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Equations for Four-Particle Scattering*

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Four-particle scattering equations are obtained that are analogous to the Lovelace or Alt-Grassberger-Sandhas equations in the three-particle case: They are equations whose kernel is connected after one iteration, in which the unknown quantities are the transition operators for the various elastic, inelastic, and rearrangement processes. The equations couple together only the transitions to the two-body channels, exactly as in the three-particle analogs, so that there are seven coupled integral equations, corresponding to the seven two-body channels (four of nucleon+triton type, and three of deuteron+deuteron type). Transition operators to the three- and four-body channels appear as integrals over the transition operators to the two-body channels.

I. INTRODUCTION

The celebrated Faddeev equations[1] for the three-particle problem have the essential property that the iterated kernel is connected (for complex energies), so that at least in principle they can be solved with ordinary numerical techniques. However, the solutions of the original Faddeev equations are not related in any very simple way to the amplitudes for the various elastic, inelastic, and rearrangement processes that can occur. For this reason, the variants of the Faddeev equations proposed by Lovelace[2] and by Alt, Grassberger, and Sandhas[3] have often been preferred in practice.

The subject of this paper is the four-particle scattering problem, and the principal aim is to obtain four-particle equivalents of the Lovelace-AGS equations, i.e., equations with connected kernels in which the quantities that appear are the transition operators for the various scattering processes. As far as we are aware, such equations have not previously been given. These equations have the advantage that they make the formulation of scattering problems much more direct, and we believe might bring the simplest four-particle problems within the range of modern computers.

The four-particle problem is considerably more complicated than the three-particle, and it is therefore hardly surprising that it has not reached the same stage of development. However, many authors[4-11] have written down connected equations for the four-particle problem, using a variety of techniques[12] to cure the disconnectedness problem. In only one case, that of the Yakubovskiĭ equations,[11] has it been proved that the solutions of the homogeneous equation at negative energies are always solutions of the Schrödinger equation. It is therefore possible for the other formulations that the homogeneous equations have additional solutions, which might cause difficulties in bound-state studies. On the other hand, the Yakubovskiĭ equations, which are sets of eighteen coupled equations, appear extremely difficult to use in any practical calculation. And if one's interest is in scattering rather than in the bound states, as in the present work, then the possible ambiguities in bound-state calculations are not of direct concern.

The four-particle equations of greatest importance for the present work are those obtained independently by Rosenberg,[7] Mitra *et al.*,[8] Takahashi and Mishima,[9] and Alessandrini.[10] These are sets of six coupled equations (one for each of the six pairs), in which the kernels are the connected parts of the amplitudes for the three-particle subsystems, and also for the subsystems consisting of two noninteracting pairs. The equations we obtain at the end of this paper (which are sets of *seven* coupled equations) are related to these equations as the Lovelace-AGS equations are to the Faddeev equations in the three-particle case. The number seven is the number of two-body channels in the four-particle scattering problem, as will be clear from the following discussion.

One of the difficulties of the four-particle problem is the great variety of channels available for initial and final states. There are essentially four different types of channel, and these are shown diagrammatically in Fig. 1. First, in Fig. 1(a) we show a two-body channel[13] of the $1+3$ type, for example, a nucleon-triton channel. Obviously, there are four channels of this sort, which we label by 1, 2, 3, or 4, according to the label on the free particle. Next, there are two-body channels of the $2+2$ or deuteron-deuteron type, as shown in Fig. 1(b). There are three channels of this sort, which we find it convenient to label according to a definite code:

channel a is $(12)+(34)$,

channel b is $(13)+(24)$,

channel c is $(14)+(23)$.

Next, there are six three-body channels, in which a single pair is bound, and which we label by the bound pair, as shown in Fig. 1(c). And finally, in Fig. 1(d) there is the four-free-particle channel, channel 0, making fourteen channels in all.

However, by analogy with the three-particle case, we would not expect the integral equations

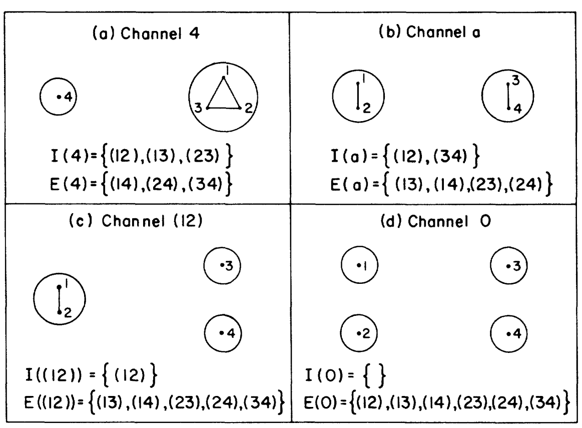

FIG. 1. Diagrammatic representations of the four different types of channel in the four-particle system. (a) Channel 4, a two-body channel of $1+3$ type; (b) channel a, a two-body channel of $2+2$ type; (c) channel (12), a three-body channel; (d) channel 0, the four-body channel. The internal interactions in a particular channel α are represented by lines. The set $I(\alpha)$ contains the pairs corresponding to internal interactions, and the set $E(\alpha)$ contains the remaining pairs, those corresponding to external interactions in channel α.

to couple together all of these channels. In fact, we shall see that the integral equations couple together only the various two-body channels, just as happens in the three-particle problem, and that the amplitudes for transitions to three- or four-body channels appear as integrals over the amplitudes for transitions to the two-body channels. It is evident, then, that the seven two-body channels play a particularly important role in the theory.

We shall use the labeling convention that the Greek letters $\alpha, \beta, \ldots$ range over all the 14 channels defined above, whereas σ (and $\sigma', \sigma'', \ldots$) ranges over only the 7 two-body channels. The letters $i, j, \ldots$ range over the six pairs $(12), (13), \ldots,$ and as in these examples, whenever a pair is written explicitly it will be enclosed in parentheses, to avoid possible confusion with repeated channel labels.

With respect to any particular channel α we find it convenient to divide the six pairs $(12), \ldots$ into two sets $I(\alpha)$, $E(\alpha)$, corresponding to what we shall call "internal" and "external" interactions in channel α: The internal interactions (shown by lines in Fig. 1) are the interactions within the separated groups in the particular channel, while the external interactions are those connecting the various groups; e.g., in channel 4 the internal interactions are $V_{(12)}$, $V_{(13)}$, $V_{(23)}$, and the external interactions are $V_{(14)}$, $V_{(24)}$, $V_{(34)}$. In channel (12) the only internal interaction is $V_{(12)}$, and all the remaining interactions are external. The sets $I(\alpha)$ and $E(\alpha)$ are given explicitly in Fig. 1 for each of the four channels illustrated there.

The integral equations introduced in this paper, like those of Refs. 7–10, have in their kernel the amplitudes for the internal scattering problem in each two-body channel σ. (The internal scattering problem in channel σ, of course, is that in which all the external interactions in channel σ are switched off.)

