Shell-Model Study of ^{4}He with Realistic Forces

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A shell-model study of the $A=4$ system previously performed with purely central forces was repeated using both the Tabakin and Sussex interactions. Agreement with the experimental spectrum improved. Some shift of the $E1$ strength to the lower of the $1-$, $T=1$ levels occurred, but the bulk of the strength remained in the state at higher energy. Over-all results achieved with the Sussex interaction were comparable to those obtained using the Tabakin potential.

A previous study[1] of the $A=4$ system using the nuclear shell model and a central force with a singlet-even and triplet-even exchange mixture to describe the nucleon-nucleon force achieved considerable success in reproducing the experimentally observed spectrum. However, while the odd-parity levels were at approximately the correct energy, they were not in the correct order. Barrett[2] had previously investigated the odd-parity levels of ^{4}He using various realistic nucleon-nucleon interactions and obtained a large measure of success with the Tabakin[3] interaction. The purpose of the present study was to reinvestigate both the even- and odd-parity spectra of ^{4}He using the Tabakin and Sussex[4] interactions in the nuclear shell model and to calculate the integrated photoabsorption cross sections from the resultant wave functions. The two sets of results would then be compared with the appropriate experimental data, with each other, and with the previous results obtained with a purely central potential.

Since second-order corrections to the nucleon-nucleon interaction can be relatively large, the two-body matrix elements employed here for both the Tabakin and Sussex interactions included the second-order corrections in the same manner as that used by Barrett.[2] In terms of relative coordinates this is

$$\langle (n'l'S)\mathcal{J}T\,|\,t\,|\,(nlS)\mathcal{J}T\rangle = \langle (n'l'S)\mathcal{J}T\,|\,V\,|\,(nlS)\mathcal{J}T\rangle + \sum_{\beta}\frac{\langle (n'l'S)\mathcal{J}T\,|\,V\,|\,\beta\rangle\langle\beta\,|\,V\,|\,(nlS)\mathcal{J}T\rangle}{W_\beta - W_0}\,, \tag{1}$$

where n is the principal quantum number, l the orbital quantum number in relative coordinates, S is the total spin, $\mathcal{J}$ the total angular momentum, and T the isospin of the states. The quantum numbers of the intermediate states $|\beta\rangle$ are n'' and l'' such that $2n''+l''=2n+l+2$ and their energies are $W_\beta = W_0 + 2\hbar\omega$, where W_0 is the energy of both the initial and final states. Excitation to intermediate states of energy differences $4\hbar\omega$ and higher were neglected. Conversion to j-j coupling was accomplished by means of well-known methods involving transformation brackets. (For pertinent formulas refer, for example, to the paper by Kuo and Brown.[5])

The present investigation used an oscillator size parameter $b=1.60$ fm ($\hbar\omega=16.2$ MeV). This was shown in Ref. 1 to produce the most nearly correct shape for the $E0$ electron inelastic scattering form factor to the first excited state of ^{4}He indicating that a larger size parameter than that characteristic of the rms charge radius of ^{4}He ($b=1.38$ fm) might better describe the excited states of ^{4}He. Additional calculations were performed with $b=1.38$ fm and with $b=1.79$ fm, a value suggested in a

TABLE I. Single-particle spectrum (MeV), $b=1.60$ fm.

Interaction	$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$	$2s_{1/2}$	$1d_{5/2}$	$1d_{3/2}$
Tabakin	−19.30	2.35	5.62	13.77	16.76	20.41
Sussex	−20.92	0.55	4.32	13.02	15.42	18.68

study of the correlation energy of ^{4}He by Kuo and McGrory[6] using an interaction derived from the Hamada-Johnston potential. In both cases agreement with experiment was considerably poorer than with $b = 1.60$ fm.

Single-particle energies were computed in j-j coupling from the expression[7]

$$E_\beta^{\text{s.p.}} = \frac{1}{2(2j_\beta+1)} \sum_{\alpha,J,T} (2J+1)(2T+1)(1+\delta_{\alpha\beta})\langle\alpha\beta JT|\, t\,|\alpha\beta JT\rangle + N\frac{\hbar\omega}{2} + \tfrac{9}{16}\hbar\omega\ , \tag{2}$$

where $E_\beta^{\text{s.p.}}$ is the single-particle energy of a nucleon in a state β, the α are single-particle states in the core (just $1s_{1/2}$ orbits for ^{4}He), the second term is the kinetic energy due to the excitation of the state β by N oscillator quanta and $\frac{9}{16}\hbar\omega$ is the "zero point" kinetic energy for a particle in a harmonic-oscillator well corrected for center-of-mass motion in ^{4}He. The last term is included only for completeness since it does not affect the energies of the particle-hole configurations which depend on differences of the single-particle energies. The added advantage of single-particle energies derived in the above manner over those obtained empirically is that the Hamiltonian now is translationally invariant, spurious states of center-of-mass motion separate from genuine excited states and the former are easily identified *a posteriori*. This method of removing spurious states has been applied in the past, for example, in shell-model calculations in oxygen.[8]

