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Effects of N^* Production on Nucleon-Nucleus Scattering

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The Glauber theory of multiple scattering is extended so as to include the contributions of N^* production and is applied to the elastic scattering of a nucleon from various nuclei for incident momenta in the range from 1.7 to 30 GeV/c. The momentum dependence of the nucleon-nucleus total cross sections is discussed. The influences of the inclusion of N^* effects on the structure of the second maximum in the differential cross section is also investigated.

I. INTRODUCTION

There is a great deal of interest in high-energy hadron-nucleus scattering. It is hoped that hadron-nucleus scattering experiments provide new information on the elementary particle interactions, as well as on the structure of nuclei. More precisely, first from the point of nuclear physics, we expect to obtain information on nuclear wave functions in general and particularly correlations which cannot be obtained from the electron scattering data. For light nucleus, the electromagnetic interaction is reasonably well described by the first Born approximation which depends only on the charge distribution in the nucleus, i.e., on a proton density function.[1] If we use hadron-nucleus scattering experiments, we may obtain some knowledge about a neutron distribution in a nucleus. Moreover, hadronic interactions – or strong interaction – are so intense that multiple collisions are quite a strong influence on the cross sections we observe. An incident particle has a large probability of interacting more than once as it passes through the nucleus, and thus it is possible to obtain information on nucleon-nucleon correlations in the nucleus, although this approach is still in a very preliminary state. Second, from the point of particle physics a nucleus is a convenient system in which rescattering of short-lived particles, or resonances, can be studied. Namely, if a resonance is produced inside a nucleus, then it has a chance to strike nucleons on its way out of the nucleus. Then, if we have a good theory of particle-nucleus interactions, we are able to extract the resonance-nucleon scattering amplitude from the nuclear production amplitude. Further, if the cross section for a particular process of particle-hydrogen collision is σ_H, then the cross section for that of particle-nucleus with mass number A may be $\sigma_A = \sigma_H A^n$, where $n > 0$. Thus if we want to examine rare production modes, we look at the production from a nucleus, and then using the theory, we may extract the production from a single nucleon.

In the hope of discovering more about the structure of nuclei, the Brookhaven group performed a series of experiments with a beam of 1.7-GeV/c protons and the elastic differential and total cross sections in H, D, ^{4}He, ^{12}C, ^{16}O, were measured with high precision.[2,3] As to nucleon-nucleus total cross sections some data have been accumulated lately in nuclei such as Be, C, Al, Cu, Cd, W, and Pb bombarded by neutron beams in the momentum range 10 to 30 GeV/c.[4-6]

The theoretical work on this problem has been carried out on the basis of the multiple-scattering model proposed many years ago by Glauber.[7] Such calculations have been mostly carried out for a simple model in which nuclei are described by a Gaussian density distribution.[8-14] Although the expression for the total cross section reproduces the experimental data fairly well in the multi-GeV/c momentum region, the total cross section does not depend on momentum in an expected way.[12-14] The experimental results show that the nucleon-nucleus total cross section falls smoothly and monotonically with increasing momentum.[4-6] As for the angular distribution of nucleons elastically scattered from the nucleus, the theoretical curve has the same general shape as that observed in the experimental data,[2] but is too low by a factor about 2–4 around the second maximum of the distribution.[9-11]

There is a certain probability that the incident particle is excited to one of its resonance states by the collision with one of the nucleons in the nucleus and finally returns to the initial state of the particle through the process of multiple scattering.[15] This means that the Glauber theory has to be extended to include the intermediate inelastic effects.[16-20] There has been an interesting paper by Harrington about these effects using the composite model for a hadron.[20] He assumed the quark-nucleon scattering amplitudes and the elastic and inelastic pion form factors. In the present paper we discuss inelastic intermediate N^* effects on the nucleon-nucleus elastic scattering, making use of the experimental information about the nucleon-nucleon elastic scattering and N^* productions. The effects of $N^*(1470)$, $N^*(1518)$, $N^*(1688)$, and $N^*(2190)$ become large with increasing momentum and lead to a reduction in the total cross section which amounts to 8% for nucleon-heavy nucleus scattering at 30 GeV/c. The presence of the intermediate $N^*(1236)$ state gives rise to a maximum at a momentum in the region 2–3 GeV/c. The inclusion of N^* states has remarkable effects around the second maximum, because it means to introduce further terms into the double scattering process.

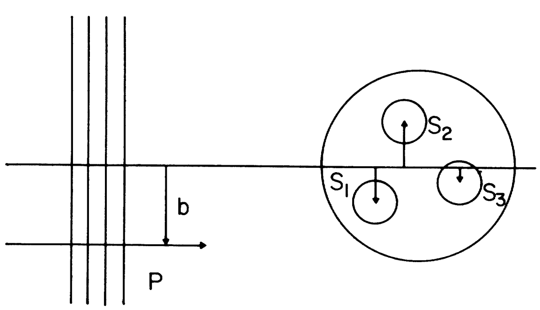

FIG. 1. Schematic picture of nucleon-nucleus scattering.

II. EXTENSION OF THE GLAUBER THEORY

For convenience of later development, the formalism of the Glauber theory is presented briefly.[7]

At sufficiently high energies the elastic scattering amplitude is approximated in the form of integrals over impact vector $\vec{b}$,

$$f(\vec{q}) = \frac{ik}{2\pi} \int e^{i\vec{q}\cdot\vec{b}} (1 - e^{i\chi(\vec{b})})\, d^2b\,, \tag{1}$$

where $\vec{k}$ is incident momentum, $\vec{q} = \vec{k} - \vec{k}'$ is the momentum transfer, and d^2b is an element of area in the impact-vector plane. This form is asymptotically correct in the near forward direction.

The scattering amplitude $f(\vec{q})$ is the two-dimensional Fourier transform of the function

$$\Gamma(\vec{b}) = 1 - e^{i\chi(\vec{b})}\,, \tag{2}$$

and by means of the inverse transformation we obtain

$$\Gamma(\vec{b}) = \frac{1}{2\pi ik} \int e^{-i\vec{q}\cdot\vec{b}} f(\vec{q})\, d^2q\,, \tag{3}$$

where the integration is over a plane perpendicular to $\vec{k}$.