To understand the reason for the occurrence of these amplitudes in the kernel, it may be useful to consider the Faddeev equations[1] for the N-body problem,

$$T_{ji} = \delta_{ji} t_j + \sum_{k \neq j} t_j G_0 T_{ki} \,. \tag{1.1}$$

Here t_j is the two-body T matrix (operating in the N-body space) for the pair j, and G_0 is the free-particle propagator,

$$G_0(s) = (s - H_0)^{-1} \,, \tag{1.2}$$

where H_0 is the kinetic energy, and s is the complex energy parameter.

If $N=3$, then the kernel of (1.1) is connected after one iteration (if s is complex), but if $N=4$ then the iterated kernel still contains disconnected terms like $t_{(12)}G_0t_{(23)}G_0$ or $t_{(12)}G_0t_{(34)}G_0$. In the first case particle 4 is not involved at all, so that the matrix element of this piece of the iterated kernel has a δ function in the momentum of particle 4. In the second case there is no interaction between the pairs (12) and (34), so that the relative momentum of the pairs is conserved, again giving a δ function. Furthermore, these difficulties are not cured by further iteration, since repetitions of $t_{(12)}G_0 \times t_{(23)}G_0$, and similarly of $t_{(12)}G_0t_{(34)}G_0$, will occur. The first corresponds to internal scatterings in channel 4, and the second to internal scatterings in channel a [see Fig. 1(b)].

One way around the disconnectedness problem is to solve the internal scattering problems in advance, so that these troublesome sequences of two-body scatterings do not appear in the iterations of the kernel. In this way we are led to the equations of Refs. 7–10, in which the amplitudes for the internal scattering problems appear explicitly in the kernel. Of course, only the parts of the amplitude that are *connected* with respect to the internal motion in the particular channel should appear in the kernel, since otherwise the disconnectedness problem would still be as bad as ever.

The reason for the special role of the two-body channels in the theory, as opposed to the three- and four-body channels of Figs. 1(c) and 1(d), becomes clear if we consider iterations of the kernel: The special property of the two-body channels is that the product of the scattering amplitudes in two *different* two-body channels is connected. This is obviously not true for the three- and four-body channels.

The situation is actually entirely analogous to that in the three-particle problem. In that case the two-body channels are $1+(23)$, $2+(31)$, and $3+(12)$, and the internal scattering problem in each channel is just the problem of the scattering of a pair. The corresponding amplitudes are just the two-body T matrices, and it is these that form the kernel of the Faddeev or Lovelace-AGS[2,3] equations.

So far we have not made any distinction between the three-particle formulations of Lovelace[2] and AGS.[3] They are very similar, in that in both cases the unknowns in the equations are the transition operators linking the various channels, but they differ in their inhomogeneous terms. The difference arises from the fact that there is a degree of choice available in the definition of the transition operators, corresponding to the fact that the physical scattering amplitudes are defined only on the energy shell. AGS[3] have exploited this freedom to obtain three-particle equations with a particularly simple inhomogeneous term. In the present work we exploit the similar freedom in the four-particle problem, with the result that our final equation [Eq. (5.26)] has almost as simple an appearance as the three-particle equations of AGS.[3]

In Secs. II and III we consider the internal scattering problem for the two types of two-body channels in Fig. 1. First, in Sec. II we consider the three-particle problem, both to establish the notations and the relations that we shall need later, and also to remind us of the various integral equations that have been used in the three-particle problem. Next, in Sec. III we briefly consider the rather simpler problem of the internal motion in channel a, i.e., the problem of two noninteracting pairs. Then in Sec. IV we establish useful notations and definitions for the four-particle problem, and rewrite in this notation the useful results of the preceding sections. The essential heart of the paper is in Sec. V, where we obtain the desired integral equations for four-particle scattering. The first step in the argument is an alternative derivation of the equations of Refs. 7–10. From there we go on to obtain integral equations for the actual transition operators between the various channels in the four-particle problem. Finally, in Sec. VI we give a brief summary of the principal results.

II. THREE-PARTICLE EQUATIONS

In this section we review the scattering problem for three particles 1, 2, 3. We use (in this section only) the usual three-body notation $(12)=3$ etc. for the three pairs, so that the interaction between particles 1 and 2 is denoted by either $V_{(12)}$ or V_3. The two-body T matrix for the pair i (regarded as an operator in the three-body space) satisfies the Lippmann-Schwinger equation,

$$t_i(s)=V_i+V_iG_0(s)t_i(s)$$

or (2.1)

$$t_i(s)=V_i+t_i(s)G_0(s)V_i\,, \quad i=1,2,3\,,$$

where

$$G_0(s)=(s-H_0)^{-1} \tag{2.2}$$

and

$$s=E+i\epsilon\,. \tag{2.3}$$

In this section H_0 denotes the three-particle kinetic energy, and E the total three-particle energy. (Later, however, we will reinterpret H_0 and E as the corresponding four-particle quantities. Operators such as G_0 and t_i will then become operators in the four-particle space, but the formal structure is otherwise unchanged.)

We denote the full T matrix for the three-particle system by $M(s)$,

$$M(s) \equiv V + VG(s)V , \tag{2.4}$$

where

$$G(s) = (s - H_0 - V)^{-1} , \tag{2.5}$$

and

$$V = \sum_{i=1}^{3} V_i . \tag{2.6}$$

Following Faddeev,[1] we decompose $M(s)$ by writing

$$M(s) = \sum_{j,i} M_{ji}(s) , \tag{2.7}$$

where

$$M_{ji} = \delta_{ji} V_j + V_j G V_i . \tag{2.8}$$

Then the amplitudes M_{ji} satisfy the Faddeev equations,[1]

$$M_{ji} = \delta_{ji} t_j + \sum_{k \neq j} t_j G_0 M_{ki} \tag{2.9}$$

or

$$M_{ji} = \delta_{ji} t_j + \sum_{k \neq i} M_{jk} G_0 t_i . \tag{2.10}$$

The multiple-scattering interpretation of M_{ji}, obtained by iterating (2.9) or (2.10), is that M_{ji} is the sum of all multiple-scattering terms (i.e., terms of the form $\cdots t_k G_0 t_l \cdots$, with $\cdots \neq k \neq l \neq \cdots$), that begin on the right with t_i, and end on the left with t_j. It is useful to keep this and similar multiple-scattering interpretations in mind, since with their aid it becomes easy to understand the relation between the various operators that arise.