The single-particle energies derived from both the Tabakin and Sussex interactions with $b = 1.60$ fm are shown in Table I. It is noteworthy that the $1s_{1/2}$-$1p_{3/2}$ and $1s_{1/2}$-$1p_{1/2}$ energy separations are very close to the empirical values used in Ref. 1. While single-particle energies determined from the Sussex interaction are an average of 1.4 MeV lower than the Tabakin energies, energy separations from the $1s_{1/2}$ state are virtually identical, the greatest difference occurring for the energy separation of the $2s_{1/2}$ state. Energy separations obtained with $b = 1.38$ fm were some 10 MeV higher. While Barrett used this latter oscillator parameter, the bulk of the energy separation of the $1s$ and $1p$ shells in his case was taken to be $\hbar\omega = 21.8$ MeV with additional terms (described in detail in Ref. 2) which split the energy levels and shifted their center of gravity being calculated from the potentials.

The present study did not include matrix elements characteristic of the random-phase approximation which had not contributed materially to the results of Ref. 1.

The energy levels of ^{4}He up to approximately 50 MeV calculated for both the Tabakin and Sussex interactions are compared with the known levels in Fig. 1. Experimental levels are those given by Werntz and Meyerhof.[9] Calculation with the Taba-

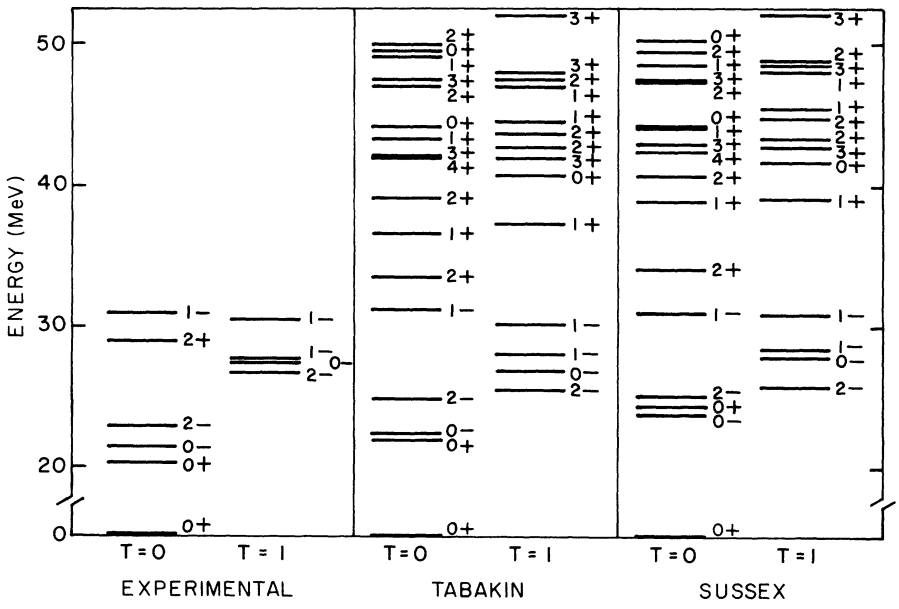

FIG. 1. Comparison of the energy levels of ^{4}He calculated using the Tabakin and Sussex interactions and $b = 1.60$ fm with the observed levels. Experimental data are taken from Fig. 1 of Ref. 9.

kin interaction provides a spectrum more in agreement with experiment than the results of Ref. 1. The first excited 0^+ state is lower in energy (22.0 MeV compared to 26.3 MeV) and the odd-parity levels in addition to having approximately the correct energy, also are in the same order as the corresponding experimental levels. However, the energy separation of the first two excited states of even parity (0^+ and 2^+, $T=0$) has increased from 8 to 11.6 MeV. The spectrum obtained with the Sussex interaction is on the average slightly higher in energy but substantially the same as the Tabakin result. The largest upward shift occurs for the first excited 0^+, $T=0$ state (2.5 MeV) and the 0^- states (about 1 MeV). The 0^-, $T=0$ state is then somewhat lower in energy than the lowest excited 0^+, $T=0$ state. Neither interaction produces a second 0^+, $T=0$ level nor one of 1^+, $T=0$ at 25.5 MeV for which there seems to be some experimental evidence.[10]