Equation (1) can be generalized to deal with high-energy particle-nucleus scattering. The phase-shift function for the nuclear scattering will depend on $\vec{b}$ and the coordinates $\vec{S}_1, \ldots, \vec{S}_A$, which are the projections of position vectors on the plane perpendicular to the incident momentum $\vec{P}$; (**Fig. 1**)

$$\Gamma(\vec{b}, \vec{S}_1, \ldots, \vec{S}_A) = 1 - e^{i\chi(\vec{b}, \vec{S}_1, \ldots, \vec{S}_A)}\,. \tag{4}$$

The scattering amplitude for a process in which the nucleus goes from an initial state $|\psi_i\rangle$ to a

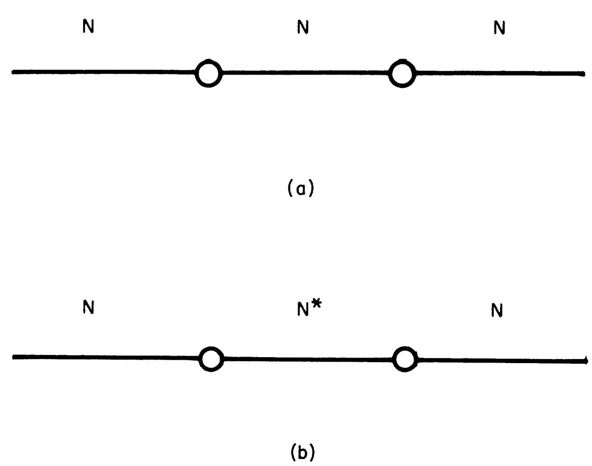

FIG. 2. The double scattering diagrams: (a) with N intermediate state, (b) with N^* intermediate state.

final state $|\psi_f\rangle$ is given by

$$F_{fi}(\vec{q})=\frac{iP}{2\pi}\int e^{i\vec{q}\cdot\vec{b}}\langle\psi_f|\Gamma(\vec{b},\vec{S}_1,\ldots,\vec{S}_A)|\psi_i\rangle\, d^2b\,, \tag{5}$$

where P is the magnitude of the incident momentum in the projectile-nucleon laboratory system.

The basic assumption of the Glauber theory is that the phase shift brought about by a particle passing through the nucleus can be written as

$$\chi(\vec{b},\vec{S}_1,\ldots,\vec{S}_A)=\sum_j \chi_j(\vec{b}-\vec{S}_j)\,, \tag{6}$$

and consequently Eq. (4) becomes

$$\begin{aligned}\Gamma(\vec{b},\vec{S}_1,\ldots,\vec{S}_A)&\\ &=1-\prod e^{i\chi_j(\vec{b}-\vec{S}_j)}\\ &=1-\prod_j[1-\Gamma_j(\vec{b}-\vec{S}_j)]\\ &=\sum_j \Gamma_j(\vec{b}-\vec{S}_j)-\sum_{j<m}\Gamma_j(\vec{b}-\vec{S}_j)\Gamma_m(\vec{b}-\vec{S}_m)+\cdots\,.\end{aligned} \tag{7}$$

Now let us restrict our discussion to the case where the incident particle is a nucleon and the resonances are nucleon resonances N^*. In order to extend the Glauber theory so as to include the effects due to diffraction production channels, it is convenient to write scattering processes of two-particle systems in the following matrix form:

$$\begin{pmatrix} N+N_j\to N+N_j & N+N_j\to N^*+N_j\\ N^*+N_j\to N+N_j & N^*+N_j\to N^*+N_j\end{pmatrix}, \tag{8}$$

where N_j represents the jth nucleon in the nucleus.

In the reaction of N^* production there is a minimum longitudinal momentum transfer which is a complex value because of the width of the resonance mass, given by

$$\Delta_m\simeq\frac{(M^*-i\eta/2)^2-m^2}{2P}\,, \tag{9}$$

where the resonance has mass M^* with width η and the incident nucleon has the laboratory momentum P. This momentum transfer makes the N^* contribution incoherent with elastic scattering for not too high energy. Taking this extra phase Δ_m into account, we introduce the following functions associated with the resonance production and its inverse processes by the jth nucleon[11]:

$$\Gamma_j^{\Phi}(\vec{b}-\vec{S}_j,Z_j)=e^{i\Delta_m Z_j}\,e^{i\Phi_j^t(\vec{b}-\vec{S}_j)} \tag{10}$$

and

$$\bar{\Gamma}_j^{\bar{\Phi}}(\vec{b}-\vec{S}_j,Z_j)=e^{-i\Delta_m Z_j}\,e^{i\bar{\Phi}_j^t(\vec{b}-\vec{S}_j)}\,, \tag{11}$$

where Z_j is the component of the jth nucleon's position vector along the axis, and $\Phi^t,\bar{\Phi}^t$ are the phase shifts associated with transverse momentum transfer and they are related to small angle scattering amplitudes, $g(q)$ and $\bar{g}(q)$, for $NN\to N^*N$ and $N^*N\to NN$ processes through the following equations:

$$g(\vec{q})=\frac{k}{2\pi i}\left(\frac{k^*}{k}\right)^{1/2}\int e^{i\vec{q}\cdot\vec{b}}\,e^{i\Phi^t(\vec{b})}\,d^2b\,, \tag{12}$$

$$\bar{g}(\vec{q})=\frac{k}{2\pi i}\left(\frac{k}{k^*}\right)^{1/2}\int e^{i\vec{q}\cdot\vec{b}}\,e^{i\bar{\Phi}^t(\vec{b})}\,d^2b\,, \tag{13}$$

where k and k^* are momenta in the NN and N^*N center-of-mass systems.

As for $N^*N\to N^*N$ elastic scattering we write the amplitude as $h(\vec{q})$ and its phase-shift function as $\Gamma^*(\vec{b})=1-e^{i\chi^*(\vec{b})}$, which can be expressed in terms of the amplitude as

$$\Gamma^*(\vec{b})=\frac{1}{2\pi ik^*}\int e^{-i\vec{q}\cdot\vec{b}}\,h(\vec{q})\,d^2q\,. \tag{14}$$

Then using Γ_j, Γ_j^{Φ}, $\Gamma_j^{\bar{\Phi}}$, and Γ_j^*, we can construct the following matrix of the phase-shift functions associated with the matrix in Eq. (8):

$$\Gamma_j^E(\vec{b}-\vec{S}_j,Z_j)=\begin{pmatrix}\Gamma_j(\vec{b}-\vec{S}_j) & \Gamma_j^{\Phi}(\vec{b}-\vec{S}_j,Z_j)\\ \bar{\Gamma}_j^{\bar{\Phi}}(\vec{b}-\vec{S}_j,Z_j) & \Gamma_j^*(\vec{b}-\vec{S}_j)\end{pmatrix}. \tag{15}$$

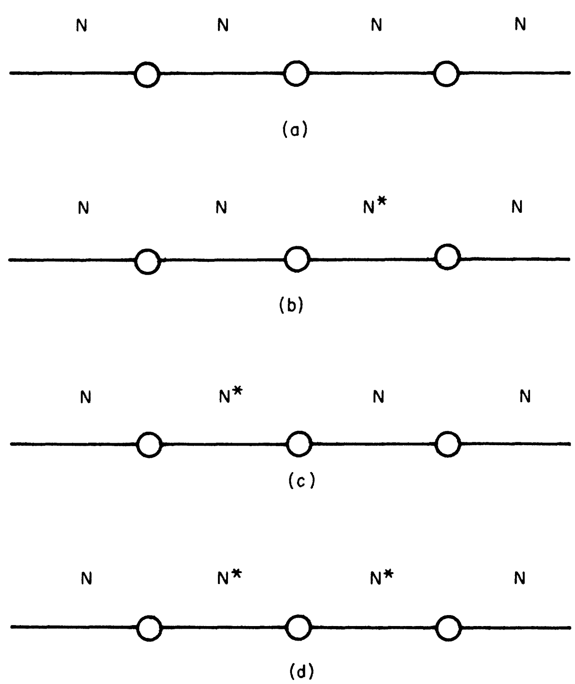

FIG. 3. Possible triple scattering diagrams.