As we noted in Sec. I, the solutions of the Faddeev equations (2.9) are not immediately related to the physical scattering amplitudes for the various elastic, inelastic, and rearrangement processes that occur in the three-particle system. We now turn to the problem of obtaining the transition operators for these physical processes, using the approach of Lovelace.[2]

The general expression for the transition operator from an initial channel i to a final channel f is given by Goldberger and Watson[14] in two alternative forms,

$$U_{fi}^{\pm} = \begin{Bmatrix} v_f \\ v_i \end{Bmatrix} + v_f G v_i , \tag{2.11}$$

where v_i and v_f are the external interactions in the initial and final channels. The physical scattering amplitude $\mathcal{T}_{fi}$ is then the matrix element of $U_{fi}^{\pm}$,

$$\mathcal{T}_{fi} = (\Phi_f, U_{fi}^{\pm}(E + i\epsilon)\Phi_i) , \tag{2.12}$$

where Φ_i and Φ_f are physical channel states in the initial and final channel, i.e., Φ_i satisfies

$$[H_0 + (V - v_i)]\Phi_i = E\Phi_i . \tag{2.13}$$

The two operators defined by (2.11) are of course different, but it is easy to prove with the aid of (2.13) and the similar equation for Φ_f that they yield the same value for the on-shell scattering amplitude $\mathcal{T}_{fi}$.

For the particular case of the three-particle problem, if the pair i is bound in the initial channel and the pair j bound in the final channel, then the transition operators (2.11) become

$$U_{ji}^{\pm} = (1 - \delta_{ji}) \begin{Bmatrix} V_i \\ V_j \end{Bmatrix} + \sum_{j \neq k \neq i} V_k + \sum_{k \neq j} V_k G \sum_{l \neq i} V_l , \tag{2.14}$$

or

$$U_{ji}^{\pm} = (1 - \delta_{ji}) \begin{Bmatrix} V_i \\ V_j \end{Bmatrix} + W_{ji} , \tag{2.15}$$

where with the aid of (2.8) we can write

$$W_{ji} = \sum_{k \neq j} \sum_{l \neq i} M_{kl} . \tag{2.16}$$

Then it follows immediately from the Faddeev equations (2.9) that the W_{ji} satisfy the equations

$$W_{ji} = \sum_{j \neq k \neq i} t_k + \sum_{k \neq j} t_k G_0 W_{ki} , \tag{2.17}$$

and hence, with the aid of the Lippmann-Schwinger equation, Eq. (2.1), that U_{ji}^{-} satisfies the Lovelace equation,[2]

$$U_{ji}^{-} = \sum_{k \neq i} V_k + \sum_{k \neq j} t_k G_0 U_{ki}^{-} . \tag{2.18}$$

Lovelace[2] also obtained a similar equation, essentially the transpose of this one, for the operator U_{ji}^{+}.

The equations of the preceding paragraph have been derived only for elastic and rearrangement collisions, but as pointed out by Lovelace,[2] they apply equally to the breakup case if we introduce an additional value of j to label the breakup channel, say $j = 0$, and define

$$V_0 = t_0 = M_{0i} = M_{j0} = 0 . \tag{2.19}$$

We follow this convention in the remainder of this section, and later on we shall use similar ideas in the four-particle problem. From Eq. (2.16), the multiple-scattering interpretation of W_{ji} for any values of j and i is that it is the sum of all multiple-scattering terms that begin with the scattering of a pair other than the pair i, and end with the scattering of a pair other than the pair j. For the particular case $j = 0$ the second of these restrictions is effectively absent.

We have seen already that there is a degree of freedom in the definition of the transition operators in Eq. (2.11), because the final scattering amplitudes are required only on the energy shell. This degree of freedom has been further exploited by AGS,[3] in order to obtain a set of integral equations with a simpler inhomogeneous term than that in the Lovelace equation (2.18). It is interesting to follow this through explicitly, because later on (in Sec. V) we shall be taking advantage of a similar degree of freedom in the four-particle problem.

The AGS transition operator corresponding to the Lovelace operators $U_{ji}^{\pm}$ is

$$U_{ji} = (1-\delta_{ji})(s-H_0) + W_{ji} . \tag{2.20}$$

Remembering that $s = E + i\epsilon$, it is very easy to show with the aid of the Schrödinger equation for the initial and final channel states that this gives the same on-shell scattering amplitude [see Eq. (2.12)] as the Lovelace operators of Eq. (2.15). Then from Eq. (2.17), we see that U_{ji} satisfies

$$U_{ji} = (1-\delta_{ji})(s-H_0) + \sum_{k\neq j} t_k G_0 U_{ki} . \tag{2.21}$$

This is the three-particle equation of AGS.[3] It has the same kernel as the Lovelace equation (2.18), but has a considerably simpler inhomogeneous term.

We conclude this section with some further relations and definitions that will prove useful in the four-particle problem. We denote by $\overline{M}_{kl}$ the connected part of M_{kl} [Eq. (2.9)], and by $\overline{W}_{ji}$ the connected part of W_{ji} [Eq. (2.16)], so that

$$\overline{M}_{kl} = M_{kl} - \delta_{kl} t_k , \tag{2.22}$$

$$\begin{aligned} \overline{W}_{ji} &= \sum_{k\neq j} \sum_{l \neq i} \overline{M}_{kl} \\ &= W_{ji} - \sum_{j\neq k\neq i} t_k . \end{aligned} \tag{2.23}$$

It is also convenient to define a quantity $\overline{M}_{k-}$,

$$\overline{M}_{k-} = \sum_{l} \overline{M}_{kl} , \tag{2.24}$$

so that $\overline{M}_{k-}$ is the sum of all connected multiple-scattering terms that end with the scattering of the pair k, but with no restriction on the beginnings. (The dash in $\overline{M}_{k-}$ serves simply as a place keeper, since obviously we could also define analogous operators $\overline{M}_{-l}$, with unrestricted endings. But we make no use of the latter in this paper, so that if desired the dash can be omitted.)

The $\overline{M}_{k-}$ and $\overline{M}_{kl}$ satisfy a relation that will be useful to us later on,

$$\overline{M}_{k-} G_0 = \left[\sum_{l\neq m} \overline{M}_{kl} + (1-\delta_{km}) t_k G_0 V_m \right] (s - H_0 - V_m)^{-1} , \tag{2.25}$$

for any $m = 1, 2, 3$. To prove it, we first split the sum in Eq. (2.24) into two parts,

$$\overline{M}_{k-} = \sum_{l\neq m} \overline{M}_{kl} + \overline{M}_{km} , \tag{2.26}$$

and then in the second term use (2.22) and the Faddeev equation (2.10), so obtaining

$$\overline{M}_{k-} = \sum_{l\neq m} \overline{M}_{kl}(1 + G_0 t_m) + (1-\delta_{km}) t_k G_0 t_m . \tag{2.27}$$

Then (2.25) follows easily with the aid of the relations

$$t_m G_0 = V_m G_m ,$$

$$G_m = G_0 + G_0 t_m G_0 ,$$

where

$$G_m = (s - H_0 - V_m)^{-1} ,$$

which are standard relations of two-particle scattering theory.

