Variation in Trinucleon Bound-State Properties with Phase-Equivalent Nucleon-Nucleon Interactions*

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We use the Faddeev formalism to investigate the variation of the trinucleon binding energy and electromagnetic form factors with phase-equivalent transformations of the Reid soft-core potential. The transformed nucleon-nucleon interactions used in our calculations are the same as, or slight variations of, those considered by Haftel and Tabakin in studies of nuclear matter. The corresponding transformed two-nucleon wave functions are essentially unchanged for nucleon separations greater than 1 fm. The variation of the trinucleon binding energy follows the same trend as (but is generally much smaller than) the corresponding variation of E_B/A for nuclear matter. The largest increase in E_B(trinucleon) is 0.19 MeV, which gives E_B(total) ≈ 6.9 MeV. The position of the minimum of the calculated $|F_{\rm ch}^{^3\rm He}(Q^2)|$ (experimental value: $Q^2 \approx 11.8^{-2}$) varies between $Q^2 = 14.4$ and 18 fm^{-2}.

I. INTRODUCTION

Two-nucleon data provide only limited information about the nucleon-nucleon interaction. Elastic scattering data and the deuteron binding energy determine only the asymptotic nucleon-nucleon wave function (or equivalently, the on-the-energy-shell nucleon-nucleon T matrix). The deuteron electromagnetic moments and form factors at low momentum transfer depend weakly on the behavior of the wave function for nucleon separations less than about 0.7 fm. The only theoretical constraint (aside from symmetry constraints) that we may presently impose with some confidence on the two-nucleon interaction is that it have a one-pion-exchange tail.

Several elegant techniques have recently been used for studying the effects on nucleon systems with $A > 2$, resulting from variations of the off-shell two-nucleon T matrix. The method of unitarily equivalent Hamiltonians[1-3] is probably the most straightforward one. Let $H = (p^2/M) + V$ be a center-of-mass two-nucleon Hamiltonian (with V having a one-pion-exchange tail) which gives a good fit to nucleon-nucleon phase shifts for $E_{\rm lab} \lesssim 350$ MeV and the deuteron properties. Now consider the unitary transformation $\tilde{H} = UHU^\dagger = (p^2/m) + \tilde{V}$, with

$$\tilde{V} = V + (U-1)H + H(U^\dagger - 1) + (U-1)H(U^\dagger - 1)\,, \tag{1.1}$$

and $\langle \vec{r}\,|U-1|\,\vec{r}\,'\rangle$ arbitrarily small for $r > r_0$ (≈1 fm). The transformed potential $\tilde{V}$ has a one-pion-exchange tail and, since the transformed-state vector $|\tilde{\psi}\rangle = U|\psi\rangle$ has $\langle \vec{r}\,|\tilde{\psi}\rangle = \langle \vec{r}\,|\psi\rangle$ for $r > r_0$, the scattering phase shifts and deuteron binding energy for $\tilde{H}$ are the same as those for H.

The method of unitarily equivalent Hamiltonians has been applied to nuclear-matter calculations in the Brueckner approximation by Miller *et al.*,[4] Coester *et al.*,[5,6] and by Haftel and Tabakin.[7] Miller *et al.*[4] considered phase-equivalent transformations (induced by radial scale distortion[2]) of a hard core $^1S_0$ nucleon-nucleon (*NN*) interaction. They found the binding energy per nucleon E_B/A to increase by as much as 2.4 MeV per nucleon. Coester *et al.*[5] used simple S-wave Yukawa interactions (with and without hard cores) and phase equivalents of these in which $U-1$ is a rank 2 (separable) operator or U is induced by distortion of the radial scale. With rank 2 forms for $U-1$, they found that E_B/A always decreased relative to the value for $U=1$, with some deviations as large as 7 MeV per nucleon. Positive and negative deviations in E_B/A up to 10 MeV per nucleon were found for the case of U induced by radial scale distortion. They also showed[6] that the large differences in E_B/A computed with different phase-equivalent potentials are not significantly reduced by the addition of three-particle-correlation corrections to the saturation curve.

Haftel and Tabakin[7] studied the effects of phase-equivalent transformations of the realistic Reid soft-core potential.[8] They used rank 1, 2 forms with exponential form factors for $U-1$. The form-factor parameters were chosen so that the calculated deuteron quadrupole moment and electromagnetic form factors did not deviate by more than the experimental uncertainties from the values given by the Reid potential. They found values of E_B/A between 1.1 and 10.7 MeV per nucleon. The untransformed Reid interaction gave $E_B/A = 10.0$ MeV/nucleon.

An alternative approach to off-shell effects has

TABLE I. Form-factor parameters and $\sin\theta$ for nucleon-nucleon interactions which are phase equivalent to the Reid soft-core potential.

