cross section. [See also C. A. Piketty and J. Procureur, Nucl. Phys. B26, 390 (1971).]
[18]This is illustrated in Fig. 4 of Ref. 5 where total cross sections resulting from two different giant-resonance models are shown, one including the small terms, the other excluding them. The pseudoscalar is negligible for $l=e$ in any case, and for $l=\mu$ contributes noticeably ($\leq 15\%$) around $E_\nu \sim 1$ GeV only in the νn reaction, due to its additional one-pion exchange form factor [see, e.g., Y. I. Azimov and V. M. Shekhter, Soviet Phys. JETP 14, 424 (1962)]. In addition, it is shown in Ref. 11 that the pseudoscalar and weak magnetic terms can be sizable at most at large angles of the emitted lepton, but are small near forward angles. Since the angular distributions are strongly forward peaked at high energies, the effect of the small terms on the total cross sections should by negligible here.
[19]H. Überall and P. Uginčius, Phys. Rev. 178, 1565 (1969).
[20]S. S. Hanna, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969).
[21]G. A. Proca and D. B. Isabelle, Nucl. Phys. A109, 177 (1968).
[22]A. Yamaguchi, T. Terasawa, K. Nakahara, and Y. Torizuka, Phys. Rev. C 3, 1750 (1971).
[23]R. E. Segel, S. S. Hanna, and R. G. Allas, Phys. Rev. 139, B818 (1965).
[24]V. Gillet and N. Vinh-Mau, Nucl. Phys. 54, 321 (1964).
[25]F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. A114, 1 (1968).
[26]The Helm-model fit in Ref. 15 is superseded by the present fit for which data up to much higher values of q has become available.
[27]H. Schmid and W. Scholz, Z. Physik 175, 310 (1965).
[28]G. R. Bishop, Phys. Letters 17, 310 (1965).
[29]G. A. Proca, Ph.D. thesis, University of Paris, 1966 (unpublished).
[30]J. W. Lightbody, Ph.D. thesis, University of Maryland, 1970 (unpublished).
[31]C. Werntz and H. Überall, Phys. Rev. 149, 762 (1966).
[32]H. L. Crannell, Phys. Rev. 148, 1107 (1966).
[33]W. Meyerhof, Phys. Rev. Letters 16, 1114 (1966).
[34]T. W. Donnelly, Phys. Rev. C 1, 833 (1970).
[35]P. Antony-Spies, P. P. Delsanto, E. Spamer, A. Goldmann, and O. Titze, Phys. Letters 31B, 632 (1970).
[36]N. Bezic, D. Brajinik, and G. Kernel, Nucl. Phys. A128, 426 (1969).
[37]The angular distribution given in Fig. 2 of Ref. 1 is in error, and is superseded by that of Fig. 6(a) of the present paper.
[38]J. S. O'Connell, T. W. Donnelly, and J. D. Walecka, Phys. Rev. C 6, 719 (1972).

PHYSICAL REVIEW C VOLUME 6, NUMBER 6 DECEMBER 1972

Comparison of Impulse-Approximation and Elementary-Particle Treatments in Nuclear β Decay*

Lloyd Armstrong, Jr., and C. W. Kim
Department of Physics, The Johns Hopkins University, Baltimore, Maryland 21218
(Received 15 June 1972)

We have compared the usual impulse-approximation and the elementary-particle treatments of nuclear processes using nuclear β decays as examples. Both treatments are shown to lead to essentially equivalent results for allowed transitions and for natural-parity forbidden transition of the types $\Delta J^{P_iP_f}=1^-$ and possibly $2^+, 3^-, \ldots$. There exist, however, some differences in detailed structures in small correction terms. In the case of unnatural-parity forbidden transitions, in particular, $J_i^{P_i}(0^-)\rightarrow J_f^{P_f}(0^+)+e^-+\bar{\nu}_e$, there exist some discrepancies between the two approaches, suggesting an important role of meson-exchange corrections in the impulse approximation.

I. INTRODUCTION

There are two alternative methods used to describe nuclear weak processes such as nuclear β decay and muon capture in nuclei. The first method is to apply the usual impulse-approximation treatment (IAT); the second[1] is to treat the nuclei as "elementary particles" (EPT). The first method involves the use of model-inspired nuclear wave functions and various other approximations such as the neglect of meson-exchange and nucleon off-mass-shell effects. The second method involves, in principle, no approximation. The nuclear structure, in this case, is contained in nuclear form factors.

The two approaches are complementary in the sense that IAT is appropriate for the study of nuclear models and structures, and EPT is convenient for the study of basic ideas in weak interactions such as the conserved-vector-current (CVC) and the partially-conserved-axial-vector current (PCAC) hypotheses. In view of the fact that IAT involves the above-mentioned approximations, the use of IAT is expected to be somewhat limited, in particular in the treatment of forbidden transitions where meson-exchange corrections are expected to be large.[2]

In this work we examine the question of validity of IAT by comparing the two approaches and using nuclear β decays as examples. Instead of using the conventional classification of β decay, we divide the transitions into three groups; allowed ($\Delta J^{P_iP_f}=0^+, 1^+$), natural-parity forbidden ($\Delta J^{P_iP_f}=1^-, 2^+, 3^-, \ldots$), and unnatural-parity forbidden ($\Delta J^{P_iP_f}=0^-, 2^-, 3^+, \ldots$) transitions, where ΔJ is the change in nuclear spin, and P_i and P_f are, respectively, the initial and final parity.

In Sec. II we review IAT results for the sake of comparison with EPT results. Self-consistent conditions on the nuclear matrix elements imposed by the CVC and PCAC hypotheses are discussed in both IAT and EPT. In Sec. III we present the EPT versions of the transitions discussed in Sec. II and compare the IAT and EPT results. The results are summarized in Sec. IV.

For the sake of simplicity we neglect the final-state interaction between the outgoing electron and the final nucleus, which modified the electron wave function. The Coulomb corrections to CVC and PCAC, however, will be taken into account.

II. IMPULSE-APPROXIMATION TREATMENT

The transition matrix element for the process $i \to f + e^- + \bar{\nu}_e$ is given by

$$\mathfrak{M} = \frac{G}{\sqrt{2}}[\bar{u}_e(\vec{p}_e)\gamma_\alpha(1+\gamma_5)v_\nu(\vec{p}_\nu)] \times\left\langle f(\vec{p}_f)\left|\int d\vec{x} J_\alpha^{(+)}(\vec{x},0)e^{i\vec{q}\cdot\vec{x}}\right| i(\vec{p}_i)\right\rangle,$$
$$J_\alpha^{(+)}(\vec{x},t) = V_\alpha^{(+)}(\vec{x},t) + A_\alpha^{(+)}(\vec{x},t), \tag{1}$$

where $G \cong 10^{-5}/m_p^2$, and $V_\alpha^{(+)}(\vec{x},t)$ and $A_\alpha^{(+)}(\vec{x},t)$ are, respectively, the charge-raising vector and axial-vector strangeness-conserving hadron weak currents. Also $\vec{p}_i$, $\vec{p}_f$, $\vec{p}_e$, and $\vec{p}_\nu$ are the momenta of the particles denoted by the subscripts, and $\vec{q} = \vec{p}_f - \vec{p}_i = -(\vec{p}_e + \vec{p}_\nu)$.