III. EQUATIONS FOR NONINTERACTING PAIRS

Here we consider the system obtained by switching off all the interactions except those that are internal in channel a [see Fig. 1(b)], i.e., all except $V_{(12)}$ and $V_{(34)}$. This is a simpler problem than the three-particle problem, but it has a very similar formal structure, so that we shall be able to treat it very briefly.

As far as possible we use the same notation as in the previous section. Thus we define amplitudes M_{ji} exactly as before, i.e., by Eqs. (2.8) and (2.5), with V given in this case by

$$V = V_{(12)} + V_{(34)} .$$

These amplitudes satisfy the same (Faddeev) integral equations (2.9) and (2.10), the only difference being that now the indices range over only the two pairs (12) and (34), so that there are only two coupled equations, and the iterative solutions for say $M_{(12)(12)}$ and $M_{(34)(12)}$ are just

$$M_{(12)(12)} = t_{(12)} + t_{(12)} G_0 t_{(34)} G_0 t_{(12)} + \cdots ,$$

$$M_{(34)(12)} = t_{(34)} G_0 t_{(12)} + t_{(34)} G_0 t_{(12)} G_0 t_{(34)} G_0 t_{(12)} + \cdots .$$

Similarly, we can define quantities W_{ji}, $\overline{W}_{ji}$, $\overline{M}_{kl}$, $\overline{M}_{k-}$ in exactly the same way as before [Eqs. (2.16), (2.23), (2.22), and (2.24)], and since the proof of Eq. (2.25) requires only the integral equation and these definitions, it too also applies to the present case.

Though this problem is formally similar to the three-particle problem, it is much easier to actually obtain the M_{ji} in this case, since they can be expressed[8,10,15] in terms of quadratures over the two-body amplitudes. The problem of two noninteracting pairs is of course algebraically equivalent (apart from its extra degrees of freedom) to that of two noninteracting particles in a static potential.

IV. FOUR-PARTICLE NOTATIONS AND RELATIONS

In the previous two sections we have discussed what is essentially the problem of the internal motion in the two types of two-body channel shown in Fig. 1, and we have defined various operators related to the scattering in the corresponding subsystems. It is now necessary to somewhat extend the definitions and the notations for these quantities, so that we can incorporate them into the four-particle problem. (The discussion of true four-particle scattering equations does not begin until the next section.)

The first extension that is needed is to recognize explicitly the existence of all four particles and all degrees of freedom, so that all of the operators of the preceding sections become operators in the full four-particle space. All that we need to accomplish this is to reinterpret H_0 and E as the total kinetic energy operator and the total energy for the four-particle system; and this we now consider done.

Obviously, we will also need labels on the operators defined in the previous sections to tell us which subsystem they refer to. More generally, on many quantities we will need labels that range over all of the fourteen channels discussed in Sec. I (in which case we recall that our convention is to label by Greek letters α, $\beta, \ldots$). We use the convenient notational device that a superscript Greek letter on any quantity refers to the *internal* motion in the particular channel. For example, we define V^α to be the total internal interaction in channel α,

$$V^\alpha = \sum_{i\in I(\alpha)} V_i \,. \tag{4.1}$$

[We recall from Sec. I that $I(\alpha)$ is the set of pairs corresponding to the internal interactions in channel α.]

With this notation, we naturally denote the full T matrix for the internal scattering in the two-body channel σ by M^σ, where [cf. Eq. (2.4)]

$$M^\sigma = V^\sigma + V^\sigma (s - H_0 - V^\sigma)^{-1} V^\sigma \,. \tag{4.2}$$

It is not necessary for us to define these amplitudes in the three- and four-body channels. (We recall our convention, that the letter σ ranges only over the seven two-body channels.)

In a similar way, corresponding to Eqs. (2.7) and (2.9) we introduce amplitudes M^σ_{ji},

$$M^\sigma = \sum_{j,i\in I(\sigma)} M^\sigma_{ji} \,, \tag{4.3}$$

which satisfy the Faddeev equations,

$$M^\sigma_{ji} = \delta_{ji} t_j + \sum_{k\neq j} t_j G_0 M^\sigma_{ki} \,, \quad i,j,k \in I(\sigma) \,. \tag{4.4}$$

The multiple-scattering interpretation of M^σ_{ji} is that it is the sum of all internal multiple scatterings in the two-body channel σ, that begin with the scattering of the pair i, and end with the scattering of the pair j. The part of M^σ_{ji} that is connected with respect to the internal motion in channel σ is [cf. Eq. (2.22)]

$$\bar{M}^\sigma_{ji} = M^\sigma_{ji} - \delta_{ji} t_j \,, \quad j,i \in I(\sigma) \tag{4.5}$$

and the corresponding operator with no restriction on the first scattering is [cf. Eq. (2.24)]

$$\bar{M}^\sigma_{j-} = \sum_{i\in I(\sigma)} \bar{M}^\sigma_{ji} \,, \quad j \in I(\sigma) \,. \tag{4.6}$$

It is also convenient to introduce generalizations of the operators $\bar{W}_{ji}$ defined in Sec. II. We can loosely characterize the $\bar{W}_{ji}$ in Sec. II as the sum of connected three-particle multiple-scattering terms that begin and end in ways appropriate to the particular channels i and j of the three-particle problem. Now we want to define similar operators that are made from the same ingredients, but that begin and end in ways appropriate to the channels of the *four*-particle problem. Specifically, we define [cf. Eq. (2.23)]

$$\bar{W}^\sigma_{\beta\alpha} = \sum_{j\in E(\beta)\cap I(\sigma)} \; \sum_{i\in E(\alpha)\cap I(\sigma)} \bar{M}^\sigma_{ji} \,, \tag{4.7}$$

so that $\bar{W}^\sigma_{\beta\alpha}$ is the sum of all connected internal multiple scatterings in the two-body channel σ, that begin with an external interaction in channel α, and end with an external interaction in channel β.

From the definition it follows that

$$\bar{W}^\sigma_{\beta\alpha} = 0 \quad \text{if } \beta=\sigma, \text{ or if } \alpha=\sigma \,. \tag{4.8}$$

For the particular case $\alpha = 0$, i.e., when α is the four-free-particle channel, the definition reduces to

$$\bar{W}^\sigma_{\beta 0} = \sum_{j\in E(\beta)\cap I(\sigma)} \; \sum_{i\in I(\sigma)} \bar{M}^\sigma_{ji} \tag{4.9}$$

$$= \sum_{j\in E(\beta)\cap I(\sigma)} \bar{M}^\sigma_{j-} \,, \tag{4.10}$$

so that in this case the first scattering on the right can be between any internal pair in channel σ, without restriction. It follows that the terms in $\bar{W}^\sigma_{\beta\alpha}$ are always a subset of those in $\bar{W}^\sigma_{\beta 0}$, and similarly of those in $\bar{W}^\sigma_{0\alpha}$.

