Quadrupole Vibrations in the Nickel Region: Structure of Odd-Mass Co Isotopes

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The odd-mass cobalt nuclei from ^{57}Co to ^{63}Co are described in the framework of the unified model. The vibrational description of even-mass nickel isotopes is discussed and the $1f_{7/2}^{-1}$, $1f_{7/2}^{-2}(0)2p_{3/2}$, $1f_{7/2}^{-2}(0)1f_{5/2}$, and $1f_{7/2}^{-2}(0)2p_{1/2}$ proton configurations are coupled to quadrupole vibrational cores. The spacings among these configurations are determined from stripping reaction data and from an analysis of binding energies, and the collective parameters are determined from experimental data on the Ni cores. Good general agreement of calculated and experimental properties is obtained without free parameters involved. For the best known isotopes, ^{57}Co and ^{59}Co, a one-to-one correspondence of the eight or ten first calculated and experimental states is established. Observed spectroscopic factors in pickup and stripping reactions and electromagnetic properties are qualitatively well reproduced for the low-lying levels in general. For the available data most of the calculated values lie within 50% of the experimental values. On the basis of this agreement some spin and parity assignments are proposed among those compatible with experimental data. The results of other models are also discussed.

I. INTRODUCTION

The low-lying states of the even-even Ni isotopes have been described in terms of admixed shell-model configurations of neutrons occupying the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbits,[1] and in terms of collective quadrupole vibrations slightly anharmonic.[2] Both models seem to be able to reproduce fairly well the energy schemes for the lowest levels of these nuclei. However, no such good agreement with experiment has been achieved in the neighboring odd Co isotopes, when the same kind of excitations have been considered. Several shell-model calculations of ^{57}Co have been done. In some of them an inert ^{48}Ca core is assumed, restricting the valence protons to the $f_{7/2}$ shell, and the valence neutrons to the $p_{3/2}$ orbit[3] or the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ orbits.[4] Perhaps the most notable disagreement of the results of these calculations with experiments, as far as energy levels are concerned, is the failure to reproduce the existence of a $\frac{1}{2}^-$ and two $\frac{3}{2}^-$ excited states observed in ^{57}Co among the low-lying levels in the vicinity of 1.5 MeV. The same failure is found when one tries to describe the structure of this and other odd-A cobalt isotopes as an $f_{7/2}$ proton hole coupled to the even-mass Ni nuclei considered as vibrational cores.[5] This difficulty has recently been overcome in a shell-model calculation[6] of the energy levels of ^{57}Co, where an inert ^{40}Ca core is assumed and the 17 external nucleons are distributed in the $f_{7/2}$, $p_{3/2}$, $f_{5/2}$, and $p_{5/2}$ orbits, with configurations of two or three particles above the $f_{7/2}$ shell.

On the other hand, a large number of experimental data other than energy levels and spin and parity assignments are now available for the low-lying levels of odd-mass Co isotopes. Thus a more complete test of theoretical models is now feasible, allowing a better understanding of the structure of these nuclei.

The purpose of the present work is to give a detailed study of odd-mass Co isotopes in the framework of the unified model. First we will be concerned with the properties of low-lying levels of even-mass Ni nuclei, compared with the shell-model and collective-model predictions. A close similarity with the properties of the quadrupole harmonic vibrational spectrum is found. Then the most important individual particle excitations at low energies in the odd-mass Co isotopes are investigated, mainly by means of data obtained from transfer reactions. The structure of these nuclei is then described as a mixture of $f_{7/2}^{-1}$, $f_{7/2}^{-2}(0)p_{3/2}$, $f_{7/2}^{-2}(0)f_{5/2}$, and $f_{7/2}^{-2}(0)p_{1/2}$ shell-model proton configurations coupled to collective quadrupole harmonic surface oscillations.

The present calculation involves several parameters such as the energy spacing between shell-model orbits, or the coupling strength in the interaction Hamiltonian for the protons and the collective vibrations, which are usually treated as free parameters of the unified model to be fixed by at least-squares fit in each nucleus. It is clear that the quality of agreement with experiments will increase with the number of free parameters. However, it might be more interesting to investigate whether reasonable agreement with experimental facts may be achieved without any direct fit of parameters. This is the approach followed

here and emphasis is put on an *a priori* estimate of all the parameters of the model when possible, as in ^{57}Co, and the most important of them in other isotopes. The sensibility of nuclear properties to the values of the parameters is also investigated.

Energy levels, spectroscopic factors in pickup and stripping reactions, static nuclear moments, electromagnetic transition probabilities, and mixing ratios are calculated and compared with experimental data. The good general agreement of theory and experiments permits the prediction of spins and other nuclear properties with some reliability. The results are also compared with the predictions of other models.

II. MODEL

A. Collective Vibrations and the Even-Mass Ni Isotopes

In the collective vibrational model[7] the equipotential surfaces for the nearly independent motion of nucleons in a central nuclear field are given by

$$r = r_0[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta\phi)]. \quad (1)$$

For small deformations from spherical symmetry the Hamiltonian becomes

$$H_c = \tfrac{1}{2}\sum_{\lambda\mu}(B_\lambda |\alpha_{\lambda\mu}|^2 + C_\lambda |\alpha_{\lambda\mu}|^2), \quad (2)$$

where B_λ and C_λ are the mass and deformability parameters, respectively. An oscillation quantum or phonon of the λ mode has angular momentum λ and parity $(-1)^\lambda$, and its energy is given by

$$\hbar\omega_\lambda = \hbar(C_\lambda/B_\lambda)^{1/2}, \quad (3)$$

where ω_λ is the collective oscillation frequency for the λ mode. In the so-called spherical nuclei, the normal sequence $0_1^+, 2_1^+$ of the lowest states in even-even nuclei may be interpreted as the beginning of a quadrupole vibrational band. Then a triplet of two-phonon states, $0_2^+, 2_2^+, 4_1^+$, is expected at twice the excitation energy of the 2_1^+ level. This is indeed observed in the even Ni isotopes from ^{58}Ni to ^{64}Ni, but the main test of the collective nature of those levels is given by their electromagnetic properties. These are summarized for $E2$ radiation in Table I for levels which may be assumed to be of collective nature. It is clear that the simple picture of pure phonon states cannot describe in detail the properties of the low-lying levels in even-mass Ni isotopes, but the experimental data[8,9] involving the electric quadrupole operator are easily understood in terms of approximately harmonic quadrupole vibrations. The $B(E2; 2_1^+ \to 0_1^+)$ values are 1 order of magnitude higher than the Weisskopf estimate, and the $B(E2)$ values for the $2_2^+ \to 2_1^+$ and $4_1^+ \to 2_1^+$ transitions are consistent with the order of magnitude expected for transitions from the two-phonon states to the one-phonon state, i.e., twice the $B(E2; 2_1^+ \to 0_1^+)$ value. On the other hand, the decay properties of the second 0^+ state are known in the ^{58}Ni isotope. The experimental $B(E2)$ values are $150 \pm 22\ e^2\,\mathrm{fm}^4$ for the 0_2^+ (2.943-MeV) $\to 2_2^+$ (2.775-MeV) transition and $(2.4 \pm 0.8) \times 10^{-3}\ e^2\,\mathrm{fm}^4$ for the 0_2^+ (2.943-MeV) $\to 2_1^+$ (1.454-MeV) transition. Thus the 0_2^+ (2.943-MeV) level cannot be a two-phonon state. The 0^+ member of the two-phonon triplet in ^{58}Ni might be the (0^+) level at 3.531 MeV, which decays exclusively to the 2_1^+ state, with a strong reduced transition probability of $75^{+30}_{-16}\ e^2\,\mathrm{fm}^4$, assuming $E2$ radiation. A regularity in the four mentioned Ni isotopes is the inhibition of the cross-over $2_2^+ \to 0_1^+$ respect to the step-over $2_2^+ \to 2_1^+ \to 0_1^+$ electric quadrupole transition.

This is easily understood in terms of the collective model because of the $\Delta N = \pm 1$ selection rule for the number of phonons. This selection rule gives also a zero quadrupole moment for the first excited state, in agreement with the measured value in ^{60}Ni.

Let us consider now the shell-model description of even-mass Ni isotopes. Generally a closed

TABLE I. Relevant $E2$ properties of some low-lying levels in even-mass Ni isotopes.

