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Structure of the Odd-Mass Gallium Isotopes with a Particle-Phonon Coupling Model

R. Almar, O. Civitarese, F. Krmpotić,* and J. Navaza
Departamento de Física, Facultad de Ciencias Exactas, Universidad de La Plata, La Plata, Argentina
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The structure of the low-lying levels in odd-mass gallium isotopes is investigated within the framework of the semimicroscopic model, assuming three protons coupled to a quadrupole vibrator. A detailed comparison with experiments is performed for ^{69}Ga.

I. INTRODUCTION

The present investigation was stimulated by the following facts:
(1) In recent years, the structure of odd-mass Ga isotopes has been studied experimentally using different methods: the nuclear reactions (^{3}He, d),[1,2] (d, n),[3,4] and (n, $n'\gamma$)[5]; $\beta^{\pm}$ decays[6-8]; nuclear resonance fluorescence[9] and other spectroscopic techniques.[10] The electric quadrupole moment and magnetic dipole moment for the ground states are also known from previous measurements.[10]
(2) Intermediate coupling between particle field and collective modes has been successfully applied to account for the structure of nuclei near closed shells.[11-32] In particular, it is a generally accepted point of view that the low-lying states in the odd-mass Cu isotopes can be described in terms of multipole structure and the mixing of multiplets.[11-18] This fact has also been confirmed in a recent comparison with a shell-model calculation using a realistic effective interaction.[33] It is the purpose of the present analysis to take a further step in the application of the semimicroscopic model and propose it for the description of the gallium odd-mass isotopes.

From the above-mentioned experiments it can be seen that the distribution of the single-particle strengths and electromagnetic properties are similar for the low-lying states in odd-mass gallium isotopes (from $A=65$ to $A=71$) even if their energy spectra change gradually as the $N=40$ subshell is filled up. This effect is due to the excitation modes not contained in the particle-vibrator picture (for example, pairing-vibrational mode).

We shall discuss mainly the ^{69}Ga isotope, whose energy spectrum is better known. Obviously, many results and conclusions will also be valid for the other gallium nuclei.

II. MODEL

Since a detailed description of the model can be found in several review articles,[31] only a brief survey of the main formulas and notation will be presented in this section. In particular, a calculation with three extracore particles has been previously performed by Alaga and Ialongo for the gold isotopes.[30]

The total Hamiltonian of the system is written as

$$H = H_0 + H_p + H_{\rm int}\,, \tag{1}$$

where H_0 represents the energy of the quadrupole harmonic vibrator and that of the extracore protons. The residual interaction between protons, H_p, is approximated by a pairing force. It will affect only the seniority-one states.[34] The interaction Hamiltonian $H_{\rm int}$ is linear in collective and particle variables and has the form

$$H_{\rm int} = -\left(\frac{\hbar\omega}{2C}\right)^{1/2} \sum_{\mu=-2}^{2} [b^{\mu\dagger} + (-)^{\mu} b^{-\mu}] \times \sum_{p} k(r_p) Y_2{}^{\mu *}(\theta_p, \phi_p)\,, \tag{2}$$

where the summation p extends over the outside protons and $k(r) = r dV(r)/dr$. The potential $V(r)$ is the average shell-model potential usually taken as a Woods-Saxon potential. The symbols $b^{\mu\dagger}$ (b^{μ}) are the operators for the creation (destruction) of the quadrupole phonon field.

The quantity $(\hbar\omega/2C)^{1/2}$, where $\hbar\omega$ is the energy of the phonon and C is the restoring force of the vibrator, measures the amplitude of the vibration-

al motion. It is related to the reduced transition probability between the first excited state and the ground state in the nucleus considered as vibrator through the relation

$$B(E2;\,2^+ \to 0^+) = \left(\frac{3}{4\pi} ZeR_0^2\right)\left(\frac{\hbar\omega}{2C}\right)^{1/2} = \left(\frac{3}{4\pi} ZeR_0^2\right)\frac{\beta}{5^{1/2}}, \tag{3}$$

where β is the deformation parameter of the core.

The matrix of H is calculated in the basis

$$|\{[(j_1 j_2)J_{12} j_3]J, NR\}IM\rangle, \tag{4}$$

in which H_0 is diagonal. Here the symbols $j_i \equiv (n_i, l_i, j_i)$ represent quantum numbers of the proton states, and the angular momenta J_{12} and J correspond to successive couplings $\vec{j}_1 + \vec{j}_2 = \vec{J}_{12}$ and $\vec{J}_{12} + \vec{j}_3 = \vec{J}$. The symbols N and R represent the phonon number and angular momentum, respectively; I and M stand for the total angular momentum of the nucleus. The eigenstate n of the total Hamiltonian is a linear combination of the basis vectors:

$$|I_n M\rangle = \sum_{\substack{j_1 j_2 j_3 \\ J J_{12} NR}} \eta_3(j_1 j_2, J_{12}, j_3, J, N, R; I_n) \times |\{[(j_1 j_2)J_{12} j_3]J, NR\}IM\rangle. \tag{5}$$

In order to calculate one-body-transfer reactions the matrix of the Hamiltonian given by Eq. (1) is also calculated for the system of two protons coupled to a vibrator in the same model space. The corresponding eigenstates with total quantum numbers I'_m and M' are expressed as

$$|I'_m M'\rangle = \sum_{\substack{j_1 j_2 J_{12} \\ NR}} \eta_2(j_1, j_2, J_{12}, N, R; I'_m) \times |[(j_1 j_2)J_{12} NR]I'M'\rangle. \tag{6}$$