Potential	$^1S_0$ α_0 (fm^{-1})	$^1S_0$ β_0 (fm^{-1})	$^3S_1$-$^3D_1$ α_0 (fm^{-1})	$^3S_1$-$^3D_1$ β_0 (fm^{-1})	$^3S_1$-$^3D_1$ α_2 (fm^{-1})	$^3S_1$-$^3D_1$ β_2 (fm^{-1})	$\sin\theta$
R (Reid)	...	...	...	...	...	...	...
1	3.0	1.2	...	...	...	...	...
1a	2.4	0.8	...	...	...	...	...
3	3.0	1.0	...	...	...	...	...
4	4.0	1.0	...	...	...	...	...
8	...	...	2.4	0.8	...	...	1
11	...	...	...	...	2.4	0.72	0
1+11	3.0	1.2	...	...	2.4	0.72	0
14	...	...	...	...	3.6	0.90	0
18	...	...	4.0	1.3	...	...	1
18a	...	...	4.0	1.5	...	...	1
18b	...	...	4.0	1.1	...	...	1
18c	...	...	4.0	1.4	...	...	1

been formulated by Baranger *et al.*[9] They derived a method for continuing the two-body T-matrix off shell without the explicit use of a potential. They showed in particular that the off-shell T matrix could be expressed in terms of a function of two variables, $\varphi(p',p)$, where $\varphi(p,p)$ is given in terms of the elastic phase shift, and that the symmetric part of $\varphi(p',p)$ may be arbitrarily specified and the antisymmetric part calculated in terms of it.

Haftel[10] has extended the analysis of Baranger *et al.* to include the case of bound states.

The main drawback of the approach of Baranger *et al.* is the difficulty of translating the one-pion-exchange constraint on the potential tail into a restriction on the off-shell T matrix. Picker, Redish, and Stephenson[11] eliminate this problem by deriving an expression for the half-off-the-energy-shell T matrix in terms of the on-shell T matrix

TABLE II. The ^3H binding energy, the ^3H, ^3He charge and magnetic radii, and the saturation momentum k_F and binding energy per nucleon E_B/A for nuclear matter, calculated using the phase-equivalent nucleon-nucleon interactions indicated in Table I. The values for k_F and E_B/A are taken from Ref. 7. Only the $[pq(00)00\,W^{A}_{1/2}{}^{-}\,\frac{1}{2}\mathcal{J}_z]$ Faddeev components were retained in solving the Faddeev equations in the "two-component" approximation. The additional $[pq(20)2\,W^{-}_{3/2}\,\frac{1}{2}\mathcal{J}_z]$ component was retained in "three-component" calculations, but its contribution to the wave functions used in form-factor calculations was dropped (see Ref. 27).

Potential	No. comp.	$R_{ch}(^3He)$ (fm)	$R_{mag}(^3He)$ (fm)	$R_{ch}(^3H)$ (fm)	$R_{mag}(^3H)$ (fm)	^3H binding energy (MeV)	E_B/A (MeV)	k_f (fm^{-1})
R	2	1.99	2.03	1.83	1.96	6.71	10.0	1.36
1	2	2.09	2.13	1.89	2.04	5.92	3.5	1.20
1a	2					6.60		
3	2	1.98	2.02	1.82	1.95	6.90	10.2	1.37
4	2	2.04	2.08	1.86	2.00	6.43	8.2	1.29
8	2	2.00	2.04	1.84	1.97	6.60		
11	2	1.99	2.03	1.83	1.96	6.70	4.3	1.10
R	3	2.04	2.08	1.86	2.00	6.37	10.0	1.36
11	3	2.04	2.08	1.86	2.00	6.19	4.3	1.10
1+11	3	2.04	2.08	1.86	2.01	5.28	1.1	1.05
14	3	2.06	2.10	1.88	2.02	6.37	9.5	1.33
18	3	2.03	2.07	1.85	1.99	6.52	10.7	1.38
18a	3					6.13		
18b	3					6.37		
18c	3					6.41		
Experimental values		1.88 ± 0.05 [a]	1.95 ± 0.11 [a]			8.49		
		1.87 ± 0.05 [b]	1.75 ± 0.10 [b]	1.70 ± 0.05 [b]	1.70 ± 0.05 [b]			

[a] Reference 24.

[b] Reference 25.

and the difference between the full scattering wave function and the phase-shifted free wave function. By using this relationship, one can construct a class of half-off-the-energy T matrices which are compatible with a specified on-shell T matrix and a prescribed local potential tail. Of course, the same end result may be achieved by the method of unitarily equivalent Hamiltonians.

