Radiative Capture of ^{3}He by ^{9}Be from 1 to 6 MeV*

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(Received 8 June 1972)

The radiative-capture reaction ^{9}Be(^{3}He, γ)^{12}C has been studied in the bombarding energy range $1.0 \leq E(^3\text{He}) \leq 6.0$ MeV. Transitions to the ground state and first two excited states were seen. Excitation curves and angular distribution measurements indicate a broad resonance near 2.55 MeV, formed by s- and d-wave capture, suggesting the presence of a 1^-, $T=1$ state of ^{12}C at 28.2 MeV. The transitions to the two 0^+ final states are strong, and show strikingly similar energy dependence. A simple interpretation of the observations in terms of a particle-hole picture of ^{12}C is presented.

INTRODUCTION

In recent years, studies of radiative capture of deuterons or ^{3}He particles[1-3] have attracted theoretical interest because of their role in estimating the importance of clustering in nuclear structure,[4] in the fine structure of the giant dipole resonance,[5,6] and in adding a further dimension to nuclear structure information at moderate excitation energies. The excitation region in ^{12}C above 26.28 MeV can be investigated through the reaction ^{9}Be(^{3}He, γ)^{12}C. (See Fig. 1.) Earlier work on this reaction has been done by Blatt and Kohler[7] (bombarding energies below 3 MeV), and Black, Jones, and Treacy[2] (up to 4.5 MeV). The latter reported 90° excitation curves for captures to the ground and first excited states (referred to below as γ_0 and γ_1, respectively). No structure attributable to ^{12}C compound-nuclear states was seen. The present work is an extension of the measurements of Ref. 7. Radiative capture has been observed to the first three states of ^{12}C; 90° excitation curves for γ_0, γ_1, and γ_2 are presented, as well as an indication of the angular distributions of these radiations. The data suggest the presence of a broad level in ^{12}C near 28.2 MeV, which dominates the captures to the two 0^+ levels (ground state and 7.65-MeV second excited state). A nonresonant contribution must be added to this resonance to explain the excitation curve for first-excited-state capture. Using a simple picture of the reaction mechanism, the results suggest important two-particle–two-hole strength in the first two 0^+ levels of ^{12}C.

EXPERIMENT

The experimental conditions common to (^{3}He, γ) studies, including the high energy and low intensity of the γ rays compared to radiations from competing reactions, dictate a high-efficiency detector with reasonable resolution above 20 MeV. A well-collimated anticoincidence-shielded NaI(Tl) detector satisfies this requirement; two such detectors were used in the present work. The data above 3 MeV were taken with the Ohio State system,[3] based on a 10-cm-diam × 15-cm-long NaI(Tl) crystal surrounded by a 10-cm-thick NE-102 plastic scintillator. The data below 3 MeV, taken at Stanford, were measured with a similar system with a 12.7-cm × 15-cm main crystal.[7] These systems, surrounded with 4 to 6 in. of lead, reduce the cosmic-ray background in the region of interest by about a factor of 1000 over a bare NaI detector. Pileup, from the prolific lower-energy γ radiation accompanying competing particle-emitting reactions, was reduced with fast electronics.[8] At $E_\gamma = 20$ MeV, the monoenergetic γ-ray line shapes produced by these detector system had a resolution of ~7%.

The ^{3}He beams were produced by 3- and 5.5-MeV Van de Graaff accelerators and were magnetically analyzed. Beryllium targets were made by evaporation of the metal onto tantalum or molybdenum backings. The target thicknesses were measured by observing the apparent width of the narrow resonance[9] at 1.08 MeV in the reaction ^{9}Be(p, γ_1)^{10}B. All targets were less than 100 keV thick for the ^{3}He energies at which they were used. Cross-section calibration was done by comparing the (^{3}He, γ) yields with the yield, from the same target, of the (p, γ_0) reaction at the 0.99-MeV resonance.[9] The detector efficiency and line shape were studied as a function of γ-ray energy, using a set of reactions which produce either monoenergetic or well-separated lines, including ^{9}Be(p, γ)^{10}B, ^{11}B(p, γ)^{12}C, and T(p, γ)^{4}He. The calibration results had to be extrapolated to the higher-energy region observed in the (^{3}He, γ) reaction; this is a major source of uncertainty in the results.