The spectroscopic factor S_j for a one-nucleon-stripping process from the initial state $|I'_m M'\rangle$ to the final state $|I_n M\rangle$ has the following form:

$$S_j(I'_m, I_n) = (2I'+1)\Bigg(\sum_{\kappa'_2, \kappa_3} (-)^{I+J+j+R}(2J+1)^{1/2} \times \begin{Bmatrix} R & I & J \\ j & J'_{12} & I' \end{Bmatrix} \eta_2(\kappa'_3, I'_m)\eta_3(\kappa_3, I_n) \times \theta_j(J'_{12}, J)\delta_{NN'}\delta_{RR'}\Bigg)^2, \tag{7}$$

where $\kappa_2 \equiv (j_1 j_2 J_{12} NR)$ and $\kappa_3 \equiv (j_1 j_2 J_{12} j_3 JNR)$ and θ_j is the usual shell-model overlap:

$$\theta_j(J'_{12}, J) = \frac{\langle\{[(j_1 j_2)J_{12}]j_3\}J \| a_j^\dagger \| (j'_1 j'_2)J'_{12}\rangle}{(2J+1)^{1/2}}. \tag{8}$$

In particular, when the core is in its ground state we shall have

$$S_j(0_1, I_n) = \Bigg[\sum_{\kappa'_2, \kappa_3}\left(\frac{2J+1}{(2j+1)(2J'_{12}+1)}\right)^{1/2} \eta_2(\kappa'_2, 0_1) \times \eta_3(\kappa_3, I_n)\theta_j(J'_{12}, J)\delta_{RR'}\delta_{NN'}\delta_{R'J'_{12}}\Bigg]^2. \tag{9}$$

It satisfies the sum rule

$$\sum_f S_j(0_1, I_f) = 1 - \frac{\langle p\rangle_j}{2j+1}, \tag{10}$$

where $\langle p\rangle_j$ is the average number of protons in the orbit (nlj) of the target nucleus, defined as

$$\langle p\rangle_j = \sum_{\kappa_2}[\eta_2(\kappa_2, 0_1)]^2(\delta_{jj_1} + \delta_{jj_2}). \tag{11}$$

The electromagnetic operators consist of a particle and a vibrator part and their matrix elements are expressed in the form

$$\langle I_i \| \mathfrak{M}(E2) \| I_f\rangle = A' e_p^{\text{eff}} + B' e_v^{\text{eff}},$$
$$\langle I_i \| \mathfrak{M}(M1) \| I_f\rangle = (C' - D')g_R + (D' - E')g_l + E' g_s^{\text{eff}}, \tag{12}$$

where e_p^{eff} is the effective proton charge, and

$$e_v^{\text{eff}} = Ze\left(\frac{\hbar\omega}{2C}\right)^{1/2} = 5^{-1/2} Ze\beta, \tag{13}$$

is the effective vibrator charge; e is the proton charge and Z is the number of protons in the vibrator. Furthermore, g_R, g_l, and g_s^{eff} are the effective gyromagnetic factors of the core, orbital, and spin part, respectively. The quantities A', B', C', D', and E' are calculated by means of the wave functions given by Eq. (4), and their explicit form is given in Ref. 31.

III. PARAMETERS AND ASSUMPTIONS

The particle-vibrator coupling constant is defined in the usual way

$$a = \langle k\rangle(\hbar\omega/8\pi C)^{1/2}, \tag{14}$$

while the radial integrals are approximated by a constant value

$$\langle r^2\rangle = \tfrac{3}{5}R_0^2,$$

with $R_0 = 1.2A^{1/3}$ fm. For the vibrator energy we take $\hbar\omega = 1.34$ MeV, which corresponds to the energy of 2_1^+ state in ^{64}Ni, and for the pairing constant we use $G = 0.35$ MeV $\approx 23/A$ MeV.

In the selection of the single-particle energies $\epsilon(j)$ and the coupling constant a we have considered the following facts observed in the reactions ^{68}Zn-$(d, n)^{69}$Ga and ^{68}Zn$(^3$He$, d)^{69}$Ga [1-4]:

(i) The energy centroids defined as

$$\bar{\epsilon}(j)=\frac{\sum_f S_j(0_1, I_f)E_f(I_f)}{\sum_f S_j(0_1, I_f)}, \tag{15}$$

i.e., the sum of the excitation energies $E_f(I_f)$ weighted by the spectroscopic factors $S_j(0_1, I_f)$, are nearly degenerate (within 20 keV) for the $p_{1/2}$ and $f_{5/2}$ proton states.
(ii) The ground state and the first two excited states take more than 65% of the corresponding strengths.

Consequently, in order to avoid strong smearing of the single-particle strengths among the higher excited states, it will be necessary to deal with a relatively weak coupling ($a \approx 0.3$–0.4 MeV). Furthermore, if the single-particle spacings $\Delta(j)=\epsilon(j)-\epsilon(\frac{3}{2})$ are not too close to the phonon energy, that situation will be favored, as can be seen from a perturbation expansion (the spreading of the reaction amplitudes is mainly due to phonon emission in the first-order diagrams).