Another approach to off-shell effects is that of Lomon[12] who used a pseudopotential version of the boundary condition model (BCM)[13] to obtain phase-equivalent nucleon-nucleon interactions, and found that they gave a large variation in E_B/A for nuclear matter. Hoenig[14] used these interactions in trinucleon calculations and found similarly large variations in the binding energy and electromagnetic form factors. There are some difficulties of interpretation of these results which require further study. Different phase-equivalent BCM interactions have different point spectra, i.e., differ with respect to singularities of the off-shell T matrix, and thus their associated Hamiltonians are not unitarily related. A unique off-shell continuation[15,16] of the BCM T matrix may be derived if certain mild analyticity constraints are imposed.[17]

Generalizations of the method of unitarily equivalent Hamiltonians have been given by Monahan, Shakin, and Thaler.[18,19] They constructed[18] phase-equivalent interactions for which the bound states are either identical or differ in some preassigned way for small nucleon separations. They also derived[19] two classes of Hamiltonians for which the energy eigenfunctions of the members of each class are identical below some cut-off energy E_c. For energies greater than E_c, the eigenfunctions of Hamiltonians of one class differ for small internucleon separations but are identical in the asymptotic region. In the other class, the eigenfunctions also differ asymptotically for energies greater than E_c.

In this paper, we use the Faddeev formalism[20] to investigate the variation of the trinucleon binding energy and electromagnetic form factors with phase-equivalent transformations of the Reid soft-core potential.[8] The transformed nucleon-nucleon interactions used in our calculations are the same as, or slight variations of, those used by Haftel and Tabakin. The comparison of our results with

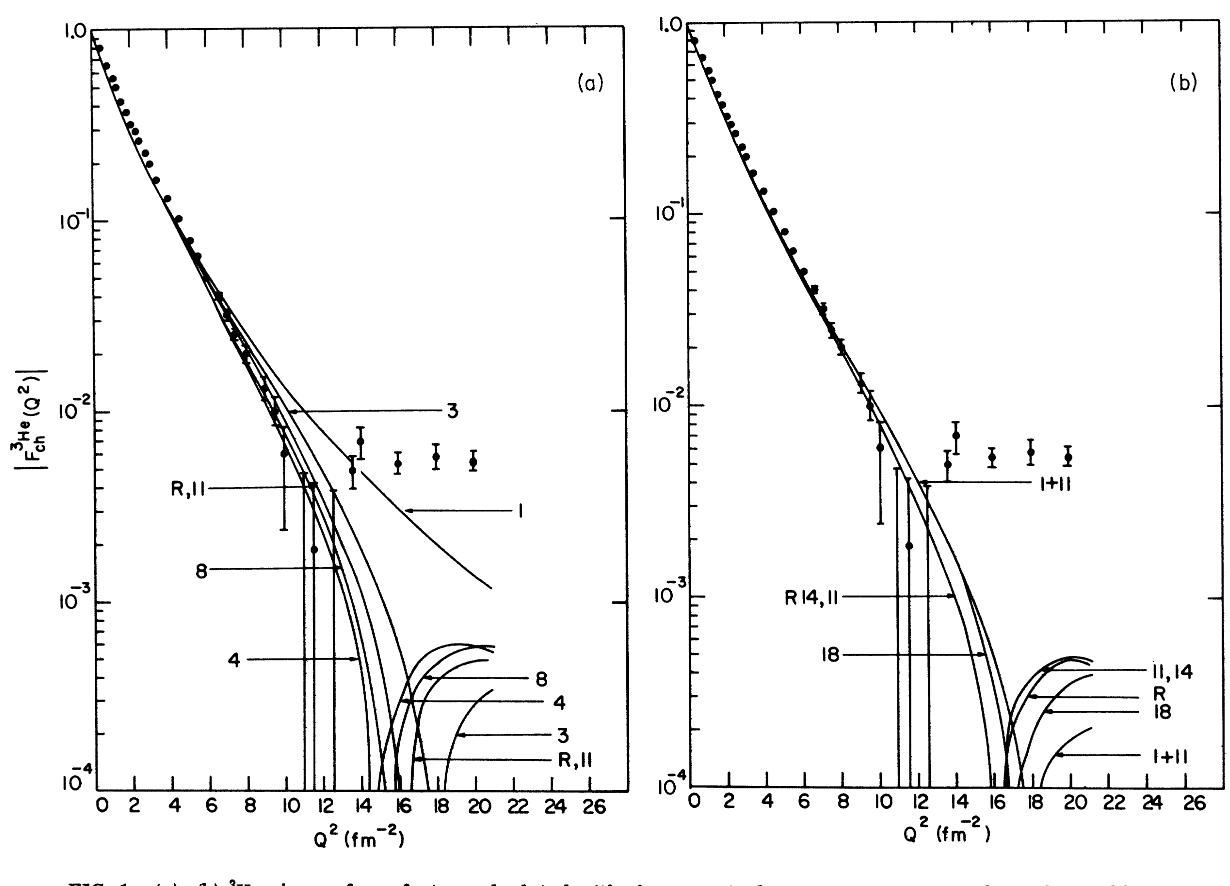

FIG. 1. (a), (b) ^{3}He charge form factor calculated with phase-equivalent interactions as indicated in Table II. The experimental points are taken from Ref. 24.

theirs should give a good indication of the relative magnitudes of off-shell effects in nuclear matter and in the trinucleon system. The sensitivity to off-shell variations of the *NN T* matrix is expected to be smaller for the properties of trinucleon systems than it is for those of nuclear matter. However, the great precision with which one can now solve the three-nucleon problem[21] (in comparison with present uncertainties in nuclear-matter calculations) partially compensates for this smaller sensitivity.