If we substitute $\Gamma_j^E(\vec{b}-\vec{S}_j, Z_j)$ into $\Gamma_j(\vec{b}-\vec{S}_j)$ in Eq. (7), we get the matrix of nuclear phase-shift functions which include the effects due to intermediate N^* production:

$$\Gamma^E(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A) = \sum_j \Gamma_j^E(\vec{b}-\vec{S}_j,Z_j) - \sum_{j<m}\Gamma_j^E(\vec{b}-\vec{S}_j,Z_j)\,\Gamma_m^E(\vec{b}-\vec{S}_m,Z_m) + \sum_{j<m<k}\Gamma_j^E\Gamma_m^E\Gamma_k^E - \cdots . \quad (16)$$

The (1, 1) element of the matrix

$$\Gamma^E(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A)$$

corresponds to the nuclear phase-shift function of nucleon-nucleus elastic scattering and the (1, 2) element corresponds to that of coherent N^* production process. It is worth noting that

$$\Gamma^E(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A)$$

depends upon the order of the longitudinal coordinates of nucleons which the incident nucleon encounters successively in the nucleus. In other words we cannot change the order of Γ_j^E and there is the constraint $Z_j < Z_m < Z_R \cdots$ in Eq. (16).

The extended amplitude of the nucleon-nucleus elastic scattering is given by

$$F^E(\vec{q}) = \frac{iP}{2\pi}\int e^{i\vec{q}\cdot\vec{b}}\left\langle \psi_i \right| (1,0) \times \Gamma^E(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A)\begin{pmatrix}1\\0\end{pmatrix}\left|\psi_i\right\rangle d^2b . \quad (17)$$

For convenience we write the amplitude $F^E(\vec{q})$ as

$$F^E(\vec{q}) = F_0(\vec{q}) + F_{N^*}(\vec{q}) , \quad (18)$$

where $F_0(\vec{q})$ is the amplitude calculated according to the ordinary Glauber theory, i.e., Eq. (5) and $F_{N^*}(\vec{q})$ represents the contribution to the elastic scattering via intermediate N^* states. Similarly we can divide the (1, 1) element of the matrix $\Gamma^E(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A)$ into two parts as

$$(1,0)\Gamma^E(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A)\begin{pmatrix}1\\0\end{pmatrix} = \Gamma_0(\vec{b},\vec{S}_1,\ldots,\vec{S}_A) + \Gamma_{N^*}(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A) , \quad (19)$$

where $\Gamma_0(\vec{b},\vec{S}_1,\ldots,\vec{S}_A)$ is just $\Gamma(\vec{b},\vec{S}_1,\ldots,\vec{S}_A)$ in Eq. (17) and $\Gamma_{N^*}(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A)$ which represents the intermediate N^* effect, is given by the following equation:

$$\Gamma_{N^*}(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A) = -\sum_{j<m}\Gamma_j^\Phi(\vec{b}-\vec{S}_j,Z_j)\,\overline{\Gamma}_m^\Phi(\vec{b}-\vec{S}_m,Z_m) + \sum_{j<m<k}(\Gamma_j^\Phi\overline{\Gamma}_m^\Phi\Gamma_k + \Gamma_j\Gamma_m^\Phi\overline{\Gamma}_k^\Phi + \Gamma_j^\Phi\Gamma_m^*\overline{\Gamma}_k^\Phi) - \cdots \quad (20)$$

together with the constraint $Z_j < Z_m < Z_k \cdots$. The diagrams of the double-scattering terms and the triple-scattering terms are shown in Fig. 2 and Fig. 3.

We use a simple model of the nucleus that nucleons in the nucleus are completely uncorrelated and the density distribution of the nuclear ground state is Gaussian:

$$|\psi_i|^2 = \left(\frac{1}{\sqrt{\pi}R}\right)^{3A}\prod_{i=1}^{A} e^{-r_i^2/R^2} . \quad (21)$$

The single parameter R is determined by the relation

$$R^2 = \tfrac{2}{3}\langle r^2\rangle , \quad (22)$$

where $\langle r^2\rangle^{1/2}$ is the root-mean-square (rms) radius of the nucleus. In the present calculations, the rms radii determined from the electron scatterings are used except in the case of a heavy nucleus where the large neutron excess may make the actual radius of the nucleus slightly larger than the electromagnetic radius. The nuclear radius of a heavy nucleus is deduced from results of Ref. 21.

As for the elastic nucleon-nucleon scattering amplitude we take the form

$$f(q) = \frac{k}{4\pi}(i+\alpha)\sigma e^{-\beta q^2/2} , \quad (23)$$

where σ is the total nucleon-nucleon cross section, and α is the ratio of the real to imaginary parts of the amplitude, which we assume to be independent of momentum transfer q. For the sake of simplicity we disregard the difference between protons and neutrons hereafter. Thus the parameters σ, α, and β are taken to be the average of those for neutrons and protons.

For the angular dependence of the N^* production amplitude we use a similar approximation as for the elastic case,

$$g(q) = g_0 e^{-\gamma q^2/2} . \quad (24)$$

Because of the principle of detailed balance, the following relation between $|g(q)|^2$ and $|\overline{g}(q)|^2$ holds:

$$(2S_N+1)^2k^2|g(q)|^2 = (2S_{N^*}+1)(2S_N+1)k^{*2}|\overline{g}(q)|^2 , \quad (25)$$

where S_N and S_{N^*} are the spin of N and N^*, respectively.