Since the $\bar{W}^\sigma_{\beta\alpha}$ play an important role in the final four-particle equations in Sec. V, it may be helpful to give some examples of Eq. (4.7). Representative examples for the two types of two-body channel σ [see Figs. 1(a) and 1(b)] are

$$\begin{aligned}
\overline{W}^4_{00} &= \overline{M}^4_{(12)-} + \overline{M}^4_{(13)-} + \overline{M}^4_{(23)-}\,,\\
\overline{W}^4_{30} &= \overline{W}^4_{a0} = \overline{W}^4_{(12)0}\\
&= \overline{M}^4_{(13)-} + \overline{M}^4_{(23)-}\\
&\equiv \overline{M}^4_{(13)(12)} + \overline{M}^4_{(13)(13)} + \overline{M}^4_{(13)(23)}\\
&\quad + \overline{M}^4_{(23)(12)} + \overline{M}^4_{(23)(13)} + \overline{M}^4_{(23)(23)}\,, \qquad (4.11)\\
\overline{W}^4_{31} &= \overline{M}^4_{(13)(12)} + \overline{M}^4_{(13)(13)} + \overline{M}^4_{(23)(12)} + \overline{M}^4_{(23)(13)}\,,\\
\overline{W}^a_{00} &= \overline{W}^a_{b0} = \overline{W}^a_{c0}\\
&= \overline{M}^a_{(12)-} + \overline{M}^a_{(34)-}\,,\\
\overline{W}^a_{10} &= \overline{W}^a_{20} = \overline{M}^a_{(12)-}\,.
\end{aligned}$$

The $\overline{W}^\sigma_{\beta 0}$ and $\overline{W}^\sigma_{\beta\alpha}$ satisfy a relation that we shall find useful in Sec. V,

$$\sum_{\beta\neq\sigma\neq\alpha} \overline{W}^\sigma_{\beta 0} G_0(s - H_0 - V^{\sigma\alpha}) = \sum_{\beta\neq\sigma\neq\alpha} \overline{W}^\sigma_{\beta\alpha} + \tau_{\beta\alpha} G_0 V^\alpha\,, \qquad (4.12)$$

for any channels β and α, where

$$\tau_{\beta\alpha} = \sum_{k\in E(\beta)\cap E(\alpha)} t_k \qquad (4.13)$$

and

$$V^{\gamma\alpha} = \sum_{i\in I(\gamma)\cap I(\alpha)} V_i\,, \quad \gamma\neq\alpha\,. \qquad (4.14)$$

The proof of Eq. (4.12) is given below.

The last equation defines $V^{\gamma\alpha}$ as the interaction that is internal in *both* channel γ and channel α. We recall that in the three-particle problem none of the interactions are internal in two different channels, so that this is one respect in which the four-particle problem is more complicated. In the four-particle problem many of the $V^{\gamma\alpha}$ vanish, for example,

$$\begin{aligned}
V^{ab} &= V^{1(12)} = V^{b(12)}\\
&= V^{(14)(12)} = V^{\gamma 0} = 0\,. \qquad (4.15)
\end{aligned}$$

In other cases, however, $V^{\gamma\alpha}$ is not zero; for example,

$$V^{21} = V^{a1} = V^{1(34)} = V^{a(34)} = V_{(34)}\,. \qquad (4.16)$$

In all such cases, $V^{\gamma\alpha}$ consists of a single pair interaction.

To prove Eq. (4.12), we first prove that for any channels β and α, and any two-body channel σ different from β and α,

$$\overline{W}^\sigma_{\beta 0} G_0(s - H_0 - V^{\sigma\alpha}) = \overline{W}^\sigma_{\beta\alpha} + \tau^\sigma_{\beta\alpha} G_0 V^{\sigma\alpha}\,, \quad \beta\neq\sigma\neq\alpha\,, \qquad (4.17)$$

where

$$\tau^\sigma_{\beta\alpha} = \sum_{k\in E(\beta)\cap E(\alpha)\cap I(\sigma)} t_k\,. \qquad (4.18)$$

This is trivial if $V^{\sigma\alpha}=0$. In the other cases, let m be the single pair that is internal to both σ and α, so that $V^{\sigma\alpha}=V_m$, and then use Eq. (4.10) and the identity (2.25) in the form

$$\overline{M}^\sigma_{k-} G_0(s - H_0 - V_m) = \sum_{l\neq m} \overline{M}^\sigma_{kl} + (1-\delta_{km})\, t_k G_0 V_m\,,$$
$$k, l, m \in I(\sigma)\,.$$

The sum over l in the first term is the sum over all l in $E(\alpha)\cap I(\sigma)$, so that on using Eqs. (4.7) and (4.18), Eq. (4.17) is obtained.

The last step of the proof of Eq. (4.12) is to sum Eq. (4.17) over all two-body channels σ different from β and α. The second term can be simplified by using Eqs. (4.14) and (4.18) and interchanging the order of summation, so obtaining

$$\sum_{\beta\neq\sigma\neq\alpha} \tau^\sigma_{\beta\alpha} G_0 V^{\sigma\alpha} = \sum_{k\in E(\beta)\cap E(\alpha)} t_k G_0 \sum_{i\in I(\alpha)} V_i m_{ki}\,,$$

where m_{ki} is the number of two-body channels σ for which the pairs k and i $(k\neq i)$ are both internal. It is easily verified that $m_{ki}=1$ for all pairs k and i. (In other words, specifying two different internal pairs k and i uniquely determines a two-body channel σ.) Then on using Eqs. (4.13) and (4.1) this term becomes $\tau_{\beta\alpha} G_0 V^\alpha$, and Eq. (4.12) is proved.

V. FOUR-PARTICLE SCATTERING EQUATIONS

Let us denote the full four-particle T matrix by $T(s)$,

$$T = V + VGV\,, \qquad (5.1)$$

where

$$G = (s - H_0 - V)^{-1}\,, \qquad (5.2)$$

and $V=\sum_i V_i$, the sum being over all six pairs in the four-particle system. We decompose T in the Faddeev manner,

$$T = \sum_{j,i} T_{ji}\,, \qquad (5.3)$$

where

$$T_{ji} = \delta_{ji} V_j + V_j G V_i\,, \qquad (5.4)$$

and then in exactly the same way as in the three-particle case, the T_{ji} satisfy the Faddeev equations

$$T_{ji} = \delta_{ji} t_j + \sum_{k\neq j} t_j G_0 T_{ki}\,. \qquad (5.5)$$

In the four-particle case, however, in contrast to the three-particle case, the iterated kernel of the Faddeev equations still contains disconnected pieces. These correspond to internal scatterings in the various two-body channels discussed in Sec. I, and give rise to δ functions in the kernel even after any number of iterations.