It should be noted that the investigation of off-shell effects described in this paper is not very comprehensive, since we have limited our class of unitarily equivalent Hamiltonians to those related by rank 1, 2 forms for $U-1$. In a future publication, we will present results obtained with $U-1$ generated by radial scale distortion and the class of Hamiltonians discussed by Monahan *et al.*[18,19]

The rank 1, 2 forms for $U-1$ used in this paper are given in Sec. II and the results of calculations are presented in Sec. III. The results are discussed in Sec. IV.

II. PHASE-EQUIVALENT NUCLEON-NUCLEON INTERACTIONS (REF. 7)

The short-range unitary operator used by Haftel and Tabakin is of the form

$$U = 1 - 2\Lambda\ , \tag{2.1}$$

where Λ is a Hermitian projection operator:

$$\Lambda^2 = \Lambda\ . \tag{2.2}$$

The transformed phase-equivalent potential $\tilde{V}$ of (1.1) may be written as

$$\tilde{V} = V - 2\Lambda V - 2V\Lambda + 4\Lambda V\Lambda - 2\Lambda T - 2T\Lambda + 4\Lambda T\Lambda\ . \tag{2.3}$$

It is assumed that $\langle\vec{r}\,|\Lambda|\,\vec{r}\,'\rangle \to 0$ faster than $1/r$ so that the bound-state energies and phase shifts of the transformed Hamiltonian are the same as those of the original Hamiltonian.

The rank 1, 2 form of $\langle\vec{r}\,|\Lambda|\,\vec{r}\,'\rangle$, which is consistent with the symmetries of the two-nucleon