The energy scale for the γ-ray spectra was established by observing the ground- and first-excited-state γ rays from the reaction ^{11}B(p, γ)^{12}C. A further calibration was obtained by short runs during which the lower-energy γ rays produced by ^{3}He bombardment of the ^{9}Be target itself were measured. A typical spectrum of this low-energy region is shown in Fig. 2. The ^{9}Be(^{3}He, $p\gamma$)^{11}B reaction produces most of the observed γ rays, although some peaks are also seen from ^{9}Be(^{3}He, $n\gamma$)^{11}C.

Data for γ rays above the region shown in Fig. 1 were recorded with the detector at 90° with respect to the beam, for ^{3}He energies from 1.0 to 6.0 MeV, in 0.5-MeV steps. Each point took from 6 to 10 h to accumulate sufficient statistical accuracy. A spectrum of the high-energy γ rays observed at a bombarding energy of 3.0 MeV is shown in Fig. 3. The line at 17.6 MeV probably comes from ^{9}Be(^{3}He, α)^{8}Be, as pointed out by Black, Jones, and Treacy.[2] The capture-γ peaks were identified by their actual energies and by observation of the dependence of these energies on

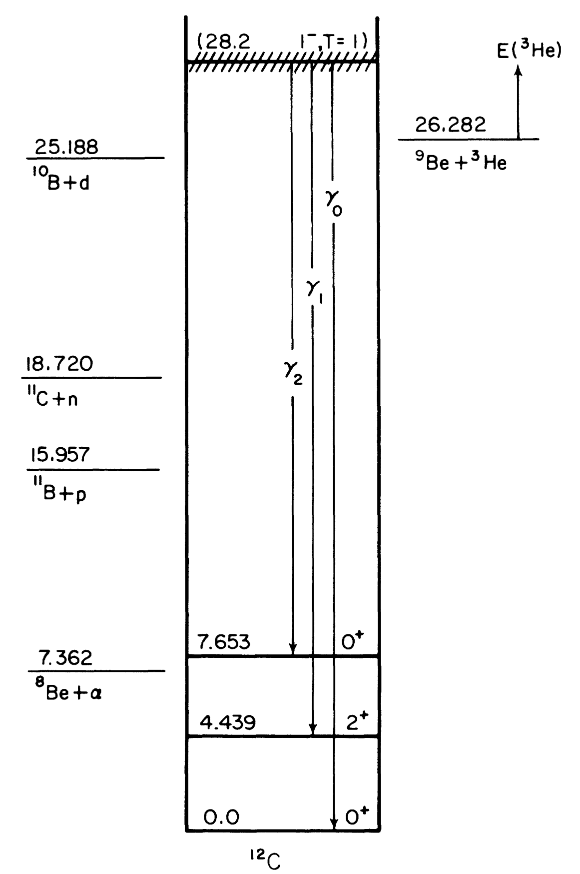

FIG. 1. Relevant energy levels of ^{12}C. The broad level at 28.2 MeV is suggested by the present work.