		$B(E2; 2_1^+ \to 0_1^+)$ ($e^2\,\mathrm{fm}^4$)	$\frac{B(E2; 2_2^+ \to 2_1^+)}{B(E2; 2_1^+ \to 0_1^+)}$	$\frac{B(E2; 4_1^+ \to 2_1^+)}{B(E2; 2_1^+ \to 0_1^+)}$	$\frac{B(E2; 2_2^+ \to 0_1^+)}{B(E2; 2_2^+ \to 2_1^+)}$	$Q(2_1^+)$ (e b)
Approximately harmonic quadrupole vibrations		$\gg 15$	≈ 2	≈ 2	≈ 0	≈ 0
^{58}Ni	Experiment	131 ± 14	1.5 ± 0.7	<4.3	$(1.8 \pm 0.3) \times 10^{-3}$	
	Shell model	117	1.71×10^{-4}	7.31×10^{-1}	7.52×10^{2}	-0.143
^{60}Ni	Experiment	169 ± 2	<2.8	>0.4	$(1.5 \pm 0.9) \times 10^{-2}$	0.00 ± 0.13
	Shell model	200	1.36	2.43×10^{-1}	2.23×10^{-2}	-0.038
^{62}Ni	Experiment	164 ± 10	$(7.0 \pm 2.9) \times 10^{-1}$		$(6.2 \pm 4.1) \times 10^{-2}$	
	Shell model	247	1.51	7.65×10^{-2}	7.64×10^{-4}	0.032
^{64}Ni	Experiment	163 ± 22				
	Shell model	226	1.14	1.67×10^{-2}	6.94×10^{-3}	0.087

^{56}Ni core has been assumed,[1] with the extra neutrons occupying the $1f_{5/2}$, $2p_{3/2}$, and $2p_{1/2}$ orbits. In some calculations realistic nucleon-nucleon potentials have been used, in other cases the effective two-body matrix elements have been obtained by a least-squares fit to the observed energy spectra of Ni isotopes. These shell-model calculations account fairly well for the energy, spin, and parity of the levels of even-mass Ni isotopes which we are here dealing with. In order to explain the strong $2_1^+ \to 0_1^+$ electric quadrupole transitions, effective neutron charges of the order of once or twice the proton charge have been used. For example, the effective charge used by Auerbach[1] was determined from a fit to the experimental $B(E2; 2_1^+ \to 0_1^+)$ values in the ^{58}Ni to ^{64}Ni even-mass isotopes. His results are given in Table I for comparison. It is seen that the neglected core excitations may be accounted for in a great extent by attaching an effective charge to the neutron, but some important disagreements with experimental values are found, especially in ^{58}Ni. In this nucleus the $B(E2; 2_2^+ \to 0_1^+)$ value calculated by Auerbach is too large by a factor 40, and the calculated value for $B(E2; 2_2^+ \to 2_1^+)$ is too short by a factor of 10^4, approximately. The calculated $B(E2; 0_2^+ \to 2_1^+)$ is too large by a factor of 2×10^4. These disagreements with experimental values have led Federman and Zamick to reconsider the calculations of Auerbach. They found that the $E2$ decay of the 2_2^+ state is very sensitive to the value of the $\langle p_{3/2} p_{1/2} J=2 \,|\, V \,|\, p_{3/2}^2 J=2 \rangle$ matrix element. Using Kuo's matrix elements, Federman and Zamick[1] obtained for the $B(E2; 2_2^+ \to 2_1^+)$ ratio the value 0.51, which is much closer than Auerbach's to the experimental value, though still far from it.

It can be said that the simplified shell-model approach of an inert ^{56}Ni core is insufficient to explain in detail the properties of the low-lying levels in the even-mass Ni nuclei. Core excitations seem to be important (first of all to account for the neutron effective charge). On the other hand there are not very striking discrepancies of the simple vibrational picture with the experimental properties discussed above. Obviously, the collective vibrational picture in its simplest form cannot deal adequately with higher levels. Neither can it account for the observed $2_2^+ \to 2_1^+$ transitions of $M1$ type. The experimental $B(M1; 2_2^+ \to 2_1^+)$ values are 1.1×10^{-2}, 9×10^{-3}, and 1.9×10^{-3} Weisskopf units for ^{58}Ni, ^{60}Ni, and ^{62}Ni, respectively.[9] These transitions are forbidden in the framework of the collective vibrational model if no anharmonicities or particle excitations are taken into account. However, when one particle or hole is coupled to the collective states in order to describe the neighboring odd-A nuclei, the collective contribution to the $M1$ properties of the resulting states is of relatively small importance. Thus the harmonic vibrational description of the even-mass Ni nuclei may be a good starting point for the description of the odd-mass Co isotopes in terms of the unified model.

B. Particle-Vibration Coupling

The unified-model theory[7] is well known and only an outline is given here for the sake of clarity in the description of calculations. For quadrupole vibrations, a nucleon outside the vibrational core moves in the potential

$$V_c(r, \theta, \phi) = V_0\left(\frac{r}{1 + \sum_\mu \alpha_{2\mu} Y_{2\mu}(\theta, \phi)}\right). \tag{4}$$

Expanding in powers of $\alpha_{2\mu}$, we get the spherically symmetric potential $V_0(r)$ plus particle-vibration interaction terms. If there are k particles outside the core, the mutual interaction of these nucleons should be taken into account. We shall write our model Hamiltonian as

$$H = \bar{H}_0 + H_c + \sum_{i=1}^{k} H_{\text{int}}(c, i), \tag{5}$$

where H_c is given by (2), the particle-vibration interaction Hamiltonian $H_{\text{int}}(c, i)$ is the leading interaction term in (4) for small deformations, and $\bar{H}_0$ is an effective single-particle shell-model Hamiltonian. We shall deal with the eigenvalues of $\bar{H}_0$ in the next section. In terms of phonon-creation and annihilation operators the total Hamiltonian (5) can be written

$$H = \bar{H}_0 + \tfrac{1}{2}\hbar\omega_2 \sum_\mu (b_{2\mu}^\dagger b_{2\mu} + b_{2\mu} b_{2\mu}^\dagger) - \sum_i r_i \frac{dV_0(r_i)}{dr_i}\left(\frac{\hbar\omega_2}{2C_2}\right)^{1/2} (b_{2\mu} + (-)^\mu b_{2-\mu}^\dagger) Y_{2\mu}(\theta_i, \phi_i). \tag{6}$$

If $|\alpha J M_J\rangle$ are the eigenstates of $\bar{H}_0$ and $|\beta R M_R\rangle$ the eigenstates of H_c, we write the basic states as $|\alpha J, \beta R; IM\rangle$ and the eigenvectors of the total Hamiltonian are then expressed as

$$|E, IM\rangle = \sum_{\substack{\alpha, J \\ \beta, R}} C_{\alpha J, \beta R}^{I; E} \,|\alpha J, \beta R; IM\rangle, \tag{7}$$

where E is the eigenvalue corresponding to this eigenstate, and $C_{\alpha J, \beta R}^{I; E}$ are expansion coefficients.

Explicit expressions for the matrix elements of the Hamiltonian (6) and for the calculation of other nucler properties such as spectroscopic factors, electromagnetic properties, etc. for some types of configurations of a few particles or holes outside closed shells can be found in the work of Gómez.[10]

C. Determination of the Parameters

The model described contains two independent collective parameters, ω_2 and C_2, plus those involved in the spherical potential $V_0(r)$ and in $\bar{H}_0$. It is desirable to obtain an *a priori* estimate of the parameters and investigate afterwards whether the agreement of calculated and experimental properties can be improved by variation of the parameters about the estimated values. Assuming that the collective parameters for each odd-mass Co isotope have approximately the same values as in its Ni isotone, $\hbar\omega_2$ is given by the excitation energy of the 2_1^+ level and C_2 is obtained from the experimental $B(E2; 2_1^+ \to 0_1^+)$ value in the Ni core.

It will be assumed that the low-energy neutron excitations may be described in terms of quadrupole vibrations, according to the considerations of Sec. II A, and the only individual particle excitations are due to the protons outside closed shells. We have then $1f_{7/2}{}^{-1}$ as the lowest energy configuration. Excitations of one proton hole to the neighboring orbits will give rise to negative-parity configurations

$$1f_{7/2}{}^{-2}2p_{3/2},\ 1f_{7/2}{}^{-2}1f_{5/2},\ 1f_{7/2}{}^{-2}2p_{1/2} \tag{8a}$$

and positive-parity configurations

$$1d_{3/2}{}^{-1},\ 2s_{1/2}{}^{-1},\ 1d_{5/2}{}^{-1}. \tag{8b}$$

The lowest positive-parity state observed in the ^{57,59,61,63}Co nuclei[8] is a $\frac{1}{2}^+$ level which decreases in excitation energy from 2.981 MeV in ^{57}Co to 2.19 MeV in ^{63}Co. Satpathy and Gujrathi[5] have shown that these states are obtained by coupling the two first configurations in (8b) to collective quadrupole vibrations.

In order to study the low-lying levels we shall consider the negative-parity configurations. If the Ni cores were described exactly one might think that the coupling of a proton hole should be sufficient for the description of odd-mass Co isotopes. The configurations with one proton excited to the orbits above the $1f_{7/2}$ should belong to the particle-hole excitations of the core. However, the description of the core in terms of quadrupole harmonic vibrations is far from complete. We have seen in Sec. II A that the low-lying levels of even-mass Ni nuclei can be described to a considerable extent in terms of the neutrons above the $1f_{7/2}$ shell, using an effective charge. On the other hand, proton-transfer reactions show that large components of $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ enter in some low-lying levels of odd-mass Co nuclei. Therefore it seems reasonable to include the proton configurations (8a) explicitly. The structure of the quadrupole phonons may contain components of proton particle-hole excitations of the type (8a), but this kind of overlap is probably small and it may be ignored in a first approximation. Its effect could perhaps be noticed in an overestimation of the matrix elements of the electric collective operator, thus a renormalization of this operator by a factor of <1 might be reasonable. In the present approach only proton states of seniority $v = 1$ will be considered, and the parameters of $\bar{H}_0$ are then reduced to the spacings between the state $f_{7/2}{}^{-1}$ and the states $f_{7/2}{}^{-2}(0)p_{3/2}$, $f_{7/2}{}^{-2}(0)f_{5/2}$, and $f_{7/2}{}^{-2}(0)p_{1/2}$. An estimate of these spacings can be obtained from the spectroscopic factors for proton-stripping reactions on even-even Fe isotopes. The centroids $E(j)$ for ^{57}Co are obtained from the spectroscopic factors measured by Rosner and Holbrow[11] according to

$$E(j) = \frac{\sum_i E_i(j) C_i{}^2 S_i(j)}{\sum_i C_i{}^2 S_i(j)}, \tag{9}$$

where $S_i(j)$ are the spectroscopic factors and C_i is the isospin Clebsch-Gordan coefficient. The level spacings $E_j = E(j) - E(f_{7/2})$ will be different from the level spacings of a single proton outside closed shells, owing to the presence of other particles outside closed shells. The centroids obtained from the reaction ^{56}Fe(^{3}He, d)^{57}Co data are given in Table II together with the total stripping strength $G(j) = \sum_i (2j+1) C_i{}^2 S_i(j)$ for an orbit j. The single-particle data from ^{48}Ca(^{3}He, d)^{49}Sc are included for comparison. The differences between the experimental $G(j)$ values and the shell-

TABLE II. Total single-particle strengths $G(j)$ in ^{57}Co and centroids of levels in ^{57}Co and ^{49}Sc.