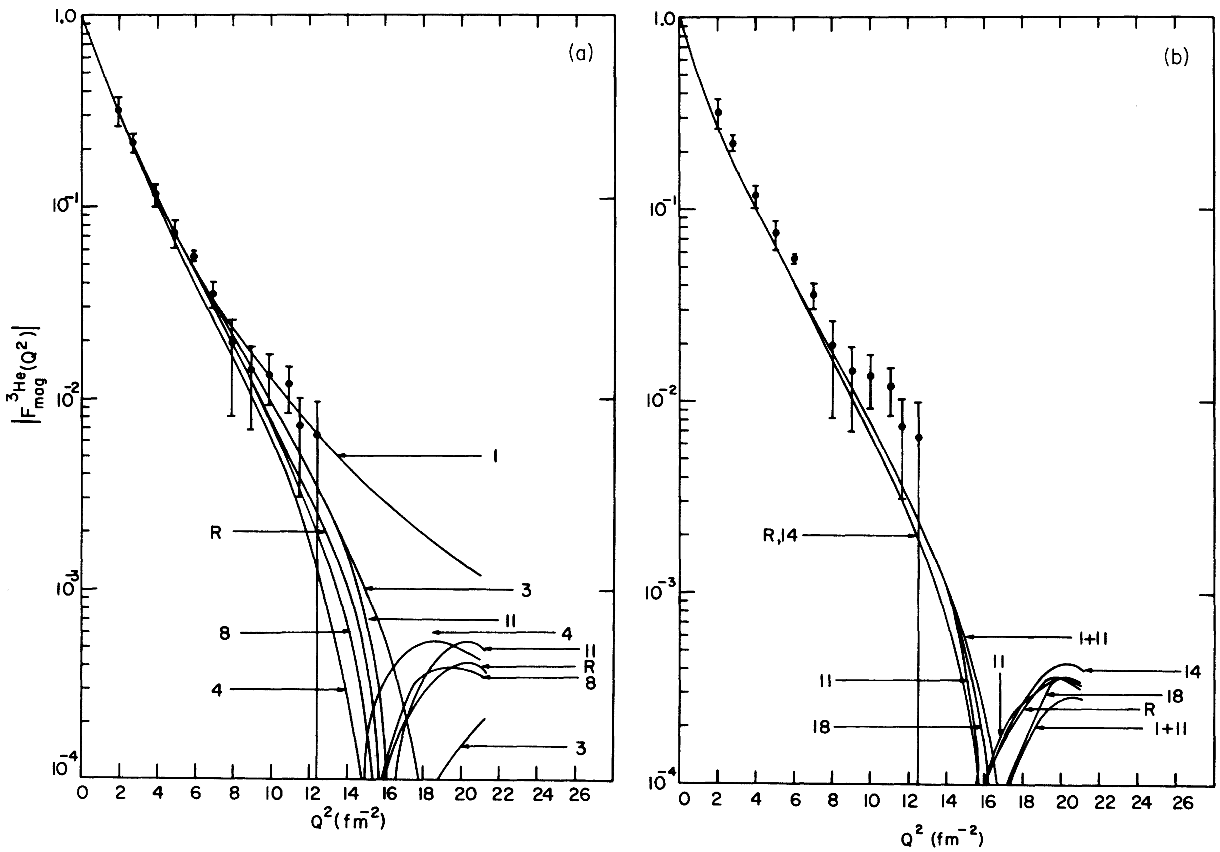

FIG. 2. (a), (b) ^{3}He magnetic form factor calculated with phase-equivalent interactions as indicated in Table II. The experimental points are taken from Ref. 24.

system, is

$$\langle\vec{r}|\Lambda|\vec{r}'\rangle = \sum_{LL'\alpha M T_3} g_L^\alpha(r) g_{L'}^\alpha(r') \times \mathcal{Y}_L^{\alpha M T_3}(\hat{r}) \mathcal{Y}_{L'}^{\alpha M T_3 \dagger}(\hat{r}') A_{LL'}^\alpha , \quad (2.4)$$

where the form factors g_L^α are real. L and L' are orbital angular momentum quantum numbers, and α denotes the quantum numbers J (total angular momentum), S (total spin), and T (total isospin). M and T_3 are, respectively, the quantum numbers for the third component of total angular momentum and total isospin. The $\mathcal{Y}_L^{\alpha M T_3}$ are normalized eigenfunctions with the indicated quantum numbers.

If we normalize the $g_L^\alpha(r)$ to unity,

$$\int_0^\infty r^2 dr\, g_L^\alpha(r) g_L^\alpha(r) = 1 , \quad (2.5)$$

then unitarity (i.e., $\Lambda^2 = \Lambda$) requires that $A_{LL} = 1$ for uncoupled channels and that

$$\sum_{L''} A_{LL''}^\alpha A_{L''L'}^\alpha = \delta_{L''L'} \quad (2.6)$$

for coupled channels (L, L', $L'' = J \pm 1$). The parametrization,

$$[A_{LL'}^\alpha] = \begin{matrix} & J-1 & J+1 \\ J-1 & \sin^2\theta & \sin\theta\cos\theta \\ J+1 & \sin\theta\cos\theta & \cos^2\theta \end{matrix} , \quad (2.7)$$

with θ real, satisfies the unitarity requirement (2.6).

The momentum-space matrix elements of Λ are

$$\langle\vec{p}|\Lambda|\vec{p}'\rangle = \frac{2}{\pi} \sum_{LL'\alpha M T_3} i^{L'-L} g_L^\alpha(p) g_{L'}^\alpha(p') \times \mathcal{Y}_L^{\alpha M T_3}(\hat{p}) \mathcal{Y}_{L'}^{\alpha M T_3 \dagger}(\hat{p}') A_{LL'}^\alpha , \quad (2.8)$$

where

$$g_L^\alpha(p) = \int_0^\infty r^2 dr\, g_L^\alpha(r) j_L(pr) . \quad (2.9)$$

We assume that the nucleon-nucleon interaction is effective only in the $^1S_0$ and $^3S_1$-$^3D_1$ states. The form factor