Continuing with this analysis it is also necessary to take into account the quadrupole moment of the ground state. Actually, it has been pointed out recently by Alaga *et al.*[32] that the quadrupole moment of the coupled system in the case of one and two extracore particles is governed by the quadrupole moment of the dominant particle configurations and the relative competition between them.

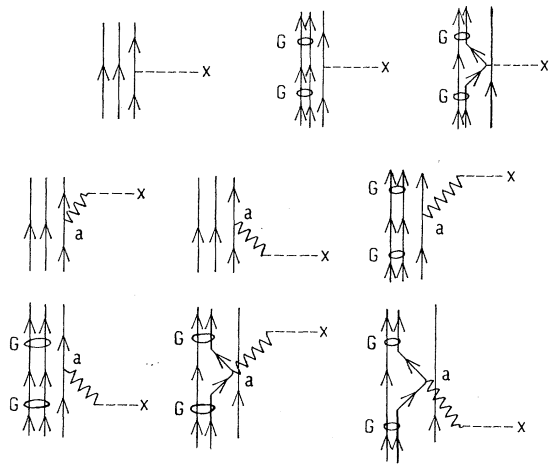

FIG. 1. Lowest-order diagrams that contribute to the electric quadrupole moment of a dominant particle state [Eq. (17)]. Time runs upwards. A line directed up represents a particle. One line running downwards would represent a hole. The quadrupole collective mode is represented by a wavy line and the operator $\mathfrak{M}(E2)$ by a dashed one. The coupling strengths a and G indicate the particle-vibrator vertices and the proton-proton interaction, respectively.

Furthermore, it is enhanced by a large effective charge,

$$e^{\text{eff}}=e_p^{\text{eff}}+\frac{5Ze\langle k\rangle}{4\pi C}, \tag{16}$$

induced by the particle-vibrator interaction. This result is a consequence of the fact that the matrix elements of the interaction and the electromagnetic operator have a similar structure, so that obviously the same situation will hold for the three extracore particles. The quadrupole moment for a predominantly seniority-one zero-phonon state, with the third-order perturbation corrections included (see Fig. 1), is

$$Q((j)^3J=j=I) = e^{\text{eff}}Q_{\text{sp}}(j)\left(1-\frac{4}{2j-1}+\frac{G^2}{4}\sum_{j'\neq j}\frac{2j'+1}{(\epsilon_j-\epsilon_{j'})^2}\right), \tag{17}$$

where $Q_{\text{sp}}(j)$ is the single-particle quadrupole moment [$Q_{\text{sp}}(\frac{3}{2})=-0.06$ b]. This last expression, together with the experimental value for the quadrupole moment of the ground state ($Q_{\text{exp}}=0.19$ b), imposes further restrictions on the single-particle spacing. Similar considerations may be used for the electromagnetic $B(E2)$ transitions, but in this case we do not have enough experimental information.

Finally, if we consider second-order diagrams contributing to the energy shifts of "bare" single-particle states together with the experimental energy spectra, we can conclude that

$$\Delta(\tfrac{1}{2})\approx\Delta(\tfrac{5}{2})\approx 1 \text{ MeV}.$$

In order to evaluate the electric quadrupole operator we assume two different effective proton charges (namely, $e_p^{\text{eff}}=e$ and $e_p^{\text{eff}}=2e$), while the vibrator effective charge is deduced from inelastic proton scattering experiments on ^{64}Ni, i.e., $e_v^{\text{eff}}=2.6e$.[35]

The effective gyromagnetic ratios must be considered more carefully. As is well known, the dipole magnetic moment and the $M1$ transitions strongly depend on the small configuration admixtures in which particles in a spin-orbit doublet are coupled to angular momentum unity. In order to obtain these components it is necessary to consider a spin-dependent interaction for the extracore particles and, simulatneously, core-excitation processes. These effects have been considered by several authors[36-38] and in the present work will be simulated by the effective gyromagnetic ratio g_s^{eff}. From the calculation performed by Mavromatis, Zamick, and Brown,[38] it can be

deduced that the value of this ratio must be $g_s^{\text{eff}} \approx 0.5 g_s$. Furthermore, for the core gyromagnetic ratios two extreme values will be considered: $g_R = 0$ and $g_R = Z/A$.

IV. RESULTS AND DISCUSSION

Keeping in mind the estimates given in the previous section for the single-particle spacings and coupling constants, several calculations for the energy spectra and the spectroscopic factors have been performed. For each total angular momentum I ($\frac{1}{2} \leq I \leq \frac{11}{2}$) and parity $\pi = -1$, the set of basis states given by Eq. (4) was generated so as to include all combinations of phonon states with $N \leq 2$ and the proton states $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ for which the diagonal unperturbed terms are less than the cutoff energy $E_{\max} = 4$ MeV. In this way the matrices are of the order of 60×60. For some cases, different truncation energies in the