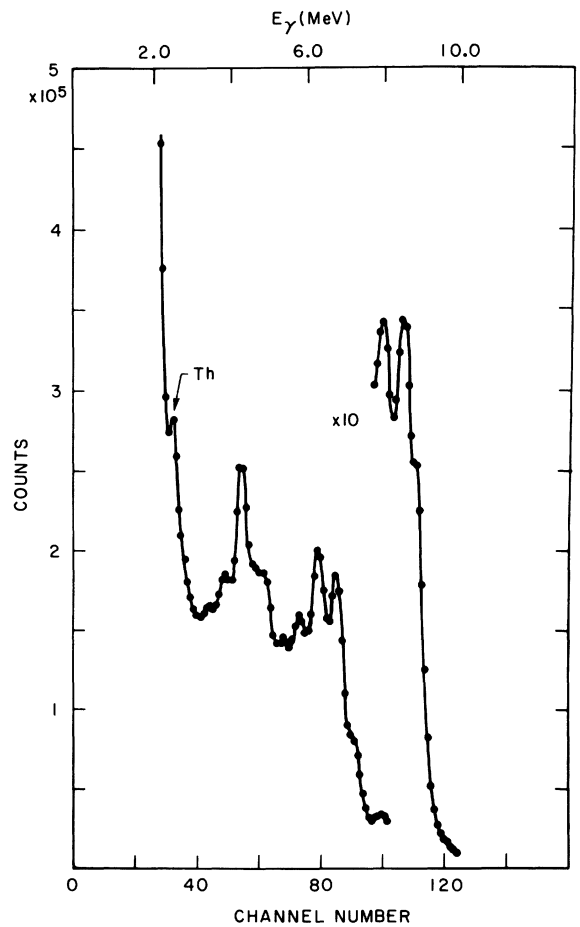

FIG. 2. Low-energy region of γ-ray spectrum for ^{3}He on ^{9}Be, recorded at E(^{3}He) = 2.5 MeV. The lines are from ^{9}Be(^{3}He, $p\gamma$)^{11}B and ^{9}Be(^{3}He, $n\gamma$)^{11}C reactions; an additional calibration line from a thorium source is also indicated.

the ^{3}He lab energy, $E_\gamma = Q + (\frac{9}{12})E(^3\mathrm{He})$. The γ-ray lines were fitted, with the line shapes obtained as indicated above, using a least-squares computer code.[10] Corrections were made for losses due to pileup rejection and random coincidences. Using the value of 36.2 μb/sr for the 90° (p, γ_0) calibration cross section,[9] and applying the extrapolated detector-efficiency function, the 90° (^{3}He, γ) excitation curves for γ_0, γ_1, and γ_2 appear as shown in Fig. 4. The error bars shown on this figure include statistical uncertainties as propagated through the least-squares fitting, the estimated uncertainty in the γ-ray detection-efficiency function, and smaller contributions from other experimental uncertainties.

Angular distribution measurements between 0 and 90° were recorded near the maximum of the broad structure seen in the excitation curves of γ_0 and γ_2. Data taken for five angles at 3.5 MeV are shown in Fig. 5. For the small angular spread subtended by the collimated detector, corrections for finite solid angle are negligible compared to the statistical uncertainties in the data. A least-squares fit was made to these data with Legendre polynomial series including terms up to $l=4$. For both γ_0 and γ_2, minimum χ^2 values were obtained for a distribution of the form $W(\theta) \propto 1 + a_2 P_2(\cos\theta)$. In the case of γ_1, the form $W(\theta) = \text{constant}$ seems not only consistent with the data, but more physically reasonable, even though inclusion of terms up to P_4 improves the fit slightly. (A positive P_4 term, as indicated by the fit, would imply f-wave capture with an $E2$ transition or d waves with an $M2$ transition; the former is unlikely due to the small penetrability factor, while the strength of an $M2$ would be much weaker than the observed value.) The angular distributions at 3.5 MeV were found to have the following values:

$$W_0(\theta) \propto 1 - (0.78 \pm 0.19) P_2(\cos\theta),$$

$$W_1(\theta) \propto 1 - (0.25 \pm 0.3) P_2(\cos\theta),$$

$$W_2(\theta) \propto 1 - (0.86 \pm 0.23) P_2(\cos\theta).$$

Additional distribution measurements were taken, covering three angles only, at 2.0 and 5.5 MeV; however, shorter runs were made, and the data obtained at these energies were sufficient to draw only qualitative conclusions. At the lower energy, all the distributions are nearly isotropic. At the higher energy, only the γ_0 data allow unambiguous interpretation; in this case, the coefficient of $P_2(\cos\theta)$ would appear to be close to -1.