In the usual IAT, one replaces $V_\alpha^{(+)}(\vec{x},0)$ and $A_\alpha^{(+)}(\vec{x},0)$ in Eq. (1) by[3]

$$\begin{aligned} V_\alpha^{(+)}(\vec{x},0) &= \sum_{a=1}^{A}\Gamma_{V,\alpha}^{(a)}\delta^{(3)}(\vec{x}-\vec{r}^{(a)}),\\ A_\alpha^{(+)}(\vec{x},0) &= \sum_{a=1}^{A}\Gamma_{A,\alpha}^{(a)}\delta^{(3)}(\vec{x}-\vec{r}^{(a)});\\ \Gamma_{V,\alpha}^{(a)} &= i\gamma_4^{(a)}\left[\gamma_\alpha^{(a)}g_V - \frac{\sigma_{\alpha\beta}^{(a)}q_\beta}{2m_p}g_M\right]\tau_+^{(a)},\\ \Gamma_{A,\alpha}^{(a)} &= i\gamma_4^{(a)}\left[\gamma_\alpha^{(a)}g_A + i\frac{2m_p}{m_\pi^2+q^2}q_\alpha g_P\right]\gamma_5^{(a)}\tau_+^{(a)}, \end{aligned} \tag{2}$$

where g_V, g_M, g_A, and g_P are, respectively, the vector, weak-magnetism, axial-vector, and induced-pseudoscalar nucleon form factors. For nuclear β decay it is sufficient to treat the form factors as constants, i.e., $g_V=1$, $g_M=3.7$, $g_A=1.23$, and $g_P=-1.23$.[4] When Eq. (2) is substituted into Eq. (1), the transition matrix element $\mathfrak{M}$ becomes

$$\mathfrak{M} = \frac{G}{\sqrt{2}}[\bar{u}_e(\vec{p}_e)\gamma_\alpha(1+\gamma_5)v_\nu(\vec{p}_\nu)] \times[\langle f|V_\alpha^{(+)}|i\rangle_{\mathrm{IAT}} + \langle f|A_\alpha^{(+)}|i\rangle_{\mathrm{IAT}}];$$
$$\langle f|V_\alpha^{(+)}|i\rangle_{\mathrm{IAT}} \equiv \left\langle\psi_f\left|\sum_{a=1}^{A}\Gamma_{V,\alpha}^{(a)}e^{i\vec{q}\cdot\vec{r}^{(a)}}\right|\psi_i\right\rangle, \tag{3a}$$
$$\langle f|A_\alpha^{(+)}|i\rangle_{\mathrm{IAT}} \equiv \left\langle\psi_f\left|\sum_{a=1}^{A}\Gamma_{A,\alpha}^{(a)}e^{i\vec{q}\cdot\vec{r}^{(a)}}\right|\psi_i\right\rangle, \tag{3b}$$

where ψ_i and ψ_f are, respectively, the initial and final nuclear wave functions; replacement of $|i\rangle$ and $|f\rangle$ by appropriate nuclear wave functions and the integration over the nucleon positions $\vec{r}^{(a)}$, implicit in Eqs. (3a) and (3b), are part of the IAT procedure.

Using the notations

$$\langle\mathcal{O}\rangle \equiv \left\langle\psi_f\left|\sum_{a=1}^{A}\mathcal{O}^{(a)}\tau_\pm^{(a)}\right|\psi_i\right\rangle, \tag{4}$$
$$a_\alpha = (\vec{a}, ia_0), \tag{5}$$

and properties of Dirac matrices, we rewrite Eqs. (3a) and (3b) as follows:

$$\langle f|V_\alpha^{(\pm)}|i\rangle_{\mathrm{IAT}} = g_V(\langle\vec{\alpha}e^{i\vec{q}\cdot\vec{r}}\rangle, i\langle e^{i\vec{q}\cdot\vec{r}}\rangle) - \frac{g_M}{2m_p}(i\vec{q}\times\langle\vec{\sigma}e^{i\vec{q}\cdot\vec{r}}\rangle + W_0\langle\gamma_4\vec{\alpha}e^{i\vec{q}\cdot\vec{r}}\rangle, -i\langle\gamma_4\vec{\alpha}e^{i\vec{q}\cdot\vec{r}}\rangle\cdot\vec{q}), \tag{6a}$$

$$\langle f|A_\alpha^{(\pm)}|i\rangle_{\mathrm{IAT}} = g_A(-\langle\vec{\sigma}e^{i\vec{q}\cdot\vec{r}}\rangle, i\langle\gamma_5 e^{i\vec{q}\cdot\vec{r}}\rangle) - \frac{2m_p g_P}{m_\pi^2+q^2}(\langle\gamma_4\gamma_5 e^{i\vec{q}\cdot\vec{r}}\rangle\vec{q}, iq_0\langle\gamma_4\gamma_5 e^{i\vec{q}\cdot\vec{r}}\rangle), \tag{6b}$$

where

$$q_0 = (p_f - p_i)_0 \cong m_f - m_i \equiv -W_0. \tag{7}$$

Next, we shall obtain self-consistent relationships among the matrix elements in Eqs. (6a) and (6b), imposed by the CVC and PCAC hypotheses. We assume the CVC hypothesis to be given by[5]

$$V_\alpha^{(\pm)}(\vec{x},t) = \pm[j_\alpha^{\mathrm{e.m.}}(\vec{x},t), I^{(\pm)}(t)], \tag{8}$$

where $j_\alpha^{\text{e.m.}}(\vec{x}, t)$ is the electromagnetic current and $I^{(\pm)}(t)$ is the isospin-raising (-lowering) operator. In the absence of electromagnetic and weak interactions $I^{(\pm)}(t)$ does not depend on time, i.e., it is a constant of motion. From Eq. (8) we have, since $\partial_\alpha j_\alpha^{\text{e.m.}}(\vec{x}, t) = 0$,

$$\partial_\alpha V_\alpha^{(\pm)}(\vec{x}, t) = \pm\left[j_0^{\text{e.m.}}(\vec{x}, t), \frac{dI^{(\pm)}(t)}{dt}\right]$$
$$= \pm i[j_0^{\text{e.m.}}(\vec{x}, t), [H, I^{(\pm)}(t)]] \,. \tag{9}$$

For practical purposes we further assume[6]

$$[H, I^{(\pm)}(t)] \cong [H_c, I^{(\pm)}(t)] \tag{10}$$

and

$$H_c \cong \text{const} + \Delta_C I^{(3)}, \tag{11}$$

where H_c is the charge-dependent part of the Hamiltonian and $I^{(3)}$ the third component of the isospin operator.

In Eq. (11), Δ_C represents the energy difference arising from the Coulomb interaction and the n-p difference, i.e.,

$$\Delta_C \cong \begin{cases} W_0 & \text{when } i \text{ and } f \text{ are members of an isomultiplet} \quad (12\text{a}) \\ \dfrac{\alpha Z}{R} - m_n + m_p & \text{when } i \text{ and } f \text{ are not members of an isomultiplet,} \quad (12\text{b}) \end{cases}$$

where α is the fine structure constant, Z the charge of the initial nucleus, and $R \cong 0.8A^{1/3}/m_\pi$ is the nuclear radius. Substituting Eqs. (10) and (11) into Eq. (9) leads to

$$\partial_\alpha V_\alpha^{(\pm)}(\vec{x}, t) = \pm i\Delta_C[j_0^{\text{e.m.}}(\vec{x}, t), [I^{(3)}, I^{(\pm)}(t)]]$$
$$= i\Delta_C[j_0^{\text{e.m.}}(\vec{x}, t), I^{(\pm)}(t)] \,, \tag{13}$$

which reduces to, when Eq. (8) is used,

$$\partial_\alpha V_\alpha^{(\pm)}(\vec{x}, t) = \pm i\Delta_C V_0^{(\pm)}(\vec{x}, t) \quad \text{for } \beta^\mp \text{ decay} \,. \tag{14}$$

Equation (14) is the modified CVC relation to be used in the following discussion; in the absence of electromagnetic interactions the right-hand side of Eq. (14) vanishes, leading to the usual CVC relation.