A natural way to avoid this problem is to solve the internal scattering problem for these two-body channels in advance, which of course is what we have already done in the preceding sections, and then try to rearrange the four-particle problem so that the kernel contains only the summed-up amplitudes for these internal scatterings. This problem has already been solved,[7-10] but it is instructive to rederive these equations, using the apparatus set up in the previous sections.

Letting σ be any one of the seven two-body channels, we treat Eq. (5.5) for the moment as an equation not for all of the T_{ji}, but just for those with j in $I(\sigma)$. That is, we rearrange the equation as

$$T_{ji}=\Big(\delta_{ji}t_j+t_jG_0\sum_{k\in E(\sigma)}T_{ki}\Big)+\sum_{\substack{k\in I(\sigma),\\ k\neq j}}t_jG_0T_{ki}\,,\qquad j\in I(\sigma)\,,\tag{5.6}$$

and for the moment treat the first two terms as inhomogeneous terms.

Then on comparing this with Eq. (4.4), which has the same kernel, but an inhomogeneous term that is just the first term of Eq. (5.6), we can write down the solution of (5.6) in terms of the M^σ_{ji},

$$T_{ji}=M^\sigma_{ji}+\sum_{l\in I(\sigma)}M^\sigma_{jl}G_0\sum_{k\in E(\sigma)}T_{ki}\,,\quad j\in I(\sigma)\,.$$

On using (4.5) and (4.6), this becomes

$$T_{ji}=\delta_{ji}t_j+\bar{M}^\sigma_{ji}+\sum_{k\in E(\sigma)}t_jG_0T_{ki}+\sum_{k\in E(\sigma)}\bar{M}^\sigma_{j-}G_0T_{ki}\,,\qquad j\in I(\sigma)\,.\tag{5.7}$$

This is valid for any initial pair i, if we adopt the natural convention that $\bar{M}^\sigma_{ji}$ vanishes when i is not in $I(\sigma)$.

Keeping i and j fixed, we now sum Eq. (5.7) over all two-body channels σ for which $j\in I(\sigma)$, i.e., over all two-body channels for which V_j is an internal interaction. Since there are three such channels for any pair j, the left-hand side and the first term of the right-hand side are simply multiplied by three. The hardest term to work out is the third term on the right-hand side. After changing the order of summation it becomes

$$t_jG_0\sum_{k\neq j}T_{ki}n_{kj}\,,$$

where n_{kj} is the number of two-body channels σ in which the interaction V_k is external, and the interaction V_j is internal. It is easily verified that this number is 2 for all $k\neq j$. Then on using Eq. (5.5) this term becomes $2T_{ji}-2\delta_{ji}t_j$, so that on solving for T_{ji} we finally obtain

$$T_{ji}=\delta_{ji}t_j+\sum_\sigma\bar{M}^\sigma_{ji}+\sum_\sigma\sum_{k\in E(\sigma)}\bar{M}^\sigma_{j-}G_0T_{ki}\,,\tag{5.8}$$

where to simplify the notation we have adopted the natural conventions that $\bar{M}^\sigma_{j-}$ vanishes if j is not in $I(\sigma)$, and $\bar{M}^\sigma_{ji}$ vanishes if either j or i is not in $I(\sigma)$.

Equation (5.8) is equivalent to the four-particle equations obtained previously in Refs. 7–10. Clearly there are six coupled equations (one for each pair j) for a fixed initial pair i. Written out explicitly, the equation for the $j=(12)$ pair and any initial pair i is

$$\begin{aligned}T_{(12)i}&=t_{(12)}\delta_{(12)i}+\bar{M}^3_{(12)i}+\bar{M}^4_{(12)i}+\bar{M}^a_{(12)i}\\&\quad+\bar{M}^3_{(12)-}G_0(T_{(13)i}+T_{(23)i}+T_{(34)i})\\&\quad+\bar{M}^4_{(12)-}G_0(T_{(14)i}+T_{(24)i}+T_{(34)i})\\&\quad+\bar{M}^a_{(12)-}G_0(T_{(13)i}+T_{(14)i}+T_{(23)i}+T_{(24)i})\,.\end{aligned}\tag{5.9}$$

Evidently Eq. (5.8) solves the disconnectedness problem, since there are no δ functions in the kernel after one iteration. (The once iterated kernel contains only terms of the form $\bar{M}^\sigma_{j-}G_0\bar{M}^{\sigma'}_{k-}G_0$, with $\sigma\neq\sigma'$.) In this respect the equations are analogous to the Faddeev equations for the three-particle system, Eq. (2.9). They are also like the Faddeev equations, as we have noted before, in that the solutions are not immediately related to the physical amplitudes for the various elastic, inelastic, and rearrangement processes that can occur. In the three-particle problem, as discussed in Sec. II, Lovelace[2] and AGS[3] have modified the integral equations so that the quantities that appear in the equations are the actual transition operators for the various scattering processes. We shall now carry out a similar procedure for the four-particle problem.

According to Eq. (2.11), the transition operator for the transition from channel α to channel β is[14]

$$U^\pm_{\beta\alpha}=\begin{Bmatrix}v_\beta\\ v_\alpha\end{Bmatrix}+v_\beta Gv_\alpha\,,\tag{5.10}$$

where v_α is the external interaction in channel α,

$$v_\alpha=\sum_{i\in E(\alpha)}V_i\,.\tag{5.11}$$

Clearly v_α is related to V^α [Eq. (4.1)], the internal interaction in channel α, by

$$v_\alpha=V-V^\alpha\,.\tag{5.12}$$

The two different operators defined by Eq. (5.10) give the same values for the physical scattering amplitudes.[14]

Using our discussion of the three-particle case in Sec. II as a guide, we rewrite Eq. (5.10) in a form analogous to Eq. (2.14),

$$U^\pm_{\beta\alpha}=(1-\delta_{\beta\alpha})\begin{Bmatrix}v_\beta-v_{\beta\alpha}\\ v_\alpha-v_{\beta\alpha}\end{Bmatrix}+v_{\beta\alpha}+v_\beta Gv_\alpha\,,\tag{5.13}$$

where $v_{\beta\alpha}$ denotes the interactions that are external in *both* channel β and channel α,

$$v_{\beta\alpha} = \sum_{i\in E(\beta)\cap E(\alpha)} V_i \,. \tag{5.14}$$

Then using Eqs. (5.11) and (5.4) we can write

$$U^{\pm}_{\beta\alpha} = (1-\delta_{\beta\alpha})\begin{Bmatrix} v_\beta - v_{\beta\alpha} \\ v_\alpha - v_{\beta\alpha}\end{Bmatrix} + X_{\beta\alpha}\,, \tag{5.15}$$

where

$$X_{\beta\alpha} = \sum_{j\in E(\beta)} \sum_{i\in E(\alpha)} T_{ji}\,. \tag{5.16}$$

