces, can, with a minimum of adjustable parameters (by some standards no free parameters) reproduce the general features of (p,d) and $(p,2p)$ reaction data. In addition some confirmation for a modified distorted-wave picture of the $(p,2p)$ reaction is obtained. The use of an energy-dependent two-proton t matrix in distorted-wave calculations of $(p,2p)$ reactions is suggested. An approximation of the momentum dependence of part of one of the Faddeev T matrices which simplifies the solution for the reaction amplitudes is partially justified. The resulting calculated cross sections are still realistic in that they tend to reproduce the over-all features of the experimental cross sections.

APPENDIX

The inhomogeneous term appearing in Eq. (14) is, from Alt, Grassberger, and Sandhas[4]

$$\langle \vec{p}_3 n|Z^{31}|m\vec{p}_1\rangle = \langle \vec{p}_3 n|\frac{1}{Z^3 - p2/2n - q2/2\mu}|m\vec{p}_1\rangle, \tag{A1}$$

which describes a state with particle 1 having momentum p_1 with respect to the 2-3 pair center of mass, which are in a state labeled by the quantum numbers m. The final state describes particle 3 having momentum p_3 with respect to the 1-2 pair center of mass which are interacting in a state with quantum numbers labeled by n.

Expanding the states of definite momentum in partial waves, we have

$$\langle L_3 M_3 p_3 l_3 m_3|Z^{31}|L_1 M_1 p_1 l_1 m_1\rangle = \int d\hat{p}_3 Y^*_{L_3 M_3}(\hat{p}_3)\, d\hat{p}_1 Y_{L_1 M_1}(\hat{p}_1)\Gamma^*_{L_3 M_3}(\bar{q}_3)\,\langle p_3\bar{q}_3|G_0(Z)|p_1 q_1'\rangle\, d\bar{q}_3\, dq_1'\,\Gamma_{L_1 M_1}(q_1), \tag{A2}$$

$$\langle p_1 q_1|G_0(Z)|p_1' q_1'\rangle = \delta(\vec{p}_1^* - \vec{p}_1')\delta(\vec{q}_1^* - \vec{q}_1')/(Z - p_1^{*2}/2n_1 - q_1^{*2}/2\mu_1). \tag{A3}$$

Coupling to states of definite J^π, all the angular integrals but one can be done analytically by repeated use of

$$|\vec{p}_1 + \vec{p}_2|^l Y_l^m(p_1+p_2) = \sum_{abm_a m_b} \delta(a+b=l)\left(\frac{4\pi}{\bar{a}}\right)^{1/2}\begin{pmatrix}2l+1\\2a\end{pmatrix}^{1/2}(p_1)^a(p_2)^b\begin{bmatrix} l & m\\ b & m_b\\ a & m_a\end{bmatrix} Y_a^{m_a}(\hat{p}_1)Y_b^{m_b}(\hat{p}_2),$$

where $\bar{a}\equiv 2a+1$, and the bracket signifies a Clebsch-Gordan coefficient of $a+b$ coupled to form a state of angular momenta l. With $l_3=0$, we obtain for a definite J and parity:

$$\langle p_3|Z^3|p_1\rangle = \sum_L \sum_{a,b}\delta(a+b=l_1)\beta_L(p_3,p_1)(p_3)^a\left(\frac{m_3 p_1}{m_2+m_3}\right)^b\left(\frac{1}{2}\right)\begin{pmatrix}2l_1+1\\2a\end{pmatrix}^{1/2}$$
$$\times(-1)^{l_1}\begin{bmatrix}J & 0\\ a & 0\\ L & 0\end{bmatrix}\begin{bmatrix}L & 0\\ b & 0\\ L_1 & 0\end{bmatrix}\begin{Bmatrix}L_1 & b & L\\ a & J & l_1\end{Bmatrix}\begin{Bmatrix}L & a & L_3\\ 0 & J & a\end{Bmatrix}\begin{vmatrix}a & 0 & a\\ J & 0 & J\\ L & 0 & L\end{vmatrix} F, \tag{A4}$$

where $F=(\bar{l}_1\bar{J}\bar{L}_1\bar{b}\bar{L}\bar{L}\bar{a}\bar{a})^{1/2}$.

The $\{\ \}$ are usual $6j$ symbols and the $|\ |$ contain a $9j$ symbol. The one integral done numerically is

$$\beta_L(p_3,p_1) = \int_{-1}^{1} dx \frac{g_0(\vec{p}_1 + m_1\vec{p}_3^*/(m_1+m_2))\, g_{l_1}(-\vec{p}_3^* - m_3\vec{p}_1/(m_2+m_3))P_L(x)}{q_3^{*l_3}q_1^{l_1}[E - p_1^2/2n_1 - (q_1^*)^2/2\mu_1]}, \tag{A5}$$

where $\vec{q}_1 \equiv -\vec{p}_3^* - m_3\vec{p}_1/(m_2+m_3)$. Here x is the angle between $\hat{p}_3$ and $\hat{p}_1$.