Single-particle state j	^{56}Fe(^{3}He, d)^{57}Co $G(j)$ Shell model	$G(j)$ Expt.	Centroid $E(j)$ (MeV)	^{48}Ca(^{3}He, d)^{49}Sc Centroid $E(j)$ (MeV)
$1f_{7/2}$	1.76	2.00	0	0
$2p_{3/2}$	2.08	3.48	1.43	3.55
$1f_{5/2}$	5.88	4.68	2.51	4.70
$2p_{1/2}$	2.04	1.58	3.00	5.9

model sum-rule predictions suggest that perhaps not all the levels with an appreciable single-particle strength have been observed, or that some of the spins assumed for higher levels might be incorrect.

The dependence on the parameters E_j for the lowest levels in general will be most sensitive to the spacing $E_{3/2}$ between $f_{7/2}{}^{-1}$ and the lowest excited configuration $f_{7/2}{}^{-2}(0)p_{3/2}$. We can estimate the value of $E_{3/2}$ for the odd-mass Co isotopes from binding energies as follows. Consider the closed-shell nucleus ^{48}Ca, and let $B(Z,N)$ represent the (ground-state) binding energy of a nucleus, with $Z=20+p$ and $N=28+n$. We shall consider only even-N nuclei and, for binding energy purposes, we assume that the ground state of a nucleus is described by a pure proton configuration, and the neutron configuration is the same for two neighboring isotones or when one proton is excited to another orbital. Thus only the proton configurations will be specified and $f_{7/2}{}^{-7}n$, for example, means the state of seven protons in the $1f_{7/2}$ orbit and n neutrons outside the ^{48}Ca core. We want to estimate the spacing

$$E_{3/2}(n)=B(f_{7/2}{}^{7}n)-B(f_{7/2}{}^{6}(0)p_{3/2}n)\,. \tag{10}$$

The term $B(f_{7/2}{}^{7}n)$ is simply $B(27,N)$. We define the proton single-particle energy $\epsilon_{7/2}=B(21,28)-B(20,28)$, and it is easily seen that

$$B(f_{7/2}{}^{6}(0)p_{3/2}n)=B(26,N)+\epsilon_{3/2}-W(p_{3/2},f_{7/2}{}^{6}(0))-W(p_{3/2},n)\,, \tag{11}$$

where $W(a,b)$ is the interaction energy of nucleons a with nucleons b. According to the assumptions above, the configuration $f_{7/2}{}^{8}(0)p_{3/2}$ is found in the Cu isotopes and we can write

$$B(29,N)-B(28,N)=\epsilon_{3/2}-W(p_{3/2},f_{7/2}{}^{8}(0))-W(p_{3/2},n)\,. \tag{12}$$

The shell-model matrix elements $W(p_{3/2},f_{7/2}{}^{8}(0))$ and $W(p_{3/2},f_{7/2}{}^{6}(0))$ are simply related and we obtain

$$E_{3/2}(N)=B(27,N)-B(26,N)-\tfrac{3}{4}B(29,N)+\tfrac{3}{4}B(28,N)+\tfrac{1}{4}\epsilon_{3/2}-\tfrac{1}{4}W(p_{3/2},n)\,. \tag{13}$$

In order to relate the last term on the right-hand side to binding energies we note that for two isotones differing by one proton, denoted by π, we have $B(Z+1,N)-B(Z,N)=\epsilon_{\pi}-W(\pi,p+n)$. Thus the function

$$\theta(Z,N)\equiv[B(Z+1,N)-B(Z,N)]-[B(Z+1,N+2)-B(Z,N+2)] \tag{14}$$

gives the interaction $W(\pi,2\nu)$ of the proton with the two neutrons of difference between $(Z+1,N+2)$ and (Z,N). Setting $Z=28$ the π proton is in the $p_{3/2}$ orbit. From the binding energies of Mattauch, Thiele, and Wapstra[12] we obtain $\theta(28,30)=-1.383$ MeV, $\theta(28,32)=-1.321$ MeV, and $\theta(28,34)=-1.326$ MeV. The interaction energy of the $p_{3/2}$ proton with the first two neutrons outside the core cannot be directly estimated from (14) because the binding energy of ^{57}Cu is not known. However, for all known binding energies in the range $Z=26-29$ and $N=28-34$, the function $\theta(Z,N)$ ranges between -0.966 and -1.580 MeV. We will assume an intermediate value $\theta(28,28)=-1.2$ MeV. The uncertainty is probably not very serious, since $W(p_{3/2},n)$ enters with a factor of $\frac{1}{4}$ in (13).

From $\epsilon_{7/2}=9.62$ MeV and Table II, one finds $E_{3/2}=6.07$ MeV. The spacing given by (13) is then $E_{3/2}=1.64$ MeV for ^{57}Co, $E_{3/2}=1.61$ MeV for ^{59}Co, and $E_{3/2}=1.62$ MeV for ^{61}Co. In spite of the raw approximations made in the above analysis the results are in good agreement with the estimation $E_{3/2}=1.43$ MeV derived from the spectroscopic factors for the ^{56}Fe(^{3}He, d)^{57}Co stripping reaction.

Concerning the radial part of $H_{\rm int}(s,i)$, the usual spherical wells $V_0(r)$, such as the square well or the Woods-Saxon potential, lead to radial matrix elements of the order of 40 MeV in absolute value, with rather small changes for different orbits. For the harmonic-oscillator well we obtain ≈ 47 MeV for diagonal matrix elements and ≈ -39 MeV for off-diagonal matrix elements in the region $A\approx 60$, for $\hbar\omega_0=41A^{-1/3}$ MeV. Here we shall use the value

$$\left\langle nl\left|r\frac{dV_0(r)}{dr}\right|n'l'\right\rangle=(-)^{n+n'}K\,, \tag{15}$$

with $K=40$ MeV.

III. RESULTS AND COMPARISON WITH EXPERIMENTAL DATA

A. Energy Levels

The one-phonon energy in the even-mass nickel isotopes from ^{58}Ni to ^{64}Ni is of the order of 1.3 MeV. In the present calculation the space of basic states is truncated so that only states which give diagonal matrix elements up to about 4 MeV are considered. This is achieved approximately by coupling the $f_{7/2}{}^{-1}$ configuration to collective states of $N\leq 3$ phonons and the excited configurations (8a) to collective states of $N\leq 1$ phonon. The Hamiltonian (6) is diagonalized in this subspace for states of total angular momentum $I=\frac{1}{2}$ to $\frac{11}{2}$. The variation of the energy spectrum with the parameters of the model has been investigated. For this purpose it is convenient to express the ener-

TABLE III. Parameters of the model.

Nucleus	$\hbar\omega$ (MeV)	η	$E_{3/2}$ (MeV)	$E_{5/2}$ (MeV)	$E_{1/2}$ (MeV)
^{57}Co (a)	1.453	2.19	1.4	2.5	3.0
^{57}Co (b)	1.453	2.19	1.6	2.4	2.6
^{59}Co (a)	1.333	2.64	1.4	2.5	3.0
^{59}Co (b)	1.333	2.64	1.6	2.4	2.6
^{61}Co	1.172	2.90	1.6	2.4	2.6
^{63}Co	1.348	2.45	1.9	2.4	2.6

gies in units of $\hbar\omega_2$ and to define a dimensionless parameter

$$\eta = \frac{K}{(2C_2\hbar\omega_2)^{1/2}}, \tag{16}$$

with K given in (15). The value of η represents a measure of the particle-vibration coupling strength in the interaction Hamiltonian $H_{\text{int}}(c,i)$. The values of η listed in Table III are deduced from the experimental $B(E2; 0_1^+ \to 2_1^+)$ values in even-mass Ni isotopes. In units of $e^2\,\text{fm}^4$ these reduced transition probabilities are[7] 657 ± 11 (^{58}Ni), 845 ± 9 (^{60}Ni), 820 ± 50 (^{62}Ni), and 810 ± 110 (^{64}Ni). The values quoted for ^{58}Ni and ^{60}Ni have been obtained from (e,e') experiments. [The $B(E2)$ values obtained from Coulomb excitation for those transitions are about 10% larger.] The energy spectrum has been plotted against the coupling strength η and the three spacing parameters E_j for fixed values of three of the parameters and a resonable range of values for the fourth one about the estimates obtained in Sec. II C and given in Table III. The variation of the energy levels with the coupling strength η is shown in Fig. 1. For $\eta = 0$ there is no particle-vibration interaction and the levels are grouped in degenerate multiplets. It can be seen that the splitting of levels with increasing η is rather smooth and in the region from $\eta \approx 2$ to $\eta \approx 4$ the sequence of levels remains practically unchanged except the first $I = \frac{1}{2}^-$ state. The excitation energy of this state decreases relatively quickly with increasing η so that, for the estimated values of $\hbar\omega_2$ and η given in Table III, one obtains the first $\frac{1}{2}^-$ level at an excitation energy of the order of 1.5 MeV, in agreement with the experimental data for the odd-mass Co isotopes. The inclusion of the configuration $f_{7/2}^2(0)p_{1/2}$ is essential for this result.