$$g_0^\alpha = C_0 e^{-\alpha_0 r}(1 - \beta_0 r) \quad (2.10)$$

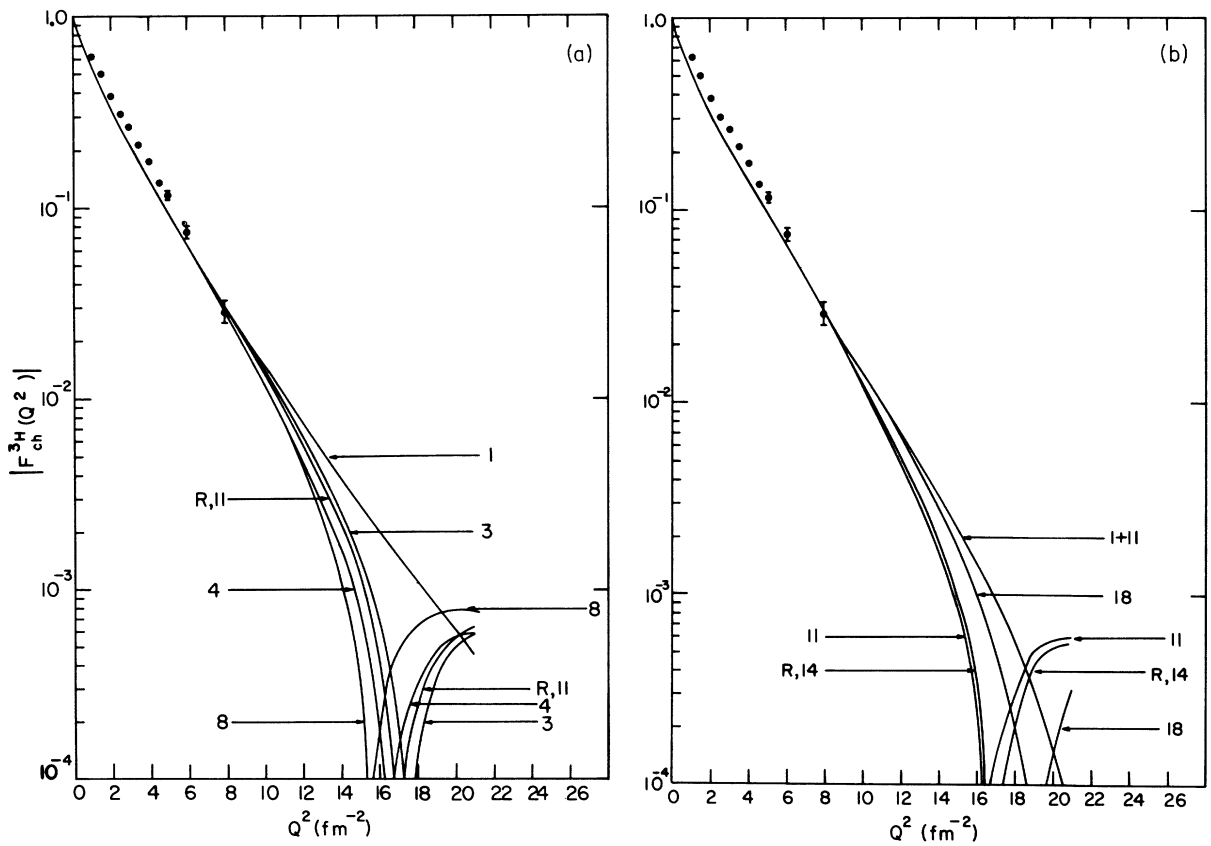

FIG. 3. (a), (b) ^{3}H charge form factor calculated with phase-equivalent interactions as indicated in Table II. The experimental points are taken from Ref. 25.

is used for the ${}^1S_0$. For the ${}^3S_1$-${}^3D_1$ states, we use

$$g_0^{\alpha}=C_0e^{-\alpha_0 r}(1-\beta_0 r)\,,$$
$$g_2^{\alpha}=C_2re^{-\alpha_2 r}(1-\beta_2 r)\,. \tag{2.11}$$

The constants C_L are determined by the normalization condition (2.5).

The parameters of the phase-equivalent interactions for which trinucleon calculations were done, are listed on Table I. We have used the same identifications as Haftel and Tabakin for interactions 1, 3, 4, 8, 11, 14, and 18. 1a and 18a,b, and c are slight variations of their interactions 1 and 18, respectively.

III. CALCULATION OF TRINUCLEON BINDING ENERGY AND ELECTROMAGNETIC FORM FACTORS

In this section, we present the results of Faddeev calculations of the trinucleon binding energy and electromagnetic form factors based on the phase-equivalent interactions given in Sec. II.

The Faddeev amplitude was expanded in components with respect to center-of-mass trinucleon basis states $|pq(Ll)\mathcal{L}W_{\mathcal{S}}^{r}\mathcal{J}\mathcal{J}_z\rangle$, where $\vec{p}=(\vec{k}_2-\vec{k}_3)/2$, $\vec{q}=(\vec{k}_2+\vec{k}_3-2\vec{k}_1)/(12)^{1/2}$, $\vec{k}_i$ = momentum of nucleon i and $W_{\mathcal{S}}^{r}$ is a normalized spin-isospin state with total spin $\mathcal{S}=\frac{1}{2}$ or $\frac{3}{2}$ and total isospin $\mathcal{J}=|\mathcal{J}_z|=\frac{1}{2}$. The index $r=A, S, +, -$ denotes, respectively, complete antisymmetry, complete symmetry mixed over-all symmetry with symmetry under 23 exchange, and mixed over-all symmetry with antisymmetry under 23 exchange.