Under these assumptions Eqs. (21) to (25), the amplitude F_0 becomes just[4]

$$F_0(q)=\tfrac{1}{2}iP\,e^{R^2q^2/4A}(R^2+2\beta)\sum_{j=1}^{A}\binom{A}{j}\frac{(-1)^{j+1}}{j}\left(\frac{\sigma(1-i\alpha)}{2\pi(R^2+2\beta)}\right)^j e^{-(R^2+2\beta)q^2/4j}\,, \tag{26}$$

and the amplitude F_{N^*} is given by

$$F_{N^*}(q)=\frac{iP}{2\pi}e^{R^2q^2/4A}\int e^{i\vec{q}\cdot\vec{b}}\int|\psi_i(\{r_j\})|^2\,\Gamma_{N^*}(\vec{b},\vec{S}_1,Z_1,\ldots,\vec{S}_A,Z_A)\prod_m dr_m d^2b$$

$$=\tfrac{1}{2}iP\,e^{R^2q^2/4A}\sum_{j=2}^{A}\sum_{m=2,4,\ldots}^{j}\left(\frac{-4g_0\bar{g}_0}{k^2(R^2+2\gamma)^2}\right)^{m/2}$$

$$\times\sum_{k=0}^{j-m}\binom{A}{j}\frac{(-1)^{j+1}}{B_{j,m,k}}\left(\frac{\sigma(1-i\alpha)}{2\pi(R^2+2\beta)}\right)^k\left(\frac{\sigma^*(1-i\alpha^*)}{2\pi(R^2+2\beta^*)}\right)^{j-k-m} e^{-q^2/4B_{j,m,k}}\,I_z(j,m,k)\,, \tag{27}$$

with

$$B_{j,m,k}=\frac{m}{R^2+2\gamma}+\frac{k}{R^2+2\beta}+\frac{j-k-m}{R^2+2\beta^*}$$

[if $\beta=\gamma=\beta^*$, then $B_{j,m,k}=j/(R^2+2\beta)$], where the term I_z represents the effect of the integration over the nuclear z coordinates. We take into account only $I_z(j,2,k)$ in the present calculation, which is, under a plausible approximation explained in the Appendix, given by

$$I_z(j,2,k)\simeq\tfrac{1}{2}e^{-\Delta_m{}^2R^2/2}\left\{1-i\left(\frac{2}{\pi}\right)^{1/2}\Delta_mR\,[1+\tfrac{1}{6}(\Delta_mR)^2+\tfrac{1}{40}(\Delta_mR)^4+\cdots]\right\}\left(1-\frac{2}{\sqrt{3\pi}}\right)^k\left(\frac{2}{\sqrt{3\pi}}\right)^{j-k-2}. \tag{28}$$

The total cross section obtained from F_0 through the optical theorem is given by

$$\sigma_0^T=2\pi(R^2+2\beta)\,\mathrm{Re}\sum_{j=1}^{A}\binom{A}{j}\frac{(-1)^{j+1}}{j}\left(\frac{\sigma(1-i\alpha)}{2\pi(R^2+2\beta)}\right)^j, \tag{29}$$

while the effects of the N^* state to the total cross section take the following form:

$$\sigma_{N^*}=2\pi\,\mathrm{Re}\sum_{j=2}^{A}\binom{A}{j}(-1)^{j+1}\sum_{m=2,4,\ldots}^{j}\left(\frac{-4g_0\bar{g}_0}{k^2(R^2+2\gamma)^2}\right)^{m/2}$$

$$\times\sum_{k=0}^{j-m}\frac{1}{B_{j,m,k}}\left(\frac{\sigma(1-i\alpha)}{2\pi(R^2+2\beta)}\right)^k\left(\frac{\sigma^*(1-i\alpha^*)}{2\pi(R^2+2\beta^*)}\right)^{j-k-m}I_z(j,m,k)\,, \tag{30}$$

where Re means the real part. Then the total cross section of the nucleon-nucleus scattering including the effects of the N^* state is

$$\sigma^T=\sigma_0^T+\sigma_{N^*}\,. \tag{31}$$

III. COMPARISON WITH EXPERIMENTS AND DISCUSSION

Let us now discuss in detail the contributions first from the $N^*(1236)$ and second from the $I=\frac{1}{2}$ resonances.

(1) The $N^*(1236)$ contribution is important in the relatively low momentum region (a few GeV/c), since the $N^*(1236)$ production cross section rapidly decreases with the increasing momentum. Up to about 3 GeV/c the effects of other resonances may be justifiably neglected because of the large mass difference, namely the large Δ_m value, so that it is sufficient to take account of only the $N^*(1236)$ contribution. From the experimental data the dependence of the $N^*(1236)$ production cross section on the incident momentum is approximately given by[22-25]

$$|g_0|^2\propto\frac{1}{P^{1.8}}\,.$$

On the other hand, the suppression due to the mass difference which is mainly brought about by the factor

$$e^{-\Delta m^2R^2/2}$$

TABLE I. The two-body parameters σ, α, and β.

P(GeV/c)	1.7	2.85	4.55	10	20	30
σ (mb)	44.0	44.0	41.9	39.3	38.5	39.3
β (GeV/c)$^{-2}$	5.45	6.30	8.00	9.62	10.58	10.68
α	−0.3	−0.17	−0.25	−0.3	−0.3	−0.2

loses its effect as the incident momentum is increased since Δ_m becomes small. Therefore the contribution of $N^*(1236)$ is expected to have its maximum effect at an incident beam momentum, which becomes larger with the nuclear radius R.

As to the phases of $g(q)$ and $\bar{g}(q)$ for the $N^*(1236)$ resonance, if the N^* production is mediated by one-pion exchange between the colliding nucleons, g_0 and $\bar{g}_0$ should be real.[26-30] In the present calculation we shall take g_0 and $\bar{g}_0$ to be real.

From the charge independence property of strong interactions, the average $N^*(1236)$ production cross section for nucleon-nucleon scattering is related to the N^{*+} production cross section for proton-proton scattering[19];

$$\frac{d\sigma_{NN\to N^*N}}{dt}=3\frac{d\sigma_{pp\to N^{*+}p}}{dt}\,. \qquad (32)$$

Since the experimental data for $pp\to N^{*+}p$ process exist at several momenta,[23-25] we get the cross section for $NN\to N^*N$ process at those momenta from Eq. (32).

At 1.7 GeV/c we estimate the $N^*(1236)$ production amplitude by assuming that the pion production amplitude in proton-nucleon collision proceeds mainly through the $N^*(1236)$ resonance.[26] Then using the pion production at 970 MeV,[22] the $N^{*++}(1236)$ production cross section in proton-proton collisions in the forward semisphere is estimated to be approximately equal to one third of

TABLE II. Nuclear radii for various nuclei.