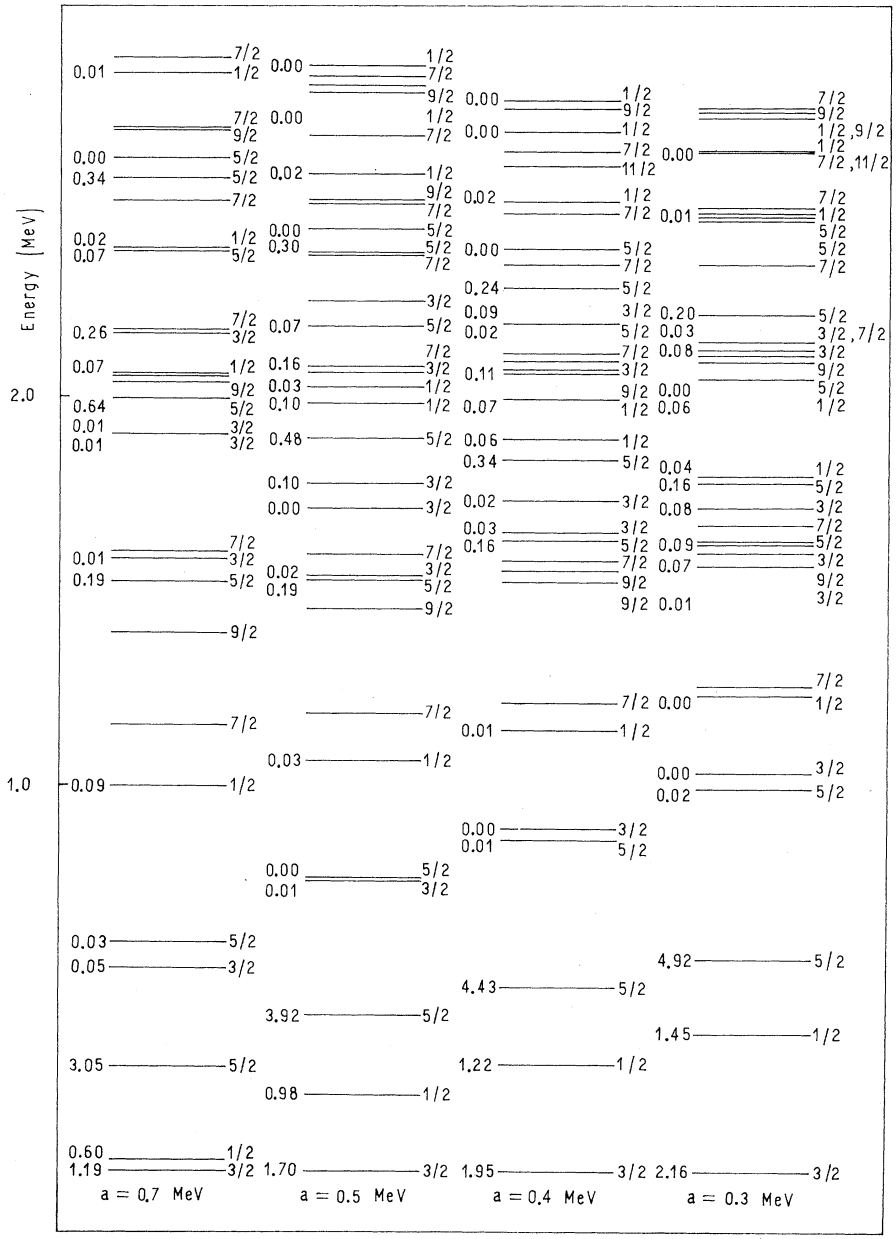

FIG. 2. The calculated energy levels and reaction strengths $(2j+1)S_j$ for the negative-parity states in ^{69}Ga for $\Delta(\frac{1}{2}) = 0.8$ MeV, $\Delta(\frac{5}{2}) = 1.0$ MeV, $G = 0.35$ MeV, and different coupling constants a.

vicinity of 4 MeV have also been considered, but no significant difference has been observed. In the evaluation of the stripping spectroscopic factors, the wave functions of zinc were obtained with the same set of parameters and for the same configuration space. Some of the results, for

$$\Delta(\tfrac{1}{2}) = 0.8 \text{ MeV}, \quad \Delta(\tfrac{5}{2}) = 1.0 \text{ MeV}, \quad \hbar\omega = 1.34 \text{ MeV}, \quad G = 0.35 \text{ MeV}, \tag{18}$$

with different values of the particle-vibrator coupling are shown in Fig. 2.[39] If we compare these results with the experimental results displayed in Fig. 3 it can be seen that the particle-phonon interaction decreases as the $N=40$ subshell is filled up.

From now on we shall limit discussion to ^{69}Ga. The calculated energy spectra and the spectroscopic factors (for $a=0.4$ MeV) are compared in Fig. 4 with experiment, as well as with the results of a recent calculation based on the quasiparticle-plus-phonon model.[40] For the wave functions of the $(\frac{3}{2})_1$, $(\frac{1}{2})_1$, and $(\frac{5}{2})_1$ states in ^{69}Ga the admixtures of the seniority-three components are