With the nucleon-nucleon interaction effective in the ${}^1S_0$ and ${}^3S_1$-${}^3D_1$ states, there are five independent components of the Faddeev amplitude. These components of the homogeneous solution of the Faddeev equations determine the trinucleon bound-state wave function.[22,23]

Because of the large amount of computer time required to solve the five coupled two-dimensional Faddeev integral equations, we have made the approximation of retaining only the $[pq(00)0W^{A}_{1/2}\frac{1}{2}\mathcal{J}_z]$, $[pq(00)0W^{-}_{1/2}\frac{1}{2}\mathcal{J}_z]$, and $[pq(20)2W^{-}_{3/2}\frac{1}{2}\mathcal{J}_z]$ Faddeev components. In previous calculations,[21] it was found that one can calculate the trinucleon binding energy E_B to within ≈0.3 MeV, the charge radius to within ≈0.04 fm, and the electromagnetic form

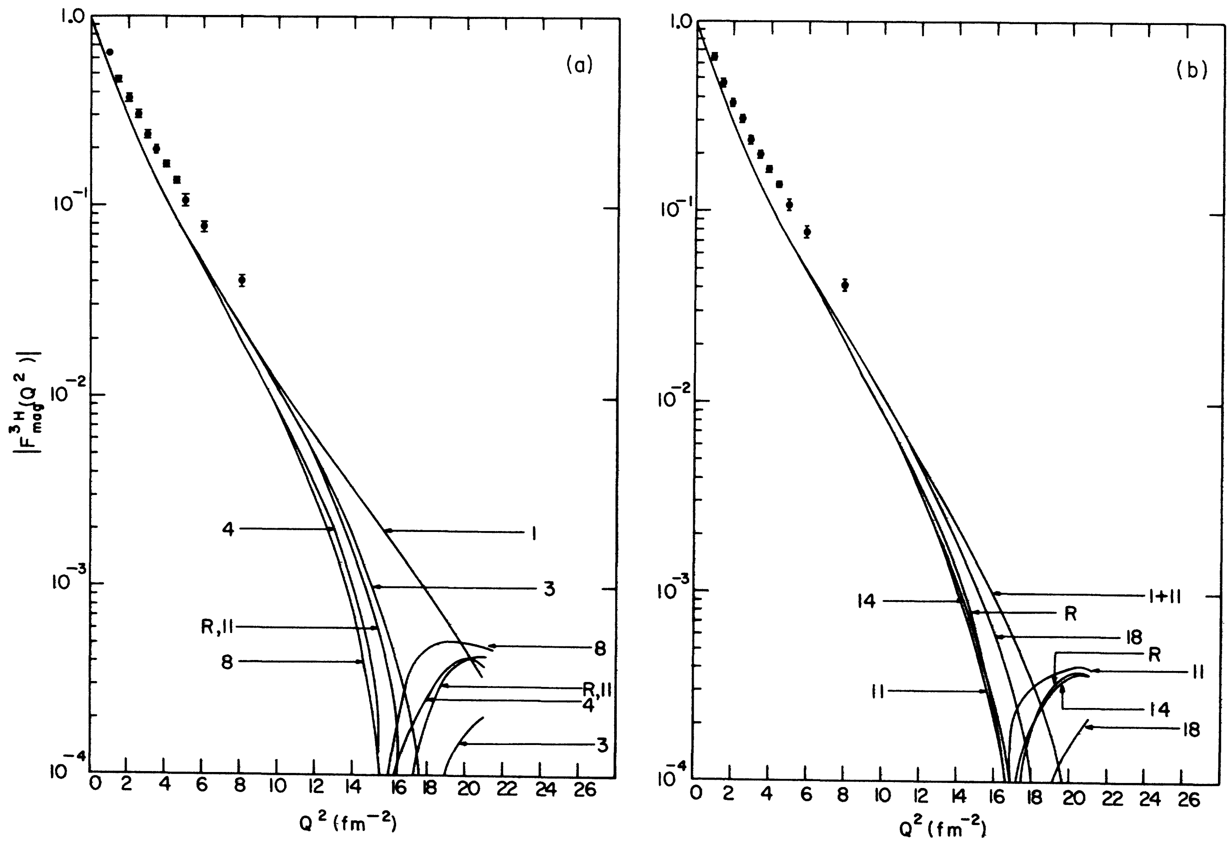

FIG. 4. (a), (b) ${}^3H$ magnetic form factor calculated with phase-equivalent interactions as indicated in Table II. The experimental points are taken from Ref. 25.

factors for momentum-transfer squared $\leq 16\ \text{fm}^{-2}$, by solving the Faddeev equations retaining only the first two of the above-mentioned components. Our method of solving the Faddeev equations is described in Ref. 21.