Integrated photoabsorption cross sections to the various levels in ^{4}He are given in Table II. The method used to calculate these cross sections is the same as that of Ref. 1 with one significant correction. Wave functions used here (as well as in Ref. 1) are not exact eigenstates of the total Hamiltonian. Harmonic-oscillator functions of $1\hbar\omega$ for the odd-parity states and of $2\hbar\omega$ for those of even parity are used. As a result, the transverse matrix elements used for the electromagnetic transitions would not be consistent with a requirement of current conservation that in the long wave-length limit these matrix elements reduce to the energy of the excited state times the Coulomb matrix element. In the present case the transverse matrix elements were multiplied by $E/N\hbar\omega$ where E is the energy of the state excited by photoabsorption and $N\hbar\omega$ is the oscillator excitation of this same state. This ensures that the correct long wave-length limit is obtained, but consistency with current conservation at energies for which the long-wave approximation is not valid is assured only if the nuclear interaction is a simple Wigner force. An exact correction is difficult to make because of the more complicated forces employed here.

Calculated total cross sections are close to the experimental value of 94 mb MeV, a figure determined by summing the ^{4}He$(\gamma, p)^3$H and ^{4}He$(\gamma, n)^3$He total cross sections reported by Gorbunov[11] to the ^{4}He$(\gamma, pn)^2$H total cross section measured by Gorbunov and Spiridonov.[12] Contributions from the ^{4}He$(\gamma, d)^2$H cross section are negligible. Using the wave functions obtained from the calculation with the Tabakin potential the ratio of the $E1$ strengths of the upper and lower 1^-, $T=1$ levels is 1.9 (a result similar to Barrett's), a considerable reduction from the 5 to 1 ratio determined in Ref. 1, but a result still indicating the bulk of the $E1$ strength in the state at higher energy. Results obtained from the Sussex interaction are similar, the only significant difference being that the same $E1$ strength ratio is 2.4 to 1. Shell-model calculations with either interaction still indicate the presence of a 2^+, $T=0$ state with appreciable quadrupole strength in the vicinity of 30 MeV excitation. Analyses[13,14] of the ^{4}He$(\gamma, d)d$ cross section as a function of energy can be interpreted on the basis of an electric quadrupole transition in which the effect of a final state d-d interaction characterized by a broad resonance at approximately 30 MeV excitation must be considered.

Gogsadze and Kopaleishvili[15] have also determined the total photoabsorption cross section to all levels in ^{4}He using the nuclear shell model. They obtain very good agreement with the experimental total cross section assuming a square-well interaction, $\hbar\omega = 18$ MeV and a ground-state wave function which considers all $2\hbar\omega$ and certain $4\hbar\omega$ excitations in addition to a closed-shell component. For the more realistic Tabakin interaction and a

TABLE II. Calculated integrated photoabsorption cross sections to individual levels in ^{4}He and their total.

Level quantum numbers	Tabakin results		Sussex results	
	Level energy (MeV)	Cross section (mb MeV)	Level energy (MeV)	Cross section (mb MeV)
$J=1^-$, $T=1$	28.2	37.0	28.6	32.8
$J=1^-$, $T=1$	30.2	71.2	31.0	78.7
$J=2^-$, $T=1$	25.6	0.47	25.9	0.51
$J=2^+$, $T=0$	33.6	0.82	34.1	0.86
$J=2^+$, $T=0$	50.0	0.02		
$J=2^+$, $T=0$	52.7	0.02		
$J=2^+$, $T=0$	59.4	0.03		
$J=2^+$, $T=1$	43.7	0.87	44.9	0.96
$J=2^+$, $T=1$	47.7	0.33	48.6	0.31
$J=2^+$, $T=1$	52.5	0.16	52.8	0.16
Total cross section		111		114

closed-shell ground state their result is too high by a factor of 2. Since they do not give contributions to individual levels in ^{4}He and details of their cross-section calculation are lacking, one can only speculate why the present calculation is not more in agreement with theirs. Correction for the difference in $\hbar\omega$ only widens the discrepancy. While it is of interest that their lower 1^-, $T=1$ state[16] has a stronger ^1P component than the upper state, which was not the case in the present calculation nor in that performed by Barrett, it is doubtful that this can account for the difference in the cross section. One can only surmise that the differences might be resolved if they have used some form of the long-wave approximation at the outset.

Our first excited state still contained about 75% of the $|1s_{1/2}^{-1}2s_{1/2}\rangle$ configuration independent of the interaction used (central, Tabakin, or Sussex). While the use of $b=1.60$ fm produces the correct shape for the $E0$ electron inelastic scattering form factor, the magnitude is still some 60% higher than the experimental data, a result identical to that found in Ref. 1.

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