Multiplying the matrix element of Eq. (14) by $e^{iq\cdot x}$ and taking the integration over d^4x, we have

$$\int d^4x \langle f | \partial_\alpha V_\alpha^{(\pm)}(\vec{x}, t) | i \rangle e^{iq\cdot x} = \pm i\Delta_C \int d^4x \langle f | V_0^{(\pm)}(\vec{x}, t) | i \rangle e^{iq\cdot x} \,. \tag{15}$$

After integration by part of the left-hand side of Eq. (15) and with use of Eq. (3a), we obtain

$$-iq_\alpha \langle f | V_\alpha^{(\pm)} | i \rangle_{\text{IAT}}\, 2\pi\delta(E_i - E_f - E_e - E_\nu) = \pm i\Delta_C 2\pi\delta(E_i - E_f - E_e - E_\nu) \langle f | V_0^{(\pm)} | i \rangle_{\text{IAT}} \,, \tag{16}$$

which reduces to, since the energy is conserved, i.e., $\delta(E_i - E_f - E_e - E_\nu) = \delta(0) \neq 0$,

$$q_\alpha \langle f | V_\alpha^{(\pm)} | i \rangle_{\text{IAT}} = \mp \Delta_C \langle f | V_0^{(\pm)} | i \rangle_{\text{IAT}} \quad \text{for } \beta^\mp \text{ decay} \,. \tag{17}$$

Note that the matrix elements in Eq. (17) are precisely the same as the ones given in Eq. (6a). Combining Eqs. (6a) and (17), we find

$$\langle \vec{\alpha} e^{i\vec{q}\cdot\vec{r}} \rangle \cdot \vec{q} + W_0 \langle e^{i\vec{q}\cdot\vec{r}} \rangle \cong \mp \Delta_C \left[\langle e^{i\vec{q}\cdot\vec{r}} \rangle + \frac{g_M}{2m_p} \langle \gamma_4 \vec{\alpha} e^{i\vec{q}\cdot\vec{r}} \rangle \cdot \vec{q} \right] . \tag{18}$$

Equation (18) is the self-consistent condition on the nuclear vector matrix elements imposed by the CVC relation (14). Importance of the modification of the CVC relation due to the electromagnetic interactions is evident from Eq. (18); the correction term (the right-hand side) is as large as the individual terms on the left-hand side. The contribution of the g_M term in Eq. (18) is negligible; hence it will be dropped in the following discussion.

The numerical value of g_P quoted below Eq. (2) is an estimate based on the application of the PCAC hypothesis to nucleon case. As in the case of the vector current, one can apply the PCAC relation directly to the nuclear axial-vector matrix element of Eq. (6b) to obtain a self-consistent condition. In this work we assume the Gell-Mann–Levy version of PCAC[7]:

$$\partial_\alpha A_\alpha^{(\pm)}(\vec{x},t)=a_\pi m_\pi{}^3\varphi_\pi^{(\pm)}(\vec{x},t)\,, \tag{19}$$

where $a_\pi = 0.94$ is the pion decay constant and $\varphi_\pi^{(\pm)}(\vec{x},t)$ is the pion field. Following the procedure given in Eq. (15), we obtain

$$-iq_\alpha\langle f|A_\alpha^{(\pm)}|i\rangle_{\mathrm{IAT}}=\frac{a_\pi m_\pi{}^3}{q^2+m_\pi{}^2}\left\langle f\left|\int d\vec{x}\,j_\pi^{(\pm)}(\vec{x},0)e^{i\vec{q}\cdot\vec{x}}\right|i\right\rangle, \tag{20}$$

where

$$(-\Box^2+m_\pi{}^2)\varphi_\pi^{(\pm)}(x)=j_\pi^{(\pm)}(x)\,.$$

The left-hand side of Eq. (20) is the IAT result, but we have not yet used IAT on the right-hand side. In contrast to the application of IAT to the matrix elements of weak currents, the use of IAT in the evaluation of strong interaction matrix elements such as the one on the right-hand side of Eq. (20) is, in general, not justified. Assuming, nevertheless, that IAT is a valid one, we can write

$$\left\langle f\left|\int d\vec{x}\,j_\pi^{(\pm)}(\vec{x},0)e^{i\vec{q}\cdot\vec{x}}\right|i\right\rangle\cong\frac{2m_p}{m_\pi}f_{\pi np}(q^2)\left\langle\psi_f\left|\sum_{a=1}^{A}i\gamma_4^{(a)}\gamma_5^{(a)}\tau_\pm^{(a)}e^{i\vec{q}\cdot\vec{r}^{(a)}}\right|\psi_i\right\rangle_{\mathrm{IAT}}, \tag{21}$$

where $f_{\pi np}(q^2)$ is the pseudovector π-n-p coupling form factor.

From Eqs. (6b), (20), and (21), we obtain[8]

$$-\langle\vec{\sigma}e^{i\vec{q}\cdot\vec{r}}\rangle\cdot\vec{q}+W_0\langle\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle+2m_p\left(\frac{q^2}{m_\pi{}^2+q^2}\right)\langle\gamma_4\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle\cong-2m_p\left(\frac{m_\pi{}^2}{m_\pi{}^2+q^2}\right)\langle\gamma_4\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle\,, \tag{22}$$

where we have used the *nucleon* Goldberger-Treiman relation,[9]

$$g_A=a_\pi f_{\pi np}(0)\cong a_\pi f_{\pi np}(q^2)\quad\text{for }|q^2|\lesssim W_0{}^2\ll m_\pi{}^2\,.$$

Equation (22) further reduces to

$$\langle\vec{\sigma}e^{i\vec{q}\cdot\vec{r}}\rangle\cdot\vec{q}-W_0\langle\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle\cong2m_p\langle\gamma_4\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle\,. \tag{23}$$

This is the self-consistent condition imposed by the PCAC relation (19) on the nuclear axial-vector matrix elements provided that the use of IAT for the strong-interaction matrix element, Eq. (21), is justified. In contrast to the case of the vector current, however, Eq. (23) does not lead to any useful relationship between nuclear matrix elements. Since we have, nonrelativistically,

$$\gamma_4^{(a)}\gamma_5^{(a)}\cong\frac{1}{2m_p}\vec{\sigma}^{(a)}\cdot\vec{q}\,, \tag{24}$$

which may be verified from

$$u(\vec{p}_f)^+\gamma_4\gamma_5u(\vec{p}_i)=\chi_f^+\frac{1}{2m_p}\vec{\sigma}\cdot(\vec{p}_f-\vec{p}_i)\chi_i\,,$$

Eq. (23) reduces to

$$W_0\langle\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle\cong0\,. \tag{25}$$

Equation (25) simply indicates that $\langle\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle\cong0$ within the accuracy of Eq. (23), i.e., in the approximation of neglecting the meson-exchange and electromagnetic corrections. To be consistent with the vector case, we consider the electromagnetic corrections to Eq. (19).

In the presence of the electromagnetic interaction, Eq. (19) is modified as, to the lowest order in e,[10]

$$(\partial_\alpha\pm ie\mathcal{A}_\alpha)A_\alpha^{(\pm)}=a_\pi m_\pi{}^3\varphi_\pi^{(\pm)}\quad\text{for }\beta^\mp\text{ decay}\,, \tag{26}$$

where $\mathcal{A}_\alpha$ is the electromagnetic vector potential. To estimate the order of magnitude of this correction, we consider only the Coulomb correction and assume for simplicity that $\mathcal{A}_\alpha$ is *effectively* given by[8]

$$\mathcal{A}_\alpha\cong\left(0,i\frac{eZ}{R}\right). \tag{27}$$

It is easy, then, to see from Eqs. (26) and (27) that Eq. (25) is modified as

$$W_0\langle\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle\cong\mp\frac{\alpha Z}{R}\langle\gamma_5e^{i\vec{q}\cdot\vec{r}}\rangle\,. \tag{28}$$

Since $RW_0\sim\alpha Z$, for β^+ decay Eq. (28) is a trivial identity and for β^- decay Eq. (28) is again equivalent to Eq. (25). At any rate, as pointed out in Ref. 8, the PCAC relation when applied to nuclear cases in IAT, does not lead to any useful relationships between nuclear matrix elements, unless meson-exchange corrections are explicitly worked out. [If we take Eq. (25) or (28) seriously, this would imply that the space part is dominant over the time part in Eq. (39b) below and $f_1(0)=0$ in Eq. (69b) below. We do not, however, consider this possibility any further.]