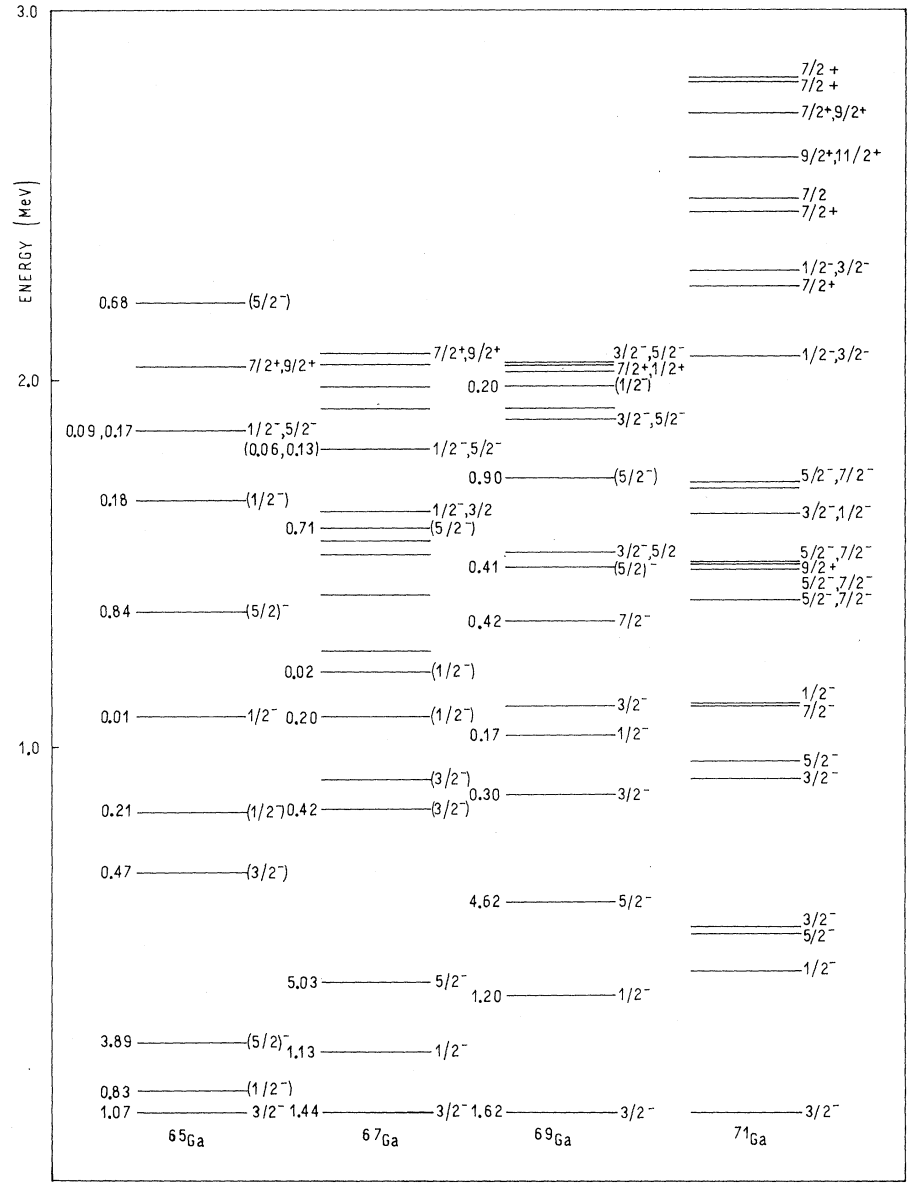

FIG. 3. The experimental energy spectra and the (d,n) stripping strengths $(2j+1)S_j$ for odd gallium isotopes (Refs. 1–10).

less than 15%. Furthermore, from Table I, we can observe a great similitude between the wave functions for the ground state in ^{68}Zn and for the low-lying states in ^{69}Ga, if we compare the corresponding seniority-zero and seniority-one amplitudes.[41] The strength functions for the ground state and the first and second excited states follow the measured values. They carry out, respectively, 75, 69, and 79% of the single-particle strength.

The next group of states [$(\frac{5}{2})_2$, $(\frac{3}{2})_2$, $(\frac{1}{2})_2$, and $(\frac{7}{2})_1$] is essentially the one-phonon seniority-one quadruplet strongly mixed with seniority-three configurations, particularly the $(\frac{5}{2})_2$ and $(\frac{1}{2})_2$ states. They can be identified with the experimental levels at 1.107, 0.872, 1.027, and 1.334 MeV, respectively. The calculated spectroscopic factor for the $(\frac{5}{2})_2$ state is consistent with the experimental result, while the theoretical strengths for the $(\frac{1}{2})_2$