Nucleus	Radius (F)
He^4	1.37
Be	1.80
C	1.93
Al	2.45
Cu	3.20
Cd	3.75
W	4.28
Pb	4.63

the elastic one,[19]

$$\sigma_{pp\to N^{*++}(1236)n}\simeq\tfrac{1}{3}\sigma_{pp\to pp}\,. \qquad (33)$$

Then the charge-independence property makes the average $N^*(1236)$ production cross section to be

$$\sigma_{NN\to N^*N}\simeq\tfrac{1}{3}\sigma_{pp\to pp}\,. \qquad (34)$$

Although the angular dependence of the $N^*(1236)$ production amplitude at 1.7 GeV/c is not so apparent, there is a very marked decrease in magnitude as t increases.[21,25] It has been shown by an experiment performed at 1.35 GeV incident energy that the angular distribution of the inelastic cross section at the $N^*(1236)$ peak is similar to that of elastic scattering.[26]

(2) At 10 to 30 GeV/c we make calculations taking into account the contribution of the isospin-$\frac{1}{2}$ resonances $N^*(1470)$, $N^*(1518)$, $N^*(1688)$, and $N^*(2190)$. The forward scattering amplitude for these processes needed in the calculations are taken from Refs. 23–25. Though in the missing mass spectrum the peak exists at about 1400 MeV, our calculations are made under the assumption that we can identify this 1400-MeV effect with the Roper P_N resonance found in phase-shift analysis at about 1470 MeV. Further we shall assume the

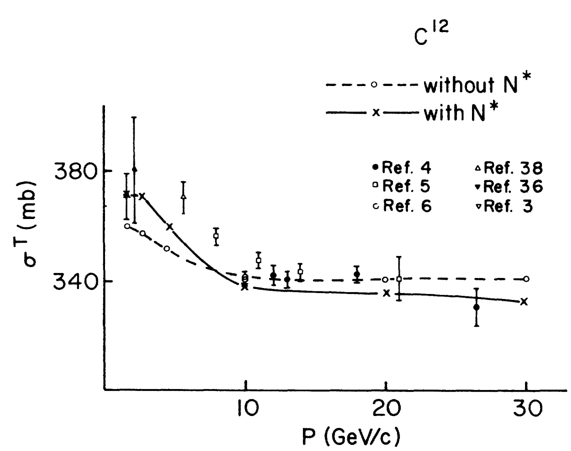

FIG. 4. Calculated and measured values of nucleon-^{12}C total cross sections.

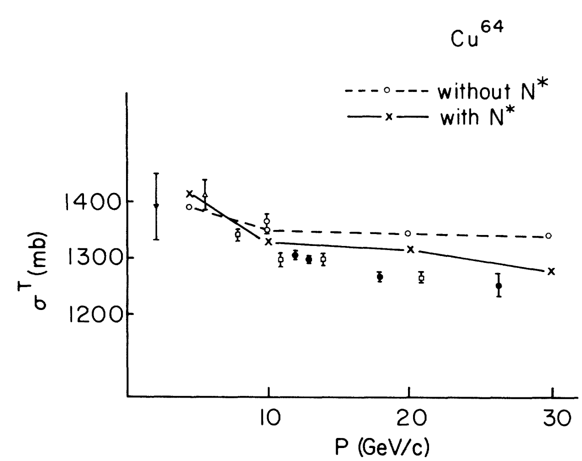

FIG. 5. Nucleon-^{64}Cu total cross sections.

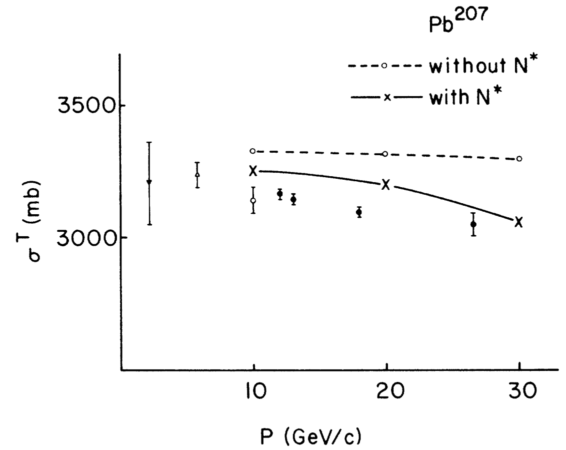

FIG. 6. Nucleon-207Pb total cross sections.

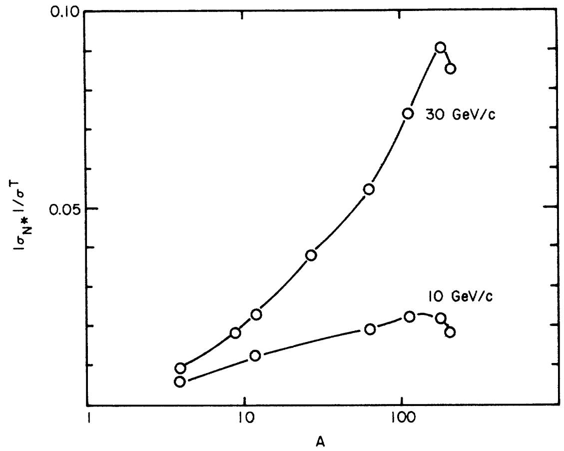

FIG. 8. "A" dependence of N^* contributions.

phases of g_0 and $\bar{g}_0$ for $I=\frac{1}{2}$ resonances to be purely imaginary.[31,32]

Since the production cross sections of these $N^*(I=\frac{1}{2})$ resonances are observed to be kept almost constant with increasing momentum, we expect that the $N^*(I=\frac{1}{2})$ contributions increase with

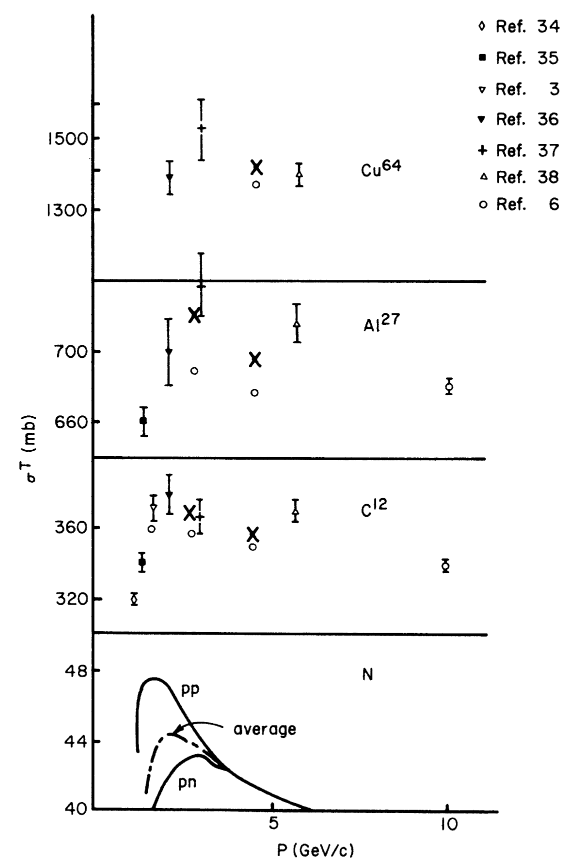

FIG. 7. Total cross sections for N, ^{12}C, ^{27}Al, and ^{64}Cu.

increasing momentum, because Δ_m becomes small and the suppression

$$e^{-\Delta_m{}^2R^2/2}$$

tends to unity.

The parameters σ, α, and β, and nuclear radius R for various nuclei which we use in numerical calculations are given in Table I and Table II.