A. $\Delta J^{P_i P_f}=0^+$

For simplicity and definiteness we take $^{14}O(0^+) \to {}^{14}N^*(0^+)+e^+ + \nu_e$, where ^{14}O and $^{14}N^*$ are members of an isotriplet.

When only the leading terms are kept, Eqs. (6a) and (6b) reduce to

$$\langle 0^+ | V_\alpha^{(-)} | 0^+\rangle_{\rm IAT} \cong g_V \langle 1\rangle \left(\frac{\langle i\vec{\alpha}\cdot\vec{r}\rangle}{\langle 1\rangle}\vec{q}, i\right), \tag{29a}$$

$$\langle 0^+ | A_\alpha^{(-)} | 0^+\rangle_{\rm IAT} = 0 . \tag{29b}$$

The CVC relation (18) becomes, with Eq. (12a),

$$\langle i\vec{\alpha}\cdot\vec{r}\rangle \vec{q}^{\,2} + W_0\langle 1\rangle \cong W_0 \langle 1\rangle , \tag{30}$$

implying that $\langle i\vec{\alpha}\cdot\vec{r}\rangle \cong 0$ to order of the accuracy of Eq. (18). This estimate is not inconsistent with the estimate[11]

$$\frac{\langle i\vec{\alpha}\cdot\vec{r}\rangle}{\langle 1\rangle} \sim (W_0 R)R \sim \frac{10^{-3}}{W_0}, \tag{31}$$

since the terms of order $\langle i\vec{\alpha}\cdot\vec{r}\rangle\vec{q}^{\,2}$ may have been dropped in Eq. (18) because of the approximations involved in Eqs. (10) and (11).

When the initial and final states are not members of an isomultiplet such as $^{66}Ga(0^+) \to {}^{66}Zn(0^+) + e^+ + \nu_e$, we have $\langle 1\rangle = 0$, so that the CVC relation (18) yields, as in the previous case, no information on $\langle i\vec{\alpha}\cdot\vec{r}\rangle$ except that it is very small. When $J_i = J_f \neq 0$, we have, in addition to Eq. (29), a contribution from the axial-vector part.

B. $\Delta J^{P_i P_f}=1^+$

We consider the decay $1^+ \to 0^+ + e^- + \bar{\nu}_e$ as an example. We find from Eqs. (6a) and (6b), keeping again only the leading terms,

$$\langle 0^+ | V_\alpha^{(+)} | 1^+\rangle_{\rm IAT} \cong \left(-i \frac{(1+g_M)}{2m_p}\vec{q}\times\langle\vec{\sigma}\rangle, 0\right), \tag{32a}$$

$$\langle 0^+ | A_\alpha^{(+)} | 1^+\rangle_{\rm IAT} \cong (-g_A\langle\vec{\sigma}\rangle, 0) , \tag{32b}$$

where we have used the relation which holds true between nucleon spinors,

$$\vec{\alpha}^{(a)} \cong \frac{1}{2m_p}[\vec{p}_i^{(a)} + \vec{p}_f^{(a)} - i\vec{q}\times\vec{\sigma}^{(a)}] .$$

The CVC relation (18) becomes trivial, since both $q_\alpha\langle 0^+ | V_\alpha^{(+)} | 1^+\rangle_{\rm IAT}$ and $\langle 0^+ | V_0^{(+)} | 1^+\rangle_{\rm IAT}$ vanish.

We remark that for transitions of the type $\Delta J^{P_i P_f} = 1^+$, $q_\alpha\langle f | A_\alpha^{(+)} | i\rangle_{\rm IAT} \neq 0$ in general, since $q_\alpha\langle f | A_\alpha^{(+)} | i\rangle_{\rm IAT} \propto \langle \gamma_4\gamma_5 e^{i\vec{q}\cdot\vec{r}}\rangle \neq 0$. It is well known that the axial-vector matrix element in Eq. (32) is subject to the meson-exchange correction. In the case of the superallowed transitions of very light nuclei, the correction is of order $5 \sim 10\%$. Very little is known about the correction for medium and heavy nuclei, but it is expected to be small compared to $\langle\vec{\sigma}\rangle$.

C. $\Delta J^{P_i P_f}=1^-, 2^+, 3^-, \ldots$

We consider the transition $1^- \to 0^+ + e^- + \bar{\nu}_e$ as an example. From Eqs. (6a) and (6b) we obtain

$$\langle 0^+ | V_\alpha^{(+)} | 1^-\rangle_{\rm IAT} \cong g_V(\langle\vec{\alpha}\rangle, i\langle i\vec{r}\rangle\cdot\vec{q}) , \tag{33a}$$

$$\langle 0^+ | A_\alpha^{(+)} | 1^-\rangle_{\rm IAT} \cong g_A(\tfrac{1}{2}\langle i\vec{\sigma}\times\vec{r}\rangle\times\vec{q}, 0) . \tag{33b}$$

The CVC relation (18) is given by[11]

$$\langle\vec{\alpha}\rangle = -\left[W_0 + \left(\frac{\alpha Z}{R} - m_n + m_p\right)\right]\langle i\vec{r}\rangle , \tag{34}$$

which fixes the ratio of the two matrix elements in Eq. (33a). The PCAC relation (23) is, in this case, a trivial one, since

$$\vec{q}\cdot(\langle i\vec{\sigma}\times\vec{r}\rangle\times\vec{q}) = 0 \tag{35}$$

or more generally

$$\langle\vec{\sigma}e^{i\vec{q}\cdot\vec{r}}\rangle\cdot\vec{q} = \langle\gamma_5 e^{i\vec{q}\cdot\vec{r}}\rangle = \langle\gamma_4\gamma_5 e^{i\vec{q}\cdot\vec{r}}\rangle = 0 . \tag{36}$$

However, it is interesting to note that in this case $q_\alpha\langle f | A_\alpha^{(+)} | i\rangle_{\rm IAT} = 0$ even in the presence of the electromagnetic interactions. In fact, for all transitions of the types $\Delta J^{P_i P_f} = 1^-, 2^+, 3^-, \ldots$, we have

$$q_\alpha\langle f | A_\alpha^{(\pm)} | i\rangle_{\rm IAT} = 0 . \tag{37}$$

This can be seen from the fact [Eqs. (20) and (21)] that

$$q_\alpha\langle f | A_\alpha^{(\pm)} | i\rangle_{\rm IAT} \propto \langle\gamma_4\gamma_5 e^{i\vec{q}\cdot\vec{r}}\rangle = 0 \quad \text{for } \Delta J^{P_i P_f} = 1^-, 2^+, 3^-, \ldots . \tag{38}$$

(Remember that the final nuclear recoil is neglected in IAT.) The electron spectrum calculated from Eq. (33), with the modifications due to the Coulomb interaction is known to be in agreement with experiment,[12] implying that the meson-exchange correction is probably insignificant in this case.