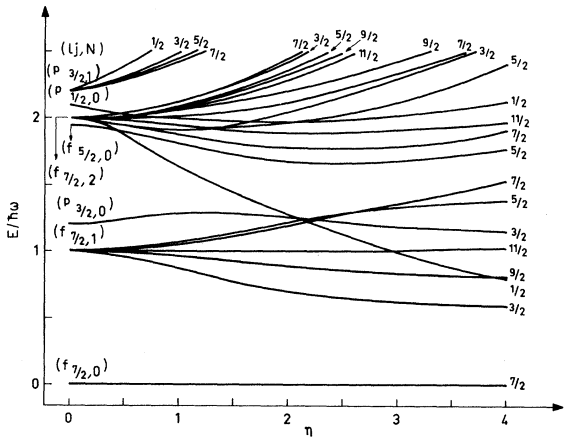

FIG. 1. Dependence of the energy eigenvalues on the coupling strength η with configuration spacings $E_{3/2} = 1.2\hbar\omega_2$, $E_{5/2} = 1.95\hbar\omega_2$, and $E_{1/2} = 2.1\hbar\omega_2$.

Only a few levels change appreciably in excitation energy as a function of the spacing parameters E_j. When $E_{3/2}$ is varied from 1 to 3 MeV, the excitation energies increase about $0.3\hbar\omega_2$ for the $(\frac{3}{2}^-)_1$, $(\frac{1}{2}^-)_1$, and $(\frac{3}{2}^-)_3$ states, $0.7\hbar\omega_2$ for the $(\frac{3}{2}^-)_2$ state, and $0.15\hbar\omega_2$ for the $(\frac{5}{2}^-)_2$ state; all the other levels below $3\hbar\omega_2$ remain at an approximately constant excitation energy. The second and third $\frac{5}{2}^-$ levels go up about $0.5\hbar\omega_2$ (relative to the ground state) when $E_{5/2}$ is changed from 1.7 to 3.7 MeV. Finally, the excitation energy of the first $\frac{1}{2}^-$ state increase $0.5\hbar\omega_2$, and that of the second $\frac{1}{2}^-$ state

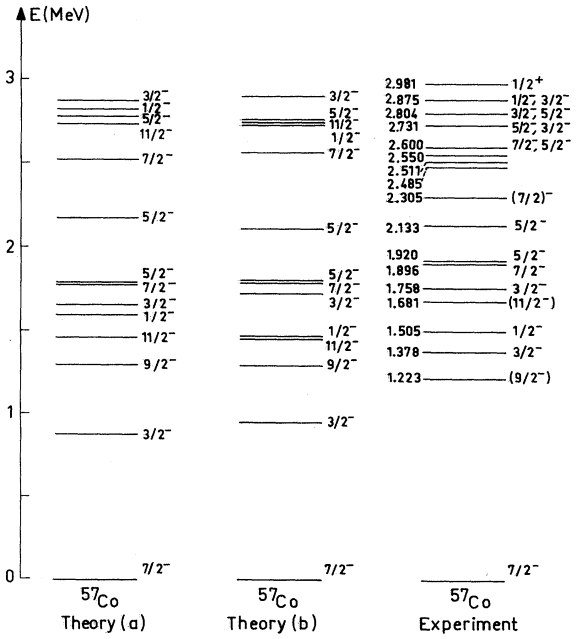

FIG. 2. Comparison between the experimental and calculated energy levels of ^{57}Co below 3 MeV. In (a) the spacing parameters are obtained from the reaction ^{56}Fe-(^{3}He, d)^{57}Co. In (b) $E_{3/2}$ is calculated from the analysis of binding energies. The values of the parameters are given in Table III.

$0.2\hbar\omega_2$, approximately, when $E_{1/2}$ is changed from 2 to 4 MeV. The effect of these parameters on the other levels is negligible.

The calculated energy spectra, corresponding to the values of the parameters given in Table III, are shown in Figs. 2–4, together with the experimental levels. The spacing parameters (a) are those deduced from the (^{3}He, d) reaction study of Rosner and Holbrow.[11] If some of the $l=1$ states assumed to be $\frac{1}{2}^-$ by these authors were $\frac{3}{2}^-$, the centroid $E(\frac{3}{2}^-)$ would increase and $E(\frac{1}{2}^-)$ would decrease. In the set of parameters (b) the spacing $E_{3/2} \approx 1.6$ MeV deduced from the analysis of binding energies is about 0.2 MeV greater than the estimation obtained from the calculated centroids, and the $E_{1/2}$ value might then be expected to be less than 3 MeV. The $E_{1/2}$ and $E_{5/2}$ values adopted in Set (b) seem to improve somehow the general agreement of calculated and experimental energy spectra in the four nuclei. However, no attempt has been made in order to find the values of the parameters which fit best the experimental energy spectrum in each isotope. The calculated and experimental levels are discussed below for each nucleus.

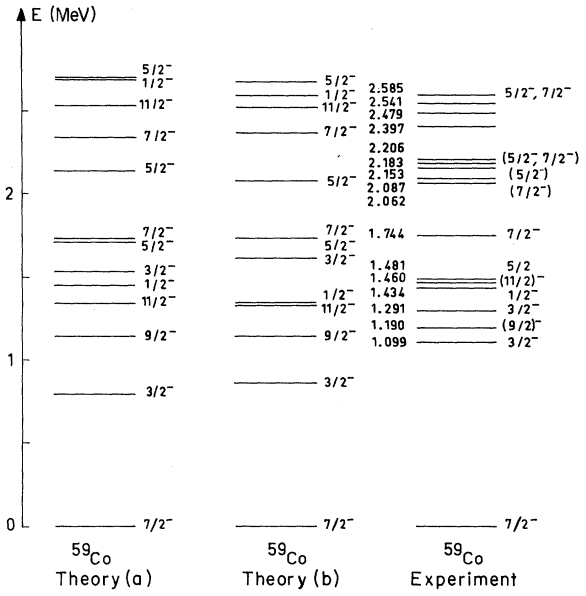

FIG. 3. Comparison between the experimental and calculated energy levels of ^{59}Co below 2.7 MeV. In (a) the values of the spacing parameters are assumed to be equal to the E_j values obtained from the reaction ^{56}Fe-(^{3}He, d)^{57}Co. In (b) $E_{3/2}$ is calculated from the analysis of binding energies. The values of the parameters are given in Table III.

Cobalt 57

The calculated and experimental[6,7,13] level schemes are shown in Fig. 2 up to 3 MeV. The spin assignments to the observed 1.223- and 1.681-

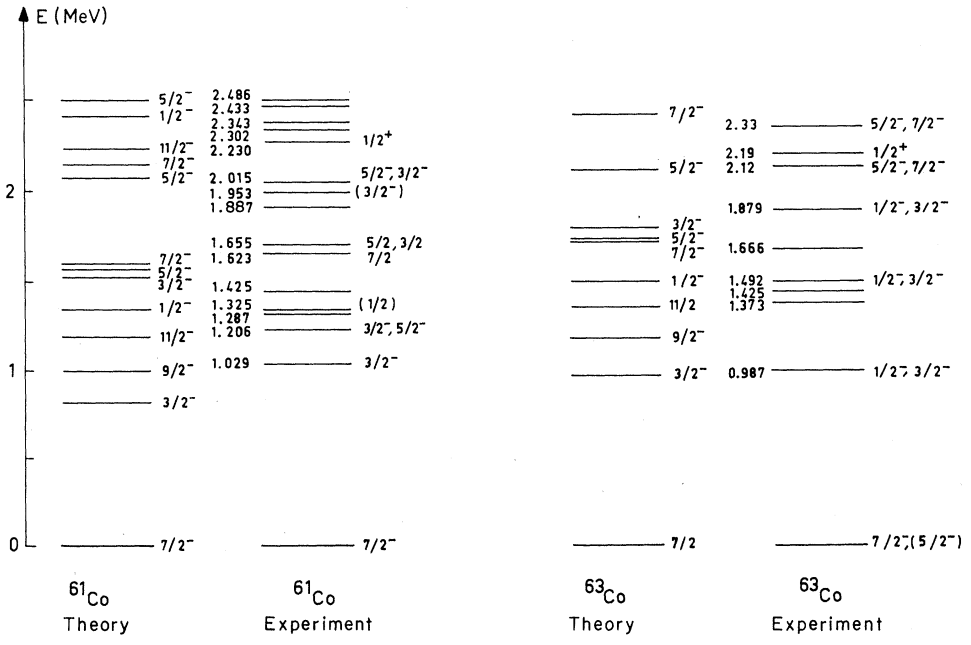

FIG. 4. The calculated and experimental energy spectra of ^{61}Co and ^{63}Co below 2.5 MeV.

MeV levels are not firmly established. The available data suggest a high spin for these states, and the $\frac{9}{2}^-$ and $\frac{11}{2}^-$ assignments are the most probable on the basis of their γ-ray properties. These spins are further supported by the similarity of these states with the corresponding $\frac{9}{2}^-$ and $\frac{11}{2}^-$ states in ^{59}Co, as pointed out by Nordhagen, Elbek, and Herskind.[14] If these assignments are right, the agreement of the present calculation with experiments is good below 2.3 MeV. All the observed states are reproduced at about the same excitation energies with the set of parameters (a) determined *a priori*. The most notable disagreement is the energy of the calculated $(\frac{3}{2}^-)_1$ state, which is about 35% below the experimental $(\frac{3}{2}^-)_1$ state. The general agreement with observed levels is slightly improved with the set of parameters (b), but, as explained above, no fundamental change is obtained from the variation of the energy spacings E_j within a reasonable range of values.

Above 2.3 MeV the model begins to fail. This is a general trend in all the isotopes studied. Actually one cannot expect the vibrational harmonic approximation to give a detailed description of levels at excitation energies of the order of $2\hbar\omega_2$ or higher, since this approximation begins to fail in the description of the core states at these energies.