The nucleon-nucleus total cross sections obtained from our calculation are shown in Figs. 4–6 together with experimental data. Without N^* contributions the total cross section is almost constant in the momentum range 10 to 30 GeV/c and there is a monotonic decrease in the 3–10-GeV/c momentum region which comes from decrease of the nucleon-nucleon total cross section. On the other hand, by including the contributions of N^*

TABLE III. Measured and calculated total cross sections of nucleon-nucleus scattering at 30 GeV/c. The total cross section using a Woods-Saxon density is also shown for Pb. The indicated uncertainty in calculated values come from errors in nucleon-nucleon total cross sections and α.

Nucleus	A	Measured total cross sections (mb) $P=26.5$ GeV/c	Calculated total cross sections (mb) $P=30$ GeV/c Without N^*	With N^*
He	4		133 ± 2.0	132 ± 2.1
Be	9	266 ± 6	269 ± 4.5	264 ± 5.3
C	12	330 ± 7	340 ± 5	332 ± 6
Al	26.9	656 ± 11	659 ± 12.5	635 ± 16
Cu	63.5	1251 ± 19	1340 ± 27	1271 ± 37
Cd	112.4	1907 ± 32	2023 ± 40	1884 ± 58
W	183.5	2720 ± 41	2873 ± 57	2634 ± 94
Pb	207.2	3044 ± 45	3302 ± 66	3044 ± 101
			3369 (Woods-Saxon)	

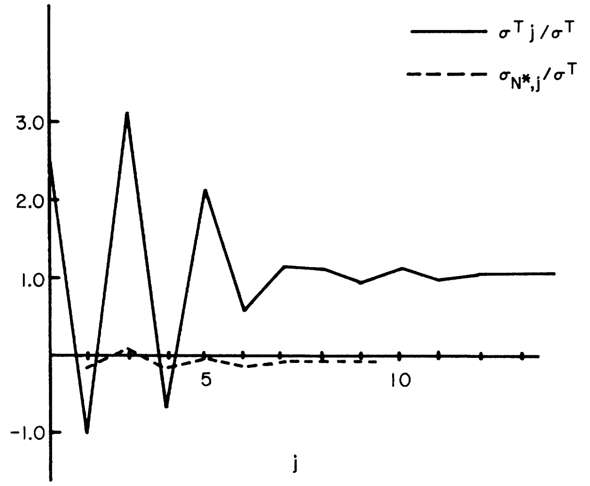

FIG. 9. Partial cross sections, we define σ_j^T and $\sigma_{N,j}^*$, which are the summations over the multiple-scattering terms up to the jth order.

state we reproduce the momentum dependence of the total cross section fairly well.

Figure 7 shows the available total cross section data for N, ^{12}C, ^{27}Al, and ^{64}Cu in the momentum range 1.2–5 GeV/c where the effects of the $N^*(1236)$ state is expected to contribute considerably.[3,33-37] The calculated values are indicated in Fig. 7 by the mark ○ (without N^*) and × (with N^*). At 4.55 GeV/c we include effects of the $N^*(I=\frac{1}{2})$ resonances as well as $N^*(1236)$ effects. The sharp peak at about 1.7 GeV/c in the proton-proton total cross section is due to the large cross section of the $N^*(1236)$ production. There are peaks also in the nucleon-nucleus total cross sections for ^{12}C, ^{27}Al, and ^{64}Cu. These peaks, however, are not sufficiently reproduced from calculations without the $N^*(1236)$ contribution. The $N^*(1236)$ contribution is large in this momentum region and it depends on the momentum of the incident particle as the nucleon-^{27}Al total cross section clearly indicates, although at 4.55 GeV/c there is a small contribution due to $N^*(1470)$. Then we may say that the peak in the nucleon-nucleus total cross section is ascribed not only to the change in magnitude of the nucleon-nucleon total cross section in this momentum region, but also to the effect of the intermediate N^* state.

Figure 8 indicates the A dependence of the N^* contributions at 10 and 30 GeV/c. There is a maximum in each curve in Fig. 8 whose A value becomes large with increasing momentum.

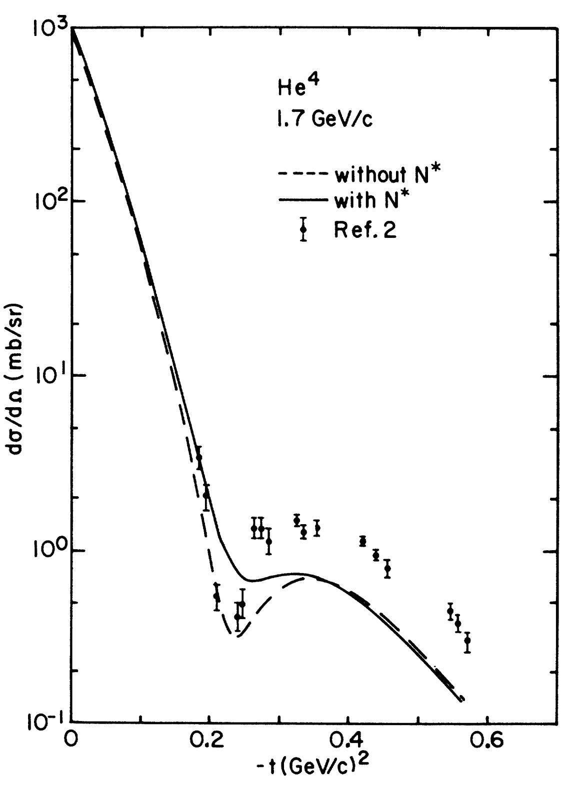

FIG. 10. Elastic nucleon-^{4}He cross section at 1.7 GeV/c.

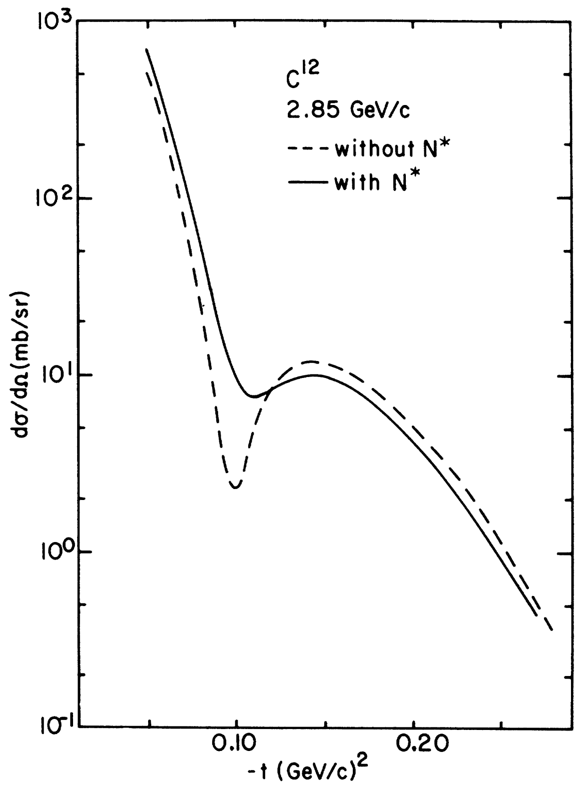

FIG. 11. Calculated differential cross section around the second maximum of elastic nucleon-^{12}C scattering.