RESULTS

The observation of a broad peak in both the γ_0 and γ_2 excitation curves suggests that a compound-

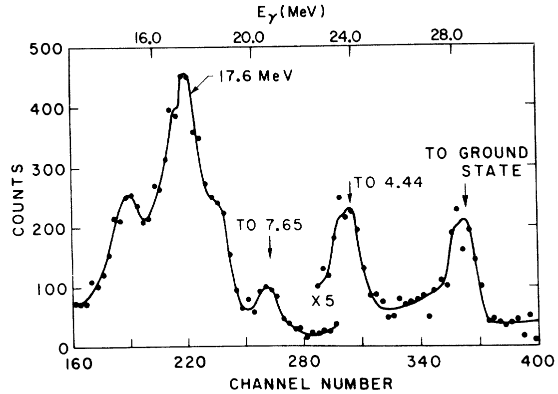

FIG. 3. Capture-γ spectrum from 3.0-MeV ^{3}He particles on ^{9}Be. The three highest-energy lines are identified as γ_0, γ_1, and γ_2.

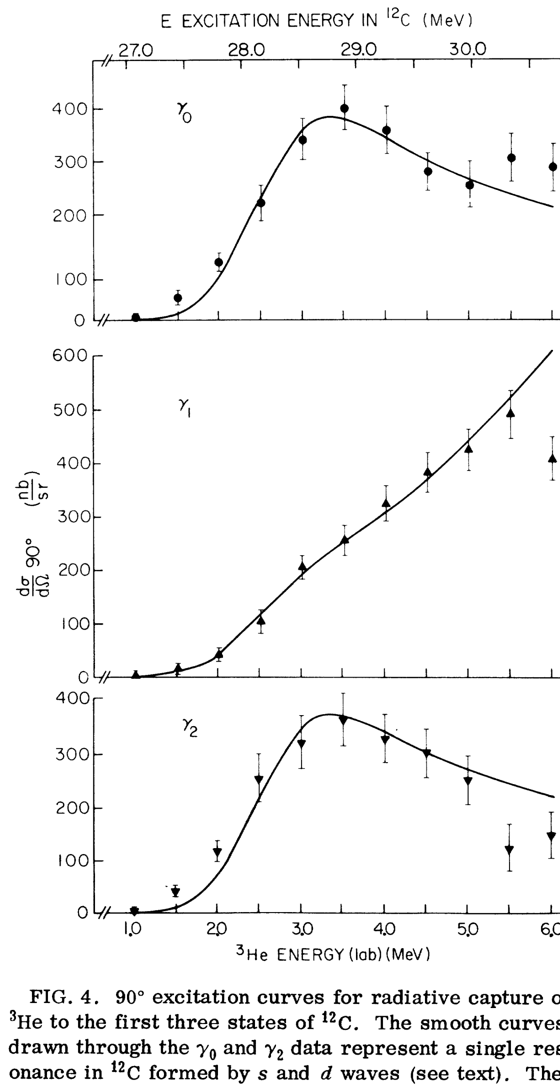

FIG. 4. 90° excitation curves for radiative capture of ^{3}He to the first three states of ^{12}C. The smooth curves drawn through the γ_0 and γ_2 data represent a single resonance in ^{12}C formed by s and d waves (see text). The curve for γ_1 includes both this same resonance and a strong nonresonant contribution.

nucleus mechanism may be important in this reaction. Accordingly, theoretical angular distributions for resonant capture to a compound state of well-defined spin and parity from incoming partial waves up to $l=3$, and outgoing radiation multipolarities up to $M2$, were calculated, utilizing the tables of Sharp *et al.*[11] No combination of a single partial wave (with or without channel-spin mixing) and a single resonant state was found which would yield a negative P_2 coefficient as large as those observed for γ_0 and γ_2. However, the experimental results can be explained if the compound state has $J^\pi = 1^-$ and is formed by coherent s and d waves (with channel spin 1). In this case, the angular distribution is given by