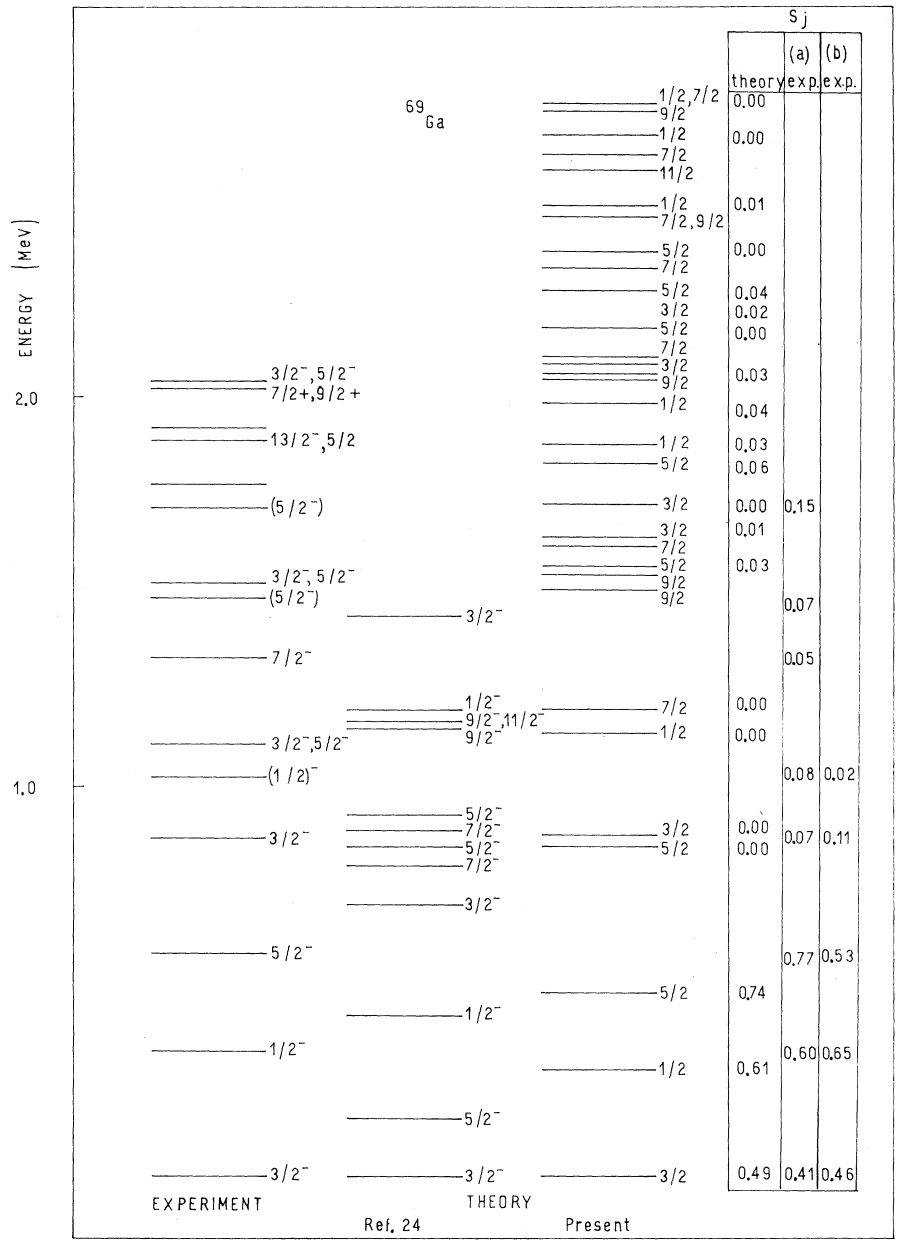

FIG. 4. Comparison of the calculated energy spectra and the spectroscopic factors of ^{69}Ga with experiment. The parameters used are given by Eq. (18). The summed transition strengths, defined by Eq. (10), for the excitations into $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ orbits are, respectively, 0.65, 0.88, and 0.94. (a) See Ref. 4; (b) see Ref. 2.

and $(\frac{3}{2})_2$ states are always too small. The $l=3$ transition to the 1.334-MeV level, observed in the (d,n) reaction,[4] clearly indicates the presence of the $f_{7/2}$ holes in the ^{68}Zn ground state. Furthermore, for the energy region between 1.5 and 2.5 MeV, our calculation does not show any state strongly excited by the stripping process, which agrees with experiment, except for the $l=3$ transition to the 1.723-MeV level.[4]

In order to understand the reason for the small value of the reaction amplitudes in the case of the first multiplet $\{|[(p_{3/2})^3\frac{3}{2}, 12]IM\rangle\}$, let us return to the perturbation method. To first order in the coupling constant the stripping strengths for $I=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ [represented by Fig. 5(c)] are

$$S_{j=I}=\frac{4\pi(2j'-1)}{(2j+1)(2j'+1)}\left(\frac{a\langle j'\|Y_2\|j\rangle}{\Delta(j)-\hbar\omega}\right)^2, \tag{19}$$

TABLE I. Calculated wave functions for the low-lying states of ^{69}Ga and for the ground state of ^{68}Zn. Each state is denoted by its spin, parity, and order of appearance. The parameters used are given by expression (18). Only those amplitudes that contribute more than 4% are listed.

$$|(\tfrac{1}{2})_1\rangle = 0.79|[(\tfrac{3}{2})^2_0\tfrac{1}{2}]\tfrac{1}{2}, 00\rangle + 0.29|[(\tfrac{5}{2})^2_0\tfrac{1}{2}]\tfrac{1}{2}, 00\rangle + 0.32|[(\tfrac{3}{2})^2_0\tfrac{5}{2}]\tfrac{5}{2}, 12\rangle + 0.21|[(\tfrac{3}{2})^2_0\tfrac{3}{2}, 12\rangle - 0.20|[(\tfrac{3}{2})^2_2\tfrac{1}{2}]\tfrac{5}{2}, 12\rangle$$

$$|(\tfrac{1}{2})_2\rangle = 0.53|[(\tfrac{3}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle + 0.50|[(\tfrac{3}{2})^2_2\tfrac{5}{2}]\tfrac{1}{2}, 00\rangle + 0.33|[(\tfrac{5}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle + 0.22|[(\tfrac{1}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle - 0.27|[(\tfrac{3}{2})^2_0\tfrac{5}{2}]\tfrac{5}{2}, 12\rangle - 0.28|[(\tfrac{1}{2}, \tfrac{5}{2})_2]\tfrac{3}{2}\tfrac{1}{2}, 00\rangle$$

$$|(\tfrac{3}{2})_1\rangle = 0.77|[(\tfrac{3}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 00\rangle + 0.39|[(\tfrac{5}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 00\rangle + 0.32|[(\tfrac{1}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 00\rangle$$