Cobalt 59

The energy levels of this isotope are well known from (p, p') experiments.[7] The low-lying levels are shown in Fig. 3. Not all the spin and parity assignments to levels below 2 MeV are well established.[7,14-17] The $\frac{5}{2}^-$ assignment to the 1.481-MeV level was suggested by Nordhagen, Elbek, and Herskind[14] as a possible candidate to the $\frac{5}{2}^-$ missing number of the so-called "one-phonon multiplet." Recently this spin has been confirmed by Coop, Graham, and Titterton,[16] but no definite evidence is available regarding the parity of the state. The level at 1.434 MeV has been observed in (τ, d) and (α, t) reactions[15] with an $l=1$ distribution, it is the only low-lying level with no γ ray to the $\frac{7}{2}^-$ ground state and it decays only to the $\frac{3}{2}^-$ states. A possible, but much less probable, spin of the levels at 1.190 and 1.481 MeV is $I=\frac{5}{2}$. Many similarities between the properties of low-lying states of ^{57}Co and ^{59}Co have been pointed out by several authors.[14,16,17]

The agreement of calculated and observed states below 2 MeV is evident. Note that in this isotope, as well as in ^{61}Co and probably in ^{63}Co, the spin and parity of the first observed state are $\frac{3}{2}^-$, as predicted by the present calculations.

Cobalt 61

The information on the levels of ^{61}Co comes from (t, α), (p, α), $(\alpha, p\gamma)$, and β-decay studies[7,16-18] The levels at 1.287 and 1.015 MeV and those shown in Fig. 4 above 2.3 MeV have recently been observed by Coop *et al.*[18] from the (p, α) reaction. According to these authors the existence of a level at 1.425 MeV should be regarded as doubtful. The errors in the energy levels observed in β-decay experiments make it difficult to decide whether some of the levels of Fig. 4 are populated from the β decay of ^{61}Fe, especially those at 1.623 and 1.655 MeV, but the states at 1.287 and 1.887 MeV have certainly not been observed in β-decay studies and they have probably a high spin. There is a similarity between the γ-ray properties of the 1.887-MeV level in ^{61}Co and the states at 1.744 and 2.062 MeV in ^{59}Co. Both states in ^{59}Co decay to the levels at 1.481 MeV, 1.190 MeV, and ground state. In ^{61}Co the 1.887-MeV level decays to the states at 1.655 MeV, 1.287 MeV, and ground state. The branching ratios for these transitions[16] are very similar to the branching ratios for the decay of the $I^\pi=\frac{7}{2}^-$ state at 1.744 MeV in ^{59}Co. We suggest $I^\pi=\frac{7}{2}^-$ for the 1.887-MeV level, $I^\pi=\frac{5}{2}^-$ for the 1.655-MeV level, and $I^\pi=\frac{9}{2}^-$ for the 1.287-MeV level, on the basis of the available data for ^{61}Co and the similarities between the spectra of ^{59}Co and ^{61}Co. A remarkable energy ratio $E(^{59}\text{Co})/E(^{61}\text{Co})\approx 1.08$ for several low-lying levels with the same spin and parity in both isotopes has been pointed out by Grench, Coop, and Menlowe[19] and Coop *et al.*[18] It seems that the low-lying $\frac{11}{2}^-$ state predicted in the present calculations does not correspond to any of the observed states in ^{61}Co. However, it was pointed out by Coop *et al.*[18] that, if such a state lies within 10 keV or so from one of the observed states, it would not have been resolved in the (p, α) or (t, α) experiments, and the probability that the state should be observed in β-decay studies is also small because of its spin.

Cobalt 63

The experimental energy spectrum of ^{63}Co shown in Fig. 4 has been obtained by Blair and Armstrong[17] from the reaction ^{64}Ni$(t, \alpha)^{63}$Co. Few conclusions can be drawn regarding the agreement of calculated and experimental energy levels. However, comparison with the spectra and spectroscopic factors of the ^{57,59,61}Co isotopes shows that the structure of the four nuclei seems to be essentially the same.

B. Wave Functions and Spectroscopic Factors

For $N \leq 3$ phonons the basic states of Eq. (7) can be labeled by the quantum numbers $|\alpha J, NR; IM\rangle$.

The eigenvectors obtained from diagonalization of the Hamiltonian matrix show that the ground state is essentially a mixture of the basic states $|f_{7/2}^{-1}; 00; \frac{7}{2}M\rangle$ and $|f_{7/2}^{-1} J = \frac{7}{2}; 12; \frac{7}{2}M\rangle$, but the excited states in general are described by a great admixture of basic states. The expansion coefficients $C^{I,E}_{\alpha J, NR}$ obtained with the set of parameters (a) for ^{57}Co are given in Table IV. The general trends concerning the main components, etc. are common for all the isotopes.

In the present model the spectroscopic factors for one-proton-transfer reactions are given by the shell-model spectroscopic factor times the square of the zero-phonon coefficient $C^{I,E}_{\alpha J, 00}$. We define the single-particle strength $G_i(j)$ for a state i as

$$G_i(j) = C^2 S_i(j) \tag{17a}$$

for pickup reactions, and

$$G_i(j) = \frac{2I+1}{2I_0+1} C_i^2 S_i(j) \tag{17b}$$

for a stripping reaction. Here I_0 is the spin of the target nucleus and I is the spin of the final state. In the present approach the neutrons are treated as a part of the vibrational core only, and the isospin Clebsh-Gordan coefficients in the calculation of single-particle strengths disappear.

The distorted-wave analysis of angular distributions from (t, α) and $(d, {}^3\mathrm{He})$ reactions show clearly that there are many similarities in the spectroscopy of the four Co isotopes. Blair and Armstrong[17] found from the (t, α) studies that for all these nuclei there is a strong $l=3$ transition to the ground state and several $l=3$ transitions to states above 1.5 MeV; two weak $l=1$ transitions to states between 1 and 2 MeV; one strong $l=0$ transition to a state between 2 and 3 MeV, one strong $l=2$ transition to a state 30 to 60 keV above the $l=0$ state, and some $l=2$ transitions to higher levels. The $I^\pi = \frac{3}{2}^-$ states observed in these pickup reactions show that there is an appreciable $p_{3/2}$ admixture in the ground state of the even-mass Ni nuclei. In the present calculations we have assumed that the $1f_{7/2}$ shell is completely full in the ground state of Ni. The $l=3$ single-particle strengths observed in the Co isotopes are compared with the calculated strengths for pickup reactions in Table V. It can be noted that in ^{57}Co and ^{63}Co the observed strength for the second and third $l=3$ groups appears in the calculated $(\frac{7}{2})_2$ state.

The ^{57}Co and ^{59}Co isotopes have been studied by $(^3\mathrm{He}, d)$ and (α, t) reactions. The experimental single-particle strengths for states below 3 MeV are given in Table VI. On the basis of excitation energy, spin, parity, and single-particle strength in stripping reactions, a one-to-one correspondence of calculated and experimental states is proposed

TABLE IV. Expansion coefficients $C^{I,E}_{\alpha J, NR} \times 10^3$ for states below 3 MeV in ^{57}Co, parameters (a) of Table III.

I^π	Energy (MeV)	$\lvert JNR\rangle$ $\lvert\frac{7}{2}00\rangle$	$\lvert\frac{3}{2}00\rangle$	$\lvert\frac{5}{2}00\rangle$	$\lvert\frac{1}{2}00\rangle$	$\lvert\frac{7}{2}12\rangle$	$\lvert\frac{3}{2}12\rangle$	$\lvert\frac{5}{2}12\rangle$	$\lvert\frac{1}{2}12\rangle$	$\lvert\frac{7}{2}20\rangle$	$\lvert\frac{7}{2}22\rangle$	$\lvert\frac{7}{2}24\rangle$	$\lvert\frac{7}{2}30\rangle$	$\lvert\frac{7}{2}32\rangle$	$\lvert\frac{7}{2}33\rangle$	$\lvert\frac{7}{2}34\rangle$	$\lvert\frac{7}{2}36\rangle$
$\frac{7}{2}^-$	0.000	839				491	−133	−27		112	−66	133	−10	42	−4	−11	16
$\frac{3}{2}^-$	0.871		−689			620	206	98	146		251	15		35	−53	16	
$\frac{9}{2}^-$	1.286					812		5			−311	454		96	−32	−129	110
$\frac{11}{2}^-$	1.454					830					237	468		82	86	−27	146
$\frac{1}{2}^-$	1.596				612		597	−326				−382			29	−125	
$\frac{3}{2}^-$	1.646		463			488	−421	−98	−120		446	322		89	−157	108	
$\frac{7}{2}^-$	1.776	−471				614	134	43		308	−353	329	−82	188	25	−86	62
$\frac{5}{2}^-$	1.797			86		851	−176	−15	−23		238	390		42	−138	85	38
$\frac{5}{2}^-$	2.172			734		187	430	−295	−224		−192	−224		−29	109	−64	−25
$\frac{7}{2}^-$	2.519	−141				16	−490	20		298	615	364	186	127	−228	172	89
$\frac{11}{2}^-$	2.749					74					724	−454		−11	266	380	−222
$\frac{5}{2}^-$	2.777			397		−421	−271	−238	−153		599	177		48	−306	159	25
$\frac{1}{2}^-$	2.818				461		−140	−364				707			−84	360	
$\frac{3}{2}^-$	2.868		−344			−281	−353	198	138		−193	715		−9	−76	257	

in Table VI. For the higher levels the assumed correspondence is only tentative. For example, in ^{57}Co the level at 2.875 MeV could correspond well to both the 2.82- and 2.90-MeV calculated levels. On the other hand the correspondence for lower levels is quite clear. For all the strong transitions observed there is a calculated state close in energy with a single-particle strength close to the experimental value. The calculated levels with small spectroscopic factors are close in energy to experimental levels with no single-particle strength detected and consistent with the calculated I values. Note, for instance, the agreement of the calculated $G_i(j)$ values for the $(\frac{5}{2})_1$ and $(\frac{5}{2})_2$ states with the experimental data. The calculated single-particle strength for the first $\frac{3}{2}^-$ state in ^{59}Co is too large. This suggests also that the estimations of the $E_{3/2}$ spacing in ^{59}Co are too small, because if $E_{3/2}$ is increased, part of the single-particle strength of the first $\frac{3}{2}^-$ state goes over to the second $\frac{3}{2}^-$ state.