$$W(\theta) = W_{00} + x^2 W_{22} + 2x W_{02},$$

where W_{00} is the distribution for s waves alone, W_{22} the distribution for d waves alone, and W_{02} is the distribution due to their interference. The quantity x^2 measures the ratio of d- to s-wave strength in the capture reaction. For the particular case of a 0^+ final state, we expect

$$W(\theta) \propto 1 - [(0.5x^2 + 1.42x)/(1+x^2)]P_2(\cos\theta).$$

If we use the approximate value of -0.8 for the experimental P_2 coefficient for both γ_0 and γ_2, there are two values of x which satisfy the requirements: $x = 1.1$ or 3.6. If either of these values is used in the formula for the distribution of radiation to a 2^+ final state,

$$W(\theta) \propto 1 - [(0.5x^2 - 0.14x)/(1+x^2)]P_2(\cos\theta),$$

values close to isotropy are found, in reasonable agreement with the γ_1 data.

It now is necessary to see if the shapes of the excitation curves can be understood using these parameters. As a first approximation, consider a resonance of Breit-Wigner form,

$$\sigma(E) \doteq \pi\lambda\!\!\!^{-2} \frac{(2J+1)}{(2j_1+1)(2J_1+1)} \frac{\Gamma(^3\text{He})\Gamma_\gamma}{(E-E_0)^2 + (\Gamma/2)^2},$$

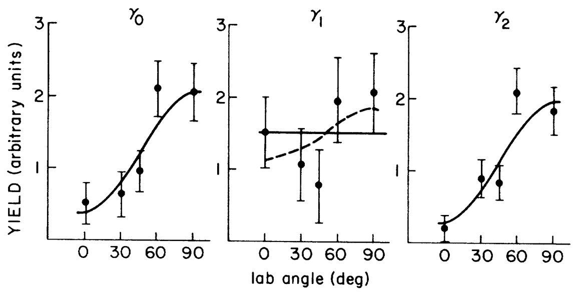

FIG. 5. Angular distributions measured at $E(^3\text{He}) = 3.5$ MeV. Curves of the form $W(\theta) = 1 + a_2 P_2(\cos\theta)$ are fitted to the data; for γ_1 an isotropic fit is also shown.

where J is the spin of the resonant state, and j_1 and J_1 are the spins of the projectile and target, respectively. If we make the approximations that the total width, Γ, is a constant, that $\Gamma_\gamma = (\text{const})E_\gamma^3$, and $\Gamma(^3\text{He}) = 2\mathcal{P}_l\gamma^2(^3\text{He})$, we have a starting point for the case of a single partial wave. Here, $\mathcal{P}_l$ is the penetrability for the lth partial wave and $\gamma^2(^3\text{He})$ is the reduced width of the resonance for ^{3}He emission. For two coherent partial waves, the numerator of the last term will contain three terms,

$$\Gamma_s\Gamma_\gamma + \Gamma_d\Gamma_\gamma + 2(\Gamma_s\Gamma_d\Gamma_\gamma\Gamma_\gamma)^{1/2},$$

assuming that the Breit-Wigner denominator remains the same and that the partial width for γ decay, Γ_γ, does not depend on which partial wave formed the state; Γ_s and Γ_d are the ^{3}He partial widths for s and d waves, respectively. To allow for different possible s- and d-wave mixtures, we introduce a parameter $y^2 = \gamma_d^2(^3\text{He})/\gamma_s^2(^3\text{He})$. By taking into account the ratio of s- to d-wave penetrabilities at 3.5 MeV, y^2 can be calculated from the value of the quantity x. For $x = 1.1$, $y^2 \approx 5$. A much larger, and rather unreasonable value results from using $x = 3.6$.

The excitation curves have thus been fitted to an equation of the form

$$\sigma(E) = \frac{\mathcal{P}_0 + y^2\mathcal{P}_2 + 2y\mathcal{P}_2\mathcal{P}_0}{(E-E_0)^2 + (\Gamma/2)^2} E_\gamma^3.$$

Fixing y^2 by the procedure indicated above, the two parameters E_0 and Γ were varied to obtain a best fit to the γ_0 and γ_2 excitation curves. For this fit, $E_0 = 2.55$ MeV and $\Gamma = 2.15$ MeV.