D. $\Delta J^{P_i P_f}=0^-, 2^-, 3^+, \ldots$

We use the transition $^{144}Pr(0^-) \to {}^{144}Nd(0^+) + e^- + \bar{\nu}_e$ as an example. From Eqs. (6a) and (6b), we have

$$\langle 0^+ | V_\alpha^{(+)} | 0^-\rangle_{\rm IAT} = 0 , \tag{39a}$$

$$\langle 0^+ | A_\alpha^{(+)}(0) | 0^-\rangle_{\rm IAT} \cong g_A(-\langle i\vec{\sigma}\cdot\vec{r}\rangle\vec{q}, i\langle\gamma_5\rangle) = g_A\langle\gamma_5\rangle\left(-\frac{\langle i\vec{\sigma}\cdot\vec{r}\rangle}{\langle\gamma_5\rangle}\vec{q}, i\right) . \tag{39b}$$

In Eq. (39b) we have neglected the contribution of the g_P term, since from Eq. (24)

$$(2m_pW_0/m_\pi^2)|\langle\gamma_4\gamma_5\rangle|\sim(W_0/m_\pi)^2\ll|\langle\gamma_5\rangle|\sim(m_\pi/m_p)\,.$$

The usual estimate[11] of the ratio $\langle i\vec{\sigma}\cdot\vec{r}\rangle/\langle\gamma_5\rangle$ based on the Ahrens-Feenberg approximation is

$$\left|\frac{\langle i\vec{\sigma}\cdot\vec{r}\rangle}{\langle\gamma_5\rangle}\right|\sim\frac{R}{RW_0}\sim\frac{1}{W_0}\,, \tag{40}$$

implying that the space and time components in Eq. (39b) are comparable in their magnitude. However, since the nuclear matrix elements in Eq. (39) are small, the meson-exchange corrections are expected to be relatively large. In this connection we wish to point out that for the transition $\Delta J^{P_iP_f}=0^-$, $q_\alpha\langle f|A_\alpha^{(\pm)}|i\rangle_{\rm IAT}\neq 0$. In fact, the relation $q_\alpha\langle f|A_\alpha^{(\pm)}|i\rangle_{\rm IAT}\neq 0$ holds true, as can be seen from Eq. (38), for all transitions of the types $\Delta J^{P_iP_f}=0^-,2^-,3^+,\ldots$.

III. ELEMENTARY-PARTICLE TREATMENT

In EPT the transition matrix element for the process $i\to f+e^-+\bar{\nu}_e$ is given by, from Eq. (1),

$$\begin{aligned}\mathfrak{M}=&\frac{G}{\sqrt{2}}(2\pi)^3\delta(\vec{p}_i-\vec{p}_f-\vec{p}_e-\vec{p}_\nu)\\ &\times[\bar{u}_e(\vec{p}_e)\gamma_5(1+\gamma_5)v_\nu(\vec{p}_\nu)]\\ &\times\langle f|V_\alpha^{(+)}(0)+A_\alpha^{(+)}(0)|i\rangle_{\rm EPT};\end{aligned} \tag{41}$$

hence the hadron part is characterized by $\langle f|V_\alpha^{(+)}(0)|i\rangle_{\rm EPT}$ and $\langle f|A_\alpha^{(+)}(0)|i\rangle_{\rm EPT}$ [compare with Eqs. (3a) and (3b)]. Note that the over-all momentum conservation was absent in IAT. The exact forms of $\langle f|V_\alpha^{(+)}(0)|i\rangle_{\rm EPT}$ and $\langle f|A_\alpha^{(+)}(0)|i\rangle_{\rm EPT}$ depend on the spins and parities of the nuclear states. In EPT the use of the CVC relation (14) and the PCAC relation (26) is straightforward. For convenience, we use, as examples, the same transitions as those used in IAT.

A. $\Delta J^{P_iP_f}=0^+$

The most general hadron matrix elements, consistent with general invariance arguments, for the transition $^{14}O\to{}^{14}N^*+e^++\nu_e$ are

$$\langle 0^+|V_\alpha^{(-)}(0)|0^+\rangle_{\rm EPT}=\frac{1}{m_i+m_f}[F_1(q^2)Q_\alpha+F_2(q^2)q_\alpha]\,, \tag{42a}$$

$$\langle 0^+|A_\alpha^{(-)}(0)|0^+\rangle_{\rm EPT}=0\,;\qquad Q_\alpha=(p_i+p_f)_\alpha\,, \tag{42b}$$

where m_i and m_f are the initial and final nuclear masses. In contrast to IAT, the final nuclear recoil effect is included in EPT.

The CVC relation (14) when applied to Eq. (42a) becomes

$$W_0F_1(q^2)+F_2(q^2)\frac{q^2}{m_i+m_f}\cong W_0F_1(q^2)-\frac{W_0^2}{m_i+m_f}F_2(q^2) \tag{43}$$

or

$$F_2(q^2)\left(\frac{q^2+W_0^2}{m_i+m_f}\right)=0\,. \tag{44}$$

This is the EPT version of Eq. (30) for the $0^+\to 0^+$ transition. Since we do not expect $F_2(q^2)$ to have a pole at $q^2=-W_0^2$, Eq. (44) yields

$$F_2(q^2)=0\,. \tag{45}$$

Taking $\vec{p}_i=0$ (i.e., $\vec{Q}=\vec{q}$), we find, from Eqs. (42a) and (45),

$$\langle 0^+|V_\alpha^{(-)}(0)|0^+\rangle_{\rm EPT}\cong F_1(q^2)\left(\frac{\vec{q}}{m_i+m_f},\,i\right). \tag{46}$$

The nuclear form factor $F_1(q^2)$ can be written as

$$F_1(q^2)=F_1(0)[1-\tfrac{1}{6}aq^2A^{2/3}m_\pi^{-2}+\cdots]\,, \tag{47}$$

where the constant a depends on the nuclei involved and is expected to be of order of unity. For $|q^2|\lesssim W_0^2\ll m_\pi^2$, it is reasonable to take $F_1(q^2)\cong F_1(0)$. In the following we denote $F_i(0)$ as F_i. We now compare Eqs. (29a) and (46). Since $1/(m_i+m_f)\sim 10^{-3}W_0^{-1}$, Eqs. (29a) and (46) are numerically the same if we identify F_1 as $g_V\langle 1\rangle$. The general agreement within the accuracy of order 10^{-3} is due to the approximate conservation of the vector current, i.e., CVC hypothesis. We observe, however, that the structures of the coefficients of $\vec{q}$ in Eqs. (29a) and (46) are different in detail, for we have

$$\frac{\langle i\vec{\alpha}\cdot\vec{r}\rangle}{\langle 1\rangle}\sim(W_0R)R\sim\frac{W_0}{m_\pi^2}A^{2/3} \tag{48}$$

and

$$\frac{1}{m_i+m_f}\sim\frac{1}{2m_p}A^{-1}\,. \tag{49}$$

Equation (49) simply represents the nuclear recoil effect which is absent in IAT and hence in Eq. (48). The disagreement is partly due to the approximate nature of the CVC relation (14).

B. $\Delta J^{P_i P_f} = 1^+$

The most general matrix elements for the process $1^+ \to 0^+ + e^- + \bar{\nu}_e$ are

$$\langle 0^+ | V_\alpha^{(+)}(0) | 1^+ \rangle_{\mathrm{EPT}} = \epsilon_{\alpha\beta\gamma\delta} q_\beta \xi_\gamma Q_\delta \left(\frac{1}{m_i + m_f} \right) \frac{F_M}{2m_p} \cong \left(i\vec{q} \times \vec{\xi} \frac{F_M}{2m_p}, 0 \right), \tag{50a}$$

$$\langle 0^+ | A_\alpha^{(+)}(0) | 1^+ \rangle_{\mathrm{EPT}} = \xi_\alpha F_A + q_\alpha \xi \cdot q \frac{F_P}{m_\pi^2} \cong (\vec{\xi} F_A, 0), \tag{50b}$$

where ξ_α is the polarization four-vector for the initial state. Comparison of Eqs. (32) and (50) shows that they can be placed in a one-one correspondence. Indeed, they are in complete agreement if we identify

$$-g_A \langle \vec{\sigma} \rangle = F_A \vec{\xi}, \tag{51}$$

$$(g_M + 1)/g_A = (F_M / F_A). \tag{52}$$

Equation (51) may be regarded as a definition of the nuclear form factor F_A within the accuracy of neglect of the meson-exchange correction. It is interesting to note that Eq. (52) holds true to the same accuracy, since the left-hand side is 3.8 and the right hand side is, e.g., 4.0 for $^{12}\mathrm{B} \to {}^{12}\mathrm{C} + e^- + \bar{\nu}_e$ and 4.2 for $^6\mathrm{He} \to {}^6\mathrm{Li} + e^- + \bar{\nu}_e$. Thus, we can conclude that IAT and EPT are in agreement to the accuracy of neglect of the meson-exchange correction in this transition.