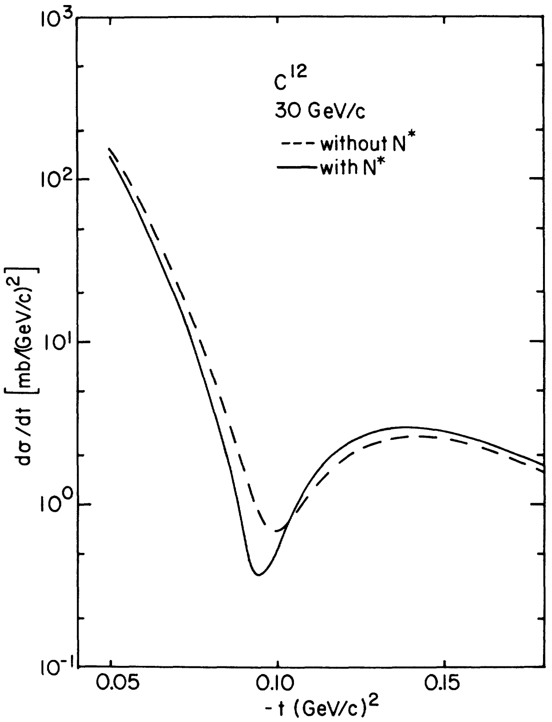

FIG. 12. Elastic differential cross section around the second maximum for N-^{12}C scattering at 30 GeV/c.

Table III compares our 30 GeV/c results with the experimental data at 26.5 GeV/c. The agreement between the calculations and measurements is very good if we include the N^* effects.

We also analyzed the Pb total cross section neglecting the N^* contribution and using the Woods-Saxon density distribution for the nucleus.[19] The quantitative results obtained are almost equal to the one with Gaussian model (see Table III).

In addition, we investigated the way in which the multiple-scattering series "converges," and this is shown in Fig. 9 for the Pb case.

In Fig. 10, we show the results of calculation for nucleon-^{4}He scattering at 1.7 GeV/c. The results are different from the previous work considering $N^*(1236)$ effects on the proton-^{4}He elastic scattering at 1 GeV,[18] in which $I_z(j, m, k)$ and the width of the resonance were not taken into account. In the present work the inclusion of the $N^*(1236)$ state increases the differential cross section at smaller angles by about 3%. $N^*(1236)$ production has little influence on the second maximum. It is, however, interesting to note that it makes the depth of the minimum shallow, because in these problems the theory always predicts a sharp minimum in the differential cross section even when only a change in slope is found experimentally.[9,10]

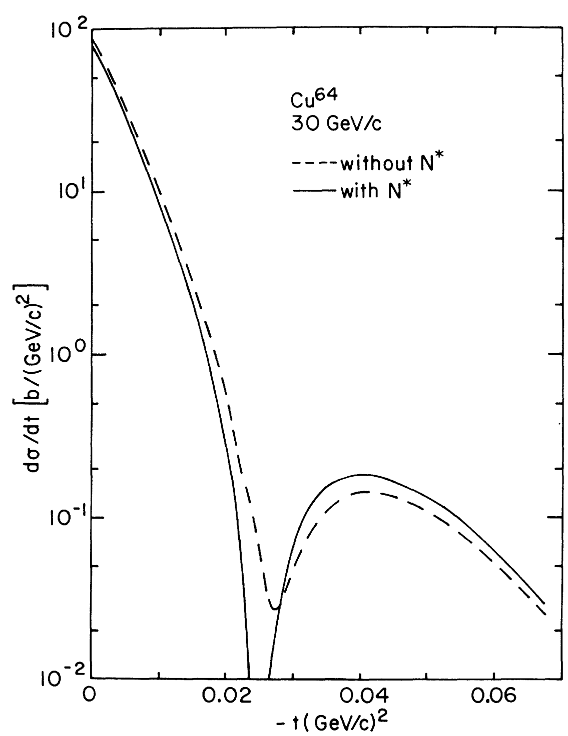

FIG. 13. Elastic differential cross section around the second maximum for N-^{64}Cu scattering at 30 GeV/c.

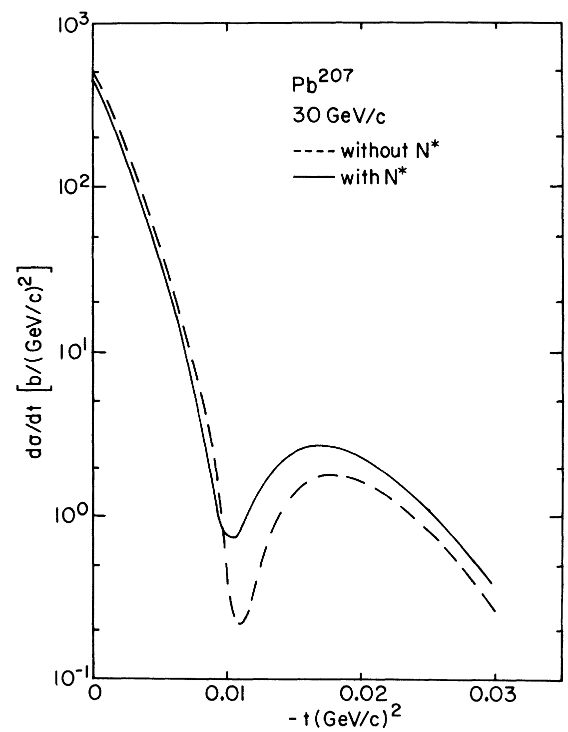

FIG. 14. Elastic differential cross section around the second maximum for N-^{207}Pb scattering at 30 GeV/c.

The reason for the small minimum in ^{4}He may be as follows. Since, at 1.7 GeV/c, $\Delta_m R(\simeq 1.3-0.3i)$ is large for $N^*(1236)$, the term $I_z(j, 2, k)$ given by Eq. (28) has a large imaginary part, which results in making the real part of the nucleon-^{4}He scattering amplitude large and the depth of the minimum shallow. A similar result is also obtained for the case ^{12}C at 2.85 GeV/c.