In order to fit the γ_1 curve, an additional term, representing a nonresonant background, was added; this term has the same energy dependence as the numerator of the expression used for γ_0 and γ_2, and is of the appropriate form for the tails of higher resonances. A fit to γ_1 with this term alone did not account well for the data, but inclusion of both the resonance and the nonresonant background, produces excellent agreement with the data.

The comparisons of these formulas with the data are shown in Fig. 4. It can be seen that up to 5 MeV, there is excellent agreement. Above this energy, additional structure may be present, which cannot be adequately explained with the single resonance and background used here. Indeed, Shay *et al.*[12] have reported preliminary work showing the existence of several strong, broad resonances in this reaction in the region of excitation of ^{12}C above 28.5 MeV. It should be noted that the rather nice fit of the assumed shape to the data does not in itself prove that s- and d-wave capture is, in fact, the reaction mode taking place. Other choices of incoming partial waves will also reproduce the general features of the data, if different values for E_0 and Γ are used; furthermore, the assumed shape is an oversimplified one, which is expected to give generally correct qualitative features only. On the other hand, it is encouraging that the fit is so good for the parameters which are necessary to explain the angular distributions at 3.5 MeV.

At higher and lower ^{3}He energies, the mixing parameter x^2 will change with the ratio of d-wave to s-wave penetrabilities, if we assume y^2 remains constant in the energy region under consideration. We can then predict that at 2.0 MeV, the coefficient of $P_2(\cos\theta)$ for captures to the 0^+ final states should be −0.49, while at 5.5 MeV, this coefficient should be −0.98. These results agree, qualitatively, with the observed distributions mentioned earlier, although the experimental γ_0 distribution at 2.0 MeV appears to be closer to isotropy than the prediction. For transitions to the 2^+ final state, all predictions indicate only small deviations from isotropy, again agreeing with observation.

Taken together, the evidence for the capture sequence considered here (mixed s and d waves, $S=1$, $J_{\rm res}=1^-$, $E1$ radiation) is quite convincing. Angular distributions at one energy (for all three final states), the energy behavior of the cross section (with E_0 and Γ held fixed for all three transitions and a nonresonant background added to γ_1), and the energy behavior of the angular distribution coefficients are all reasonably well satisfied with these parameters.

By using the measured 90° differential cross sections and the angular distributions, the total cross sections at $E_0=2.55$ MeV can be found. Using the simple Breit-Wigner shape, we can then find a value for the quantity $(2J+1)\Gamma_{^3\mathrm{He}}\Gamma_\gamma/\Gamma^2$ for each transition. If we take $J=1$, as seems most likely, and we use as the largest possible value for $\Gamma_{^3\mathrm{He}}$ the center-of-mass total width, we find a lower limit for the partial radiative width, Γ_γ, for transition to each of the final states. (In calculating Γ_γ for γ_1, that portion of the 2.55-MeV cross section attributed to nonresonant capture has been subtracted.) The results are summarized in Table I, where comparisons are also made with the Weisskopf single-particle estimates. The strengths are all within the average range for $E1$ transitions, as tabulated for lower-energy γ rays by Skorka, Hertel, and Retz-Schmidt.[13]

TABLE I. Transition strengths for ^{9}Be(^{3}He, γ)^{12}C resonance at E (lab) = 2.55 MeV, width Γ (c.m.) = 1.6 MeV.

	$E_{\rm final}$ (MeV)	$J^\pi{}_f$	E_γ (MeV)	$\sigma_{\rm TOT}$ (μb) [a]	Γ_γ (eV) [b]	$\Gamma_\gamma/\Gamma_{\gamma w}$
γ_0	0.0	0^+	28.19	2.7	≥11.8	1.5×10^{-3}
γ_1	4.44	2^+	23.75	1.1 [c]	≥4.6	0.9×10^{-4}
γ_2	7.56	0^+	20.63	2.6	≥11.3	3.6×10^{-3}

[a] The total resonant cross sections at 2.55 MeV are calculated assuming angular distributions identical to those measured at 3.5 MeV.