In Eq. (50b) we have neglected the contribution of the F_P term as in the case of IAT. When Eqs. (26) and (27) are applied directly to Eq. (50b), we have

$$F_A(q^2) + \frac{q^2}{m_\pi^2} F_P(q^2) \cong \frac{a_\pi m_\pi^2}{q^2 + m_\pi^2} f_{\pi if}(q^2) + C F_P(q^2), \tag{53}$$

where

$$C \equiv \left(\frac{\alpha Z}{R W_0} \right) \left(\frac{W_0}{m_\pi} \right)^2 \tag{54}$$

and

$$\langle f | j_\pi(0) | i \rangle = i \frac{f_{\pi if}(q^2)}{m_\pi} \xi \cdot q. \tag{55}$$

The second term on the right-hand side in Eq. (53) represents the Coulomb correction. Since we have $|F_A| \sim |F_P|$ in the normalization of F_P as given in Eq. (50b) [see Eq. (60) below] and $\alpha Z / R W_0 \sim 1$, the correction term is small compared to the leading term, i.e., F_A, but does effect the estimate of F_P, since the correction term is of the same order as that of the F_P term on the left-hand side in Eq. (53).

First, the Goldberger-Treiman relation is given by, setting $q^2 = 0$ in Eq. (53),

$$F_A(0) = a_\pi f_{\pi if}(0) + C F_P(0). \tag{56}$$

Solving Eq. (53) for F_P, we obtain

$$F_P(q^2) = -\frac{m_\pi^2}{m_\pi^2 + q^2} F_A(q^2) \left\{ 1 + \frac{1}{(q^2/m_\pi^2 - C)} \left[(1 + C) - \frac{a_\pi f_{\pi if}(q^2)}{F_A(q^2)} \right] \right\}. \tag{57}$$

Since $F_A(q^2)$ and $f_{\pi if}(q^2)$ are *nuclear* form factors, we can write[13]

$$\begin{aligned} f_{\pi if}(q^2) &\cong f_{\pi if}(0)[1 + a q^2 R^2] \cong f_{\pi if}(0)[1 + a q^2 A^{2/3} m_\pi^{-2}], \\ F_A(q^2) &\cong F_A(0)[1 + b q^2 A^{2/3} m_\pi^{-2}], \quad \text{for } |q^2| \lesssim W_0^2 \ll m_\pi^2, \end{aligned} \tag{58}$$

where a and b are constants of order unity which may vary from nucleus to nucleus. Substituting Eqs. (56) and (58) into Eq. (57) and neglecting terms of order $(W_0/m_\pi)^4$, we obtain

$$F_P(q^2) \cong -\frac{m_\pi^2}{m_\pi^2 + q^2} F_A(q^2) \left\{ 1 + \frac{1}{(q^2/m_\pi^2 - C)} \left[C\left(1 + \frac{F_P(0)}{F_A(0)}\right) + (b - a) A^{2/3} m_\pi^{-2} q^2 \right] \right\}. \tag{59}$$

In the absence of the Coulomb correction, i.e., $C = 0$, Eq. (59) reduces to

$$F_P(q^2) \cong -\frac{m_\pi^2}{m_\pi^2 + q^2} F_A(q^2)[1 + (b - a) A^{2/3}]. \tag{60}$$

In the case of $a=b$, which is realized when $F_A(q^2)$ and $f_{\pi if}(q^2)$ have the same q^2 dependence, Eq. (60) becomes the well-known Nambu PCAC relation[14]

$$F_P(q^2) = -\frac{m_\pi^{\,2}}{m_\pi^{\,2}+q^2}F_A(q^2)\,. \tag{61}$$

We note that a combination of IAT and the *nucleon* PCAC relation also leads to Eq. (61).[15]

In the absence of the Coulomb correction, the ratio of the contributions of the F_A and F_P terms in the matrix element is given by, from Eqs. (50b) and (60),

$$\left|\left(\frac{m_e W_0}{m_\pi^{\,2}}\right)\frac{F_P}{F_A}\right| \cong \left(\frac{m_e W_0}{m_\pi^{\,2}}\right)|1+(b-a)A^{2/3}|\,. \tag{62}$$

Since $W_0 \lesssim 15$ MeV for nuclear β decay, the above ratio is always small, provided that the factor $(b-a)$ is not unusually large. For example, the ratio in Eq. (62) is $\sim 10^{-3}$ for $W_0 = 15$ MeV, $A=12$, and $(b-a)=1$.

In the presence of the Coulomb correction, the $(b-a)A^{2/3}$ term in Eq. (62) is to be modified. From a glance at Eq. (57) and from the fact that $-W_0^{\,2}<q^2<0$ and $C\sim(W_0/m_\pi)^2$, it is clear[16] that the Coulomb correction does not change the term $(b-a)A^{2/3}$ by an order of magnitude. Thus, we neglect the F_P term unless $F_A(0)=0$ or $(b-a)$ is unusually large.

C. $\Delta J^{P_i P_f} = 1^-, 2^+, 3^-, \ldots$

The most general matrix elements for the process $1^- \to 0^+ + e^- + \bar{\nu}_e$ in EPT are

$$\langle 0^+|V_\alpha^{(+)}(0)|1^-\rangle_{\mathrm{EPT}} = \xi_\alpha F_1(q^2) + Q_\alpha \xi\cdot q\frac{F_2(q^2)}{(m_i+m_f)2m_p} + q_\alpha \xi\cdot q\,\frac{F_3(q^2)}{m_\pi^{\,2}}\,, \tag{63a}$$

$$\langle 0^+|A_\alpha^{(+)}(0)|1^-\rangle_{\mathrm{EPT}} = -i\,\epsilon_{\alpha\beta\gamma\delta}\,\xi_\beta\, q_\gamma Q_\delta \frac{F_A(q^2)}{(m_i+m_f)m_\pi} \cong \left(\vec{\xi}\times\vec{q}\,\frac{F_A}{m_\pi},0\right). \tag{63b}$$

The F_P term is absent in Eq. (63b). Direct application of Eq. (14) to Eq. (63a) yields

$$F_1(q^2) + W_0\frac{F_2(q^2)}{2m_p} + \frac{q^2}{m_\pi^{\,2}}F_3(q^2) \cong -\Delta_C\left(\frac{F_2(q^2)}{2m_p} - W_0\,\frac{F_3(q^2)}{m_\pi^{\,2}}\right), \tag{64}$$

where we have used the fact that $\xi_0 = 0$, independent of the spin projection when the nucleus with spin one is at rest. Rewriting Eq. (64), we have

$$\left[F_1(q^2) + (W_0+\Delta_C)\frac{F_2(q^2)}{2m_p}\right] = -\frac{F_3(q^2)}{m_\pi^{\,2}}[q^2 - W_0\Delta_C]\,. \tag{65}$$

Since it is unlikely for the left-hand side to have a zero at $q^2 = W_0\Delta_C$, Eq. (65) implies that $F_3(q^2)$ should have a pole at $q^2 = W_0\Delta_C$, which is in contradiction with experiment. Thus we have from Eq. (65)

$$F_3(q^2) = 0\,,\quad F_1(q^2) = -(W_0+\Delta_C)\frac{F_2}{2m_p}\,. \tag{66}$$

Substituting Eq. (66) into Eq. (63a), we obtain

$$\langle 0^+|V_\alpha^{(+)}(0)|1^-\rangle_{\mathrm{EPT}} \cong \left(F_1\vec{\xi},\, i\vec{q}\cdot\vec{\xi}\,\frac{F_2}{2m_p}\right),$$

with

$$F_1 \cong -\left[W_0 + \left(\frac{\alpha Z}{R} - m_n + m_p\right)\right]\frac{F_2}{2m_p}\,, \tag{67}$$

which is to be compared with the IAT result given in Eqs. (33a) and (34); they are in agreement if we identify $F_1\vec{\xi} = g_V\langle\vec{\alpha}\rangle$. The agreement between the two approaches for the vector matrix elements is ensured by the CVC hypothesis and will, of course, hold true for all transitions of the types $\Delta J^{P_i P_f} = 1^-, 2^+, 3^-, \ldots$.