C. Static Moments and Electromagnetic Transitions

The electromagnetic multipole operators for the coupled system of collective vibrations and k extracore particles can be written as

$$M(\lambda,\mu)=\sum_{i=1}^{k} M_i(\lambda,\mu)+M_c(\lambda,\mu), \tag{18}$$

where λ is the multipole order, $M_i(\lambda,\mu)$ is a single-particle operator, and $M_c(\lambda,\mu)$ is a collective operator.

If the nuclear shape is given by (1) when r_0 is equal to the uniform radius R_0, assuming a uniform charge distribution and volume conservation for small collective oscillation, one obtains in leading order

$$M_c(E\lambda,\mu)=\frac{3}{4\pi}ZeR_0^{\lambda}\alpha^*_{\lambda\mu}. \tag{19}$$

The electric quadrupole operator is then given by

TABLE V. Single-particle strength $G_i(j)$ of low-lying $l=3$ states in pickup reactions.

	Level (MeV)		I^π		$G_i(j)$ Experiment [a]			
Nucleus	Expt.	Theory (b)	Expt.	Theory	(d,τ)	(t,α)	Theory (a)	Theory (b)
^{57}Co	0	0	$\frac{7}{2}^-$	$\frac{7}{2}^-$	5.50	5.53	5.63	5.63
	1.896	1.80	$\frac{7}{2}^-$	$\frac{7}{2}^-$	1.40	1.37	1.77	1.81
	2.305		$\frac{7}{2}^-$, $(\frac{5}{2}^-)$		0.84	0.20		
	2.600	2.55	$\frac{7}{2}^-$, $\frac{5}{2}^-$	$\frac{7}{2}^-$		0.07	0.16	0.12
^{59}Co	0	0	$\frac{7}{2}^-$	$\frac{7}{2}^-$	5.94	6.61	5.04	5.05
	1.744	1.73	$\frac{7}{2}^-$	$\frac{7}{2}^-$	1.07	1.01	2.04	2.09
	2.062		$(\frac{7}{2}^-)$		0.89	0.80		
	2.183, 2.206	2.37	$(\frac{7}{2}^-, \frac{5}{2}^-)$	$\frac{7}{2}^-$		0.20	0.25	0.20
^{61}Co	0	0	$\frac{7}{2}^-$	$\frac{7}{2}^-$		4.91		4.75
	1.623, 1.655	1.57	$\frac{7}{2}^-$, $\frac{5}{2}^-$, $\frac{3}{2}^-$	$\frac{7}{2}^-$		0.83		2.24
	2.887	2.12	$\frac{7}{2}^-$, $\frac{5}{2}^-$	$\frac{7}{2}^-$		0.15		0.23
^{63}Co	0	0	$\frac{7}{2}^-$, $\frac{5}{2}^-$	$\frac{7}{2}^-$		6.93		5.27
	2.12	1.71	$\frac{7}{2}^-$, $\frac{5}{2}^-$	$\frac{7}{2}^-$		1.36		2.04
	2.33		$\frac{7}{2}^-$, $\frac{5}{2}^-$			1.12		
	2.93	2.43	$\frac{7}{2}^-$, $\frac{5}{2}^-$	$\frac{7}{2}^-$		0.25		0.12

[a] The (t,α) data are from Ref. 17, the ^{58}Ni(d,τ) data from G. Mairle, G. T. Kaschl, H. Link, M. Mackh, U. Schmidt-Rohr, G. J. Wagner, and P. Turek, Nucl. Phys. A134, 180 (1969), and the ^{60}Ni(d,τ) data are an average from Ref. 17 and G. Mairle, G. T. Kaschl, H. Link, M. Mackh, U. Schmidt-Rohr, G. J. Wagner, and P. Turek, Nucl. Phys. A134, 180 (1969).

$$M(E2,\mu)=\sum_{i=1}^{k} e_i r_i^2 Y_{2\mu}(\theta_i,\phi_i) + \frac{3}{4\pi} ZeR_0^2 \left(\frac{\hbar\omega_2}{2C_2}\right)^{1/2} [b_{2\mu}^{+} + (-)^{\mu} b_{2-\mu}] . \quad (20)$$

In analogy with the approximation (15) we use for the radial matrix elements the estimate

$$\langle nl|r^2|n'l'\rangle = (-)^{n+n'} \tfrac{3}{5} R_0^2 , \quad (21)$$

with $R_0 = 1.2A^{1/3}$. The parameter C_2 is related to the coupling strength η by Eq. (16). No effective proton charge is assumed.

The collective contribution to the magnetic dipole operator can be expressed as a term propor-

TABLE VI. Single-particle strength $G_i(j)$ in stripping reactions and assumed correspondence of calculated and experimental states in ^{57}Co and ^{59}Co.

Nucleus	Level (MeV) Expt.	Level (MeV) Theory (b)	I^{π} Expt.	I^{π} Theory	$G_i(j)$ Experiment (Refs. 11 and 15) (τ, d)	$G_i(j)$ Experiment (Refs. 11 and 15) (α, t)	$G_i(j)$ Theory (a)	$G_i(j)$ Theory (b)
^{57}Co	0	0	$\frac{7}{2}^-$	$\frac{7}{2}^-$	1.80	1.36	1.41	1.41
	1.223	1.28	$(\frac{9}{2}^-)$	$\frac{9}{2}^-$			0	0
	1.378	0.95	$\frac{3}{2}^-$	$\frac{3}{2}^-$	1.80		1.90	1.60
	1.505	1.47	$\frac{1}{2}^-$	$\frac{1}{2}^-$	0.72		0.75	0.96
	1.681	1.45	$(\frac{11}{2}^-)$	$\frac{11}{2}^-$			0.	0.
	1.758	1.72	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0.30		0.86	1.09
	1.896	1.78	$\frac{7}{2}^-$	$\frac{7}{2}^-$			0.44	0.45
	1.920	1.80	$\frac{5}{2}^-$	$\frac{5}{2}^-$			0.04	0.11
	2.133	2.11	$\frac{5}{2}^-$	$\frac{5}{2}^-$	2.00		3.23	3.58
	2.305	2.76	$\frac{7}{2}^-, (\frac{5}{2}^-)$	$\frac{5}{2}^-$	0.70		0.95	0.56
	2.485	2.75 [a]	$(\frac{11}{2}^-)$	$\frac{11}{2}^-$			0	0
	2.511	3.32 [a]	$(\frac{13}{2}^-)$	$\frac{13}{2}^-$			0	0
	2.550							
	2.600	2.56	$\frac{7}{2}^-, \frac{5}{2}^-$	$\frac{7}{2}^-$			0.04	0.03
	2.731	3.50	$\frac{5}{2}^-, \frac{3}{2}^-$	$\frac{5}{2}^-$			0.29	0.20
	2.804	2.90	$\frac{3}{2}^-, \frac{5}{2}^-$	$\frac{3}{2}^-$			0.47	0.43
	2.875	2.74	$\frac{1}{2}^-, \frac{3}{2}^-$	$\frac{1}{2}^-$	0.39		0.43	0.32
^{59}Co	0	0	$\frac{7}{2}^-$	$\frac{7}{2}^-$	1.36	1.52	1.26	1.26
	1.099	0.88	$\frac{3}{2}^-$	$\frac{3}{2}^-$	0.44	0.32	1.63	1.35
	1.190	1.14	$(\frac{9}{2}^-)$	$\frac{9}{2}^-$			0	0
	1.291	1.61	$\frac{3}{2}^-$	$\frac{3}{2}^-$	1.36	1.04	0.91	1.12
	1.434	1.34	$(\frac{1}{2}^-)$	$\frac{1}{2}^-$	0.74	0.58	0.68	0.87
	1.460	1.33	$(\frac{11}{2}^-)$	$\frac{11}{2}^-$			0	0
	1.481	1.73	$\frac{5}{2}^{(-)}$	$\frac{5}{2}^-$			0.00	0.01
	1.744	1.73	$\frac{7}{2}^-$	$\frac{7}{2}^-$			0.51	0.52
	2.062		$(\frac{7}{2}^-)$					
	2.087	2.08	$(\frac{5}{2}^-)$	$\frac{5}{2}^-$	2.28		2.90	3.33
	2.153							
	2.183							
	2.206	2.37	$(\frac{5}{2}^-, \frac{7}{2}^-)$	$\frac{7}{2}^-$			0.06	0.05

[a] High spins have recently been observed for the states at 2.49, 2.51, and 2.55 MeV (Ref. 23).

TABLE VII. Magnetic dipole moments.

Nucleus	State	μ/μ_0 Theory (b) $g_s=5.585$	$g_s=3.50$	Experiment (Ref. 8)
^{57}Co	$(\frac{7}{2}^-)_1$	5.49	4.52	4.62
^{59}Co	$(\frac{7}{2}^-)_1$	5.43	4.48	4.649
	$(\frac{3}{2}^-)_2$	2.88	2.20	1.90 ± 0.35
^{61}Co	$(\frac{7}{2}^-)_1$	5.39	4.45	
^{63}Co	$(\frac{7}{2}^-)_1$	5.46	4.50	

TABLE VIII. Electric quadrupole moments.