In Figs. 11–14 we show effects of the N^* states on the elastic differential cross section around the second maximum for ^{12}C at 2.85 GeV/c and for ^{12}C, ^{64}Cu, and ^{207}Pb at 30 GeV/c, though experimental data do not exist yet for them. It is to be noted that at 2.85 GeV/c the $N^*(1236)$ reduces the second maximum and at 30 GeV/c increases the maximum. The reason for this is that we took the phases of $g(q)$ and $\bar{g}(q)$ real for the $N^*(1236)$ and imaginary for $I=\frac{1}{2}$ resonances. We see also in Figs. 12–14 that the $N^*(I=\frac{1}{2})$ contributions to the maximum become large as the mass of the nucleus increases and for Pb it makes the maximum higher by the factor 1.5.

IV. CONCLUSION

From an analysis of the numerical calculations we see that the ordinary Glauber model which takes only elastic intermediate states into account fails to reproduce the momentum dependence of the nucleon-nucleus total cross section, even if we include the momentum dependence of σ_{NN}^T and α. On the other hand, the inclusion of the intermediate N^* states is important for high-energy nucleon-nucleus scattering and well describes the energy dependence of the total cross section, as is shown in the figures.

It is also pointed out that the details of the second maximum in the differential cross section are extremely sensitive to the phase and the magnitude of the N^* production amplitudes. Then, without taking N^* contribution into account, we cannot expect to get new information about the nuclear structure which is considered to have remarkable effects on the second maximum.

Harrington suggested that inelastic contributions in π-d scattering are negligible near the forward direction but are important at large angles.[20] He ignored mass differences of the intermediate states, whereas mass differences cause a suppression of the contributions from the inelastic intermediate states as shown in the present calculations. This suppression is large especially in the case of the deuteron because of the large average distance between a neutron and a proton, and at the energies we considered the inelastic effects may be small even at large angles.

The main usefulness of the Gaussian form is that it allows a simple calculation of the high-energy cross section on the basis of the Glauber theory. Such wave functions, however, are somewhat unrealistic. In considering the second maximum in the differential cross section it is quite necessary to take a more complicated expression than a simple Gaussian.[10] It seems that quantitative results for total cross section, however, are not very sensitive to the nuclear model and our predictions for N^* contributions could serve as a rather severe test of the multiple-scattering theory.

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APPENDIX

In this Appendix the detailed derivation of the function $I_z(j,m,k)$ is given. As the first step let us restrict our attention to $I_z(2,2,0)$:

$$I_z(2,2,0)=\left(\frac{1}{\sqrt{\pi}R}\right)^2\int_{-\infty}^{\infty}\int_{z_1}^{\infty} e^{-z_1^2/R^2}e^{i\Delta_m(z_1-z_2)} \times e^{-z_2^2/R^2}\,dz_2\,dz_1\,. \tag{A1}$$

Making the coordinate transformations according to $z=z_2-z_1$ and $z'=z_1+\frac{1}{2}z$ and integrating over the z' coordinate, we get

$$I_z(2,2,0)=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{\pi}R}e^{-\Delta_m{}^2R^2/2}\int_0^{\infty}e^{-(z+i\Delta_m R^2)^2/2R^2}\,dz\,. \tag{A2}$$

Under the transformation $w=z+i\Delta_m R^2$, the integral becomes

$$I_z(2,2,0)=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{\pi}R}e^{-\Delta_m{}^2R^2/2}\int_{0+i\Delta_m R^2}^{\infty+i\Delta_m R^2}e^{-w^2/2R^2}\,dw, \tag{A3}$$

where integration is performed along the line which begins from the point $w_0=i\Delta_m R^2$ in the complex plane and is parallel to the real axis. Since the function $\exp(-w^2/2R^2)$ is analytic and vanishes for $\mathrm{Re}\,w\to\infty$, then the integration along the above line is equal to the integration along the positive real axis with the additional integration along the line from w_0 to 0:

$$I_z(2,2,0)=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{\pi}R}\,e^{-\Delta_m{}^2R^2/2}\int_0^\infty e^{-z^2/2R^2}\,dz\;+\int_{w_0}^0 e^{-w^2/2R^2}\,dw\,. \tag{A4}$$

The second integral can be approximately obtained by using the power-series expansion of the exponential function and we finally obtain

$$I_z(2,2,0)=\tfrac{1}{2}e^{-\Delta_m{}^2R^2/2}\left\{1-i\frac{\sqrt{2}}{\sqrt{\pi}}\Delta_mR\,[1+\tfrac{1}{6}(\Delta_mR)^2+\tfrac{1}{40}(\Delta_mR)^4+\cdots]\right\}. \tag{A5}$$

Similarly we can calculate $I_z(3,2,1)$ given by

$$I_z(3,2,1)=2\times\frac{1}{(\sqrt{\pi}R)^3}\int_{-\infty}^\infty\int_{z_1}^\infty\int_{z_2}^\infty e^{-(z_1{}^2+z_2{}^2+z_3{}^2)/R^2}\,e^{i\Delta_m(z_1-z_2)}\,dz_3dz_2dz_1\,, \tag{A6}$$

and the result of the integral is approximately written by

$$\begin{aligned}I_z(3,2,1)&=I_z(2,2,0)-\frac{1}{\sqrt{3\pi}}e^{-\Delta_m{}^2R^2/2}\left(1-\frac{\sqrt{\pi}}{\sqrt{2}}\Delta_mR+\cdots\right)\\ &\simeq I_z(2,2,0)\left\{1-\frac{2}{\sqrt{3\pi}}+i\frac{2}{\sqrt{3\pi}}\left[\left(\frac{\pi}{2}\right)^{1/2}-\left(\frac{2}{\pi}\right)^{1/2}\right]\Delta_mR-\cdots\right\}\\ &\simeq I_z(2,2,0)\left(1-\frac{2}{\sqrt{3\pi}}\right),\end{aligned} \tag{A7}$$

since Δ_mR is small at high energy, we take only $1-2/\sqrt{3\pi}$ and neglect other terms. On the other hand, $I_z(3,2,0)$ is related to $I_z(3,2,1)$ by the equation $I_z(3,2,0)=I_z(2,2,0)-I_z(3,2,1)$, so that we get approximately

$$I_z(3,2,0)\simeq\frac{2}{\sqrt{3\pi}}I_z(2,2,0)\,. \tag{A8}$$

As for the higher-order terms, $I_z(z+k,2,k)$ and $I_z(z+k,2,0)$, we put them roughly equal to

$$I_z(2+k,2,k)\simeq\left(1-\frac{2}{\sqrt{3\pi}}\right)^kI_z(2,2,0) \tag{A9}$$

and

$$I_z(2+k,2,0)\simeq\left(\frac{2}{\sqrt{3\pi}}\right)^kI_z(2,2,0) \tag{A10}$$

for simplicity and neglect other dependence on k. Since the number k corresponding to the multiple scattering which appreciably contributes to the nucleon-nucleus scattering is no more than 6 even if the nucleus is as heavy as Pb, the results of our calculation may be hardly changed by more precise calculations of $I_z(z+k,2,k)$ and $I_z(z+k,2,0)$.

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