Comparison of Eqs. (33b) and (63b) shows that the matrix elements of the axial-vector current are of the same form in the two approaches and that the vector and axial-vector matrix elements are related as

$$\langle i\vec{\sigma}\times\vec{r}\rangle/\langle\vec{\alpha}\rangle \cong 2(g_V/g_A)(F_A/F_1)m_\pi^{\,-1}\,. \tag{68}$$

As mentioned already, the meson-exchange correction to this case is probably not important, at least not necessary to explain experimental data. This implies that the meson-exchange corrections would not change Eq. (68) significantly.

The divergencelessness of the axial-vector current in the matrix element is self-evident from Eq. (63b), since $\epsilon_{\alpha\beta\gamma\delta}q_\alpha\xi_\beta q_\gamma Q_\delta=0$. In general, in EPT, the matrix element of $(\partial_\alpha \pm ie\mathfrak{A}_\alpha)A_\alpha^{(\pm)}=a_\pi m_\pi{}^3\varphi_\pi^{(\pm)}$ for the transitions $\Delta J^{P_iP_f}=1^-, 2^+, 3^-, \ldots$ vanish when the nuclear recoil effect is neglected. Whether or not the fact that the axial-vector current is "effectively" conserved has any relevance to the absence of significant meson-exchange corrections in IAT remains to be seen.

D. $\Delta J^{P_iP_f}=0^-, 2^-, 3^+, \ldots$

The EPT version of Eq. (39) is given by

$$\langle 0^+|V_\alpha^{(+)}(0)|0^-\rangle_{\rm EPT}=0\,, \tag{69a}$$

$$\langle 0^+|A_\alpha^{(+)}(0)|0^-\rangle_{\rm EPT}=\left(\frac{1}{m_i+m_f}\right)[f_1(q^2)Q_\alpha+f_2(q^2)q_\alpha]\,. \tag{69b}$$

Applying Eq. (26) to Eq. (69b), we obtain

$$f_1(q^2)+\frac{q^2}{(m_i+m_f)W_0}f_2(q^2)\cong a_\pi\frac{m_\pi{}^2}{m_\pi{}^2+q^2}f_{\pi if}(q^2)-C'f_1(q^2)+C'\left(\frac{W_0}{m_i+m_f}\right)f_2(q^2)\,, \tag{70}$$

where

$$C'=\frac{\alpha Z}{RW_0}\sim 1 \tag{71}$$

and

$$\langle 0^+|j_\pi(0)|0^-\rangle\equiv i\frac{q\cdot Q}{m_\pi(m_i+m_f)}f_{\pi if}(q^2)=i\frac{W_0}{m_\pi}f_{\pi if}(q^2)\,. \tag{72}$$

Following the procedure given in Eqs. (56)–(59), we obtain[17]

$$\begin{aligned}f_2(q^2)=&-\frac{m_\pi{}^2}{m_\pi{}^2+q^2}f_1(q^2)(1+C')\left[\frac{W_0(m_i+m_f)}{m_\pi{}^2}\right]\\&\times\left\{1+\frac{1}{[(q^2/m_\pi{}^2)-C]}\left[C\left(1+\frac{[m_\pi{}^2/W_0(m_i+m_f)]f_2}{(1+C')f_1}\right)+(b-a)A^{2/3}m_\pi{}^{-2}q^2\right]\right\},\end{aligned} \tag{73}$$

where C was defined in Eq. (54).

In the absence of the Coulomb correction, i.e., $C=C'=0$, Eq. (73) reduces to

$$f_2(q^2)\cong-\frac{m_\pi{}^2}{m_\pi{}^2+q^2}f_1(q^2)\left[\frac{W_0(m_i+m_f)}{m_\pi{}^2}\right][1+(b-a)A^{2/3}], \tag{74}$$

whence the ratio of the contributions of the f_2 and f_1 terms in the transition matrix element is given by

$$\left|\frac{m_e}{(m_i+m_f)}\left(\frac{f_2}{f_1}\right)\right|\cong\left(\frac{m_eW_0}{m_\pi{}^2}\right)|1+(b-a)A^{2/3}|\,. \tag{75}$$

The right-hand side of Eq. (75) is in agreement with that of Eq. (62). As discussed already, the Coulomb correction does not change Eq. (75) by an order of magnitude. The ratio in Eq. (75) is, e.g., $\sim 10^{-2}$ for $W_0=3.5$ MeV, $A=144$, and $|b-a|=1$; and $\sim 10^{-4}$ for $W_0=3.5$ MeV, $A=144$, and $a=b$.

Rewriting Eq. (69b), we have

$$\begin{aligned}\langle 0^+|A_\alpha^{(+)}(0)|0^-\rangle_{\rm EPT}&=f_1\left((m_i+m_f)^{-1}\left(1+\frac{f_2}{f_1}\right)\vec{q},\,i\left(1-\frac{W_0}{m_i+m_f}\,\frac{f_2}{f_1}\right)\right)\\&\cong f_1\left((m_i+m_f)^{-1}\left(\frac{f_2}{f_1}\right)\vec{q},\,i\right),\end{aligned} \tag{76}$$

which is to be compared with Eq. (39b).[18] After identifying $f_1 = g_A\langle\gamma_5\rangle$, Eqs. (39b) and (76) are of the same form. However, there is a significant difference between the values of the coefficients of $\vec{q}$. As mentioned already, the space and time components in Eq. (39b) are comparable in their magnitude. On the other hand, the space part in Eq. (76) is, in general, smaller than the time part, as can be seen from Eq. (75). In particular, if IAT is valid in estimating the nuclear pseudoscalar form factor f_2, we have $a = b$ in Eq. (75) and the contribution of the f_2 term and hence the space part are negligibly small, as mentioned below Eq. (75).

The difference between Eqs. (39b) and (76) is then to be interpreted as due to the absence of the meson-exchange correction in IAT. To bring Eqs. (39b) and (76) to an agreement, the meson-exchange correction is expected to modify Eq. (39b) or Eq. (40) in such a way that the space part becomes smaller than the time part. It is possible only when the meson-exchange correction is as large as the leading nuclear matrix elements. Detailed calculation of the meson-exchange correction is necessary to verify that this is indeed the case. At any rate, the above discussion casts some doubt on the previous IAT estimate[19] of the electron spectrum shape for the transition $0^- \to 0^+ + e^- + \bar{\nu}_e$, since the shape factor is sensitive to the ratio of the space and time parts.

IV. SUMMARY

We have reviewed briefly the IAT results for nuclear β decay and compared them with the EPT results.[20] Implications of applying the CVC and PCAC relations to nuclear matrix elements have been discussed in detail in both IAT and EPT. In contrast to the CVC relation, the PCAC relation does not lead to any useful relationship between nuclear matrix elements, the only prediction being that of the value of g_P. In EPT, application of the CVC and PCAC relations leads to relationships between *nuclear* form factors.