Nucleus	State	$Q(e\,\mathrm{b})$ Theory	Experiment [a]
^{57}Co	$(\frac{7}{2}^-)_1$	0.34	
^{59}Co	$(\frac{7}{2}^-)_1$	0.39	0.38 ± 0.04
^{61}Co	$(\frac{7}{2}^-)_1$	0.40	
^{63}Co	$(\frac{7}{2}^-)_1$	0.37	

[a] Reference 7 and W. J. Childs and L. S. Goodman, Phys. Rev. 170, 50 (1968).

tional to the collective angular momentum R. Following the assumption that R is made up of orbital angular momenta, we use for the collective gyromagnetic factor the estimate $g_R = Z/A$. The magnetic dipole operator for the coupled system is then written as

$$M(M1,\mu) = \left(\frac{3}{4\pi}\right)^{1/2}\left[\sum_{i=1}(g_s s_\mu + g_l l_\mu)_i + g_R R_\mu\right]\mu_0 , \tag{22}$$

where g_s and g_l are the spin and orbital angular momentum gyromagnetic factors, respectively, and μ_0 is the nuclear magneton.

The ground-state wave functions, and therefore the ground-state nuclear moments, are found to be very little sensitive to the values of the spacing parameters E_j. The calculated values for the sets of parameters (a) and (b) differ by 1% or less. The deviations from the Schmidt value ($5.79\mu_0$ for an $f_{7/2}$ proton) due to the particle-vibration coupling are in the right direction, but they are too small. It has been pointed out by several authors that, in unified-model calculations, an effective spin gyromagnetic factor of the order of $g_s \approx 3.5$ improves the agreement of calculated and experimental values. As shown in Table VII, this is also true for the three magnetic moments measured in these nuclei.

The electric quadrupole moments for the ground states are given in Table VIII. The calculated value for ^{59}Co is in good agreement with the experimental value, and may be compared with the shell-model quadrupole moment $Q = 0.087\ e$ b for the pure $f_{7/2}^{-1}$ configuration.

It has been pointed out in Sec. III A that the most sensitive level to the value of the spacing parameter $E_{3/2}$ is the $(\frac{3}{2}^-)_2$ state. In the limit of zero coupling strength, this state is the pure $f_{7/2}^{-2}(0)p_{3/2}$ configuration with a negative quadrupole moment. For $\eta = 2.64$ in ^{59}Co we obtain $Q(\frac{3}{2}^-)_2 = +0.047\ e$ b in case (a) and $Q(\frac{3}{2}^-)_2 = +0.014\ e$ b in case (b). For the same coupling strength and $E_{3/2} \gtrsim 1.7$ MeV the calculated quadrupole moment of the $(\frac{3}{2}^-)_2$ state becomes negative because the $f_{7/2}^{-2}(0)p_{3/2}$ configuration is dominant.

The experimental information on γ transition probabilities in the odd-mass cobalt isotopes is very scarce, except for ^{59}Co. This nucleus is stable and has been studied by Coulomb excitation.[14,20] The calculated $B(E2)\uparrow$ values are compared with the experimental values in Table IX. The $B(E2)$ Weisskopf estimate is of the order of 12 $e^2\,\mathrm{fm}^4$, and the good qualitative agreement of the theoretical and experimental values is mainly due to the contribution from the collective part of the electric quadrupole operator. Again the difference between the calculated values for the spacings (a) and (b) is small (7% or less), except for the second $\frac{3}{2}^-$ state, with $B(E2)\uparrow = 29\ e^2\,\mathrm{fm}^4$ in case (a). In the paper of Alkazov, Erokhina, and Lemberg[20] the value $B(E2)\uparrow = (1.2\pm0.2)\times10^2\ e^2\,\mathrm{fm}^4$ for a level at 1.29 MeV was reported. This measurement has not been included in Table IX because the observed meanlife of the $\frac{3}{2}^-$ state at 1.291 MeV is[7] $\tau = 0.85\pm0.03$ nsec, and this implies $B(E2)\uparrow = 0.13\ e^2\,\mathrm{fm}^4$. This reduced transition probability is far from the calculated $B(E2)\uparrow$ value for the $(\frac{3}{2}^-)_2$ state, but this disagreement can be eas-

TABLE IX. Calculated and experimental $B(E2)\uparrow$ values in ^{59}Co.

$I_i^\pi \to I_f^\pi$	E_γ (MeV)	$B(E2)$ $(e^2\,\mathrm{fm}^4\times10^{-1})$ Theory (b) $\sigma=1$	$\sigma=0.7$ [a]	Experiment (Ref. 8)
$(\frac{7}{2}^-)_1 \to (\frac{3}{2}^-)_1$	1.10	9.6	5.2	7 ± 2 [b]
$(\frac{7}{2}^-)_1 \to (\frac{9}{2}^-)_1$	1.19	31.2	16.1	19 ± 3
$(\frac{7}{2}^-)_1 \to (\frac{3}{2}^-)_2$	1.29	2.0	0.9	
$(\frac{7}{2}^-)_1 \to (\frac{11}{2}^-)_1$	1.46	28.8	14.7	12 ± 2
$(\frac{7}{2}^-)_1 \to (\frac{5}{2}^-)_1$	1.48	4.7	2.1	
$(\frac{7}{2}^-)_1 \to (\frac{7}{2}^-)_2$	1.74	8.2	3.7	

[a] σ is in a renormalization factor for the collective charge.

[b] Average value from Refs. 14 and 20.

ily understood in the framework of the model. In the discussion of the energy levels and spectroscopic factors of ^{59}Co we have seen that the agreement of the calculated and experimental values should be improved if η is decreased and $E_{3/2}$ is increased from the estimated values of Table III. These changes of the parameters would cooperate in a very fast decrease of the $(\frac{7}{2}^-) \rightarrow (\frac{3}{2}^-)_2$ transition probability, because the contributions of the particles and the core to the amplitude of this transition are out of phase and would come close to each other in absolute value. On the other hand the same change of the parameters η and $E_{3/2}$ would decrease much more smoothly the $B(E2)\uparrow$ values for the $(\frac{3}{2}^-)_1$, $(\frac{9}{2}^-)_1$, and $(\frac{11}{2}^-)_1$ states; thus it is clear that the general quantitative agreement of the calculated and experimental properties could be easily improved by a fit of the parameters of the model. As an example, if we relax the assumption that the collective parameters of the electric quadrupole operator in Eq. (19) should have approximately the same value in ^{59}Co as in the ^{60}Ni core, we can renormalize the operator (19) by a factor σ. The results for $\sigma = 0.7$ are given in Table IX, and it can be seen that much better agreement with the experimental $B(E2)$ values is achieved.

Another important test of the model is given by the $E2/M1$ mixing ratios of γ rays from the low-lying levels to the ground state. The differences between the mixing ratios calculated with the sets of spacing parameters (a) and (b) are negligible. The results for Set (b) are compared to experimental data in Table X. The agreement of theory and experiments is quite good both when the free proton g_s value and $\sigma = 1$ are assumed, and when the effective g_s and σ values obtained from magnetic moments and $B(E2)$ values, respectively, are used in the calculation.

IV. COMPARISON WITH OTHER MODELS

Several models based on the idea of particle-core coupling have been proposed for the odd-mass Co nuclei, especially for ^{59}Co. A comparative scheme is shown in Fig. 5. The simplest model (I) of Lawson and Uretsky[21] is based on the center-of-gravity theorem. If an $f_{7/2}$ hole is coupled to the ^{60}Ni core, five states of $I^\pi = \frac{1}{2}^-$ to $\frac{11}{2}^-$ result from the $I^\pi = 2^+$ first excited state of the core. The theorem states that the center of gravity of the quintuplet, $E = E_i(2I_i + 1)/5$, should lie at the same excitation energy in ^{59}Co as the 2_1^+ state in ^{60}Ni. Lawson and Uretsky proposed tentative spin as-

TABLE X. Mixing ratios of γ rays from levels below 2 MeV to the ground state.

			Theory (b) $\lvert\delta(E2/M1)\rvert$		Experiment[b]	
Nucleus	Assumed I^π	E_γ (MeV)	$g_s = 5.585$ $\sigma = 1$	$g_s = 3.50$ $\sigma = 0.7$[a]	E_γ (MeV)	$\delta(E2/M1)$
^{57}Co	$\frac{9}{2}^-$	1.28	0.31	0.30	1.223	-0.23 ± 0.03
	$\frac{3}{2}^-$	0.95	Pure $E2$		1.378	Pure $E2$
	$\frac{1}{2}^-$	1.47	No $M1$ or $E2$		1.505	No γ ray
	$\frac{11}{2}^-$	1.45	Pure $E2$		1.681	Pure $E2$
	$\frac{3}{2}^-$	1.72	Pure $E2$		1.758	Pure $E2$
	$\frac{7}{2}^-$	1.78	0.62	0.56	1.896	
	$\frac{5}{2}^-$	1.80	0.25	0.21	1.920	0.24 ± 0.03
^{59}Co	$\frac{3}{2}^-$	0.88	Pure $E2$		1.099	Pure $E2$
	$\frac{9}{2}^-$	1.14	0.28	0.27	1.190	-0.25 ± 0.05
	$\frac{3}{2}^-$	1.61	Pure $E2$		1.291	Pure $E2$
	$\frac{1}{2}^-$	1.34	No $M1$ or $E2$		1.434	No γ ray
	$\frac{11}{2}^-$	1.33	Pure $E2$		1.460	Pure $E2$
	$\frac{5}{2}^-$	1.73	0.20	0.18	1.481	
	$\frac{7}{2}^-$	1.73	0.62	0.56	1.744	$0.89^{+0.25}_{-0.17}$

[a] σ is a renormalization factor for the collective charge.
[b] References 13 and 16 and B. J. O'Brien and G. E. Coote, Nucl. Phys. A153, 593 (1970).

signments to the observed levels on the basis of best fulfillment of the theorem (the spin of the first excited state was believed to be $\frac{5}{2}$ at that time). The experimental data show that the center-of-gravity theorem in general cannot be used to determine spins.