In Table II, we give the ^{3}H binding energy, the ^{3}H, ^{3}He charge, and magnetic radii, and the nuclear matter parameters k_F and E_B/A calculated by Haftel and Tabakin, for the phase-equivalent interactions indicated in Table I. The trinucleon electromagnetic form factors calculated with these interactions are plotted in Figs. 1 through 4 along with the experimental values of McCarthy *et al.*[24] and Collard *et al.*[25] The analytic forms of Janssens *et al.*[26] were used for the nucleon electromagnetic form factors. In the calculation of the trinucleon form factors, the contributions to the wave functions from the $[pq(20)2W^{-}_{3/2}\tfrac{1}{2}\tfrac{1}{2}\mathscr{J}_z]$ components of the Faddeev amplitudes were dropped.[27]

IV. DISCUSSION OF RESULTS

We see from the results given in Sec. III that the properties of ^{3}H, ^{3}He are generally much less sensitive to variations of the off-shell nucleon-nucleon T matrix than are those of nuclear matter. For example, the phase-equivalent interaction (1+11) gives a value of E_B/A for nuclear matter which is about 10 times smaller than the value given by the Reid potential; on the other hand, (1+11) gives a value for the ^{3}H binding energy which is only about 20% less than that given by the Reid potential. In all cases for which both nuclear-matter and trinucleon calculations have been done, the variations of $E_B(^3\text{H})$ and E_B/A (nuclear matter) from their values for the Reid potential are of the *same* sign.

There are considerable variations in the ^{3}H, ^{3}He electromagnetic form factors for $Q^2 \gtrsim 8\ \text{fm}^{-2}$, but all of the minima of $|F_{\text{ch}}^{^3\text{He}}(Q^2)|$ are substantially larger than the experimental value[24] of $Q^2 \approx 11.8\ \text{fm}^{-2}$. The variation of the low-Q^2 behavior of the form factors is represented by variations of a few percent in the charge and magnetic radii.

An analysis somewhat similar to the one reported in this paper was done by Hadjimichael and Jackson[28] for phase-equivalent interactions R, 3, 4, 14, and 18. They calculated the ^{3}H binding energy and wave function using a variational technique[29,30] with harmonic-oscillator basis states. Unfortunately, they did not include a sufficiently large number of terms in their wave functions and had to make very rough extrapolations for the values of the binding energies. Thus a detailed comparison of their results with ours is not very meaningful.

Another study of phase equivalent nucleon-nucleon interactions in trinucleon systems, based on a simple Yukawa form for the "untransformed" interaction, has been reported by Haftel.[31]

As was mentioned in the Introduction, our present investigation of off-shell effects in the trinucleon system is very limited, and must be greatly expanded. Only after one has a clear picture of the variation of trinucleon parameters with a broad class of phase-equivalent and phase-semi-equivalent[19] nucleon-nucleon interactions, will we be able to quantitatively assess the importance of relativistic effects and three-nucleon forces.

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Effects of N^* Production on Nucleon-Nucleus Scattering

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The Glauber theory of multiple scattering is extended so as to include the contributions of N^* production and is applied to the elastic scattering of a nucleon from various nuclei for incident momenta in the range from 1.7 to 30 GeV/c. The momentum dependence of the nucleon-nucleus total cross sections is discussed. The influences of the inclusion of N^* effects on the structure of the second maximum in the differential cross section is also investigated.

I. INTRODUCTION

There is a great deal of interest in high-energy hadron-nucleus scattering. It is hoped that hadron-nucleus scattering experiments provide new information on the elementary particle interactions, as well as on the structure of nuclei. More precisely, first from the point of nuclear physics, we expect to obtain information on nuclear wave functions in general and particularly correlations which cannot be obtained from the electron scattering data. For light nucleus, the electromagnetic interaction is reasonably well described by the first Born approximation which depends only on the charge distribution in the nucleus, i.e., on a proton density function.[1] If we use hadron-nucleus scattering experiments, we may obtain some knowledge about a neutron distribution in a nucleus. Moreover, hadronic interactions – or strong interaction – are so intense that multiple collisions are quite a strong influence on the cross sections we observe. An incident particle has a large probability of interacting more than once as it passes through the nucleus, and thus it is possible to obtain information on nucleon-nucleon correlations in the nucleus, although this approach is still in a very preliminary state. Second, from the point of particle physics a nucleus is a convenient system in which rescattering of short-lived particles, or resonances, can be studied. Namely, if a resonance is produced inside a nucleus, then it has a chance to strike nucleons on its way out of the nucleus. Then, if we have a good theory of particle-nucleus interactions, we are able to extract the resonance-nucleon scattering amplitude from the nuclear production amplitude. Further, if the cross section for a particular process of particle-hydrogen collision is σ_H, then the cross section for that of particle-nucleus with mass number A may be $\sigma_A = \sigma_H A^n$, where $n > 0$. Thus if we want to examine rare production modes, we look at the production from a nucleus, and then using the theory, we may extract the production from a single nucleon.

In the hope of discovering more about the structure of nuclei, the Brookhaven group performed a series of experiments with a beam of 1.7-GeV/c protons and the elastic differential and total cross sections in H, D, ^{4}He, ^{12}C, ^{16}O, were measured with high precision.[2,3] As to nucleon-nucleus total cross sections some data have been accumulated lately in nuclei such as Be, C, Al, Cu, Cd, W, and Pb bombarded by neutron beams in the momentum range 10 to 30 GeV/c.[4-6]

The theoretical work on this problem has been carried out on the basis of the multiple-scattering