These four-particle operators $X_{\beta\alpha}$ are directly analogous to the three-particle operators W_{ji} in Eqs. (2.15) and (2.16). Our next step is to find the equations satisfied by the $X_{\beta\alpha}$, just as in the three-particle case it was to find the equations satisfied by the W_{ji}. These equations are easily obtained from Eq. (5.8). With the aid of Eqs. (4.7), (4.10), and (4.8) they become

$$X_{\beta\alpha} = \tau_{\beta\alpha} + \sum_{\beta\neq\sigma\neq\alpha} \overline{W}^{\sigma}_{\beta\alpha} + \sum_{\sigma\neq\beta} \overline{W}^{\sigma}_{\beta 0} G_0 X_{\sigma\alpha}\,, \tag{5.17}$$

where

$$\tau_{\beta\alpha} = \sum_{i\in E(\beta)\cap E(\alpha)} t_i\,.$$

Corresponding to the two inhomogeneous terms in Eq. (5.17), we can split $X_{\beta\alpha}$ into two parts,

$$X_{\beta\alpha} = X'_{\beta\alpha} + X''_{\beta\alpha}\,, \tag{5.18}$$

where $X'_{\beta\alpha}$ and $X''_{\beta\alpha}$ satisfy

$$X'_{\beta\alpha} = \tau_{\beta\alpha} + \sum_{\sigma\neq\beta} \overline{W}^{\sigma}_{\beta 0} G_0 X'_{\sigma\alpha}\,, \tag{5.19}$$

$$X''_{\beta\alpha} = \sum_{\beta\neq\sigma\neq\alpha} \overline{W}^{\sigma}_{\beta\alpha} + \sum_{\sigma\neq\beta} \overline{W}^{\sigma}_{\beta 0} G_0 X''_{\sigma\alpha}\,. \tag{5.20}$$

We shall find this separation useful below. In multiple-scattering language, the distinction between $X'_{\beta\alpha}$ and $X''_{\beta\alpha}$ is that $X''_{\beta\alpha}$ contains all terms that are part of terms of form $\overline{W}^{\sigma}_{\beta 0} G_0 \overline{W}^{\sigma'}_{\sigma 0} G_0 \cdots \overline{W}^{\sigma'''}_{\sigma''\alpha}$ (with $\beta\neq\sigma\neq\sigma'\neq\cdots\neq\sigma''\neq\sigma'''\neq\alpha$), whereas $X'_{\beta\alpha}$ contains all terms that cannot be written in this way because they have a single two-body T matrix left over at the right-hand end. It is not too hard to see that these are the only possibilities.

Our final task is to find equations similar to Eq. (5.17) for the full transition operator from channel α to channel β [see Eq. (5.15)]. We recall, however, that there is a degree of freedom at this stage, corresponding to the fact that the physical scattering amplitude is only required on the energy shell. We also recall that in the three-particle case this freedom was usefully exploited by AGS to obtain an equation with a remarkably simple inhomogeneous term [Eq. (2.21)]. In a similar way, we will exploit this freedom in the four-particle case to make the inhomogeneous term as simple as possible.

As in the three-particle case, it turns out to be better not to use either of the definitions in Eq. (5.15), but instead to obtain a new expression by using the Schrödinger equation for the initial or final asymptotic state. The physical asymptotic state Φ_α in channel α satisfies

$$(H_0 + V^{\alpha})\Phi_\alpha = E\Phi_\alpha\,, \tag{5.21}$$

where V^α is the total internal interaction in channel α [see Eq. (4.1)]. Now for any $\beta\neq\alpha$ we can write

$$\begin{aligned} v_\beta - v_{\beta\alpha} &\equiv \sum_{i\in E(\beta)} V_i - \sum_{i\in E(\beta)\cap E(\alpha)} V_i \\ &= \sum_{i\in E(\beta)\cap I(\alpha)} V_i \\ &= V^{\alpha} - V^{\beta\alpha}\,, \quad \beta\neq\alpha\,, \end{aligned}$$

where $V^{\beta\alpha}$ is the interaction that is internal in *both* channels β and α [see Eq. (4.14) and related discussion], so that from (5.21) we obtain

$$(v_\beta - v_{\beta\alpha})\Phi_\alpha = (E - H_0 - V^{\beta\alpha})\Phi_\alpha\,, \quad \beta\neq\alpha\,. \tag{5.22}$$

We therefore define a transition operator $U_{\beta\alpha}$, analogous to the AGS operator in the three-particle problem, by

$$U_{\beta\alpha} = (1-\delta_{\beta\alpha})(s - H_0 - V^{\beta\alpha}) + X_{\beta\alpha}\,. \tag{5.23}$$

The Schrödinger equation (5.22) then assures us that $U_{\beta\alpha}$ has the same on-shell limit as $U^{\pm}_{\beta\alpha}$ [Eq. (5.15)] when $s = E + i\epsilon$ and $\epsilon\to 0+$.

The desired equations for $U_{\beta\alpha}$ are obtained by substituting (5.23) into (5.17), the equations for $X_{\beta\alpha}$. The only difficulty is in simplifying the resulting inhomogeneous term. To simplify this term we use Eq. (4.12), which was derived previously for just this purpose. The equations then become

$$\begin{aligned} U_{\beta\alpha} = {} & (1-\delta_{\beta\alpha})(s - H_0 - V^{\beta\alpha}) + \tau_{\beta\alpha}(1 - G_0 V^{\alpha}) \\ & + \sum_{\sigma\neq\beta} \overline{W}^{\sigma}_{\beta 0} G_0 U_{\sigma\alpha}\,. \end{aligned} \tag{5.24}$$

A feature of this equation, compared to Eq. (5.17), is that there are no longer any three-particle amplitudes in the inhomogeneous term. There is also another simplifying feature, however, that may not be apparent at first sight, namely, that in any practical calculation *the second inhomogeneous term in Eq. (5.24) disappears*. This is because one solves in practice not for the operator $U_{\beta\alpha}$, but for $U_{\beta\alpha}\Phi_\alpha$, where Φ_α is a physical asymptotic state in channel α. But for such a state it is an obvious consequence of the Schrödinger equation (5.21) that

$$(1 - G_0 V^{\alpha})\Phi_\alpha = 0\,, \tag{5.25}$$

in the limit $\epsilon \to 0+$. Therefore the second inhomogeneous term vanishes when it operates on Φ_α.