$$|(\tfrac{3}{2})_2\rangle = 0.46|[(\tfrac{3}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle - 0.38|[(\tfrac{3}{2})^2_2\tfrac{1}{2}]\tfrac{3}{2}, 00\rangle - 0.29|[(\tfrac{5}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 00\rangle - 0.31|[(\tfrac{1}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 00\rangle + 0.26|[(\tfrac{5}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle + 0.25|[(\tfrac{1}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle - 0.30|[(\tfrac{3}{2})^2_0\tfrac{1}{2}]\tfrac{1}{2}, 12\rangle$$

$$|(\tfrac{5}{2})_1\rangle = 0.78|[(\tfrac{3}{2})^2_0\tfrac{5}{2}]\tfrac{5}{2}, 00\rangle + 0.23|[(\tfrac{5}{2})^2_0\tfrac{5}{2}]\tfrac{5}{2}, 00\rangle + 0.22|[(\tfrac{1}{2})^2_0\tfrac{5}{2}]\tfrac{5}{2}, 00\rangle + 0.24|[(\tfrac{3}{2})^2_0\tfrac{1}{2}]\tfrac{1}{2}, 12\rangle - 0.24|[(\tfrac{3}{2})^2_0\tfrac{5}{2}]\tfrac{5}{2}, 12\rangle$$

$$|(\tfrac{5}{2})_2\rangle = 0.57|[(\tfrac{3}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle - 0.58|[(\tfrac{3}{2})^2_2\tfrac{1}{2}]\tfrac{5}{2}, 00\rangle + 0.28|[(\tfrac{1}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle + 0.22|[(\tfrac{5}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle + 0.23|[(\tfrac{3}{2})^2_0\tfrac{1}{2}]\tfrac{1}{2}, 12\rangle$$

$$|(\tfrac{7}{2})_1\rangle = 0.68|[(\tfrac{3}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle - 0.45|[(\tfrac{3}{2})^2_2\tfrac{5}{2}]\tfrac{7}{2}, 00\rangle + 0.29|[(\tfrac{5}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle + 0.22|[(\tfrac{1}{2})^2_0\tfrac{3}{2}]\tfrac{3}{2}, 12\rangle$$

$$|(0)_1\rangle = 0.78|(\tfrac{3}{2})^2_0, 00\rangle + 0.38|(\tfrac{5}{2})^2_0, 00\rangle + 0.30|(\tfrac{1}{2})^2_0, 00\rangle - 0.23|(\tfrac{3}{2})^2_2, 12\rangle - 0.21|(\tfrac{3}{2}, \tfrac{1}{2})_2, 12\rangle$$

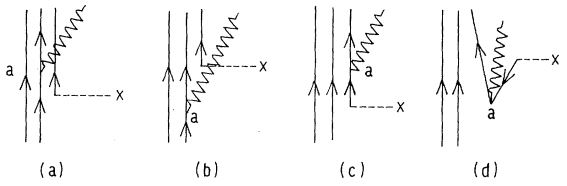

FIG. 5. Lowest-order graphs describing the stripping process from the ground state in Zn to the states of the first multiplet in Ga. The convention for particles and holes used here is that of Fig. 1 (see the corresponding caption). In the present case, however, the dashed line indicates the particle creation operator.

with $j'=\frac{3}{2}$. Figures 5(a) and 5(b) contribute only for $I=\frac{3}{2}$, but they are out of phase and cancel themselves. The spectroscopic strength is furthermore reduced owing to the strong admixture of the seniority-three one-phonon states, which appear through second-order graphs and renormalize the amplitudes of the wave functions. The perturbation treatment allows us also to estimate the spectroscopic factor for the $(\frac{7}{2})_1$ state. In first order its magnitude is again given by Eq. (19) and corresponds to Fig. 5(d). With $\Delta(f_{7/2})=-4$ MeV and $a=0.4$ MeV the estimate is too small to account for the $l=3$ strength of the 1.334-MeV level.

Let us remark that the present model of pairing force and harmonic vibration obviously implies too great a regularity. A more realistic interaction for the extracore particles, as well as the influence of the particle-hole structure of the phonons,[42,43] will probably rearrange appreciably the single-particle strengths for higher states.

The electromagnetic properties have also been calculated for several sets of parameters. In Tables II–IV we present the results for the set used in the calculation of the spectroscopic factors, which also give quite satisfactory results for the energy spectra. With the proton effective charge $e_p^{\rm eff}=2e$ and the effective gyromagnetic

TABLE II. Experimental and theoretical magnetic dipole moment and the electric quadrupole moment for the ground states in units of μ_N and b, respectively.

	Experimental [a]			Theoretical
	^{67}Ga	^{69}Ga	^{71}Ga	
μ	1.850 ± 0.001	2.016 02	2.561 61	1.99 [b]
Q	0.217 ± 0.009	0.19 ± 0.01	0.12 ± 0.01	0.10, [c] 0.14 [d]

[a] Reference 10.
[b] $g_l=1$, $g_R=0$, $g_s^{\rm eff}=0.4g_s$.
[c] $e_p^{\rm eff}=e$, $e_v^{\rm eff}=2.4e$.
[d] $e_p^{\rm eff}=2e$, $e_v^{\rm eff}=2.4e$.