The expression for $\langle p_2|Z^{23}|p_3\rangle$ can be obtained from the above by a cyclic permutation of subscripts (123) → (312).

For the $\langle p_1|Z^{12}|p_2\rangle$ case, one considers the numerical integrals

$$\beta_L(p_1,p_2)=\int_{-1}^{1}dx\,\frac{g_{l_1}(\vec{p}_2+m_2/(m_2+m_3)\vec{p}_1^{\,*})\,g_{l_2}(-\vec{p}_1^{\,*}-m_1\vec{p}_2/(m_1+m_3))P_L(x)}{q_2^{l_2}q_1^{*l_1}[E-p_2^{\,2}/2n_2-q_2^{\,2}/2\mu_2]}\,. \tag{A6}$$

Now $\vec{q}_2=-\vec{p}_1^{\,*}-m_1\vec{p}_2/(m_1+m_3)$, so in the limit of m_1/m_3 and m_2/m_3 both going to zero, that is of m_3 large, the only x dependence is in the Legendre polynomial $P_L(x)$. x is here the angle between $\hat{p}_1$ and $\hat{p}_2$. Therefore only the term with $L=0$ does not vanish in this limit, and we obtain

$$\langle p_1|Z^{12}|p_2\rangle=\beta_0(p_1,p_2)(p_1)^{l_2}(p_2)^{l_1}(\tfrac{1}{2})(\bar{l}_1)^2(\bar{l}_2)^{3/2}(-1)^{l_1}\begin{Bmatrix}l_1&0&l_1\\l_2&J&l_2\end{Bmatrix}\sum_{J'}(-1)^{J'}(J')$$
$$\times\begin{Bmatrix}l_1&J'&l_2\\l_1&J&l_2\end{Bmatrix}\begin{vmatrix}l_2&l_1&J'\\l_2&0&l_2\\0&l_1&l_1\end{vmatrix}\delta(L_1=l_2)\delta(l_1=L_2)\,. \tag{A7}$$

If m_3 is not so large that m_1 and m_2 can not be neglected in comparison, then

$$\langle p_1|Z^{12}|p_2\rangle=\sum\delta(a+b=l_2)\delta(a'+b'=l_1)\beta_L(p_1,p_2)(p_1^*)^a\left(\frac{m_2p_1^*}{m_2+m_3}\right)^{a'}\left(\frac{m_1p_2}{m_1+m_3}\right)^{b}(p_2)^{b'}$$
$$\times\left(\frac{1}{2}\right)\binom{\bar{l}_2}{2a}^{1/2}\binom{\bar{l}_1}{2a'}^{1/2}(-1)^{l_1+l_2+a}\begin{bmatrix}L_6&0\\a'&0\\L_1&0\end{bmatrix}\begin{bmatrix}L_6&0\\a&0\\L&0\end{bmatrix}\begin{bmatrix}L_7&0\\b&0\\L_2&0\end{bmatrix}\begin{bmatrix}L_7&0\\b'&0\\L&0\end{bmatrix}(\bar{l}_1\bar{l}_2\bar{L}_1\bar{L}_2\bar{b}\,\bar{b}'\,\bar{L}\,\bar{L})^{1/2}$$
$$\times\begin{Bmatrix}L_2&b&L_7\\a&J&l_2\end{Bmatrix}(-1)^{J'}\bar{J}'\begin{Bmatrix}L_7&J'&L_1\\l_1&J&a\end{Bmatrix}\begin{vmatrix}a&l_1&J'\\L_6&a'&L_1\\L&b'&L_7\end{vmatrix}. \tag{A8}$$

The summation is over a, b, a', b', L, L_6, L_7, and J'.

If m_1/m_3 and m_2/m_3 approach zero, then $L=0$, $a'=0$, $b=0$,

$a=l_2$, and $b'=l_1$,

and Eq. (A7) results. The δ functions in Eq. (A7) come from the above Clebsch-Gordan coefficients which basically enforce conservation of parity selection rules.

The presence of these δ functions is what requires, in Eq. (14′) the quantum numbers of k to be identical to the quantum numbers of element i. The initial state need not have the same quantum numbers as the final state, of course, as L_1 need not equal l_1. L_1 is the angular momentum of particle 1, the free particle, with respect to the interacting pair (2, 3) which have orbital angular momentum l_1. These δ functions do simplify the structure of the integral equations and lead to the approximation Eq. (17).

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*Permanent address: University of Georgia, Athens, Georgia 30601.

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