[b] The lower limits on partial radiative widths are calculated assuming $J_{\rm res}=1$ and $\Gamma_{^3\mathrm{He}}=\Gamma$.

[c] The nonresonant part of the cross section contributes an additional 0.9 μb at this energy, according to the fit to the data described in the text.

DISCUSSION

There have been several shell-model calculations of ^{12}C states in the region of excitation considered here.[14] However, these all include only one-particle–one-hole states. The ^{9}Be(^{3}He, γ) reactions, on the other hand, would be expected to excite 3p-3h configurations.[5] If the three nucleons of the incoming ^{3}He particle excite such a state, with three ($2s$, $1d$) particles [there are already three (1p) holes in the $1p_{3/2}$ subshell in the ^{9}Be nucleus], the most direct $E1$ transition would be to a 2p-2h configuration. In this case, a single nucleon would be making the transition, with $\Delta l=\pm1$ and the appropriate parity change. The transitions to the 0^+ states will conform to the L-S coupling selection rules only for channel spin $S=1$, an assumption made above. It can also be noted that, for transitions of the strength seen in this reaction, the inhibition of $\Delta T=0$ $E1$ transitions in self-conjugate nuclei rules out $T=0$ as the isospin of the resonance; thus we appear to have a 3p-3h, 1^-, $T=1$ state at 28.2 MeV. The final states, in this picture, should have strong admixtures of 2p-2h states, with the two particles in the s-d shell. For the 0^+ state in ^{12}C at 7.65 MeV, this is the structure proposed by Cohen and Kurath,[15] in order to explain the lack of agreement with other energy levels in the $1p$ shell. The partial width for γ decay to the 0^+ ground state is similar in magnitude to that to the excited 0^+ state; within the framework of the above model, this suggests a strong 2p-2h component in the ground state, as well. Such a component, which could arise from configuration mixing between the two 0^+ states, is not out of the question; the ground state of ^{12}C is considered to be deformed,[16] and thus would not be entirely a closed subshell state.

According to the ratios of the lower-limit Γ_γ values to the single-particle estimates (Table I), γ_2 carries about 2.4 times the single-particle strength of γ_0. Thus, as a rough approximation, we expect that, if the two lowest 0^+ states are indeed mixed, the second excited state carries 2.4 times more of the 2p-2h configuration than the ground state. The 7.65-MeV state thus would contain some 70% of this configuration.

Since, according to the model of Gillet, Melkanoff, and Raynal[5] there should be an interference between the 3p-3h components presumably seen in the present reaction and the 1p-1h states making up the giant dipole resonance as seen in (γ, n), (γ, p), or (p, γ) reactions, it is of interest to look for correlations between the present data and the results of measurements of the latter reactions. The most recent data[17,18] for the ^{11}B$(p, \gamma_0)^{12}$C reaction show an indication of a shallow dip in the 28.2-MeV region of ^{12}C. However, there is no way to tell from these data whether this is an actual interference effect. (As has been recently pointed out[19] in connection with such interference interpretations of ^{16}O giant-dipole-resonance data, great care must be exercised in comparing the excitation curves produced by different incoming channels; the coincidence of a maximum in the cross section for one channel at the same excitation as a dip in another is not sufficient evidence for this effect.) Such an effect would probably be much stronger in the ^{11}B$(p, \gamma_2)^{12}$C channel, since according to the picture we have been using here, there would be a weaker transition to this state from a 1p-1h than from the 3p-3h configuration. The proton-radiative-capture reaction to this state may be too weak to measure, however; Brassard *et al.*[17] report measurements only of γ_0, γ_1, and γ_3. The same resonance seen in the present experiment may also be identified with structure seen in ^{9}Be(^{3}He, $n_{0,1}$)^{11}C.[20] Again, a detailed study would have to be done before such a conclusion could be drawn with confidence.