Since the second inhomogeneous term in (5.24) never appears in practice, it may seem cleaner to leave it out from the start. Thus we are led to define a new transition operator $\tilde{U}_{\beta\alpha}$, which satisfies

$$\tilde{U}_{\beta\alpha} = (1-\delta_{\beta\alpha})(s - H_0 - V^{\beta\alpha}) + \sum_{\sigma\neq\beta} \overline{W}^{\sigma}_{\beta 0} G_0 \tilde{U}_{\sigma\alpha}, \tag{5.26}$$

and which obviously gives the same physical amplitudes as $U_{\beta\alpha}$. We can write down the formal solution of Eq. (5.26) with the aid of Eqs. (5.24) and (5.19),

$$\tilde{U}_{\beta\alpha} = U_{\beta\alpha} - X'_{\beta\alpha}(1 - G_0 V^{\alpha}). \tag{5.27}$$

Then on using (5.23) and (5.18) we obtain

$$\tilde{U}_{\beta\alpha} = (1-\delta_{\beta\alpha})(s - H_0 - V^{\beta\alpha}) + X'_{\beta\alpha} G_0 V^{\alpha} + X''_{\beta\alpha}, \tag{5.28}$$

which now replaces Eq. (5.23) as the formal definition of the off-shell transition operator.

Equation (5.26) is our final set of four-particle scattering equations. The transformations carried out in the previous paragraph remind us that the inhomogeneous term in the equations is far from unique, because of the great freedom that exists in defining the off-shell extension of the scattering amplitudes, but Eq. (5.26) is much the simplest one we have found. This equation has in fact a striking similarity to the three-particle equations of AGS[3] [Eq. (2.21)], particularly if we remember that the $V^{\beta\alpha}$ [the interactions that are internal in both channels β and α, Eq. (4.14)] all vanish in the three-particle case.

VI. SUMMARY

The principal result is Eq. (5.26), a set of four-particle scattering equations for the operator $\tilde{U}_{\beta\alpha}$, where $\tilde{U}_{\beta\alpha}$ is a conveniently defined off-shell extension of the transition operator from channel α to channel β. Here α and β can be channels of any of the four types discussed in Sec. I, and shown diagrammatically in Fig. 1. Thus Eq. (5.26) gives the transition operators for all possible processes in the four-particle system, including those to three- or four-body final states.

However, the summation index σ in Eq. (5.26) ranges only over the seven two-body channels [four of them of the $1+3$ type shown in Fig. 1(a), and three of the $2+2$ type shown in Fig. 1(b)], so that for any fixed initial channel α, the only transition operators that appear on the right-hand side are the operators $\tilde{U}_{\sigma\alpha}$ for transitions to these seven channels.

Therefore the first step in the calculation of transitions from channel α to any final channel β is to solve the coupled integral equations for the $\tilde{U}_{\sigma\alpha}$. If β is a three-body channel, as in Fig. 1(c), or is the four-free-particle channel of Fig. 1(d), the transition operator $\tilde{U}_{\beta\alpha}$ is then obtained by a second application of Eq. (5.26), where now the quantities on the right-hand side are all known. This two-step procedure is directly analogous to the way three-particle breakup is calculated in the Lovelace[2] or AGS[3] formulation of the three-particle problem. The second step is of course not necessary if β is one of the two-body channels, so that the formulation is more direct for the case of two-body final-state channels.

It may be helpful to write out the coupled integral equations for $\tilde{U}_{\sigma\alpha}$ explicitly. For the particular case $\alpha = 1$, corresponding to particle 1 incident on a bound state of 2, 3, 4, the equations are

$$\begin{pmatrix} \tilde{U}_{11} \\ \tilde{U}_{21} \\ \tilde{U}_{31} \\ \tilde{U}_{41} \\ \tilde{U}_{a1} \\ \tilde{U}_{b1} \\ \tilde{U}_{c1} \end{pmatrix} = \begin{pmatrix} 0 \\ s-H_0-V^{21} \\ s-H_0-V^{31} \\ s-H_0-V^{41} \\ s-H_0-V^{a1} \\ s-H_0-V^{b1} \\ s-H_0-V^{c1} \end{pmatrix} + \begin{pmatrix} 0 & w_{12} & w_{13} & w_{14} & w_{1a} & w_{1b} & w_{1c} \\ w_{21} & 0 & w_{23} & w_{24} & w_{2a} & w_{2b} & w_{2c} \\ w_{31} & w_{32} & 0 & w_{34} & w_{3a} & w_{3b} & w_{3c} \\ w_{41} & w_{42} & w_{43} & 0 & w_{4a} & w_{4b} & w_{4c} \\ w_{a1} & w_{a2} & w_{a3} & w_{a4} & 0 & w_{ab} & w_{ac} \\ w_{b1} & w_{b2} & w_{b3} & w_{b4} & w_{ba} & 0 & w_{bc} \\ w_{c1} & w_{c2} & w_{c3} & w_{c4} & w_{ca} & w_{cb} & 0 \end{pmatrix} G_0 \begin{pmatrix} \tilde{U}_{11} \\ \tilde{U}_{21} \\ \tilde{U}_{31} \\ \tilde{U}_{41} \\ \tilde{U}_{a1} \\ \tilde{U}_{b1} \\ \tilde{U}_{c1} \end{pmatrix}, \tag{6.1}$$

where we have used the notation

$$w_{\beta\sigma} = \overline{W}^{\sigma}_{\beta 0} \tag{6.2}$$

to simplify the writing of the kernel.

The ingredients $V^{\beta\alpha}$ and $\overline{W}^{\sigma}_{\beta 0} \equiv w_{\beta\sigma}$ in the equations are defined by Eqs. (4.14) and (4.9). Loosely, $V^{\beta\alpha}$ is the interaction (if any) that is internal in both channels β and α, and $\overline{W}^{\sigma}_{\beta 0}$ is the connected part of the three-particle amplitude for the internal scattering in channel σ, with the restriction that the last interaction on the left is an external interaction in channel β. The classification into internal and external interactions with respect to a particular channel is discussed in Sec. I. We observe that the existence of interactions that are internal in two different channels β and α is one respect in which the four-particle problem (and in general the N-particle problem for $N \geqslant 4$) is essentially more complicated than the three-particle problem.

We have not discussed the consequences of particle identity in this paper, but by analogy with the three-particle case, we can anticipate important practical simplifications if some or all of the particles are identical.

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[13]Following the usual convention in the three-particle problem, we use the term channel to describe the grouping of the particles, not just the physically possible initial and final states. Thus channel 4, for example, is defined as the grouping 4+(123), independently of whether or not there is a bound state of 1, 2, 3.

[14]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964), Sec. 4.1. Note that their notation is different from that in Eq. (2.11), and in particular that their use of a superscript minus sign has a quite different significance. Our notation follows Lovelace (Ref. 2).

[15]A word of warning: Mitra *et al.* (Ref. 8) assume without comment, in proving their Eq. (12), that there are no bound states of the pair kl. Also, the factor $\langle \vec{p}_{ij}'' | t_{ij}^{+}(E - Q'^2 - p_{kl}^2) | \vec{p}_{ij}' \rangle$ is missing from the integrand of the last term of their Eq. (12).