ratio $g_s^{\text{eff}} = 0.4 g_s$ it was possible to account for the experimental values of the electric quadrupole moment and magnetic dipole moment of the ground state, respectively, (see Table II).[44,45] Furthermore, the $B(E2)$ transition probability between the first excited state and the ground state is in fair agreement with the measured one, $B(E2; (\frac{1}{2})_1 \rightarrow (\frac{3}{2})_1)_{\text{exp}} = 160$ fm^4. Theoretical and measured half-lives for the lowest states are compared in Table IV. The agreement is satisfactory except for the 0.574-MeV level, which decays only through the $l=2$ transition to the ground state. The discrepancy can be attributed then to the fact that the present interaction is not able to remove the forbiddenness in the magnitude contribution to the total transition probabilities. Concerning this point, it is interesting to discuss also the branching ratios associated with the first multiplet. Particularly, we want to compare: (i) transitions from a state of the multiplet to the ground state with (ii) transitions from the same state of the multiplet to states with different single-particle structure (first and second excited states). Let us define the ratio

$$R(I_m, I_n) = \frac{P(I_m, I_n)}{P(I_m, (\frac{3}{2})_1)} ,$$

where $P(I_m, I_n)$ is the total $(E2+M1)$ transition probability between the states I_m and I_n. Then, on the basis of the discussion presented in Ref. 32 for the $B(E2)$ transition, we can expect the ratio R to always be a small quantity. The results are presented in Table V and compared with experiment. The agreement is quite satisfactory.

TABLE III. Theoretical $B(E2)$ and $B(M1)$ values for transitions between low-lying states in ^{69}Ga.

Transition	$B(E2)$ (fm^4) $e_p^{\text{eff}}=2e$	$e_p^{\text{eff}}=e$	$B(M1)$ (μ_N^2) $g_l=1,\ g_R=0,\ g_s^{\text{eff}}=0.4g_s$
$(\frac{1}{2})_1 \rightarrow (\frac{5}{2})_1$	82	45	0.110
$(\frac{5}{2})_1 \rightarrow (\frac{3}{2})_1$	28	16	0.000
$(\frac{7}{2})_1 \rightarrow (\frac{3}{2})_1$	348	258	
$(\frac{3}{2})_2 \rightarrow (\frac{3}{2})_1$	370	228	0.052
$(\frac{5}{2})_2 \rightarrow (\frac{3}{2})_1$	555	340	0.032
$(\frac{1}{2})_2 \rightarrow (\frac{3}{2})_1$	256	179	0.004
$(\frac{3}{2})_3 \rightarrow (\frac{3}{2})_1$	8	10	0.007
$(\frac{5}{2})_3 \rightarrow (\frac{3}{2})_1$	0.00	0.00	0.001
$(\frac{5}{2})_1 \rightarrow (\frac{1}{2})_1$	330	123	
$(\frac{3}{2})_2 \rightarrow (\frac{1}{2})_1$	411	227	0.054
$(\frac{5}{2})_2 \rightarrow (\frac{1}{2})_1$	132	74	
$(\frac{3}{2})_3 \rightarrow (\frac{1}{2})_1$	110	71	0.001
$(\frac{5}{2})_3 \rightarrow (\frac{1}{2})_1$	68	52	
$(\frac{7}{2})_1 \rightarrow (\frac{5}{2})_1$	80	44	0.009
$(\frac{3}{2})_2 \rightarrow (\frac{5}{2})_1$	58	30	0.006
$(\frac{5}{2})_2 \rightarrow (\frac{5}{2})_1$	0.8	0.4	0.003
$(\frac{1}{2})_2 \rightarrow (\frac{5}{2})_1$	220	118	
$(\frac{3}{2})_3 \rightarrow (\frac{5}{2})_1$	75	43	0.00
$(\frac{5}{2})_3 \rightarrow (\frac{5}{2})_1$	237	147	0.006

TABLE IV. Theoretical and experimental results for the half-lives for the low-lying states in ^{69}Ga.

I_n	E_{exp} (MeV)	τ (psec) Theory	Exp.	Ref.
$(\frac{1}{2})_1$	0.318	10	<100	10
$(\frac{5}{2})_1$	0.574	250	5	9
$(\frac{3}{2})_2$	0.872	0.80	0.21	9

V. CONCLUSION

A semi-microscopic calculation with pairing force in the $(1f_{5/2}, 2p_{3/2}, 2p_{1/2})$ space plus two quadrupole phonons is performed for odd-mass gallium isotopes. Keeping in mind the approximations made in the present paper, the agreement with experiment may be considered satisfactory. Several problems are encountered in a detailed comparison with data, but the results are found to agree with the gross structure of the observed level spectra, electromagnetic moments, and one-proton stripping strengths. This encourages further work to improve the present description by considering a more realistic residual interaction (in particular, the neutron-proton short-range part of the interaction) and by taking into account additional degrees of freedom. Further experimental investigations, especially on $E2$ transition rates, should help to clarify the effects of different excitation modes.

TABLE V. Theoretical and experimental values for the branching ratios for the total $(E2+M1)$ transition probabilities between the low-lying states in ^{69}Ga.

$I_n \rightarrow I_m$	$R(I_n \rightarrow I_m)$ Theory	Exp. [a]
$(\frac{3}{2})_2 \rightarrow (\frac{1}{2})_1$	0.099	0.062
$(\frac{5}{2})_2 \rightarrow (\frac{1}{2})_1$	0.019	0.012
$(\frac{5}{2})_2 \rightarrow (\frac{5}{2})_1$	0.005	0.006
$(\frac{7}{2})_1 \rightarrow (\frac{5}{2})_1$	0.073	0.056

[a] See Ref. 9.

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