The simple picture of the ^{9}Be(^{3}He, γ)^{12}C reaction mechanism used here appears to be consistent both with the data and with theoretical understanding of the lower states of ^{12}C. It would be of great interest now to see further experimental studies of this reaction at higher energies, for more thorough comparison to ^{11}B$(p, \gamma)^{12}$C data; any clear correlations between features in these two reactions should shed additional light on the reaction mechanisms and the nuclear structure in this region of excitation.

ACKNOWLEDGMENTS

The authors wish to thank Professor Peter Paul for valuable discussions, and Dr. N. G. Puttaswamy, Dr. A. M. Young, and Dr. S. C. Ling for help in taking the data.

Note added: In a preliminary report of some of this work[21] the γ_1 data were described as consistent with the presence of a resonance peaking at about 5.5 MeV. We have taken a more conservative view of the data in the present paper, since measurements at somewhat higher energies would be needed to definitively confirm such a suspicion. Such measurements have recently been performed by Linck and Kraus,[22] and the resonant character of γ_1 is clear in their work. The magnitude of the cross sections obtained in the present work is also confirmed by Linck and Kraus, as well as by Warburton *et al.*[23]

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*Work supported in part by the National Science Foundation.

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PHYSICAL REVIEW C VOLUME 6, NUMBER 5 NOVEMBER 1972

Gel'fand-Levitan-Unitarity-Transform Formalism for Direct Extension of the Two-Nucleon T Matrix off the Energy Shell*

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The unitary-transform method of Coester *et al.* is modified, for uncoupled partial waves in which there are no bound states, so that empirical phase shifts rather than a potential fitted to them may be used as the basic input. This is accomplished by invoking the Gel'fand-Levitan inverse scattering formalism to generate a complete orthonormal set of scattering wave functions from the phase shifts. The result is a convenient formal framework for analyzing the uncertainties in the off-energy-shell behavior of the two-nucleon interaction. Variations in the off-energy-shell T matrix arising from changes in the phase shifts, as well as those due to different short-range nonlocalities, may be studied directly using the method presented here.

I. INTRODUCTION

The unitary-transform method of Coester *et al.*[1] provides an elegant and straightforward procedure for studying the arbitrariness in the two-nucleon T matrix off the energy shell (hereafter called the off-shell T) once the on-energy-shell T matrix (on-shell T) has been specified. As such, it has already been applied in several calculations to investigate the dependence of multinucleon observables on specifics of the two-nucleon interaction.[2] However, because this scheme takes as its basic input a potential fitted to the empirical nucleon-nucleon elastic scattering phase shifts, the resulting off-shell T's are related only indirectly to the available data. Moreover, reliance on a parametrized potential introduced at the outset is a disadvantage in the following practical sense: The elastic scattering phase shifts at high energies are unknown and almost certainly unknowable. It is therefore important to determine the sensitivity of the off-shell T to variations in these ambiguous quantities. A calculation which adopts a particular potential commits itself to a fixed set of high-energy phase shifts, and a different potential must be introduced in order to change them. Not only does this entail cumbersome recalculation, but it also introduces additional uncertainties because it is unlikely that the second potential gives the same fit to the empirical low-energy phase shifts as the first one. Of course, since the low-energy phase shifts are not known to arbitrary accuracy, it is of interest to test the sensitivity of the off-shell T to changes in these quantities as well. However, the uncontrollable differences which result from the *ad hoc* substitution of one potential for another do not seem well suited to such studies.

In this paper, we present a pedestrian remedy for the above difficulties. We eliminate the input potential by merging the unitary-transform method with the inverse scattering theory of Gel'fand and Levitan,[3] which generates a complete orthonormal set of scattering wave functions directly from the phase shifts. The resulting formalism provides a complete framework for analyzing the sources of uncertainty in the off-energy-shell behavior of the two-nucleon interaction, assuming that this interaction is well represented by an en-