Satpathy and Gujrathi[5] have applied the unified model to the odd-mass Co isotopes, considering only an $f_{7/2}$ hole for negative-parity states and including vibrational states up to three phonons (positive-parity states due to hole excitations were also calculated). In these calculations the particle-vibration coupling strength was treated as a free parameter (and also the single-hole level spacings) for each isotope. The levels obtained for ^{59}Co below 2.5 MeV are shown in Fig. 5 (II), and the results for the other isotopes are very similar. The agreement of calculated and experimental levels is poor, but the results for ground-state moments and spectroscopic factors in pick-up reactions are similar to the results of the present calculations. Model (III) is a variant of the same model where a dipole-dipole term of the type $\vec{j}\cdot\vec{R}$ is included in the particle-vibration interaction.[22] The energy spectrum is then improved and good agreement of calculated and experimental electromagnetic properties is obtained for the states connected by dashed lines in Fig. 5.

Several shell-model calculations for ^{57}Co have been reported. A comparative scheme of energy spectra is shown in Fig. 6. In Model (II) the seven protons outside the $Z=20$ shell are placed in the $1f_{7/2}$ orbital, and the two neutrons outside $N=28$ are placed in the $2p_{3/2}$ orbit.[3] The two-body matrix elements are determined by the effective interaction method. Model (III) is similar, but the two valence neutrons are distributed in the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbits.[4] The agreement of calculated and experimental levels is very poor.

In the work of Gatrousis *et al.*[6] an inert ^{40}Ca core is assumed and the 17 valence nucleons are distributed in the $p-f$ shell as follows:

$$\left.\begin{matrix} f_{7/2}{}^{14},\ j_1,\ j_2,\ j_3 \\ f_{7/2}{}^{15},\ j_1,\ j_2 \end{matrix}\right\} j_1,\ j_2,\ j_3 = p_{3/2},\ p_{1/2},\ f_{5/2}.$$

The matrix elements derived by Kuo and Brown from the Hamada-Johnston free two-nucleon potential are used for the two-body part of the shell-model residual interaction. The spacings of single-particle orbits are treated as free parameters and are determined by a fitting process to the experimental low-lying levels. The resulting energy scheme is shown in Fig. 6 (I). The qualitative agreement with experimental levels is good below 2 MeV, except for an extra $\frac{9}{2}^-$ state calculated. The results of the present calculation up to about 2.3 MeV can be favorably compared, since no free parameters are involved. Some additional states would be obtained in the proximity of 3 MeV of excitation if higher seniority states were included in the present model, while the spectrum below 2 MeV would not be seriously modified.

The vectors of Gatrousis *et al.* have up to several hundreds of components and the single-proton strength is spread over a large number of states, so that the considerable single-particle strengths observed in pickup and stripping reactions for several low-lying levels of ^{57}Co are not explained in the model. It should be interesting to compare the

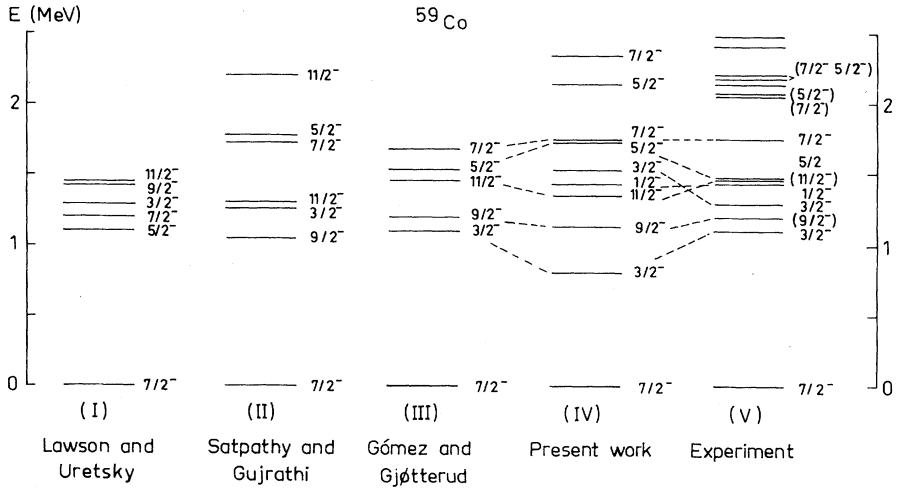

FIG. 5. Comparison of the spectra predicted by various hole-core coupling models with the present results and the experimental energy levels of ^{59}Co.

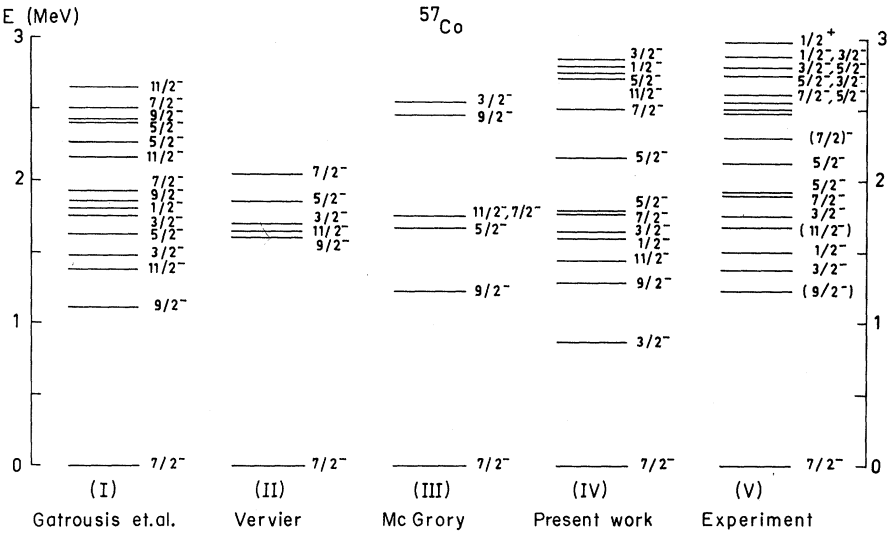

FIG. 6. Comparison of the experimental energy levels of ^{57}Co with various shell-model predictions and with the present results.

electromagnetic properties calculated with the shell-model wave functions and those obtained in the present model, but the shell-model results have not been reported as yet.

V. CONCLUSIONS

The structure of odd-mass Co nuclei can be understood to a large extent as a result of single-proton excitations and collective vibrations. The collective parameters of the model are estimated from experimental data of the even-mass Ni isotopes; the spacings between pure shell-model configurations are estimated from the measured cross sections in one-proton transfer reactions and from binding energies. Energy levels and level properties are then calculated without free parameters involved, and good qualitative agreement with experimental data is found. Several of the difficulties found in formerly reported calculations have been overcome in the present model. Microscopic properties as single-particle strengths and collective properties as large quadrupole moments and $B(E2)$ values are explained in a consistent manner in the present model. The calculated $E2/M1$ mixing ratios are also in satisfactory agreement with experimental data. For the best known isotopes, ^{57}Co and ^{59}Co, a one-to-one correspondence can be established between the lowest calculated and experimental states. Consequently the model predicts several spin and parity assignments which are consistent with the observed properties, but not definitely determined from experiments.

The author believes that the unified model of collective vibrations and individual particle motion is a useful instrument for the investigation of nuclear structure if the proper excitations are considered. It is hoped that some experiments will be encouraged in order to establish definitely whether one $\frac{9}{2}^-$ and one $\frac{11}{2}^-$ low-lying state exist in the odd-mass Co isotopes, and whether the two lowest $\frac{3}{2}^-$ and the two lowest $\frac{5}{2}^-$ states have quite different properties, as predicted in the present study. From the results of experiments which are now in progress[23] and recently reported γ-ray branching ratios, a broader survey of electromagnetic properties in odd-mass Co isotopes will be obtained. A more detailed test of the present model will then be possible and the convenience of some refinements may be considered.

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Decay of ^{71}As

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The singles γ-ray spectrum of ^{71}As was studied using 20- and 40-cm^3 Ge(Li) detectors. γ-γ coincidence data were taken using 7-mm and 20-cm^3 Ge(Li) detectors in conjunction with a 4096 multichannel analyzer with digital gates. Of the 73 γ rays observed, 43 are placed in a decay scheme with 14 excited states along with log*ft* values for 12 of these.

I. INTRODUCTION

The levels of ^{71}Ge have been populated primarily through the ^{71}Ga$(p, n\gamma)$[1] and ^{70}Ge(d, p)[2,3] reactions. While there is 2.01 MeV for decay from ^{71}As (62 h) to ^{71}Ge, most of the decay (87% according to our study) takes place to the first two levels at 175 and 199 keV. The relatively low intensities of γ rays from higher states have until now precluded their resolution with NaI(Tl) detectors. Previous work on ^{71}As decay has reported decay to the 175- and 199-keV states only.[4]

It has been our purpose to resolve the low-intensity γ rays using a Ge(Li) detector. Because of low intensity of all γ rays except the 175-keV γ ray, we performed only one γ-γ coincidence ex-