For allowed and natural-parity forbidden transitions, no apparently significant discrepancy between the two approaches has been found. There exist, however, some differences in detailed structures in small correction terms. The general agreement in the vector part is attributed to the CVC hypothesis. Meson-exchange corrections in the axial-vector part are not important in the allowed transitions (which is well known to be $5 \sim 10\%$ of $\langle\vec{\sigma}\rangle$) and do not seem to play significant roles in the natural-parity forbidden transitions. It is interesting to note in this connection that for the natural-parity forbidden transitions the axial-vector current is "effectively" conserved, i.e., the matrix element of the divergence of the axial-vector current between the initial and final states vanish.

In the case of unnatural-parity forbidden transitions, at least in the case of $0^- \to 0^+ + e^- + \bar{\nu}_e$, it is shown that neglecting of the meson-exchange correction leads to serious discrepancies between the two approaches in the axial-vector current. To reach an agreement between the results in the two approaches, it is necessary to introduce a significant amount of meson-exchange corrections in the matrix elements. In contrast to the case of natural-parity forbidden transitions, the axial-vector current in this case is not "effectively" conserved. Whether or not the fact that the axial-vector current is "effectively" conserved has any relevance to the absence of significant meson-exchange corrections in IAT remains to be seen.

In conclusion, in view of the foregoing discussion, the IAT results previously used for analysis of the electron energy spectrum shape for the transition $0^- \to 0^+ + e^- + \bar{\nu}_e$ should be reexamined. An EPT calculation of the electron spectrum shape for the transition $0^- \to 0^+ + e^- + \bar{\nu}_e$, including the effects of the final-state Coulomb interaction and the possibility that $\langle\gamma_5\rangle \cong 0$ or $f_1(0) \cong 0$ will be published elsewhere.

---

*Research supported in part by the National Science Foundation.

[1]C. W. Kim and H. Primakoff, Phys. Rev. 139, B1447 (1965); 140, B566 (1965).

[2]See, for example, M. Chemtob and M. Rho, Nucl. Phys. A163, 1 (1971).

[3]In Eq. (2), meson-exchange effects and off-mass-shell effects of nucleons inside the nuclei are neglected. In the following, the term "meson-exchange corrections" refers to all the corrections neglected in Eq. (2).

[4]The numerical value of g_P is based on the normalization $(2m_p/m_\pi^2)q_\alpha\gamma_5$.

[5]We have assumed the isotriplet-vector-current hypothesis, generalized to the case of arbitrary t. To obtain the modification due to the electromagnetic interaction we follow, for example, M. Morita, M. Yamada, J.-I. Fujita, A. Fujii, H. Ohtsubo, R. Morita, K. Ikeda, Y. Yokoo, M. Hiro-Oka, and K. Takahashi, Progr. Theoret. Phys. Suppl. (Kyoto) No. 48, 41 (1971). For a slightly different approach see R. J. Blin-Stoyle and S. K. Nair, Advan. Phys. 15, 494 (1966).

[6]The usual Ahrens and Feenberg approximation may replace this assumption.

[7]M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).

[8]A relation similar to Eq. (22) [or Eq. (28)] was pre-

viously discussed by F. Krmpotic and D. Tadic, Phys. Letters 21, 680 (1966). Their interpretation, however, is different from ours. See, also, the second paper in Ref. 5.

[9]M. J. Goldberger and S. B. Treiman, Phys. Rev. 110, 1178 (1958).

[10]S. Adler, Phys. Rev. 139, B1638 (1965). This approach was followed for the vector current in the second paper in Ref. 5.

[11]See the first paper in Ref. 5, and references therein. See, also, E. J. Konopinski, *The Theory of Beta Radioactivity* (Clarendon Press, Oxford, England, 1966).

[12]J. Sodermann and A. Winther, Nucl. Phys. 69, 369 (1965).

[13]We have assumed that $F_A(0) \neq 0$ and $f_{\pi i f}(0) \neq 0$.

[14]Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[15]See the second paper in Ref. 1.

[16]For β^+ decay, C is negative so that a more careful analysis is necessary.

[17]We have assumed that $f_1(0) \neq 0$ and $f_{\pi i f}(0) \neq 0$. It should be noted that if $f_1(0) = 0$, an entirely different result follows.

[18]Note that in Eq. (76) the contribution of the f_2 term appears to be $[W_0/(m_i + m_f)](f_2/f_1)$, but when combined with the lepton part, it is $[m_e/(m_i + m_f)](f_2/f_1)$.

[19]See T. Nagarajan, M. Ravindranath, and K. Venkata Reddy, Nuovo Cimento 3A, 699 (1971) for the latest list of references.

[20]A preliminary discussion of these results appears in Phys. Letters 41B, 39 (1972).

PHYSICAL REVIEW C VOLUME 6, NUMBER 6 DECEMBER 1972

Study of ^{15}N States by the ^{14}N(d, p) Reaction

A. Amokrane, M. Allab, H. Beaumevieille, and B. Faid
Institut d'Etudes Nucléaires d'Alger, Algérie

and

O. Bersillon, B. Chambon, D. Drain, and J. L. Vidal
Institut de Physique Nucléaire, 69-Villeurbanne, France

(Received 1 August 1972)

The bound states of ^{15}N have been studied by the ^{14}N(d, p) reaction. Absolute differential cross sections were measured at $E_d = 3$ MeV for the 9.05-, 9.152 + 9.155-, 9.22-, 9.76-, and 10.07-MeV states and at $E_d = 3.6$ MeV for the 9.76-, 9.83-, 9.93-, 10.07-, and 10.45-MeV states in ^{15}N. The target was natural nitrogen confined in a differentially pumped gas target. Distorted-wave Born-approximation analysis and Hauser-Feshbach calculations indicate that $l_n = 1$ is involved in the formation of the 9.22-, 9.76-, and probably 10.45-MeV states, whereas $l_n = 2$ transfer is associated with the 9.155-MeV level. An $l_n = 0$ transfer was discerned for the angular distribution associated with the 9.05-MeV state and $l_n = 0 + 2$ for the angular distribution associated with the 10.07-MeV and probably the 9.93-MeV ^{15}N state. Spectroscopic factors have been obtained. The correspondence between mirror levels in ^{15}N and ^{15}O and those predicted within the framework of the weak-coupling model is discussed.

I. INTRODUCTION

In recent papers,[1,2] the negative- and positive-parity states for $A = 15$ have been investigated in a weak-coupling model. The over-all agreement with experimental energies, structure information from direct reactions, and electromagnetic transitions is good for all the known levels below 10 MeV. In addition, several predictions have been made and a one-to-one correspondence between all levels of the $A = 15$ nuclei below 10-MeV excitation energy is suggested. This implies spin and parity assignments of $\frac{5}{2}^+$ for the $J = \frac{5}{2}$ member of the 9.16-MeV doublet, $\frac{1}{2}^-$ for the 9.22-MeV level, and $\frac{3}{2}^-$ for the other member of the 9.16-MeV doublet.

The experimental spectra[3] for ^{15}N and ^{15}O and the positive- and negative-parity states below 10.5 MeV predicted by Lie *et al.*[1,2] are shown in Fig. 1.

The present work was undertaken to complement experimental information on neutron transfer to ^{15}N states between 9.05 and 10.45 MeV and to clarify the spin-parity assignment of these states. Many experimental studies have previously been reported on the ^{14}N(d, p) reactions and corresponding information is compiled in Ref. 3. In particular, a recent investigation[4] of the ^{14}N(d, p) reaction was devoted to levels up to 10.80 MeV excitation in ^{15}N. However, very few experimental data have been reported on the levels in ^{15}N near 9 MeV excitation, probably owing to experimental difficulties associated with the contaminants present in solid targets which obscured these levels. In the